MIXTURES, PROFIT AND LOSS

Notes / Rough Work

Mixture (Alligation)

These types of questions are very important and involve the theory of ratio, proportion, percentages, profit and loss.

Generally questions on mixtures are based on either simple mixtures or compound mixtures. Simple mixtures involve mixing of only two items, while compound mixtures may involve more than two items.

Alligation helps us

- To find the mean or average values of mixtures, when the prices of two or more ingredients and the proportion in which they are to be mixed are given.
- To find the proportion in which the ingredients at given prices must be mixed to produce a mixture at a given price.

When quantities at two different prices are mixed in a certain ratio, then our aim is to find the price of the final mixture.

It is given by the formula

☞ $Qc \times Pc + Qd \times Pd = (Qc + Qd) \times Pm$

$Qc \times Pc + Qd \times Pd = Qc \times Pm + Qd \times Pm$

$Qd\,(Pd - Pm) = Qc\,(Pm - Pc)$

or $\frac{Qc}{Qd} = \frac{Pd - Pm}{Pm - Pc}$ or

☞ $\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{\text{Dearer Price} - \text{Mean Price}}{\text{Mean Price} - \text{Cheaper Price}}$

where Pd and Qd means the cost price and quantity of the dearer item; Pc and Qc means the cost price and quantity of the cheaper item and mean price Pm means the price of the final mixture.

In some questions, profit and loss condition is given along with some other data.

Notes / Rough Work

E1. In what proportion must a grocer mix one kind of tea at Rs.45 per kg with another at Rs.40 per kg so that the final mixture costs him Rs.41.50 per kg?

Sol. Mean price = Rs.41.50.

Dearer price = Rs.45.

Cheaper price = Rs.40.

Putting the values in the formula

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{45 - 41.5}{41.5 - 40} = \frac{3.5}{1.5} = \frac{7}{3}.$$

He should mix 7 parts of tea costing Rs.40 per kg with 3 parts of tea costing Rs.45 per kg to get the final mixture.

E2. How many kg of tea selling at Rs.10.40 per kg should be mixed with tea selling at Rs.8.80 per kg to make a mixture 15 kg at Rs.146.40?

Sol. Cost per kg of resulting mixture = $\frac{146.40}{15}$ = Rs.9.76

Putting the values in the formula

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{10.40 - 9.76}{9.76 - 8.80} = \frac{0.64}{0.96} = \frac{2}{3}.$$

$\therefore$ The two varieties should be mixed in the proportion 3 : 2. (Dearer : Cheaper = 3 : 2)

In 15 kg of mixture there should be 9 kg of tea @ Rs.10.40 and 6 kg of tea @ Rs.8.80.

E3. In what proportion must sugar costing Rs.14 per kg and Rs.17 per kg be mixed so that 20% profit is earned by selling the mixture at Rs.18 per kg?

Sol. Mean price = 18/1.2 = Rs.15.

Dearer price = Rs.17.

Cheaper price = Rs.14.

Putting the values in the formula

$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} = \frac{17 - 15}{15 - 14} = \frac{2}{1}.$$

One should mix 2 parts of the cheaper sugar with 1 part of the costlier sugar to get the final mixture.

E4. Gold is 19 times as heavy as water and copper 9 times. In what ratio should these metals be mixed so that the mixture may be 15 times as heavy as water?

Sol.

	Gold		Copper
Resultant	19 —	15	— 9
Proportion	6 —		— 4

is Gold : Copper = 15 – 9 : 19 – 15 = 6 : 4 = 3 : 2.

E5. A mixture of 70 litres of wine and water contains 10% of water. How much water must be added to make water 37% of the resulting mixture?

Sol. The mixture contains $\left(\frac{10}{100}\right)70 = 7$ litres of water.

$\Rightarrow$ It contains (70 – 7) = 63 litres of wine.

Let x litres of water be added. $\therefore \frac{7 + x}{70 + x} = \frac{37}{100} \Rightarrow x = 30$ litres.

E6. In two alloys, the ratio of zinc to tin are 3 : 2 and 2 : 3. If 7 kg of the first alloy and 21 kg of the second alloy are mixed together to form a new alloy, then what will be the ratio of zinc and tin in the new alloy?

Notes / Rough Work

Sol.

	Zinc	**Tin**
First alloy (I)	3	2
Second alloy (II)	2	3
Quantity mixed	**Zinc**	**Tin**
From I	4.2	2.8
From II	8.4	12.6
Total	**12.6**	**15.4**

Hence, the ratio of zinc to that of tin = $\frac{12.6}{15.4} = \frac{9}{11}$.

Three ingredients: Number of proportions unlimited

Now, let us understand the alligation rule of three ingredients with the help of a couple of examples.

E7. In what proportion should three kinds of wheat at Rs.1.27, Rs.1.29 and Rs.1.32 per kg be mixed to produce mixture worth Rs.1.30 per kg?

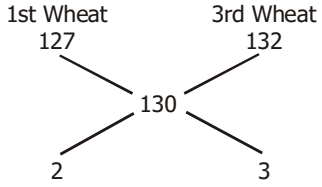
Sol.

1st wheat	2nd wheat
127 P	129 P
3rd wheat	Mean Price
132 P	130 P

Here, the first two prices are less than the mean price while the third price is more.

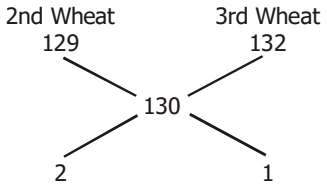
We first find the proportion in which wheat at 127 P and 132 P must be mixed to produce a mixture at 130 P.

(i)



The proportion is 2 : 3

We next find the proportion in which wheat at 129 P and 132 P must be mixed to produce a mixture at 130 P.



The proportion is 2 : 1

Now, in whatever proportion these two mixtures are mixed, the price of the resulting mixture will always be 130 P per kg because both mixtures cost 130 P/kg. Now 5 kg of the first mixture is composed of 2 kg of wheat at 127 P and 3 kg of wheat at 132 P, and 3 kg of second mixture is composed of 2 kg of wheat at 129 P and 1 kg of wheat at 132 P. Hence, 5 + 3 or 8 kg of the resulting mixture is composed of 2 kg at 127 P, 2 kg at 129 P and (3 + 1) or 4 kg at 132 P. Hence, the required proportion is 2 : 2 : 4 or 1 : 1 : 2

Take another case:

If we use (say) 4 kg of the first wheat we must use 6 kg of the third wheat. Again, if we use (say) 10 kg of the second wheat, we must use 5kg of third wheat. There is thus another proportion.

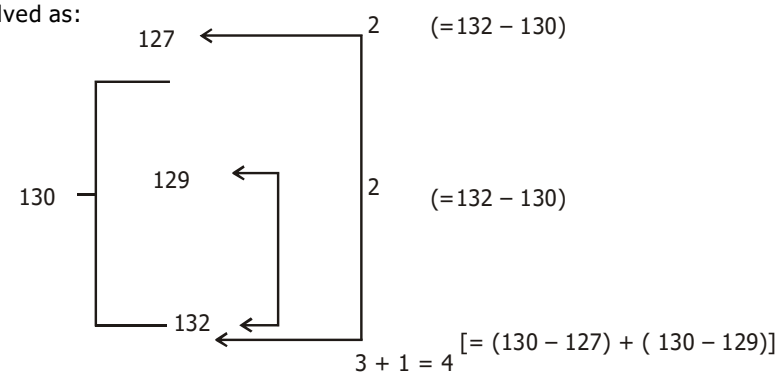
1st	2nd	3rd
4 kg	10 kg	6 + 5 = 11 kg

or 4 : 10 : 11

In fact, we can use any number of kg of the first or second wheat as long as we use the necessary corresponding number of kg of the third. Hence, the number of proportions is unlimited.

Note: The above calculations can be simplified further. For this observe the following rule.

Rule: Reduce the several prices to one denomination (like, Rs.1.24, Rs.1.31, Rs.1.20 can be written as 124, 131 and 120) and place them under one another in order of magnitude, the least being uppermost. Set down the mean price to the left of the prices. Link the prices in pairs so that the prices greater and lesser than the average price go together. Then find the difference between each price and the mean price and place it opposite the price with which it is linked. These differences will give the required answer. For example, the above example can be solved as:



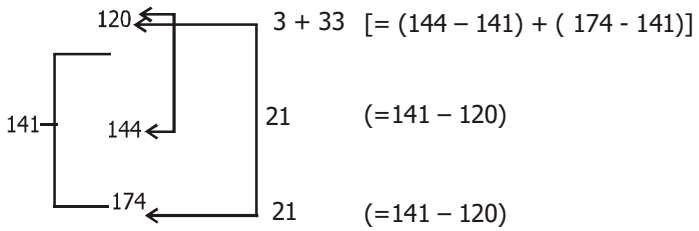
$\therefore$ The required proportion is 2 : 2 : 4 or 1 : 1 : 2

E8. In what ratio must a person mix three kinds of wheat costing him Rs 1.20, Rs 1.44 and Rs 1.74 per kg, so that the mixture may be worth Rs 1.41 per kg?

Sol.

1st Wheat	2nd Wheat	3rd wheat
120	144	174

Following the above rule, we have,



Therefore, the required ratio = 36 : 21: 21 = 12 : 7 : 7

Note: Try to get the other ratios which satisfy the conditions.

Important

If a vessel contains "x" litres of milk and if "y" litres is withdrawn and replaced by water, then again "y" litres of mixture is withdrawn and replaced by water and the operation is repeated "n" times in all, then

☞ $\dfrac{\text{Milk left in the vessel after } n^{th} \text{ operation}}{\text{Initial quantity of milk in the vessel}} = \left[\dfrac{x-y}{x}\right]^n$

Notes / Rough Work

E9. Ten gallons of wine is drawn from a vessel. It is then filled with water. Again 10 gallons of the mixture is drawn and the vessel is again filled with water. The quantity of wine now left in the vessel to that of water bears the ratio of 49 : 32. How much liquid does the vessel hold?

