PART : MATHEMATICS

1. Find number of words by using all letters of the word "DAUGHTER" such that no two vowels come together

(1) 5200 (2) 7200 (3) 14400 (4) $3 \times \lfloor 5$

Ans. **(3)**

Sol.

D	G	H	T	R

Number of ways of arrangement of consonants = $\lfloor 5$

Now there are 6 gaps between these consonants.

So, number of ways of arrangement of three vowels A, U, E = ${}^6P_3$

So total number of words = $\lfloor 5 \times {}^6P_3 = 120 \times 6 \times 5 \times 4 = 120 \times 120 = 14400$

2. Find sum of all rational terms in expansion of $(1 + 2^{1/3} + 3^{1/2})^6$

(1) 144 (2) 612 (3) 720 (4) 562

Ans. **(2)**

Sol. General term = $\dfrac{\lfloor 6}{\lfloor r_1 \lfloor r_2 \lfloor r_3} \times 2^{\frac{r_2}{3}} \times 3^{\frac{r_3}{2}}$; $0 \le r_1, r_2, r_3 \le 6$ and $r_1 + r_2 + r_3 = 6$.

For rational term : $r_2 = 0 \to r_1 + r_3 = 6$ $\begin{cases} r_3 = 0, r_1 = 6 \\ r_3 = 2, r_1 = 4 \\ r_3 = 4, r_1 = 2 \\ r_3 = 6, r_1 = 0 \end{cases}$

$r_2 = 3 \to r_1 + r_3 = 3$ $\begin{cases} r_3 = 0, r_1 = 3 \\ r_3 = 2, r_1 = 1 \end{cases}$

$r_2 = 6 \to r_1 + r_2 = 0$ $(r_1 = 0, r_2 = 0$

Sum of all rational terms =

$\dfrac{\lfloor 6}{\lfloor 0 \lfloor 0 \lfloor 6} 2^0.3^0 + \dfrac{\lfloor 6}{\lfloor 0 \lfloor 2 \lfloor 4} 2^0.3^1 + \dfrac{\lfloor 6}{\lfloor 0 \lfloor 4 \lfloor 2} 2^0.3^2 + \dfrac{\lfloor 6}{\lfloor 0 \lfloor 6 \lfloor 0} 2^0.3^3 + \dfrac{\lfloor 6}{\lfloor 3 \lfloor 0 \lfloor 3} 2^1.3^0 + \dfrac{\lfloor 6}{\lfloor 3 \lfloor 2 \lfloor 1} 2^1.3^1 + \dfrac{\lfloor 6}{\lfloor 6 \lfloor 0 \lfloor 0} 2^2.3^0$

$= 1 + 45 + 135 + 27 + 40 + 360 + 4 = 612$

3. If for an AP, if first term is 3 and sum of first four terms is equal to $\frac{1}{5}$ of the sum of next four terms, then the sum of first 20 terms is:

(1) –540 (2) –1080 (3) 2016 (4) 4080

Ans. **(2)**

Sol. a = 3, let common difference = d.

4. Value of sin70º (cot10º cot70º – 1) is:

(1) 2 (2) 1 (3) $\frac{1}{2}$ (4) 3

Ans. **(2)**

Sol. $\sin 70^\circ \left(\dfrac{\cos 10^\circ}{\sin 10^\circ}\dfrac{\cos 70^\circ}{\sin 70^\circ} - 1\right)$

$\sin 70^\circ \left(\dfrac{\cos 70^\circ \cos 10^\circ - \sin 70^\circ \sin 10^\circ}{\sin 70^\circ \sin 10^\circ}\right) = \dfrac{\cos(70^\circ + 10^\circ)}{\sin 10^\circ} = \dfrac{\cos 80^\circ}{\sin 10^\circ} = \dfrac{\sin 10^\circ}{\sin 10^\circ} = 1$

5. Value of $\cos^{-1}\left[\dfrac{12}{13}\cos x + \dfrac{5}{13}\sin x\right]$ is, if $x \in \left(\dfrac{\pi}{2}, \dfrac{3\pi}{4}\right)$

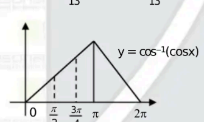
(1) $x + \tan^{-1}\dfrac{12}{5}$ (2) $x - \tan^{-1}\dfrac{12}{5}$ (3) $x - \tan^{-1}\dfrac{5}{12}$ (4) $x + \tan^{-1}\dfrac{5}{12}$

Ans. **(3)**

Sol. $\cos^{-1}\left[\dfrac{12}{13}\cos x + \dfrac{5}{13}\sin x\right]$

$\cos^{-1}[\cos(x - \phi)]$

Let $\cos\phi = \dfrac{12}{13}$ and $\sin\phi = \dfrac{5}{13}$, so $\tan\phi = \dfrac{5}{12}$



$= x - \phi$

$= x - \tan^{-1}\dfrac{5}{12}$

6. If function $f(x) = \begin{cases} \dfrac{2}{x}\{\sin(k_1 + 1)x + \sin(k_2 + 1)x\} & x < 0 \\ 4 & x = 0 \end{cases}$ is continuous at x = 0, then value of $k_1^2 + k_2^2$ is

