MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. If the 5th, 6th and 7th term of the binomial expansion of $(1 + x^2)^{n+4}$ are in A.P. Then the greatest binomial coefficient in the expansion of $(1 + x^2)^{n+4}$ is

(1) 10 (2) 35

(3) 25 (4) 14

Answer (2)

Sol. Binomial coefficient of $T_{r+1} = {}^{n+4}C_r$

${}^{n+4}C_4, {}^{n+4}C_5, {}^{n+4}C_6$ in A.P.

$\Rightarrow \dfrac{(n+4)!}{4!\,n!} + \dfrac{(n+4)!}{6!(n-2)!} = 2\dfrac{(n+4)!}{(n-1)!\,5!}$

$\Rightarrow$ Solving $n = 3$

$\Rightarrow$ Greatest binomial coefficient in the expansion of $(1 + x^2)^{n+4} = (1 + x^2)^7$

$\Rightarrow {}^7C_3 = {}^7C_4 \Rightarrow \dfrac{7!}{3!\,4!} = \dfrac{7\times6\times5}{6} = 35$

2. If A is 3 × 3 matrix such that det(A) = 2. Then $\det(adj(adj(adj(adjA))))$ is

(1) 2^{32} (2) 2^{16}

(3) 2^8 (4) 2^{12}

Answer (2)

Sol. $\det\underbrace{\left(adj\left(adj\left(adj\left(adjA\right)\right)\right)\right)}_{r\text{ times}} = |A|^{(n-1)^r}$

As $|A| = 2$

$\therefore \det\left(adj\left(adj\left(adj\left(adjA\right)\right)\right)\right) = |A|^{(3-1)^4}$

$= |A|^{2^4} = |A|^{16}$

$= 2^{16}$

3. Evaluate

$\lim\limits_{x\to0} \operatorname{cosec}x.\left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}\right)$

(1) 0 (2) 1

(3) $\dfrac{1}{2\sqrt5}$ (4) $-\dfrac{1}{2\sqrt5}$

Answer (4)

Sol. $\lim\limits_{x\to0} \dfrac{\left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}\right)\left(\sqrt{2\cos^2 x + 3\cos x} + \sqrt{\cos^2 x + \sin x + 4}\right)}{(\sin x)\left(\sqrt{2\cos^2 x + 3\cos x} + \sqrt{\cos^2 x + \sin x + 4}\right)}$

$\dfrac{1}{2\sqrt5}\lim\limits_{x\to0} \dfrac{2\cos^2 x + 3\cos x - \cos^2 x - \sin x - 4}{\sin x}$

$\dfrac{1}{2\sqrt5}\lim\limits_{x\to0} \left[\dfrac{\cos^2 x + 3\cos x - \sin x - 4}{\sin x}\right]$

Apply L' Hopital

$\dfrac{1}{2\sqrt5}\lim\limits_{x\to0} \left[\dfrac{-2\cos x\sin x - 3\sin x - \cos x}{\cos x}\right]$

$= -\dfrac{1}{2\sqrt5}$

4. If $\vec a = \hat i + 2\hat j + 3\hat k$, $\vec b = 3\hat i + \hat j - \hat k$ and $\vec c$ is coplanar with $\vec a$ and $\vec b$. Also $\vec a\cdot\vec c = 5$ and $\vec c$ is perpendicular to $\vec b$. Then $|\vec c|$ is

(1) 18 (2) 16

(3) $\dfrac{\sqrt5}{14}$ (4) $\sqrt{\dfrac{11}{6}}$

Answer (4)

Sol. $\vec a = \hat i + 2\hat j + 3\hat k$

$\vec b = 3\hat i + \hat j - \hat k$

$\vec c$ coplanar with $\vec a$ and $\vec b$ (Let $\vec c = a_1\hat i + a_2\hat j + a_3\hat k$)

$\Rightarrow \quad (\vec{a}\times\vec{b})\cdot\vec{c}=0$

$(-5\hat{i}+10\hat{j}-5\hat{k})\cdot(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})=0$

$\Rightarrow \quad -5a_1 + 10a_2 - 5a_3 = 0$

$\Rightarrow \quad \boxed{a_1 + a_3 = 2a_2}$ ...(i)

$\vec{c}\cdot\vec{b}=0$ and $\vec{a}\cdot\vec{c}=5$

$\boxed{3a_1 + a_2 - a_3 = 0}$ ...(ii)

