Vedantu

JEE-Main-23-01-2025 (Memory Based)
[MORNING SHIFT]
Math

Question: How many words can be formed from the word DAUGHTER such that any vowels are not together
Options:
(a) 14444
(b) 14400
(c) 10044
(d) 12400
Answer: (b)

DGHTR AUE

$5! \times {}^6C_3 \times 3!$

$= 120 \times 20 \times 6$

$= 14400$

Question: Two biased dice are tossed. Die 1 has 1 on two faces, 2 on two faces, 3 and 4 on other faces, while die 2 has 2 on 2 faces, 4 on 2 faces and 1 and 3 on other faces. Then the probability that when throwing these dice we get sum 4 or 5.
Options:
(a) 3/7
(b) 2/3
(c) 4/9
(d) 8/9
Answer: (c)

1 on 2 2 on 2
2 on 2 4 on 2
3 on 1 1 on 1
4 on 1 3 on 1
Sum (4 or 5)
= P(1, 3 or 4) + P(2, 2 or 3)
+P(3, 1 on 2) + P (4, 1)

$$\frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{6} \times \frac{1}{2} + \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{12} + \frac{1}{36} = \frac{16}{36} = \frac{4}{9}$$

Question: If for an arithmetic progression if first term is 3 and sum of first four terms is equal to ⅕ of the sum of next four terms, then the sum of first 20 terms is
Options:
(a) 1080
(b) 364

(c) -1080
(d) -364
Answer: (c)

$S_4 = \frac{1}{5}(S_8 - S_4)$

$5S_4 = S_8 - S_4$

$6S_4 = S_8$

$6 \times \frac{4}{2}[6+3d] = \frac{8}{2}[6+7d]$

$\Rightarrow 12(6+3d) = 4(6+7d)$

$3(6+3d) = (6+7d)$

$2d = 6 - 18$

$d = -6$

Now $S_{20} = \frac{20}{2}[6+19(-6)]$

$= 10 \times 6(1-19)$

$= 60 \times -18 = -1080$

Question: If f(x) is continuous at x = 0, where $f(x) = \begin{cases} \frac{2}{x}(\sin(k_1+1)x + \sin(k_2+1)x) & x<0 \\ 4 & x=0 \\ \frac{2}{x}\log\left[\frac{k_2x+1}{k_1x+1}\right] & x>0 \end{cases}$
Then $k_1^2 + k_2^2$ is
Options:
(a) 1
(b) 2
(c) 0
(d) 4
Answer: (b)

$LHL_{x=0} = \lim_{x \to 0^-} 2\left[\frac{\sin(k_1+1)x}{x} + \frac{\sin(k_2+1)x}{x}\right]$

$= 2[k_1+1+k_2+1] = 2(k_1+k_2+1)$

$RHL_{x=0} = \lim_{x \to 0^+} \frac{2}{x}\log\left(\frac{k_2x+1}{k_1x+1}\right)$

$= \lim_{x \to 0^+} \frac{2}{x}\left(\frac{k_2x+1}{k_1x+1} - 1\right)$

$= \lim_{x \to 0^+} \frac{2}{x}\left[\frac{x(k_2-k_1)}{k_1x+1}\right]$

$RHL_{x=0} = 2(k_2 - k_1)$

$f(0) = 4$

$\Rightarrow 2(k_2-k_1) = 4 = 2(k_1+k_2+2)$

$k_2 - k_1 = 2 \qquad k_2 = 1$

$k_1 + k_2 = 0 \qquad k_1 = -1$

$k_1^2 + k_2^2 = 2$

Vedantu

Question: (a(b - c)x^2 + b(c - a)x + c(a - b) = 0 has equal roots a + c = 15; b = $\frac{36}{5}$ find a^2 + c^2 = ?
Options:
(a) 108
(b) 225
(c) 117
(d) 333
Answer: (c)