6. If function $f(x)=\begin{cases}\frac{2}{x}\{\sin((k_1+1)x)+\sin((k_2+1)x)\} & x<0\\ 4 & x=0\\ \frac{2}{x}\ell n\left(\frac{k_2x+1}{k_1x+1}\right) & x>0\end{cases}$ is continuous at $x=0$, then value of $k_1^2+k_2^2$ is equal to –

(1) 6 (2) 2 (3) 4 (4) 8

Ans. **(2)**

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Sol. $f(0)=4$

$f(0^-)=\lim\limits_{h\to 0}\frac{2\{\sin(k_1+1)h+\sin(k_2+1)h\}}{h}$

$=\lim\limits_{h\to 0}\frac{2\{(k_1+1)\cos(k_1+1)h+(k_2+1)\cos(k_2+1)h\}}{1}$

$=2(k_1+1+k_2+1)=2(k_1+k_2+2)$

$f(0^+)=\lim\limits_{h\to 0}\frac{2\{\ell n(k_2h+1)-\ell n(k_1h+1)\}}{h}$

$=\lim\limits_{h\to 0}\frac{2\left\{\frac{k_2}{k_2h+1}-\frac{k_1}{k_1h+1}\right\}}{1}$

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$=\lim\limits_{h\to 0}\frac{2\{(k_1+1)\cos(k_1+1)h+(k_2+1)\cos(k_2+1)h\}}{1}$

$=2(k_1+1+k_2+1)=2(k_1+k_2+2)$

$f(0^+)=\lim\limits_{h\to 0}\frac{2\{\ell n(k_2h+1)-\ell n(k_1h+1)\}}{h}$

$=\lim\limits_{h\to 0}\frac{2\left\{\frac{k_2}{k_2h+1}-\frac{k_1}{k_1h+1}\right\}}{1}$

$=2(k_2-k_1)$

Now $2(k_1+k_2+2)=4$

$k_1+k_2=0$ ...(i)

and $2(k_2-k_1)=4$

$k_2-k_1=2$ ...(ii)

So $k_1=-1$ and $k_2=1$

So $k_1^2+k_2^2=1+1=2$

7. A relation defined on set A = {1, 2, 3, 4}; then how many ordered pairs are added to R = {(1, 2), (2, 3), (3,3)} so that it becomes equivalence?

Ans. **(7)**

Sol. For equivalence it must be transitive, symmetric and reflexive all.

For reflexive → (1, 1), (2, 2), (4, 4)

For symmetric → (2, 1), (3, 2)

For transitive → (1, 3), (3, 1)

Total 7 pairs has to be added to make it's an equivalence relation.

8. Find value of λ for which system of equation:

$(\lambda-1)x+(\lambda+2)y+(\lambda-1)z=0$

$\lambda x+(\lambda-1)y+(\lambda+1)z=0$

$(\lambda-1)x+(\lambda+1)y+(\lambda+2)z=0$ has infinite solution.

(1) 0 (2) $\frac{2}{11}$ (3) 2 (4) $\frac{3}{11}$

Ans. **(2)**

9. There are two biased dice such that, for first dice two faces show 1, 2 faces show 2, one face show 3 and one face show 4. For second dice one face show 1, two faces show 2, one face show 3, and two faces show 4. Then find probability of getting sum 4 or 5, when dice are thrown together.

(1) $\frac{5}{9}$ (2) $\frac{4}{9}$ (3) $\frac{2}{9}$ (4) $\frac{8}{9}$

Ans. **(2)**

Sol. First dice have 1, 1, 2, 2, 3, 4

Second dice have 1, 2, 2, 3, 4, 4

Now P (sum 4 or sum 5) =P (sum 4) +P (sum 5)

$= P(1, 3) + P(2, 2) + P(3, 1) + P(1, 4) + P(2, 3) + P(3, 2) + P(4, 1)$

$= \frac{2}{6}\times\frac{1}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{1}{6}$

$= \frac{2+4+1+4+2+2+1}{36} = \frac{16}{36} = \frac{4}{9}$

10. If $\left|\frac{\bar{z}-i}{2\bar{z}+i}\right| = \frac{1}{3}$ represent a circle whose centre is C and area of triangle whose vertices are (0, 0), C and $(\alpha, 0)$ is 11 then find α^2.

Ans. **(100)**

Sol.

$3|\bar{z}-i| = |2\bar{z}+i|$

$3|x - i(y+1)| = |2x + i(1-2y)|$

$\Rightarrow 9(x^2 + (y+1)^2) = 4x^2 + (1-2y)^2$

$\Rightarrow 5x^2 + 5y^2 + 22y + 8 = 0$

Centre $C\left(0, \frac{-11}{5}\right)$

Area $\Delta = \left|\frac{1}{2}.\frac{11}{5}.\alpha\right| = 11$

$|\alpha| = 10$

$\alpha^2 = 100$

11. If both roots of quadratic equation

$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal and $a + c = 15$, $b = 2/15$ then value of $a^2 + c^2$ is:

(1) 217 (2) 223 (3) 213 (4) 211

Ans. **(2)**

Sol. Clearly one root is one

$\therefore$ Product of roots =1

$\frac{c(a-b)}{a(b-c)} = 1$

$ac - bc = ab - ac$

$2ac = b(a+c)$

$2ac = \frac{2}{15}\times 15$

$ac = 1$

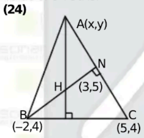
Now $a^2 + c^2 = (a+c)^2 - 2ac$

$= (15)^2 - 2 = 223$

12. Two vertices of triangle are (–2, 4) and (5, 4) and its orthocentre is (3, 5) and centroid is (c, d) then the value of c +3d is:

Ans. **(24)**

Sol.