$(a_1 + 2a_2 + 3a_3) = 5$ ...(ii)

Solving (i), (ii) and (iii)

$a_1=\frac{1}{6},\ a_2=\frac{2}{3},\ a_3=\frac{7}{6}$

$\Rightarrow \quad \vec{c}=\frac{1}{6}\hat{i}+\frac{2}{3}\vec{j}+\frac{7}{6}\vec{k}$

$\Rightarrow \quad |\vec{c}|=\sqrt{\frac{11}{6}}$

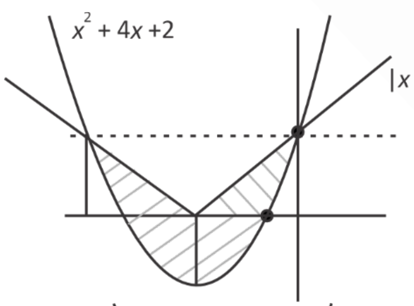
5. The area of the region bounded by $S(x, y)$ such that $S = \{(x, y) : x^2 + 4x + 2 \le y \le |x + 2|\}$ is (in sq. units)

(1) $\frac{24}{5}$ (2) 5

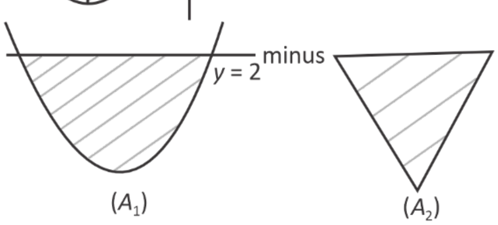
(3) $\frac{20}{3}$ (4) 7

Answer (3)

Sol. $x^2 + 4x + 2 \le y \le |x + 2|$



Required area =



$$A_1=\int_{-4}^{0}[2-(x^2+4x+2)]dx-\frac{1}{2}\times4\times2$$

$$=\left(\frac{-x^3}{3}-2x^2\right)\Bigg|_{-4}^{0}-4$$

$$=0-\left(\frac{64}{3}-32\right)-4$$

$$=32-\frac{64}{3}-4=\frac{20}{3}$$

6. If $\frac{dy}{dx}+\left(\frac{x}{1+x^2}\right)y=\frac{\sqrt{x}}{\sqrt{1+x^2}}$; $y(0) = 0$, then $y(1)$ is

(1) $\frac{2}{3}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\frac{\sqrt{2}}{3}$ (4) $\sqrt{\frac{2}{3}}$

Answer (3)

Sol. $\frac{dy}{dx}-\left(\frac{x}{1+x^2}\right)y=\frac{\sqrt{x}}{\sqrt{1+x^2}}$

If $= e^{\int\frac{x}{1+x^2}dx}=e^{\frac{1}{2}\ln(1+x^2)}=\sqrt{1+x^2}$

Solution will be $y\sqrt{1+x^2}=\int\frac{\sqrt{x}}{\sqrt{1+x^2}}\cdot\sqrt{1+x^2}\,dx$

$y\sqrt{1+x^2}=\frac{2x^{3/2}}{3}+c$

$y(0) = 0 \Rightarrow c = 0$

$y=\frac{2}{3}\frac{x^{3/2}}{\sqrt{1+x^2}}$

Now $y(1)=\frac{\sqrt{2}}{3}$

7. If α and β are real numbers such that $\sec^2(\tan^{-1}(\alpha)) + \text{cosec}^2(\cot^{-1}(\beta)) = 36$ and $\alpha + \beta = 8$, then $(\alpha^2 + \beta)$ is $(\alpha < \beta)$

(1) 23 (2) 14

(3) 24 (4) 27

Answer (2)