Sol. Initially there was only wine in the vessel. Hence, finally, the total quantity of wine and water should be equal to the initial quantity of wine. Let the initial quantity of wine be X gallons.

Applying the formula:

$$\frac{\text{Wine left in the vessel after } n^{th} \text{ operation}}{\text{Initial quantity of wine in the vessel}} = \left[\frac{X-10}{X}\right]^2$$

$= \frac{49}{49+32} = \frac{49}{81} \Rightarrow \frac{X-10}{X} = \frac{7}{9}$. Solving, we get X = 45 gallons.

E10. Nine litres are drawn from a cask full of wine and it is then filled with water. Nine litres of the mixture are drawn and the cask is again filled with water. The quantity of wine now left in the cask to that of water in it bears the ratio of 16 : 9. How much does the cask hold?

Sol. $\frac{A}{Q} = \left[1 - \frac{q}{Q}\right]^n$

A = quantity of wine in the final mixture. Q = volume of cask.

q = quantity removed.

n = number of times the operation is repeated.

$\Rightarrow \frac{A}{Q} = \frac{16}{16+9} = \frac{16}{25}$. Here q = 9, n = 2. $\Rightarrow \frac{16}{25} = \left[1 - \frac{9}{Q}\right]^2 \Rightarrow$ Q = 45 litres.

Mini Revision Test # 01

DIRECTIONS: *Answer the following questions.*

1. In what ratio must tea at Rs.124 per kg be mixed with tea at Rs.144 per kg, so that the mixture is worth Rs.129 per kg?
2. In all, 500 students appeared for an examination. Out of these, 70% of the girls and 75% of the boys passed the examination. If the total pass percentage is 72%, how many girls appeared in the examination?
3. Find the ratio in which water should be mixed with milk to gain 20% by selling it at cost price.
4. How many kg of sugar worth Rs.8 per kg should be mixed with 80 kg of sugar worth Rs.10 per kg, so that the mixture is worth Rs.9.25 per kg?
5. In what ratio should silver at Rs.6 per gm be mixed with silver at Rs.4.50 per gm to obtain a mixed variety of silver worth Rs.4.90 per gm?
6. A barrel contains $33\frac{1}{3}\%$ of kerosene in a mixture of petrol and kerosene. What is the ratio of kerosene to that of petrol?
7. Five litres of diesel is drawn from a vessel and it is then filled with spirit. The quantity of diesel left to that of spirit is in the ratio of 10:15. How much does the vessel hold?
8. If a mixture of Rs.10 per kg rice is to be prepared by mixing two varieties, one at Rs.6 per kg and other at Rs.14 per kg, then what should be the ratio of the two varieties?
9. Six litres of milk is withdrawn from a vessel and replaced with water. This procedure is repeated twice. If the ratio of milk to water is 16:9, then how much does the vessel hold?
10. A barrel of capacity 40 litres, containing wine is diluted by replacing the wine with spirit twice. If the ratio of wine to spirit is 16:20, then what is the quantity of wine withdrawn each time?

Notes / Rough Work

Challenge Problems # 01

DIRECTIONS: *Answer the following questions.*

1. Cambay Services has three departments I, II and III. The average salary of the three departments is Rs.8,300, Rs.7,600, Rs.8,500 respectively. The average salary of I and II is Rs.7,900 and average salary of II and III is Rs.8,200. What is the average salary of all the employees in all the department? ***(Q. code - 110505001)***

   (1) Rs.8,220 (2) Rs.8,150
   (3) Rs.7,980 (4) Rs.8,040

2. Two bags of equal volume contain rice mixed with gravel. In the first bag, there is twice as much rice as gravel. In the second, there is three times as much gravel as rice. The two bags are mixed in the ratio 3 : 2. One kg of rice is added to one kg of this mixture. Find the percentage of gravel in the final mixture. ***(Q. code - 110505002)***

   (1) 45% (2) 33%
   (3) 50% (4) 25%

3. Tushar bought two cars. He then sold the first car at 10% profit and the second one at 25% profit. The selling price of the second car is 25% more than the selling price of the first car. What is the approximate profit percent on both the cars together? ***(Q. code - 110505003)***

   (1) 17.85% (2) 18.36%
   (3) 16.19% (4) Can't be determined

Notes / Rough Work

Profit and loss

The following basic terms are used while solving problems on profit and loss. Hence, you are expected to understand each of these terms very clearly.

Cost price (C.P.): The price at which an article is bought.

Selling price (S.P.): The price at which an article is sold.

Profit

When an article is sold for more than its cost we say there is a **profit** or **gain**.

☞ Profit or gain = S.P – C.P

Loss

When an article is sold for less than its cost, we say there is a **loss.**

☞ Loss = C.P – S.P

E11. A gift item worth Rs.375 is sold by a shopkeeper with a profit of Rs.36. Find the selling price of the gift item.

Sol. The cost price of the gift item = Rs.375. Gain = Rs.36.

Now, S.P = C.P + Gain. So, S.P = Rs.375 + Rs.36 = Rs.411.

E12. Gajendra bought a fan for Rs.1350. He found that the fan was defective. So, he sold it at a loss of Rs.160. Find the selling price of the fan.

Sol. Cost price of the fan = Rs.1350.

Loss = Rs.160.

S.P = C.P – Loss.

So, S.P = Rs.1350 – Rs.160 = Rs.1190.

E13. Saurabh purchased a watch for Rs.1260 and sold it to Sachin for Rs.1320. Sachin sold it to Parthiv for Rs.1400. Who gained more and by how much?

Sol. *For Saurabh*

The cost price of the watch = Rs.1260.

The selling price = Rs.1320.

So, the profit = Rs.1320 – Rs.1260 = Rs.60.

For Sachin

The cost price of the watch = Rs.1320.

The selling price of the watch = Rs.1400.

So, the profit = Rs.1400 – Rs.1320 = Rs.80.

Clearly, Sachin gained more than Saurabh.

The difference in profits

= Rs.80 – Rs.60 = Rs.20.

Thus, Sachin gained Rs.20 more than Saurabh.

Profit and loss percent

Profit or loss is also expressed as a percentage of the cost price. To calculate the profit or loss percent we use the following formulae :

$$\text{Profit percent} = \left(\frac{\text{Profit}}{\text{C.P}} \times 100\right)\% .$$

$$\text{Loss percent} = \left(\frac{\text{Loss}}{\text{C.P}} \times 100\right)\% .$$

E14. An article worth Rs.120 is sold for Rs.145. What is the profit percent?

Sol. C.P of the article = Rs.120.

S.P of the article = Rs.145. So, profit = Rs.145 – Rs.120 = Rs.25.

$$\therefore \text{ Profit percent} = \left(\frac{\text{Profit}}{\text{C.P}} \times 100\right)\% = \left(\frac{25}{120} \times 100\right)\% = \left(\frac{125}{6}\right)\% .$$

Notes / Rough Work

POINT TO REMEMBER

Profit expressed as a percentage of cost price will be always greater than profit expressed as a percentage of selling price.

Loss expressed as a percentage of cost price will be always less than loss expressed as a percentage of selling price.

E15. The cost price of a video cassette player is Rs.12000. If it is sold for Rs.11040, then what is the loss percent?

Notes / Rough Work

Sol. C.P of the video cassette player= Rs.12000.

S.P of the video cassette player = Rs.11040.

So, the loss = Rs.12000 – Rs.11040 = Rs.960.

$\therefore$ Loss percent = $\left(\frac{960}{12000} \times 100\right)\%$ = 8%.

E16. If the S.P of 10 articles is the same as the C.P of 11 articles, find the gain percent.

Sol. Let the C.P of 1 article be Re.1.

$\therefore$ C.P of 10 articles = Rs.10.

$\therefore$ C.P of 11 articles = Rs.11.

$\therefore$ S.P of 10 articles = C.P of 11 articles = Rs.11.

$\therefore$ Gain % $= \frac{(11-10)}{10} \times 100$ = 10%.

Imp.

Profit and loss are always calculated with CP as the base.

$$\text{Selling Price} = \frac{\text{Cost Price} \times (100 + \text{Gain}\%)}{100}$$

$$\text{Selling Price} = \frac{\text{Cost Price} \times (100 - \text{Loss}\%)}{100}$$

E17. A man loses 10% by selling a book for Rs.144. What should be his selling price to gain 15%?

Sol. S.P = Rs.144. Loss = 10%.

$\therefore$ C.P = Rs.(100 × 144)/(100 – 10) = Rs.160.

Gain expected = 15%.

$\therefore$ S.P = Rs.(115 × 160)/100 = Rs.184.

E18. Find the cost price of an article, if, by selling the same for Rs.450, you incur a loss of 10%. What must be its selling price to gain 10%?

Sol. C.P = Rs.x, S.P = Rs.450.

$\therefore$ (90x/100) = 450 $\Rightarrow$ x = 500.

C.P = Rs.500.

For 10% profit,

S.P = (110/100)500 = Rs.550.

E19. A man sold an article for Rs.1800 and thereby made a profit of 20% on the outlay. How much did the article cost him?

Sol. Using the formula, $1800 = \frac{\text{Cost price} \times (100 + 20)}{100}$.

$\Rightarrow \text{Cost price} = 1800 \times \frac{100}{120}$ = Rs.1500.

E20. A man bought oranges at the rate of 4 for a rupee and sold them at a profit of 40 percent. How many oranges would a customer get for Rs 7?

Sol. Cost price of one orange = 100/4 = 25 paise.

Selling price of one orange

$= \frac{100+40}{100} \times \frac{100}{4} = 35$ paise .

$\Rightarrow$ For Rs 7 i.e. for 700 paise, the customer will get 700/35 = 20 oranges.

E21. An article costing Rs.210 is to be sold at a price 20% more than the cost price. What is the selling price of the article?