$a + c = 15 \quad b = \frac{36}{5}$

$\frac{c(a-b)}{a(b-c)} = 1 \rightarrow ac - bc = ab - ac$

Assuming roots are repeated

$2ac = b(a + c)$

$= \frac{36}{5} \times 15$

$a^2 + c^2 = 15^2 - 108$

$= 117$

Question: value of $\cos^{-1}\left[\frac{12}{13}\cos x + \frac{5}{13}\sin x\right]$ is $\left(x \in \left[\frac{\pi}{2}, \pi\right]\right)$
Options:
(a) $x + \tan^{-1}\frac{12}{13}$
(b) $x - \tan^{-1}\frac{12}{13}$
(c) $x - \tan^{-1}\frac{5}{12}$
(d) $x + \tan^{-1}\left(\frac{4}{5}\right)$
Answer: (c)

$= \cos^{-1}[\cos(x - \alpha)] \qquad tan\alpha = \frac{5}{12}\alpha \in 1^{st}qu$

$= x - \alpha$

$= x - \tan^{-1}\frac{5}{12}$

Question: If for the system of linear equations having infinite solutions (λ -4)x + (λ -2)y +λz =0
2x +3y +5z = 0
x +2y +6z =0 then λ2 + λ is

Options:
(a) 9
(b) 81
(c) 90
(d) 162
Answer: (c)

$$\begin{vmatrix} \lambda-4 & \lambda-2 & \lambda \\ 2 & 3 & 5 \\ 1 & 2 & 6 \end{vmatrix} = 0$$

$$\rightarrow \begin{vmatrix} \lambda-4 & 6-\lambda & 6-2\lambda \\ 2 & -1 & -4 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$4(\lambda-6)+(6-2\lambda)=0$

$2\lambda-18=0$

$\lambda=9$

$\lambda^2+\lambda=90$

Question: A relation defined on set A = {1,2,3,4}, then how many minimum ordered pairs are added to R {(1,2,),(2,3), (3,3))} so that it becomes an equivalence relation ?
Options:
(a) 4
(b) 5
(c) 6
(d) 7
Answer: (d)

The given set is $A=\{1,2,3,4\}$, and the relation $R=\{(1,2),(2,3),(3,3)\}$. To make R an equivalence relation, it must satisfy reflexivity, symmetry, and transitivity.

1. **Reflexivity**: Add $(1,1),(2,2),(4,4)$.
2. **Symmetry**: Add $(2,1),(3,2)$ (to match $(1,2)$ and $(2,3)$).
3. **Transitivity**:
   - $(1,2)$ and $(2,3)$ imply $(1,3)$.
   - $(2,3)$ and $(3,2)$ imply $(2,2)$ (already added for reflexivity).
   - $(3,2)$ and $(2,1)$ imply $(3,1)$.

Total pairs to add:

$$\{(1,1),(2,2),(4,4),(2,1),(3,2),(1,3),(3,1)\}.$$

Number of pairs added: 7.

Question: The sum of all rational terms in the expansion of $\left(1+2^{\frac{1}{3}}+3^{\frac{1}{2}}\right)^6$ **is**
Options:
(a) 612
(b) 373
(c) 400
(d) 180
Answer: (a)

$\left(1+2^{\frac{1}{3}}+3^{\frac{1}{2}}\right)^6$

$\frac{6!}{a!b!c!}1^a2^{\frac{b}{3}}3^{\frac{c}{2}}; a+b+c=6$

$b \rightarrow 0,3,6$

$c \rightarrow 0,2,4,6$

a	b	c
6	0	0
4	0	2
2	0	4
0	0	6
3	3	0
1	3	2
0	6	0

$\frac{6!}{6!}+\frac{6!}{4!2!}3+\frac{6!}{2!4!}+\frac{6!}{6!}3^3+\frac{6!}{3!3!}2^1+\frac{6!}{3!2!}2!3!+\frac{6!}{6!}2^2$

$=1+45+135+27+40+360+4$

$=612$

Question: $(7+e^{2x})dy-(1+2e^{2x})(y+3)dx=0$, **y(0) = 5 then** $y(\ln^2)$
Solution :

$\frac{dy}{y+3}=\frac{1+2e^{2x}}{7+e^{2x}}dx=\frac{e^{-2x}}{7e^{-2x}+1}+\frac{2e^{2x}}{7+e^{2x}}dx$