Sol. Let $\tan^{-1}\alpha = A \Rightarrow \tan A = \alpha$

$\cot^{-1}\beta = B \Rightarrow \cot B = \beta$

$\sec^2 A + \text{cosec}^2 B = 36$

$(1 + \tan^2 A) + (1 + \cot^2 B) = 36$

$\Rightarrow \quad 1 + \alpha^2 + 1 + \beta^2 = 36$

$\Rightarrow \quad \alpha^2 + \beta^2 = 34$

$\alpha + \beta = 8$

$(\alpha + \beta)^2 = 34 + 2\alpha\beta = 64$

$\Rightarrow \quad \alpha\beta = 15$

$\Rightarrow$ α, β are roots of equation

$x^2 - 8x + 15 = 0$

$(x - 3)(x - 5) = 0$

$\Rightarrow \quad x = 3, 5$

$\Rightarrow \quad \alpha = 3, \beta = 5$

$\Rightarrow \quad \alpha^2 + \beta = 14$

8. Two persons *A* and *B* throws a pair of dice alternatively. For *A* to win he should throw sum of 5 before *B* throws sum of 8. If *A* throws first, then the probability that *A* wins, is

(1) $\frac{8}{19}$ (2) $\frac{9}{19}$

(3) $\frac{8}{17}$ (4) $\frac{9}{17}$

Answer (2)

Sol. For sum 5, (1, 4), (2, 3) (3, 2), (4, 1) $\Rightarrow P(A) = \frac{4}{36}$

For sum 8, (2, 6), (3, 5) (4, 4), (5, 3), (6, 2) $\Rightarrow P(B) = \frac{5}{36}$

$\Rightarrow \quad P(\bar{A}) = \frac{32}{36}, P(\bar{B}) = \frac{31}{36}$

P(A wins) =

$P(A) + P(\bar{A})P(\bar{B})P(A) + P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(A) + \ldots$

$$= \frac{P(A)}{1 - P(\bar{A})P(\bar{B})} = \frac{\frac{4}{36}}{1 - \frac{32}{36}\cdot\frac{31}{36}} = \frac{9}{19}$$

9. For a distribution of 10 observations, $\sum_{i=1}^{10} x_i = 55$ and $\sum_{i=1}^{10} x_i^2 = 328$. If the observations 4 and 5 are replaced by 6 and 8 respectively, then the new variance is

(1) 2.5 (2) 2.7

(3) 3.4 (4) 3.6

Answer (2)

Sol. $x_1 + x_2 \ldots + x_8 + 4 + 5 = 55,$

$x_1^2 + x_2^2 \ldots + x_8^2 + 16 + 25 = 328$

$\mu = \frac{x_1 + x_2 \ldots + x_8 + 6 + 8}{10} = 6,$

$x_1^2 + x_2^2 + \ldots + x_8^2 + 36 + 64 = 387$

$\Rightarrow \sigma^2 = \frac{387}{10} - 36 = \frac{387 - 360}{10} = \frac{27}{10} = 2.7$

10. If *S* be the set of 10 distinct primes and let *A* be the set of products of two or more elements from the set *S*. If $P = \{(x, y) : x \in S$ and $y \in A$ and y is divided by $x\}$. Then $n(P)$ is equal to

(1) 5110 (2) 5000

(3) 5220 (4) 5420

Answer (1)

Sol. $S = \{P_1, P_2, P_3 \ldots, P_{10}\}$

A = {product of distinct elements of S}

If $P = \{(x, y) : x \in S, y \in A$ and y is divided by $x\}$ then $n(P)$ is

$\Rightarrow$ since $x \in S$.

Let $x = P_i \in S$., selected from S in $\left(10_{C_1}\right)$ ways.

Now, $x|y$ iff y contains P_i in product.

$\Rightarrow$ (P_i) is there and at rest one other element as a product

$\Rightarrow \quad \left(10_{C_1}\right)\cdot\left[(1)\cdot(2^9 - 1)\right] = 10\cdot(511) = 5110$

11. If $I(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx$, $m, n > 0$, then $I(9, 14) + I(10, 13)$ is equal to

(1) $I(1, 13)$

(2) $I(9, 1)$

(3) $I(9, 13)$

(4) $I(19, 29)$

Answer (3)

Sol. Beta function

$$\beta(p,q)=\int_0^1 x^{p-1}(1-x)^{q-1}dx=\frac{(p-1)!(q-1)!}{(p+q-1)!}p,q\in I$$

$$\Rightarrow \quad I(9,14)=\frac{8!\ 13!}{22!}$$

$$I(10,\ 13)=\frac{9!\cdot 12!}{22!}$$

$$\Rightarrow \quad I(9,\ 14)+I(10,\ 13)=\frac{1}{22!}(8!\ 13!+9!\cdot 12!)$$

$$\Rightarrow \quad =\frac{1}{22!}8!\cdot 12!(13+9)=\frac{8!\ 12!}{21!}$$

$$=\frac{(9-1)!\,(13-1)!}{(9+13-1)!}=I(9,\ 13)$$

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The number of 3 digit numbers which is divisible by 2 and 3 but not divisible by 4 and 9?