Sol. $SP = \frac{210 \times (100+20)}{100} \Rightarrow SP = 21 \times 12 = 252$.

Alternatively, SP = 210 × 1.2 = Rs.252.

E22. A man sold an article for Rs.80 and thereby made a profit of 25%. How much did the article cost him?

Sol. Using the formula, $80 = \frac{CP \times (100+25)}{100} \Rightarrow CP = \frac{8000}{125} = \text{Rs. } 64$.

E23. A person purchases 50 dozen eggs at Rs.4 per dozen. Of these, 40 eggs were found broken. At what price should he sell the remaining eggs in order to make a profit of 5%?

Sol. C.P of 50 dozen eggs = 50 × 4 = Rs.200.

$\therefore$ S.P = (105/100)200 = Rs.210.

Eggs remaining = (50)12 – 40 = 560 = 560/12 = 140/3 dozen.

$\therefore$ S.P of 1 dozen eggs = 210(3/140) = Rs.4.50.

Concept of same selling price with profit or loss

When two products are sold at the same price in such a way that on one of the products we earn a profit and on the other we incur a loss, but the percentage of profit or loss is the same, then for the combined transaction the net result would be a loss. In such cases the selling price is immaterial. There is always a loss in such transactions.

Loss % = $\left(\frac{\text{common gain or loss\%}}{10}\right)^2$

E24. Two articles are sold at Rs.198 each such that a profit of 10% is made on the first while a loss of 10% is incurred on the other. What would be the net profit or loss on the two transactions combined?

Sol. Article I : Profit = 10%, selling price = Rs.198.

$\Rightarrow$ Cost price = 198/1.1 = Rs.180.

Article II : Loss = 10%, selling price = Rs.198.

$\Rightarrow$ Cost price = 198/0.9 = Rs.220.

$\therefore$ Total cost price = Rs.180 + Rs.220 = Rs.400.

Total selling price = 2 × 198 = Rs.396.

Clearly, on the two transactions together, we have a loss of Rs.400 – Rs.394 = Rs.4.

Notes / Rough Work

POINT TO REMEMBER

When two articles are sold at the same selling price, one at X% profit and other at X% loss, then there is always a loss. This loss, in rupees, is given by:

$$\frac{2 \times X^2 \times S.P}{100^2 - X^2}$$

The concept of false weight

Notes / Rough Work

If a seller sells his goods at cost price, but uses false weight, then the profit earned is given by the formula:

$$\text{Gain \%} = \left(\frac{\text{Error}}{\text{True value} - \text{Error}} \times 100\right)\% .$$

E25. A fruit vendor sells apples at cost price but uses false weight of 960 gm instead of 1 kg. Find the gain %.

Sol. In this transaction, there is an error of 40 gm.

$\therefore$ By the above formula,

$$\text{Gain \%} = \left(\frac{40}{960} \times 100\right)\% = 4\frac{1}{6}\% .$$

Mini Revision Test # 02

DIRECTIONS: *State if the following statements are True or False.*

1. The S.P of an article is Rs.650. The C.P is Rs.520 if the profit made is 26%.
2. If S.P = Rs.400, Loss = 20%, then C.P = Rs.500.
3. A man sells 6 mangoes at a price which is equal to the C.P of 8 mangoes. His profit is 20%.
4. The C.P of a pen is Rs.10. At a profit of 12.5%, its S.P will be Rs.22.50.
5. Mary purchased a table for Rs.200 and paid Rs.40 for transportation. She sold it to a neighbour for Rs.300 and earned a profit of 25%.

DIRECTIONS: *Answer the following questions.*

6. A man buys a pen for Rs.25 and sells it for Rs.23. Find his loss percent.
7. "A" sells a car to "B" at 10% profit. "B" sells it to "C" at 20% loss. If the C.P for "A" was Rs.5000, then find the C.P for "C."
8. A man sells 320 bananas at the cost price of 400 bananas. What is his gain percent?
9. A man sold 250 chairs and had a gain equal to S.P of 50 chairs. What is his profit percent?
10. What is the loss percent if a man loses Rs.10 on selling an article for Rs.100?

Challenge Problems # 02

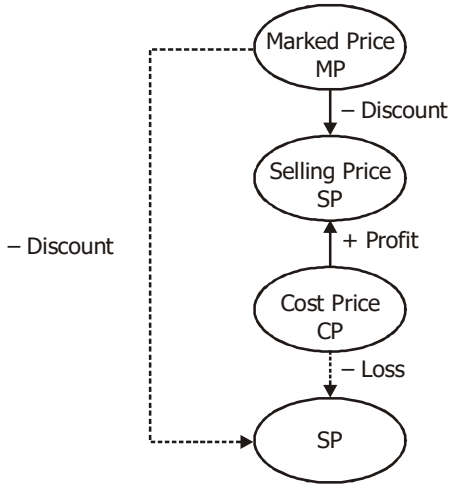
DIRECTIONS: *Answer the following questions.*

1. The price of tea falls by 10%. How many kilograms can be bought for the same money which was sufficient to buy nine kg before the reduction?
   (Q. code - 110506001)
   (1) 10 kg (2) 11 kg
   (3) 9.9 kg (4) 8.1 kg

2. Ram bought 25 TVs and VCRs for Rs.2,05,000. He sold 80% of the TVs and 12 VCRs for a profit of Rs.40,000. Each TV was marked up by 20% over cost and each VCR was sold at a profit of Rs.2,000. The remaining TVs and 3 VCRs couldn't be sold. What is Ram's overall profit or loss?
   (Q. code - 110506002)
   (1) Rs.15,000 profit (2) Rs.40,000 profit
   (3) Rs.1,000 profit (4) Rs.1,000 loss

3. Two-fifth of the voters promised to vote for P and the rest promised to vote for Q. Of these, on the last day, 15% of the voters went back of their promise to vote for P and 25% of voters went back on their promise to vote for Q. P lost by what percentage of total votes.
   (Q. code - 110506003)
   (1) 2 (2) 5
   (3) 10 (4) It was a tie

The concept of marked price (MP)



Some discounts are given at times on the marked price, which is generally higher than the cost price of the item. **Discounts** are concessions that are offered while purchasing an item. Once a discount is offered, the actual money paid by a customer is lower than the marked price. The money actually paid or the money received by the shopkeeper happens to be the selling price.

Imp.

The amount of discount is the difference between the marked price and the selling price. The rate of discount is usually given as a fraction or as a percentage.

Use the formula of the percent problems: $P = R \times B$.

P stands for the part or discount, R is the rate, and B, the base, is the original price.

Notes / Rough Work

E26. In a shop, a table listed at Rs.1600 was marked 15% off. What was the selling price of the table?

Sol. P = R × B = 0.15 × Rs.1600 = Rs.240.

This is the amount of discount or the amount that should be subtracted from the original price.

∴ Selling price of the table = Rs.1600 – Rs.240 = Rs.1360.

E27. If a television set priced at Rs.10000 was sold for Rs.9200, then what was the discount percentage?

Sol. Amount of discount = Rs.10000 – Rs.9200 = Rs.800.

Discount = Rate × Original price

Rs.800 = Rate × Rs.10000. $\Rightarrow \text{Rate} = \frac{800}{10000} = 0.08 = 8\%$.

E28. An article costing Rs.200 is marked 25% higher than its C.P and is sold at a discount of 10% for cash payment. A customer is ready to pay the full amount on the spot. What is the shopkeeper's percentage profit?

Sol. CP = Rs.200. MP = Rs.200 × 1.25 = Rs.250.

Discount = $\frac{10}{100} \times 250$ = Rs.25.

SP = MP – Discount = 250 – 25 = Rs.225.

Shopkeeper's profit = Rs.25.

Percentage profit = $\frac{25}{200} \times 100 = 12.5\%$.

E29. A dealer allows 10% discount on the list price of certain articles and yet makes a profit of 25% on each article. Find the cost price of an article whose listed price is Rs.50.

Sol. List price = Rs.50.

Selling price = (90/100)50 = Rs.45.

C.P = Rs.x.

Gain % = 25. ∴ SP = (125/100)x.

(125/100)x = 45 ⇒ x = 36. ⇒ C.P = Rs.36.

Successive discounting

When an item is discounted more than once, it is called successive discounting.

E30. In a store, a dress tagged at Rs.800 was offered at a discount of 12.5%. When it did not sell at the lower price, an additional discount of 10% was offered. What was the final selling price?

Sol. Discount = Rate × original price.

∴ First discount = 0.125 × Rs.800 = Rs.100.

⇒ Selling price after first discount = Rs.800 – Rs.100 = Rs.700.

Also, second discount = 0.10 × Rs.700 = Rs.70.

∴ Final selling price = Rs.700 – Rs.70 = Rs.630.

Notes / Rough Work

POINT TO REMEMBER

Same article sold at different prices

If an article costing Rs.C is sold at price P_1 and P_2 making a profit or loss of X% and Y% respectively on the sale, then the following formula will hold true.

$$\frac{P_1}{100+X} = \frac{C}{100} = \frac{P_2}{100+Y}$$

E31. In another store, an identical dress was marked at Rs.850. When it did not sell, it was discounted 25% all at once. Is the final selling price lower or higher than that in the above example?

Sol. Discount = Rate × original price

= 0.25 × Rs.850 = Rs.212.50.

∴ Final selling price = Rs.850 – Rs.212.50 = Rs.637.50.

This selling price is higher than the one in the above example, where two successive discounts were given.

E32. Which is more favourable to buyer and by how much: A discount series of 20%, 15%, 10% or the discount series of 25%, 12%, 8% on the list price of Rs.700?

Sol. The cost is Rs.700.

Case 1	**Case 2**
After 20% = Rs.560	After 25% = Rs.525
After 15% = Rs.476	After 12% = Rs.462
After 10% = Rs.428.40	After 8% = Rs.425.04

Second discount series is better.