BC $\Rightarrow$ parallel to x – axis

AH $\Rightarrow$ parallel to y – axis

so x =3

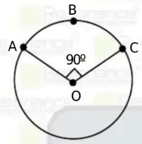
$M_{AC}\times M_{BN} \Rightarrow -1$

$\frac{4-y}{5-3}\times\frac{5-4}{3+2} = -1$

$= (4-y) = -10$

$\Rightarrow y = 14$

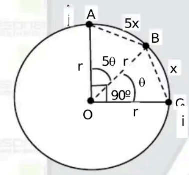
13. In the given figure, $\frac{AB}{BC} = \frac{1}{5}$, $\overrightarrow{OC} = \alpha\overrightarrow{OA} + \beta\overrightarrow{OB}$; find $\alpha + \sqrt{2}(\sqrt{3}-1)\beta$



(1) $(\sqrt{3}+1)\frac{3}{2\sqrt{2}}$ (2) $(\sqrt{3}-1)\frac{5}{2\sqrt{2}}$ (3) $(\sqrt{3}-1)\frac{3}{2\sqrt{2}}$ (4) $(\sqrt{3}+1)\frac{5}{2\sqrt{2}}$

Ans. **(3)**

Sol.



$5\theta + \theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{12}$

$\overrightarrow{OC} = r\cos\theta\hat{i}$

$\overrightarrow{OA} = r\hat{j}$

$\overrightarrow{OB} = r\cos\theta\hat{i} + r\sin\theta\hat{j}$

$\overrightarrow{OC} = \alpha\overrightarrow{OA} + \beta\overrightarrow{OB}$

$r\cos\theta\hat{i} = \alpha(r\hat{j}) + \beta(r\cos\theta\hat{i} + r\sin\theta\hat{j})$

$r\cos\theta = \beta r\cos\theta$

$\beta = 1$

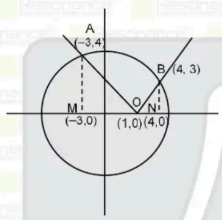
$\alpha r + r\sin\theta.\beta = 0$

14. Area of the larger region bounded by curves $y = |x-1|$ and $x^2 + y^2 = 25$ is:

(1) $\left(\frac{75\pi}{4} + \frac{1}{2}\right)$ (2) $\left(\frac{75\pi}{4} - \frac{1}{2}\right)$ (3) $\left(\frac{25\pi}{4} + \frac{1}{2}\right)$ (4) $\left(\frac{25\pi}{4} - \frac{1}{2}\right)$

Ans. **(1)**

Sol. Area of shaded of region = area of circle – area AOB (unshaded)



$\therefore$ Area of AOB $= \int_{-3}^{4}\sqrt{25-x^2}$ – Area of ΔAOM – Area of ΔONB

$= \int_{-3}^{4}\sqrt{25-x^2} - \frac{1}{2}\times 4\times 4 - \frac{1}{2}\times 3\times 3$

$= \int_{-3}^{4}\sqrt{25-x^2} - 8 - \frac{9}{2}$ ...(i)

$= \left[\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{-3}^{4} - \frac{25}{2}$

$= \frac{25\pi}{4} - \frac{1}{2}$

$\therefore$ Required area $= \pi(5^2) - \left(\frac{25\pi}{4} - \frac{1}{2}\right) = \frac{75\pi}{4} + \frac{1}{2}$

15. Given $f(x) = \ell nx$ and $g(x) = \dfrac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$, then the domain of f(g(x)) is :

(1) $(0, \infty)$ (2) $(1, \infty)$ (3) R (4) $(-\infty, 0)$

Ans. **(3)**

Sol. $f(g(x)) = \ell n(g(x))$ $\Rightarrow$ $g(x) > 0$

$$g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1} > 0$$

$\Rightarrow$ $2x^2 - 2x + 1 > 0 \ \forall \ x \in R$ $(\because a > 0, D < 0)$

Now, $x^4 - 2x^3 + 3x^2 - 2x + 2 = x^4 - 2x^3 + 2x^2 + x^2 - 2x + 2$

$x^4 - 2x^3 + 2x^2 + x^2 - 2x + 2 = (x^2 + 1)(x^2 - 2x + 2) > 0 \ \forall \ x \in R$

$x^4 - 2x^3 + 3x^2 - 2x + 2 > 0$

$g(x) > 0 \ \forall \ x \in R$

Therefore, domain of f(g(x)) is R.