Answer (125)

Sol. Total number divisible by 6

102, 108, ... 996

So total number = 150

Number divisible by 36

108, ..., 972

Total number = 25

Required number = (divisible by 6) – (divisible by 36)

= 150 – 25 = 125

22. The product of all real roots of equation $(x^2 - 9x + 11)^2 - (x - 4)(x - 5) = 2$ is

Answer (99)

Sol. $(x^2 - 9x + 11)^2 - (x - 4)(x - 5) = 2$

$(x^2 - 9x + 11)^2 - (x^2 - 9x + 20) = 2$

Let $x^2 - 9x + 11 = t$

$t^2 - (t + 9) = 2$

$t^2 - t - 11 = 0$

$$t=\frac{1\pm\sqrt{1+44}}{2}=\frac{1\pm\sqrt{45}}{2}$$

$$x^2-9x+11=\frac{1+\sqrt{45}}{2}$$

$$x^2-9x=\frac{\sqrt{45}-21}{2}$$

$$D_1:81-4\left(\frac{\sqrt{45}-21}{2}\right)>0$$

Similarly,

$$x^2-9x+11=\left(\frac{1-\sqrt{45}}{2}\right)$$

$$\Rightarrow \quad D_2:81-4\left(\frac{-21-\sqrt{45}}{2}\right)>0$$

$\Rightarrow$ All roots are real

$\Rightarrow$ Product of roots $=\left(\dfrac{\text{Constant term}}{1}\right)$

$$=\left(\frac{121-(20)-2}{1}\right)=\frac{121-22}{1}=\boxed{99}$$

23. If $S_n=\dfrac{1}{2}+\dfrac{1}{6}+\dfrac{1}{12}+\ldots n$ terms. The sum of first six terms in A.P. with first term equal to $-p$ and common difference p is $\sqrt{2026.\,S_{2025}}$. The absolute value of difference between 20th and 15th terms in A.P. is

Answer (25)

Sol. $S_n=\dfrac{1}{1.2}+\dfrac{1}{2.3}+\dfrac{1}{3.4}+\ldots\dfrac{1}{n(n+1)}$

$$=\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$=1-\frac{1}{n+1}=\frac{n}{n+1}\Rightarrow S_{2025}=\frac{2025}{2026}$$

For A.P. $\dfrac{6}{2}(-2p+(6-1)p)=\sqrt{2026\times\dfrac{2025}{2026}}=45$

$\Rightarrow\ 3p=\dfrac{45}{3}\Rightarrow p=5$

Now for A.P. $|a_{20}-a_{15}|=|19\times5-14\times5|=25$

24. If f(x) satisfies the functional equation $f(x)+6f\left(\dfrac{1}{x}\right)=\dfrac{35}{3x}-\dfrac{7}{2},\ x\in R-\{0\}$ and if $\lim\limits_{x\to0}\left(\dfrac{1}{\alpha x}+f(x)\right)$ exist finitely and is equal to β, then $(\alpha-2\beta)$ is

Answer (2)

Sol. $f(x)+6f\left(\dfrac{1}{x}\right)=\dfrac{35}{3x}-\dfrac{7}{2}$

$$f\left(\frac{1}{x}\right)+6f(x)=\frac{35x}{3}-\frac{7}{2}$$

$$36f(x)+6f\left(\frac{1}{x}\right)=70x-21$$

$$35f(x)=(70x-21)-\left[\frac{35}{3x}-\frac{7}{2}\right]$$

$$35f(x)=70x-\frac{35}{3x}-\frac{35}{2}$$

$$\Rightarrow\quad f(x)=2x-\frac{1}{3x}-\frac{1}{2}$$

$$\lim_{x\to0}\left(\frac{1}{\alpha x}+2x-\frac{1}{3x}-\frac{1}{2}\right)=\beta$$

Limit exist finitely iff

$\alpha=3\Rightarrow\beta=-\dfrac{1}{2}$

$$(\alpha-2\beta)=3-2\left(-\frac{1}{2}\right)=4$$

25.

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