$\ln|y+3|=-\frac{1}{2}\ln\left|1+7e^{-2x}\right|+\ln\left|7+e^{2x}\right|+C$

$\ln 8=-\frac{1}{2}\ln|8|+\ln 8+c \qquad c=\frac{3}{2}\ln 2$

$\ln(y+3)=-\frac{1}{2}\ln\left|1+7.\frac{1}{4}\right|+\ln|7+4|+\frac{3}{2}\ln 2$

$=-\frac{1}{2}\ln\frac{11}{4}+\ln 11+\frac{3}{2}\ln 2$

$=\frac{1}{2}\ln\left(\frac{121\times 4}{11}\times 8\right)=\frac{1}{2}\ln(11\times 32)$

$y+3=4\sqrt{22}$

$y=4\sqrt{22}-3$

Question: The range of values of a for which $5x^3-15x-a=0$ **has 3 distinct solutions is** (α,β) **then** $\beta-2\alpha$ **is**
Options:
(a) 10
(b) 20
(c) 30
(d) 40
Answer: (c)

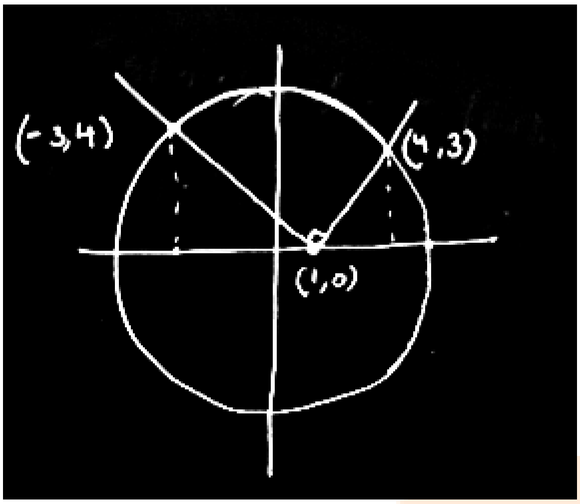
$$f'(x) = 15x^2 - 15 = 0$$
$$x = \pm 1$$
$$f(1)f(-1) = -(10 + a)(10 - a) < 0$$
$$a \in (-10, 10)$$
$$\beta - 2\alpha = 30$$

Question: Find the value of sin70º(cot10º cot70º - 1)
Options:
(a) 0
(b) 1
(c) 2
(d) 3
Answer: (b)

$$\sin 70^o(\cot 10^o \cot 70^o - 1)$$
$$\cot 80 = \frac{(\cot 10 \cot 70^o - 1)}{\cot 10 + \cot 70}$$
$$= \sin 70^o \left[\frac{1}{\sqrt{3}}(\cot 10 + \cot 70)\right] cot 80$$
$$= \sin 70 \left[\frac{\sin 80}{\sin 10 \sin 70}\right] \frac{\cos 80}{\sin 80}$$
$$= 1$$

Question: Area of the larger region bounded by curves y = |x - 1| and $x^2 + y^2 = 25$ is
Solution :

$$A = 25\pi - \int_3^1 \sqrt{25 - x^2} - (1 - x)dx$$
$$- \int_{-3}^1 \sqrt{25 - x^2} - (x - 1)dx$$
$$= 25\pi - \int_3^4 \sqrt{25 - x^2}\, dx + 8 + \frac{9}{2}$$
$$= 25\pi - \left[\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{-3}^4 + \frac{25}{2}$$
$$-25\pi - \left[(6 + 3) + \frac{25}{2}\left(\frac{\pi}{2}\right)\right] + \frac{25}{2} = \frac{75\pi}{4} + \frac{7}{2}$$



Question:f(x) = log$_e$x $g(x)=\frac{x^4-2x^3+3x^2-2x+2}{2x^2-2x+1}$ **Then find the domain of fog(x).**
Options:
(a) Q
(b) R
(c) C
(d) N
Answer: (b)
$\frac{x^4-2x^3+3x^2-2x+2}{2x^2-2x+1}>0$

$x^4-2x^3+3x^2-2x+2>0$

$\rightarrow\left(x^2+1\right)\left(x^2+2\right)-2x\left(x^2+1\right)>0$

$\left(x^2+1\right)\left[x^2-2x+2\right]>0$

So $x\in R$.