E33. The list price of a watch is Rs.160. A retailer pays Rs.122.40 for it. He gets successive discounts of 10% and another rate which is illegible. What is the second discount rate?

Sol. List price = Rs.160.

Price after 10% discount = (90/100) 160 = Rs.144.

Second discount rate = x%.

Price after second discount = (100 – x)(144/100)

∴ (100 – x)(144/100) = 122.40 ⇒ x = 15 ⇒ Second discount rate = 15%.

E34. A sells a watch to B at a gain of 20% and B sells it to C at a loss of 10%. If C pays Rs.216, what does it cost A?

Sol. Let the watch cost A Rs.x.

A sells it to B for Rs.(120/100)x = Rs.1.2x.

B sells it to C for Rs (90/100)1.2x = Rs.1.08x.

Now, 1.08x = 216 ⇒ x = 200.

∴ A bought the watch for Rs.200.

Notes / Rough Work

Mini Revision Test # 03

DIRECTIONS: *Fill in the blanks.*

1. Successive discounts of 5% and 10% are equivalent to a single discount of .....
2. The selling price of a book marked at Rs.200 after being offered a discount of 5% is .....
3. A fruit vendor uses a false weight of 450 gm instead of 500 gm. His profit margin is .....
4. A property seller sold two plots of equal dimensions at Rs.300000 each. He made a profit of 10% on one of them but lost 10% on the other. His net profit or loss is .....%.
5. A petrol pump owner makes a profit of 5% by using some false metering devices. He saves ..... ml on a litre.
6. A television set worth Rs.20000 was sold for Rs.18000, the discount offered was .....%.
7. A camera was marked 50% higher than its cost price. If a discount of 50% is offered the profit or loss is .....
8. A table marked at Rs.2000 was offered two successive discounts of 10% each. The net payable price is .....
9. In the above problem, if a discount of 20% is offered instead of two successive discounts, the price of the table is .....
10. A calculator worth Rs.500 is offered at a discount. If Mr. X purchases it for Rs.475, then the discount offered is .....

Challenge Problems # 03

DIRECTIONS: *Answer the following questions.*

1. There were two different copper alloys, the first containing 40 percent less copper than the second. When these were melted together, the resulting alloy contained 36 percent of copper. Determine the percentage of copper in the first and second alloys, if it is known that there was 6 kg of copper in the first alloy and 12 kg in the second. ***(Q. code - 110507001)***

   (1) 35%, 75% (2) 20%, 60%
   (3) 30%, 45% (4) None of these

2. Two drums D1 and D_2 have a mixture of oil and water in the ratio 2 : 3 and 3 : 4 respectively. Five litres from D_1 and 6 litres from D_2 are mixed to form a new mixture D_3. Now, 9 litres of oil is added to this mixture. What is the final ratio of oil to water? ***(Q. code - 110507002)***

   (1) 2 : 1 (2) 3 : 2
   (3) 19 : 9 (4) 32 : 45

For Q.3 and Q.4

A vessel contains a mixture of milk and water. The volume of the vessel is 100 litres. The concentration of milk in the vessel is given by C(n) = A × C(n – 1) where A, n are natural numbers and n is the number of times a certain volume of the solution is replaced by the same volume of water. C(0) = 80%.

3. If for some n, $\frac{C(n)}{C(n-2)} = \frac{1}{4}$, then what is the value of A? ($n \geq 2$).

   (1) 1/2 (2) 1/4 ***(Q. code - 110507003)***
   (3) 2 (4) None of these

4. If C(n) = 5% and A = 1/2, then n is ***(Q. code - 110507004)***

   (1) 16 (2) 4
   (3) 2 (4) None of these

Notes / Rough Work

WORK & TIME

Work and Time

If a man can do a piece of work in X days (or any unit of time), then the work done by him in one day will be 1/X of the total work.

E1. Sujeet can finish a piece of work by himself in 16 days. Calculate the amount of work done by him in 1 day and in 13 days.

Sol. Sujeet, working alone for 16 days completes 1 unit of work.
Sujeet, working alone for 1 day completes 1/16 units of work.
Sujeet, working alone for 13 days completes 13/16 units of work.

- If one man, X, can complete the work in A days and another man Y in B days, when they work together, they will complete the work in $\frac{AB}{A+B}$ days .

  X can complete the work in A days. So in 1 day, he will do 1/A of the work.
  Y can complete the work in B days. So in 1 day, he will do 1/B of the work.
  Total work done by both in 1 day = (1/A) + (1/B) = (A + B)/AB.
  Hence the total time required to do the work
  = AB/(A + B) days.

E2. A can do a piece of work in 30 days while B alone can do it in 40 days. In how many days can A and B working together complete it?

Sol. A's 1 day work $= \frac{1}{30}$ and B's 1 day work = $\frac{1}{40}$.

$\therefore$ (A + B)'s 1 day work $= \left(\frac{1}{30} + \frac{1}{40}\right) = \frac{7}{120}$.

$\therefore$ Both together will finish the work in $\frac{120}{7} = 17\frac{1}{7}$ days.

Notes / Rough Work

E3. A can do a piece of work in 25 days and B can do it in 20 days. They work together for 5 days and then A goes away. How many more days will B require to finish the work?

Notes / Rough Work

Sol. A and B together complete $\left(\frac{1}{25}+\frac{1}{20}\right)=\frac{9}{100}$ of the work in a day.

$\therefore$ In 5 days they will complete $5\left(\frac{9}{100}\right)=\frac{9}{20}$ of the work.

Remaining work = $1-\frac{9}{20}=\frac{11}{20}$.

$\frac{1}{20}$ work is completed by B in a day.

$\therefore$ $\frac{11}{20}$ work will be completed by B in $\frac{20\times 11}{20}$ = 11 days.

E4. Amar can finish a piece of work alone in 12 days. Atul alone, can finish the same work in 24 days. If both of them work together, then in how many days, the work will be finished?

Sol. Applying the formula, $\frac{12\times 24}{(12+24)}=\frac{288}{36}=8$. The work will be finished in 8 days.

E5. A can build a wall in the same time in which B and C together can do it. If A & B together could do it in 10 days & C alone in 15 days, in what time could B alone do it ?

(1) 30 days (2) 24 days
(3) 60 days (4) 27 days

Sol. Given that

$\frac{1}{A}=\frac{1}{B}+\frac{1}{C}$ ..... (i)

$\frac{1}{A}+\frac{1}{B}=\frac{1}{10}$ ..... (ii)

$\frac{1}{C}=\frac{1}{15}$ ..... (iii)

(i) can also be written as

$\frac{1}{A}-\frac{1}{B}=\frac{1}{C}=\frac{1}{15}$ ..... (iv)

subtracting (iv) from (ii) we get

$\frac{2}{B}=\frac{1}{10}-\frac{1}{15}=\frac{1}{30}\therefore B=60$

$\therefore$ B alone can do it is 60 days. **Ans.(3)**

E6. Ram and Shyam together can finish a piece of work in 8 days. Both started the work together. After 3 days, Ram fell ill. Shyam alone finished the remaining work in 15 days. How many days would Ram take to finish the work alone?

Sol. Let Ram & Shyam take x & y days respectively to finish the work. $\therefore$ $\frac{1}{x}+\frac{1}{y}=\frac{1}{8}$.

In 3 days, $\frac{3}{8}$ of the work is done. Shyam completes $1-\frac{3}{8}=\frac{5}{8}$ of the work in 15 days.

$\therefore$ he can complete the whole work alone in $\left(\frac{8}{5}\right)$ 15 = 24 days.

$\therefore$ y = 24 $\Rightarrow$ $\frac{1}{x}=\frac{1}{8}-\frac{1}{24}=\frac{1}{12}$.

$\therefore$ Ram alone will take 12 days to finish the work.

➢ If three men, A, B, C can complete the work in P, Q, R days respectively while working alone, they together will take $\frac{PQR}{[PQ+QR+PR]}$ days to finish it.

E7. A can do a piece of work in 10 days, B in 12 days and C in 15 days. They all start the work together, but A leaves the work after 2 days and B leaves 3 days before the work is completed. How many days did the work last?

Sol. A, B & C work together for 2 days, C works alone for 3 days. B & C finish the remaining work together.

A, B & C finish in 2 days : $2\left(\frac{1}{10}+\frac{1}{12}+\frac{1}{15}\right)=\frac{1}{2}$ of the work.

C finishes in 3 days : $3\left(\frac{1}{15}\right)=\frac{1}{5}$ of the work.

$\Rightarrow$ Remaining work i.e. $\frac{1}{2}-\frac{1}{5}=\frac{3}{10}$ is done by B & C together. For 1 work, B and C take $\frac{60}{9}$ days. For $\frac{3}{10}$ work, they will take $\frac{60}{9}\times\frac{3}{10}=2$ days.

$\Rightarrow$ Work lasted for 2 + 3 + 2 = 7 days.

Alternate Solution :

Let the work be finished in X days. Then the following equation can be formed :

$$\frac{2}{10}+\frac{X-3}{12}+\frac{X}{15}=1$$

Solving the equation, X = 7 can be obtained. Thus the work will be completed in 7 days.

E8. A and B can do a piece of work in 24 days, B and C can do a piece of work in 30 days. C and A can do a piece of work in 40 days. How long would they all take to do the same work? In what time can each do it separately?

Sol. Let A, B, C do x, y, z amount of work in one day respectively.

$\Rightarrow$ x + y = $\frac{1}{24}$, y + z = $\frac{1}{30}$, x + z = $\frac{1}{40}$.

Adding all 3, we get

2x + 2y + 2z = $\frac{1}{24}+\frac{1}{30}+\frac{1}{40}=\frac{1}{10}$.

$\Rightarrow$ Working together, they will complete the work in 20 days. Also A alone = $\frac{1}{20}-\frac{1}{30}=\frac{1}{60}\Rightarrow 60$ days.