Question: $f(x)=\int\frac{dx}{(x-1)^{\frac{11}{13}}(x+15)\frac{15}{13}}$ **and** $I(37)-I(24)=\frac{1}{4}\left(\frac{1}{b^{\frac{1}{13}}}-\frac{1}{c^{\frac{1}{18}}}\right)$ **the value of 3(b + c)**
Options:
(a) 9
(b) 4
(c) 27
(d) 39
Answer: (d)

$$\int \frac{dx}{(x-1)^2\left[\left(\frac{x+15}{x-11}\right)^{\frac{15}{3}}\right]}$$

$$\frac{x+15}{x-11} = t$$

$$\Rightarrow \frac{(x-11)1-(x+15)1}{(x-1)^2}$$

$$\frac{dx}{(x-1)^2} = \frac{-1}{26}$$

$$\Rightarrow \frac{-1}{26}\int t^{\frac{-15}{13}} at = \frac{-1}{26} \cdot \frac{t^{\frac{-2}{13}}}{\frac{2}{b}}$$

$$I = \frac{1}{4}\left(\frac{r-11}{x+15}\right)^{\frac{2}{13}} + C$$

$$I(37) = \frac{1}{4}\left[\frac{26}{52}\right]^{\frac{2}{13}} = \frac{1}{4}2^{\frac{2}{13}} = \frac{1}{4} \cdot 4^{\frac{1}{3}}$$

$$I(24) = \frac{1}{4}\left[\frac{13}{39}\right]^{\frac{2}{13}} = \frac{1}{4} \cdot 3^{\frac{2}{13}} = \frac{1}{4} \cdot 9^{\frac{1}{3}}$$

$$b = 4_1 C = 9$$

$$3(b+c) = 39$$

Question: Two vertices of a triangle are (-2, 4) and (5, 4) and its orthocenter is (3, 5). The area of triangle is 35 and centroid is (c, d) then c + 2d is equal to

Options:

(a) 10/3

(b) 20/3

(c) 40/3

(d) 50/3

Answer: (d)

Now, Area of ΔABC

$$= \frac{1}{2} AB \times CP = 3.5$$

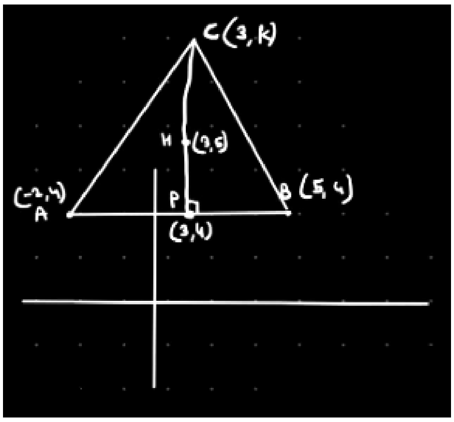
$$\frac{1}{2} \times 7 \times (k-4) = 35$$

$$k = 1y$$

Now centroid $\left(\frac{3+5-2}{3}, \frac{4+4+14}{3}\right)$

$$\left(2, \frac{22}{3}\right)$$

$$c + 2d = 2 + \frac{44}{3} = \frac{50}{3}$$



Question: $\int_{e^2}^{e^4} \frac{e^{\left(\ln^2 x+1\right)^{-1}}}{e^{\left(\ln^2 x+1\right)^{-1}} + e^{\left((6-\ln x)^2+1\right)^{-1}}} \frac{dx}{x}$

Options:

(a) 4

(b) 2

(c) 0

(d) 1

Answer: (d)

$$\int_{e^2}^{e^4} \frac{e^{\left(\ln^2 x+1\right)^{-1}}}{e^{\left(\ln^2 x+1\right)^{-1}} + e^{\left((6-\ln x)^2+1\right)^{-1}}} \frac{dx}{x}$$

$$\ln x = t \quad \begin{matrix}4\\2\end{matrix}$$

$$\frac{1}{n}dn = dt$$

$$\int_{2}^{4} \frac{e^{\left(t^2+1\right)^{-1}}}{e^{\left(t^2+1\right)^{-1}} + e^{\left((6-t)^2+1\right)^{-1}}} dt$$

$$t \to 6-t$$

$$2I = \int_{2}^{4} 1 dt$$

$$2I = (4-2)$$

$$I = 1$$

Question: Let Q be the foot of perpendicular from point P(3,4,5) the line $\frac{x-1}{2} = \frac{y+3}{-5} = \frac{z-5}{2}$. Find the area of ΔPQR given that R (1,-2,3).