B alone $=\frac{1}{20}-\frac{1}{40}=\frac{1}{40}\Rightarrow 40$ days.

C alone = $\frac{1}{20}-\frac{1}{24}=\frac{1}{120}\Rightarrow 120$ days.

Notes / Rough Work

POINT TO REMEMBER

If A can do a piece of work in a days and B can do it in b days, then working together they can complete the work in $\frac{ab}{a+b}$ days.

E9. A and B could do a piece of work in 12 days. B and C together do it in 15 days. If A is twice as good a workman as C, find in what time B alone will do it?

Notes / Rough Work

Sol. Let A take x days to do a piece of work, $\therefore$ C takes 2x days to do the same work. Let B take y days to do the same work.

$\therefore \quad \frac{1}{x}+\frac{1}{y}=\frac{1}{12}$, $\frac{1}{2x}+\frac{1}{y}=\frac{1}{15}$.

Solving these equations, we get y = 20.

$\Rightarrow$ B can complete the work in 20 days.

E10. To complete a task, it was calculated that

(a) A, B and C would take 15 days to complete it. After one-third of the task was complete, A left and it was calculated that the task would take 4 days more for completion. After two-third of the task was complete, B left and it was assessed that the task would take 2 days more yet again for completion. C could do the whole task alone in

(1) 27 days (2) 30 days

(3) 33 days (4) 36 days

Sol. (a) Let in one day, A, B, C do 1/a, 1/b and 1/c parts of work respectively. Then

$$\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=\frac{1}{15}$$

i.e. one-third of the work is done in 5 days. Now

$$\left(\frac{1}{b}+\frac{1}{c}\right)(10+4)=\frac{2}{3}$$

$$\Rightarrow \left(\frac{1}{b}+\frac{1}{c}\right)7=\frac{1}{3}$$

i.e. next one-third of work is done in 7 days. Now

$$\frac{1}{c}(7+2)=\frac{1}{3}$$

($\because$ the next one-third of work is done in 9 days)

$$\Rightarrow \frac{1}{c}=\frac{1}{27}$$

$\therefore$ c can alone do the work in 27 days. **Ans.(1)**

Imp.

If A is twice as good as B, he will take half the time B takes to finish the same work. If both of them work together and complete a given work, A will get double the amount B gets, although both are working for same number of days, because wages are always in direct proportion to the skill.

Number of days taken to complete the work varies inversely with the skill.

The above relations or statements may vary with some specified condition in any particular question.

E11. Two persons A and B can complete a work in 10 and 15 days respectively, working separately. In how many days will the work be completed, if they both work together. If they received Rs.300 as payment, then what are their respective shares?

Sol. In 1 day A does 1/10 of the work,

B does 1/15 of the work.

$\Rightarrow$ Total number of days required to finish the work

= (10 × 15) / (10 + 15) = 6 days.

Ratio in which work is done is 1/10 : 1/15.

(Multiply both by the LCM of 10 and 15)

i.e., 1/10 × 30 : 1/15 x 30 = 3 : 2.

i.e., A's share is 3/5 of the total wages.

B's share is 2/5 of the total wages.

Total wages are Rs.300.

$\Rightarrow$ A's share is 3/5 × Rs.300 = Rs.180 and

B's share is 2/5 × Rs.300 = Rs.120.

E12. A, B and C can do a piece of work in 18 days, 27 days and 36 days respectively. They start working together. After working for 4 days. A goes away and B leaves 7 days before the work is finished. Only C remains at work from beginning to end. In how many days was the whole work done?

(1) 17 days (2) 18 days

(3) 16 days (4) None of these

Sol. Let the whole work was done in x days. So according to the given condition.

$\frac{4}{18}+\frac{x-7}{27}+\frac{x}{36}=1$ or $\frac{48+8(x-7)+6x}{216}=1$

or 14x = 216 + 8 = 224, x = 16 days. **Ans.(3)**

E13. (a) Three persons can do a work in x, y and z days when working in pairs. They can do the same work if all three do it together in

(1) $\frac{x+y+z}{3}$ (2) $\frac{x+y+z}{2}$

(3) $\frac{2}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$ (4) $\frac{3}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$

Sol. (a) Let them do $\frac{1}{A},\frac{1}{B}$ and $\frac{1}{C}$ parts of work in one day. Then

$\frac{1}{A}+\frac{1}{B}=\frac{1}{x},\frac{1}{B}+\frac{1}{C}=\frac{1}{y}$ and $\frac{1}{C}+\frac{1}{A}=\frac{1}{z}$

and they can together do the work in

$\frac{1}{1/A+1/B+1/C}$, i.e. $\frac{2}{1/x+1/y+1/z}$ days . **Ans.(3)**

Notes / Rough Work

POINT TO REMEMBER

If A can do a piece of work in a days, B can do it in b days and C can do it in c days then working together they can complete the work in $\frac{abc}{ab+bc+ca}$ days.

Mini Revision Test # 01

DIRECTIONS: *Answer the following questions.*

1. A can do 3/4 of a work in 12 days. In how many days can he finish 1/2 of work?
2. Vicky alone does a piece of work in 2 days and Vinny does it in 6 days. In how many days will the two do it together?
3. Som and Vivek can finish a work in 16 days while Som can do the same work in 24 days. In how many days can Vivek alone finish the same work?
4. Munish is two times faster than Raj. If Munish and Raj can complete a job in 12 days, how long will it take for Raj alone to complete the same job?
5. 5 men can do a piece of work in 20 days of 8 hours each. In how many days of 10 hours each can 8 men do it?
6. If Uday and Anish can finish a job in 12 days, then how much time will Anish take while working alone? (Assuming that Uday and Anish are equally efficient)
7. Rakesh and Namit finish a job in 48 hrs. If Rakesh is twice as efficient as Namit, then what is the time taken by Rakesh while working alone?
8. 6 men consume 8 kg of rice in 2 days. How long will the same ration last, if one more person joins the group?
9. If 'A' can paint a fence 100 metres long in 6 days, then how much time will both 'A' and his friend 'B' take, if 'B' is thrice as efficient as 'A'?
10. If I can complete a piece of work in 24 hours, then how much time I will take, if I am assisted with two men – one of them twice and the other thrice as efficient as I am?

Challenge Problems # 01

DIRECTIONS: *Answer the following questions.*

1. 2 men, 3 women and 4 children can do a work in 6 days; 3 men, 4 women and 2 children can do the same work in 3 days; 4 men, 2 women and 3 children can do the same work in 2 days. Then a man, a woman and a child can do the same work in ***(Q. code - 110605001)***

   (1) 8 days (2) 9 days (3) 12 days (4) Data Inconsistent

2. A, B, C and D can finish a work together in a certain number of integral days. However they work two in a day and it is found that the work gets completed when (A, B), (B, C) and (C, D) worked for respectively 5, 9 and 4 days or 7, 6 and 5 days. They couldn't have all together done the work in

   (Q. code - 110605002)

   (1) 8 days (2) 9 days (3) 10 days (4) 11 days

3. A and B can do a work alone exactly in 20 and 25 days respectively. However when they work together, they do 25% more work than is expected. If they work for a few days alone and for a few days together (both being integers only), then the work could not have been completed in exactly

   (Q. code - 110605003)

   (1) 10 days (2) 14 days (3) 16 days (4) 17 days

4. The efficiency of work done by A gets reduced by 10% on each successive days he works consecutively while his efficiency springs back to normal if he takes the rest for one day. For doing a work which takes a long time to complete he should take rest after working consecutively for every ..... to complete the work in minimum time. ***(Q. code - 110605004)***

   (1) 1 day (2) 2 days (3) 4 days (4) 10 days

5. 2 men and 3 women can do a work exactly in 20 days. If 3 men and 1 women can do the work in less than 20 days, then 1 men and 5 women can do the same work in ***(Q. code - 110605005)***

   (1) 20 days (2) less than 20 days
   (3) more than 20 days (4) cannot be determined

Notes / Rough Work

Pipes and Cisterns

Notes / Rough Work

The same principle of Time & Work is employed to solve the problems on Pipes & Cisterns. The only difference being that in this case, the work done is in terms of filling or emptying a cistern (tank) and the time taken is the time taken by a pipe or a leak (crack) to fill or empty a cistern respectively.

Unless otherwise specified, the amount of work done, i.e., filling or emptying a cistern is generally taken as unity (One – 1). Also, if it is given that a pipe can fill a cistern in 10 min, then it implies that the pipe **alone** can fill the cistern in 10 min.

Generally, the time taken to fill a cistern is taken as positive and the time taken to empty a cistern is taken as negative.

A pipe connected with a cistern is called an inlet pipe or an outlet pipe, accordingly as it fills it or empties it respectively.

Some important formulae

If an inlet pipe can fill a cistern in X hours, the part filled in 1 hour is $\frac{1}{X}$.

If an inlet pipe can fill a tank in X hours and an outlet pipe empties the full tank in Y hours, then the net part filled in 1 hour when both the pipes are opened = $\frac{1}{X} - \frac{1}{Y}$.

$\Rightarrow$ In 1 hour, the part filled (or emptied) = $\frac{1}{X} - \frac{1}{Y}$.

☞ Time required to fill or empty the tank = $\frac{X \times Y}{X \sim Y}$ hours.

(X ~ Y indicates [X – Y] or [Y – X], whichever is positive).

Important

If X > Y, then an empty tank can never be filled. Similarly if X < Y, then a full tank can never be emptied.

E14. Three pipes A, B and C can fill a cistern in 10, 12 and 15 hours respectively, while working alone. If all the three pipes are opened together, then find the time taken to fill the cistern.

Sol. The pipes can fill the cistern in 10, 12 and 15 hours respectively.

When the three pipes are opened together, the time taken to fill the cistern = $\frac{10 \times 12 \times 15}{10 \times 12 + 12 \times 15 + 10 \times 15} = 4$ hours.

POINT TO REMEMBER

If 10 men can do a work in 10 days, then 1 man will complete the same work in 100 days (and not in 1 day).

E15. A pipe can fill 3/4 of the cistern in 12 min. In how many minutes, it can fill 1/2 of the cistern?

Notes / Rough Work

Sol. Using the formula,

$$\frac{t_1}{p_1} = \frac{t_2}{p_2} \Rightarrow \frac{12}{\frac{3}{4}} = \frac{t_2}{\frac{1}{2}} \Rightarrow t_2 = 8 \text{ min.}$$

E16. Two pipes A and B can separately fill a tank in 12 minutes and 15 minutes respectively. Both the pipes are opened together but 4 minutes after the start, pipe A is turned off. How much time will it take to fill the tank ?

(1) 11 min (2) 12 min
(3) 6 min (4) 8 min

Sol. Part filled by A in a minute $= \frac{1}{12}$.

Part filled by B in a minute $= \frac{1}{15}$

$\therefore$ The part filled in a minute $= \frac{1}{12} + \frac{1}{15} = \frac{3}{20}$.

The part filled in 4 minutes $= \frac{4 \times 3}{20} = \frac{3}{5}$

Remaining Part = 1- 3/5 = 2/5
Time taken by B to complete 2/5 of the work = 2/5 × 15 = 6 min. **Ans.(3)**

Working simultaneously and alternately

Lets understand the difference between three pipes working simultaneously and alternatively on the same cistern.

E17. There are pipes A, B & C. A & B are inlet pipes which can fill the cistern in 6 min. and 8 min. respectively, whereas C can empty the full cistern in 12 min. Find the time taken to fill the cistern in the following cases

(a) Three pipes are opened simultaneously.

(b) Three pipes are opened in alternate min. starting from A followed by B & C.

Ans. (a) 24/5 min. (b) 13 min

Work equivalence concept is also used for solving many questions. We use different units for measuring work. For example, generally work is measured in man days, man hours etc. But there can be other units also like man minutes. At times when some machines are used, we can also use machine hours etc.

E18. 4 men of equal efficiencies can finish a piece of work by working 10 hrs a day in 7 days. How many men will be required to finish the same work if they work 3.5 hrs a day and the work is to be completed in 10 days?

Sol. The total work to be done

= 4 × 10 × 7 = 280 man hours.

Let X men be required to finish this job, then

280 = X × 3.5 × 10 $\Rightarrow$ X = 8.

So, 8 men are required.

E19. A certain number of men can finish a piece of work in 60 days. If there were 8 men more, the work could be finished in 10 days less. Find the original number of men.

Notes / Rough Work

Sol. Let the original number of men be x.

x men can complete the work in 60 days and (x + 8) men can complete the work in 50 days.

$\Rightarrow (x + 8) : x = 60 : 50.$

$\Rightarrow \frac{(x+8)}{x} = \frac{60}{50}$. $\therefore x = 40.$

$\therefore$ the original number of men = 40.

E20. 3 men or 5 women can do a job in 12 days. How long will 6 men and 5 women take to finish the job?

Sol. 3 men = 5 women.

6 men + 5 women = 6 men + 3 men = 9 men.

Also 3 men finish a work in 12 days.

Let 6 men and 5 women i.e., 9 men finish the work in x days.

$9x = 3(12) \Rightarrow x = 4$ days.

Applying the concept of variations

Most of the questions given in this chapter based on work and time can be solved using the concept of variations. The following points should be kept in mind while solving the questions by this method.

- Number of persons employed to do the work is directly proportional to the amount of work done. (More the number of persons employed, more the work done).
- The number of days is directly proportional to the work done. (More the number of days for which a work was done, more shall be the total amount of work done).
- The number of persons employed is inversely proportional to the number of days required to finish a work. (More the number of persons employed, less will be the time required to finish the work).

Similarly, many such variations can be obtained to solve questions on this pattern.

E21. 12 men can build a wall 100 metres long, 3 metres high and 0.5 metre thick in 25 days. In how many days will 20 men build a wall 60 metres long, 4 metres high and 0.25 metre thick?

Sol.

Men	Days	Length	Height	Thickness
12	25	100	3	0.5
20	X	60	4	0.25

$\frac{25}{X} = \frac{20}{12} \times \frac{100}{60} \times \frac{3}{4} \times \frac{0.50}{0.25} \Rightarrow X = 6.$

The concepts used to solve this question are

- Men and days are inversely proportional to each other.
- Length, height and thickness of the wall are directly proportional to the number of days.

Challenge Problems # 02

1. 3 men and 4 women can do a work exactly in 5 days. 4 men and 2 women can complete the work on fifth day. Then 2 men and 6 women can complete the same work on ***(Q. code - 110606001)***

   (1) 4 th day (2) 5 th day

   (3) 6 th day (4) cannot be determined

2. A, B, C, D, E are 5 labourers (men, women and children) who can together do a work in 180/7 days. If A, B and C can do the work in 36 days and A, D, E can do the work in 45 days, then a man, a woman and a child can do the same work in ***(Q. code - 110606002)***

   (1) 39 days (2) 40 days

   (3) 42 days (4) cannot be determined

   (Assume that all kinds of labourers are accounted for and nobody takes more than 180 days working alone)

3. A, B are two supply pipes while C is a drain pipe to a tank. A and B can fill the tank alone in few hours which sum is 10 hrs. When A and C are opened together, it takes 1 hour more than what B alone takes to fill the tank while when B and C are opened together it takes 1 hour less than what A alone takes to fill the tank. What is the time in which the tank will get filled if A, B and C are opened simultaneously? ***(Q. code - 110606003)***

   (1) 4 hrs (2) 5 hrs

   (3) 6 hrs (4) none of these

4. A, B, C and D do a work in the following manner: A and B start doing it and after a quarter of work is complete in 24 days, C replaces A and work together with B to complete another quarter of work in 12 days; at this point, D replaces B and work together with C to complete another quarter of work in 10 days and now A replaces C to complete the remaining work together with D. The last quarter of work is done in about

   (Q. code - 110606004)

   (1) 17 days (2) 18 days

   (3) 20 days (4) 22 days

5. Four persons A, B, C and D can do a work in 10, 20, 40 and 80 days respectively, but they do the work in pairs. To complete the work in integral days each pair will work twice except one pair which is working only once. The pair is .... ***(Q. code - 110606005)***

   (1) A, B (2) A, C

   (3) B, D (4) A, D

Notes / Rough Work

TIME, SPEED AND DISTANCE

Speed

If the location of an object changes with time then it is said to be in motion. A bus running on the road, an ant crawling on the floor, etc. are all examples of objects in motion because the locations of these objects keep on changing with respect to their surroundings.

Let an object move from point A to point B through a given path, then the actual length of the path traced by the object is called the distance travelled by that object.

Note: Speed is scalar whereas velocity is a vector quantity. Since vector theory is not in the syllabus, in the present context, speed and velocity will be considered synonymous.

The rate at which any moving body covers a particular distance is called its speed.

☞ $\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$

If the distance is constant, Speed $\alpha \frac{1}{\text{Time}}$.

$\text{Time} = \frac{\text{Distance travelled}}{\text{Speed}}$.

If time is constant, Distance α Speed.

Distance travelled = Time × Speed.

If speed is constant, Distance α Time.

We can say that for a constant distance travelled, speed is inversely proportional to the time taken. This can be explained by a simple example. If a person travels at a speed of 25 kmph, he needs 4 hours to complete a distance of 100 km. Travelling at a speed of 50 kmph, he needs 2 hours to complete the same journey.

Notes / Rough Work

Uniform speed

If an object covers equal distances in equal intervals of time, howsoever small the interval may be, then its speed is called uniform speed.

Variable speed

If an object travels unequal distances in equal intervals of time, then its speed is called variable speed. In this case the speed changes from interval to interval.

Units of measurement

- Time is measured in seconds (sec), minutes (min) or hours (hrs).
- Distance is usually measured in metres (m), kilometres (km), miles, yards or feet.
- Speed is usually measured in metres per second (m/s), kilometres per hour (kmph) or miles per hour (mph).

Conversion of units

- 1 hour = 60 minutes = 60 × 60 seconds.
- 1 kilometre = 1000 metres.
- 1 kilometre = 0.6214 miles.
  1 mile = 1.609 kilometre.
  i.e. 8 kilometres ≈ 5 miles.
- 1 yard = 3 feet.
- $\frac{\text{km}}{\text{hr}} = \left(\frac{5}{18}\right)$ m/s (To convert kmph to m/s we multiply by $\frac{5}{18}$).
- $\frac{\text{m}}{\text{s}} = \left(\frac{18}{5}\right)$ km/hr (To convert m/s to kmph we multiply by $\frac{18}{5}$).
- $\frac{\text{km}}{\text{hr}} \approx \frac{5\,\text{miles}}{8\,\text{hr}}$
- $\frac{\text{miles}}{\text{hr}} = \frac{22}{15}\frac{\text{ft}}{\text{sec}}$

E1. A train runs at the rate of 45 km per hour. What is its speed in metres per second?

Sol. 45 x 5/18 = 12.5 m/sec.

E2. A motor car takes 50 seconds to travel 500 metres. What is its speed in km per hour?

Sol. Speed = (500/50) m/sec = 10m/sec = 10 × (18/5) km/hr
= 36 km/hr.

Average speed

Average speed of a body is defined as the ratio of the total distance covered to the total time taken.

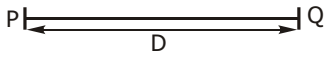
Notes / Rough Work

If a body moves with a speed of s_1, s_2, s_3 ... in the time t_1, t_2, t_3 ... respectively, then the average speed of the body is given as

☞ Average Speed $= \dfrac{s_1 \times t_1 + s_2 \times t_2 + \ldots}{t_1 + t_2 + \ldots}$.

E3. If A goes from P to Q at S_{PQ} km/hr and comes back from Q to P at S_{QP} km/hr, then what is his average speed during the entire journey?