Solution :

$L : \frac{x-1}{2} = \frac{y+3}{-5} = \frac{z-5}{2} = \lambda$

$(2\lambda+1, -5\lambda-3, 2\lambda+5)$ $\;\bar{b} = 2\hat{i} - 5\hat{j} + 2\hat{k}$

Now

$\overline{PQ} = (2\lambda - 2)\hat{i} + (-5\lambda - 7)\hat{j} + 2\lambda\hat{k}$

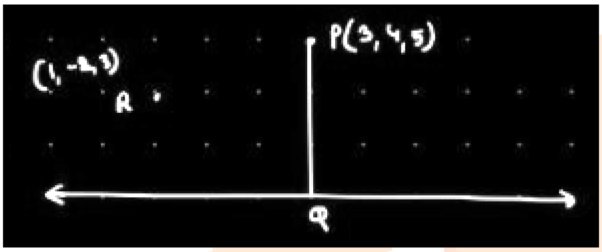
Now $\overline{PQ} \perp \bar{b}$

$\Rightarrow \overline{PQ} \cdot \bar{b} = 0$

$2(2\lambda - 2) - 5(-5\lambda - 7) + 4\lambda = 0$

$33\lambda + 31 = 0$

$\Rightarrow \lambda = \frac{-31}{33}$



$Q\left(\frac{-29}{33}, \frac{-204}{39}, \frac{103}{33}\right)$

Now $\overline{RP} = 2\hat{i} + 6\hat{j} + 2\hat{k}$

$\overline{RQ} = \frac{-62}{33}\hat{i} - \frac{138}{33}\hat{j} + \frac{4}{33}\hat{k}$

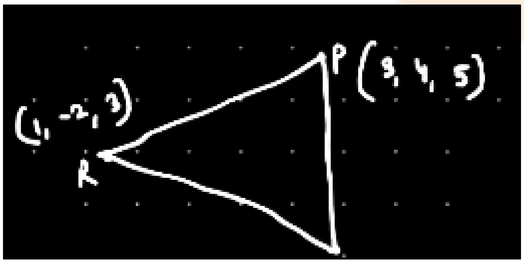
Now area of $\Delta PQR = \frac{1}{2}\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 2 \\ \frac{-62}{33} & \frac{-138}{33} & \frac{4}{3} \end{vmatrix}\right|$

$= \frac{1}{33}\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 1 \\ -62 & -138 & 4 \end{vmatrix}\right|$

$= \frac{1}{33}\left|150\hat{i} - 66\hat{j} - 48\hat{k}\right|$

$= \frac{1}{33}\sqrt{22500 + 4356 + 2304}$

$= \frac{1}{3}\sqrt{29160}$



Question: If $\left|\frac{z}{z+i}\right| = 2$ **represents a circle with centre P then distance of P from D is (where D:(1, 5))**

Solution :

$Let\ z = x + iy$

$\left|\frac{x+iy}{x+i(y+1)}\right| = 2$

$\Rightarrow \sqrt{x^2 + y^2} = 2\sqrt{x^2 + (y+1)^2}$

$x^2 + y^2 = [x^2 + y^2 + 2y + 1]$

$\Rightarrow 3x^2 + 3y^2 + 8y + 4 = 0$

$\Rightarrow x^2 + y^2 + \frac{8}{3}y + \frac{4}{3} = 0$

Center$\left(0, \frac{-4}{3}\right)$

Now,P$\left(0, \frac{-4}{3}\right)$and $D(1,5)$

$PD = \sqrt{1 + \left(\frac{-4}{3} - 5\right)^2} = \sqrt{1 + \frac{361}{9}}$

$= \sqrt{\frac{370}{9}}$