Sol.



$S_{PQ} = \dfrac{D}{t_1}$ and $S_{QP} = \dfrac{D}{t_2}$.

Total time taken = $t_1 + t_2$.

$= \dfrac{D}{S_{PQ}} + \dfrac{D}{S_{QP}} = D\left[\dfrac{S_{PQ} + S_{QP}}{S_{PQ} \times S_{QP}}\right]$.

Average speed = Total distance covered ÷ Total time taken.

☞ $S_{Average} = \dfrac{2D}{D\left[\dfrac{S_{PQ} + S_{QP}}{S_{PQ} \times S_{QP}}\right]} = \dfrac{2\left(S_{PQ} \times S_{QP}\right)}{S_{PQ} + S_{QP}}$.

E4. If Tushar goes from Delhi to Noida at a speed of 30 km per hour and comes back at a speed of 70 km per hour, then what is his average speed during the entire journey?

Sol. $S_{Average} = \dfrac{2 \times 30 \times 70}{30 + 70} = 42$ kmph.

E5. A bird flying 400 km covers the first 100 km at the rate of 100 km/hr, the second 100 km at 200 km/hr, the third 100 km at the rate of 300 km/hr and the last 100 km at the rate of 400 km/hr. Determine the average speed of the bird.

Sol. Total time taken

= 100/100 + 100/200 + 100/300 + 100/400.

= 1 + 1/2 + 1/3 + 1/4 = 25/12 hours.

$\therefore$ Average speed = $\dfrac{400}{(25/12)}$ = 192 kmph.

E6. If Arun rides his scooter at a speed of 20 kmph, then he reaches his office 5 minutes late. If he rides at 30 kmph, then he reaches 5 minutes early. Find the distance between his home and office.

Sol. Let t be the time he usually takes to reach his office.

When he drives at 20 kmph, then $D = 20 \times (t + \dfrac{5}{60})$.

When he drives at 30 kmph, then $D = 30 \times (t - \dfrac{5}{60})$.

The distance (D) remains constant.

$\therefore \dfrac{20\,(60t + 5)}{60} = \dfrac{30\,(60t - 5)}{60}$.

Notes / Rough Work

$\Rightarrow 120t + 10 = 180t - 15.$

$\Rightarrow 60t = 25 \Rightarrow t = \frac{5}{12}$.

$\therefore$ We get t = 5/12 hours and distance = 10 km.

Alternative solution: Let D be the distance between his home and office.

$\therefore \frac{D}{20} - \frac{D}{30} = \frac{10}{60}$. (Total time difference = 5 + 5 = 10 min)

$\Rightarrow \frac{D}{60} = \frac{10}{60} \Rightarrow D = 10\,\text{km}.$

E7. A man travelled a distance of 61 km in 9 hours partly on foot at the rate of 4 km per hour and partly on bicycle at the rate of 9 km per hour. Find the distance travelled on foot.

Sol. Let the distance travelled on foot = x km.

$\therefore$ Distance travelled on bicycle = (61 – x) km.

Time taken to travel on foot = $\frac{x}{4}$.

Time taken to travel on bicycle = $\frac{61 - x}{9}$.

$\therefore \frac{x}{4} + \frac{(61 - x)}{9} = 9 \Rightarrow x = 16$ km.

E8. A starts from a place at 3 pm at 2.5 km per hour. B follows him from the same place at 5 pm, walking at 3 km per hour. If A takes some rest on the way and is overtaken by B after he has covered 24 km, then A took rest for

(1) 20 minutes (2) 24 minutes

(3) 30 minutes (4) 36 minutes

Sol. Distance travelled by A in 2 hrs (from 3 pm to 5 pm) = $2.5 \times 2 = 5$ km.

Let x be the time for which A took rest.

Now, time taken by B for 24 km = Time taken by A for 19 km $\Rightarrow$ 24/3 = Actual time + Rest time

$\Rightarrow 8 = (19/2.5) + x \Rightarrow x = 0.4$ hrs = 24 min. **Ans.(2)**

E9. The distance between two towns, T1 and T2, is 200 km. A motorcycle rider starts from T1 towards T2 at 9.00 am at a speed of 20 km/hr. Another rider starts from T2 towards T1 at 10.00 am at the speed of 30 km/hr. The time when they cross each other is

(1) 1.00 pm (2) 6.08 pm

(3) 11.00 am (4) 1.36 pm

Sol. Suppose they meet X hours after the first rider has started from town T1.

Distance travelled by first rider in X hours = 20X km.

Distance travelled by second rider in (X–1) hours = 30(X–1) km.

Total distance travelled = 200 km $\Rightarrow 200 = 20X + 30(X-1)$.

$\Rightarrow$ X = 23/5 hours = 4.6 hours $\Rightarrow$ The required time = 9.00 am + 4.6 hours. = 13.36 hrs or 1.36 pm. **Ans.(4)**

Notes / Rough Work

POINT TO REMEMBER

If a man changes his speed in the ratio m : n, then the ratio of time taken becomes n : m.

E10. Walking at 5/7th of his usual rate, a man reaches the market 16 minutes late. Find his usual time taken to reach the market.

Notes / Rough Work

Sol. If the man is walking at $\frac{5}{7}$th of his usual speed, then the time taken should become $\frac{7}{5}$ times the original time required. Thus, we can say that the man is going to require $\frac{2}{5}$th more time as compared to his normal time (at his normal speed)

(as $\frac{7}{5}T - 1T = \frac{2}{5}T$). Thus, $\frac{2}{5}T \longrightarrow 16$,

$1T \longrightarrow X$

On solving, we get X = 40 min.

E11. Walking at $\frac{12}{11}$th of his usual rate, a man takes 5 minutes less to reach his office. Find his usual time taken to reach the office.

Sol. If the man is walking at $\frac{12}{11}$th of his usual speed, then the time taken should become $\frac{11}{12}$th time the original time required. Thus, we can say that the man will take $\frac{1}{12}$th less time as compared to the standard time required,

as $1T - \frac{11}{12}T = \frac{1}{12}T$.

Thus, $\frac{1}{12}T \longrightarrow 5$ and $1T \longrightarrow X$

On solving, we get X = 60 min.

E12. A train running between two towns arrives at its destination 10 minutes late when it goes at 60 km per hour and 16 minutes late when it goes at 40 km per hour. Determine the distance between the two towns.

Sol. Let the distance between the two towns be x km.

In the first case, time taken = x/60 hrs.

In the second case, time taken = x/40 hrs.

$\therefore \frac{x}{40} - \frac{x}{60} = \frac{(16-10)}{60} = \frac{6}{60} \Rightarrow x = 12$ km.

E13. Two cars start from one point and move along two roads at right angles to each other. If their speeds are 36 km/hr and 48 km/hr respectively, find the distance between the two cars 15 seconds after the start.

Sol. Distance covered by the first car in 15 sec

$= 36\left(\frac{15}{3600}\right) = 0.15$ km.

Distance covered by the second car in 15 sec

$= 48\left(\frac{15}{3600}\right) = 0.2$ km.

$\therefore$ The distance between them is

$\sqrt{[(0.15)^2 + (0.2)^2]} = 0.25$ km = 250 m.

E14. A train has 320 km to run. After going 1/5th of the distance, the engine breaks down and it can only run the remaining part of the journey at 3/4th of the original speed. If it arrives 2 hrs 40 min late, what was its original speed?

(1) 24 km/hr (2) 32 km/hr
(3) 48 km/hr (4) 64 km/hr

Sol. Let the original speed of the train= x km/hr . Actual time taken by the train for 320 km = $\frac{320}{x}$ hrs. Delay = 2 hrs 40 min = $\frac{8}{3}$ hrs.

From the question, $\frac{\frac{320}{5}}{x} + \frac{320\left(1-\frac{1}{5}\right)}{\frac{3}{4}x} = \frac{320}{x} + \frac{8}{3}$.

$\Rightarrow \frac{64}{x} + \frac{1024}{3x} = \frac{320}{x} + \frac{8}{3} \therefore x = \frac{256}{8} = 32$ km/hr. **Ans.(2)**

Concept of relative speed

Let us try to understand this concept by taking a practical example.

Consider a person A standing on a platform waiting for a train. In the meantime, a different train passes the same platform and takes t minutes to cross A.

If A tries to run towards this train such that both of them keep on moving in opposite directions then the train will definitely take a time t_{less} which is less than t min to cross this person A.

On the other hand, if the person runs in the same direction in which the train is moving, then the train will require a time t_{more} which is more than t min.

Thus, the basic concept of relative speed is that, when two objects or bodies move in the same direction, the relative speed of one body with respect to the other is the difference between the speeds of the two. And when the two bodies are moving in the opposite direction, the relative speed of one body with respect to the other is the sum of the speeds of the two.

E15. A train, 140 metres long, moves at the rate of 36 kmph. How long will it take to pass a certain telegraph pole? (The height of the pole is 2 m).

Sol. Direct formula gives,

Time = $\frac{140 \text{ m}}{10 \text{ m/sec}}$ = 14 sec.

Note : 36 kmph = $36 \times \frac{5}{18} = 10$ m/sec. The height of the post is superfluous information.

Notes / Rough Work

POINTS TO REMEMBER

- If a man is standing and a train of length A km, running at a speed Y km/hr, crosses the standing man, then the time taken by the train to cross the man

  $= \frac{A}{Y}$ hrs.

- If a man is running at a speed of X km/hr in the same direction in which a train is running at a speed Y km/hr, then (Y – X) km/hr is called the speed of the train relative to the man. Also, if the train has the length A km, then time taken by the train to cross the man

  $= \frac{A}{(Y-X)}$ hrs.

- If a man is running at a speed of X km/hr in the direction opposite to which a train is running at a speed of Y km/hr, then (Y + X) km/hr is called the speed of the train relative to the man. Also if the train has the length A km, then time taken by the train to cross the man

  $= \frac{A}{(Y+X)}$ hrs.

- If a train of length L_1 km running at a speed of X km/hr crosses a bridge of length L_2 km, then the time taken by the train to cross the bridge

  $= \frac{L_1 + L_2}{X}$ hrs.

E16. The distance between two cities A and B is 80 km. A motorcycle rider starts from A towards B at 7 am at a speed of 10 km/hr. Another motorcyclist starts from B towards A at 8 am at a speed of 25 km/hr. At what time will they cross each other?

Sol. In one hour, the earlier rider covers a distance of 10 km.

The distance between the two = 80 – 10 = 70 km.

The two riders approach each other with a relative speed of 10 + 25 = 35 kmph.

The time taken = 70/35 = 2 hrs.

$\therefore$ They meet each other at (8 + 2) = 10 am.

E17. Two trains start at the same time from two stations and proceed towards each other at the rate of 20 km/hr and 25 km/hr respectively. When they meet, it is found that one train has travelled 80 km more than the other. Find the distance between the two stations.

Sol. In one hour, one train travels 5 km more than the other.

$\Rightarrow$ The trains meet in 80/5 = 16 hours.

$\Rightarrow$ Distance = (20 + 25) × 16 = 720 km.

E18. A train 150 m long, travelling at 75 km per hour overtakes another train travelling in the same direction at 45 km per hour. In how many seconds does the first train pass a passenger sitting in the second train? If the first train passes the second completely in 30 seconds, then find the length of the second train.

Sol. The two trains move at a relative speed of

(75 – 45) = 30 kmph.

The first train passes the passenger in

$\frac{150 \times 18}{30 \times 5}$ = 18 seconds.

Let the length of the second train be X metres.

$\frac{X + 150}{1000 \times 30} = \frac{30}{3600} \Rightarrow X = 100$ m.

E19. Two trains start at the same time from Aligarh and Delhi and proceed towards each other at the rate of 16 km/hr and 21 km/hr respectively. When they meet, it is found that one train has travelled 60 km more than the other. The distance between the two stations is

(1) 445 km (2) 444 km

(3) 440 km (4) 450 km

Sol. Both the trains are coming closer to each other by (21 – 16), i.e., 5 km every hour. $\therefore$ They would meet after 60/5 = 12 hrs.

Hence, the distance between the 2 stations = (21 + 16) × 12 = 37 × 12 = 444 km. **Ans.(2)**

E20. In a car race, two-third of B's rate is equal to three-fourth of A's rate. If the distance is 1080 km and the speed of A is given as 120 kmph, how long would B take to finish the race?

(1) 12 hrs (2) 9 hrs

(3) 10 hrs (4) 8 hrs

Sol. We have $\frac{2}{3}$ of B $= \frac{3}{4}$ of A and A = 120 kmph.

Thus speed of B = 135 kmph.

Hence, to cover 1080 km, B would take (1080 ÷ 135) = 8 hours. **Ans.(4)**

Notes / Rough Work

Mini Revision Test # 01

DIRECTIONS: *Answer the following questions.*

1. What is the length of a bridge, which a man riding at 15 km/hr can cross in 5 minutes?
2. Mukesh walks 10 km in 5 hours. How much time will he take to travel 28 km?
3. If a cyclist covers 11 km in 3 hours, find the distance covered in 5 hours.
4. A man walks at the rate of 5 km/hr for 6 hours and at 4 km/hr for 12 hours. Find the average speed of the man.
5. Walking $\frac{3}{4}$ th of his usual rate, a man is $1\frac{1}{2}$ hours late. Find the usual time he takes to reach the same destination.
6. What is the ratio of the speeds of two trains, one travelling at 45 kmph and the other at 10 m/s?
7. A railway passenger counts the telegraph posts as he passes them. If they are 50 m apart and the train is going at a speed of 60 kmph, then how many posts will he pass per minute?
8. A 200 m long train is moving at a speed of 60 kmph. How long will it take to cross a telephone pole?
9. A 180 m long train is moving at a speed of 54 kmph. How long will it take to pass a tunnel 720 m long?
10. A train running at 30 m/sec takes 30 seconds to cross a platform 600 m long. What is the length of the train?

Challenge Problems # 01

1 A and B walk from X to Y, a distance of 27 km at 5 km/hr and 7 km/hr, respectively. B reaches Y and immediately turns back, meeting A at Z. What is the distance from X to Z ?
(1) 25 km (2) 22.5 km (3) 24 km (4) 20 km

2 A man travels three-fifths of a distance AB at speed 3a, and the remaining at speed 2b. If he goes from A to B and returns at speed 5c in the same time, then:
(1) $\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$ (2) a + b = c
(3) $\frac{1}{a}+\frac{1}{b}=\frac{2}{c}$ (4) None of these

3 A train travelled from Town A to Town B at a uniform speed. If the speed of the train had been increased by 18 kmph then the time taken for the journey would have been 3 hours less. If the speed had been reduced by 9 kmph then the time taken for the journey would have been 3 hours more. The speed of the train and the distance between Town A and Town B are
(1) 36 kmph, 124 km (2) 18 kmph, 324 km
(3) 16 kmph, 224 km (4) 36 kmph, 324 km

4 A train X departs from station A at 11.00 am for station B, which is 180 km away. Another train Y departs from station B at 11.00 am for station A. Train X travels at an average speed of 70 kms/hr and does not stop anywhere until it arrives at station B. Train Y travels at an average speed of 50 km/hr, but has to stop for 15 minutes at station C, which is 60 kms away from station B enroute to station A. Ignoring the lengths of the trains, what is the distance, to the nearest km, from station A to the point where the trains cross each other ?
(1) 112 (2) 118 (3) 120 (4) None of these

5 In a race, one boat is rowed over the course at an average pace of 4 yards per second. Another moves over the first half of the course at the rate of $3\frac{1}{2}$ yards per second and over the last half at $4\frac{1}{2}$ yards per second, reaching the winning post 15 seconds later than the first boat. Find the time taken by the second boat.
(1) 16 min. (2) 18 min. (3) 81 min. (4) 27 min.

Notes / Rough Work

Boats and streams

Notes / Rough Work

A few important terminologies

The following terms will be used quite often while discussing problems on boats and streams.

Stream: A river which is flowing.

Upstream: Going against the flow.

Downstream: Going with the flow.

Still water: Speed of water is zero (as in a lake).

If the speed of a boat (or man) in still water be X km/hr and the speed of the stream (or current) be Y km/hr, then

Speed of boat with the stream (or downstream or v_{down})
= (X + Y) km/hr.

Speed of boat against the stream (or upstream or v_{up})
= (X – Y) km/hr.

We have X = [(X + Y) + (X – Y)]/2
and Y = [(X + Y) – (X – Y)]/2.

E21. A boat is rowed 28 km down a river in 4 hours and 12 km up in 6 hours. Find the speed of the boat in still water and the speed of the river.

Sol. Downstream speed is $\frac{28}{4}$ = 7 km/hr.

Upstream speed is $\frac{12}{6}$ = 2 km/hr.

Speed of boat in still water

$= \frac{1}{2} \times$ [Downstream speed + Upstream speed].

$= \frac{1}{2}(7 + 2) = 4.5$ km/hr.

Speed of current

$= \frac{1}{2} \times$ [Downstream speed – Upstream speed].

$= \frac{1}{2}(7 - 2) = 2.5$ km/hr.

E22. A person rows a kilometre down the stream in 10 minutes and up the stream in 30 minutes. Find the speed of the stream.

Sol. Let the speed upstream and speed downstream be S_u and S_d respectively.
S_u = (1000/30)(1/60) = 5/9 m/s.
S_d = (1000/10)(1/60) = 5/3 m/s.
Let the speed of the boat and the current be x m/s and y m/s respectively.
$\therefore$ x + y = 5/3, x – y = 5/9.
Solving these two equations simultaneously, we get
y = 5/9 m/s = 5/9 (18/5) = 2 kmph.
Hence, the speed of the current = 2 kmph.

POINT TO REMEMBER

Boat's speed in still water

$= \frac{v_{down} + v_{up}}{2}$.

Speed of current

$= \frac{v_{down} - v_{up}}{2}$.

E23. A man rows 18 km down a river in 4 hours with the stream and returns in 12 hours. Find his speed in still water and also the speed of the stream.

Notes / Rough Work

Sol. Speed with the stream = $\frac{18}{4}$ = 4.5 kmph.

Speed against the stream = $\frac{18}{12}$ = 1.5 kmph.

$\Rightarrow$ Speed of the stream = $\frac{1}{2}$ (4.5 – 1.5) = 1.5 kmph.

Speed in still water = 4.5 – 1.5 = 3 kmph.

E24. A boat goes 30 km upstream and 44 km downstream in 10 hours. It also goes upstream 40 km and downstream 55 km in 13 hours. Find the speed of the boat and the stream.

Sol. Let X km/hr be the speed of the boat in still water and Y km/hr be the speed of the stream.

$\therefore$ The speed of the boat upstream = (X – Y) km/hr and

the speed of the boat downstream = (X + Y) km/hr.

$\therefore \frac{30}{X-Y} + \frac{44}{X+Y} = 10$ .... (1) $\frac{40}{X-Y} + \frac{55}{X+Y} = 13$ .... (2)

Solving, we get X = 8 and Y = 3.

$\therefore$ The speed of the boat is 8 km/hr and the speed of the stream is 3 km/hr.

Mini Revision Test # 02

DIRECTIONS: *Answer the following questions.*

1. A man can row 6 km/hr in still water. It takes him twice as long to row up as to row down the river. What is the rate at which the stream flows?
2. The speed of a boat in still water is 15 km/hr and the speed of the current is 3 km/hr. Find the distance travelled downstream in 15 minutes.
3. A man can row with the stream at a speed of 20 kmph and against the stream at 5 kmph. What is the man's speed in still water?
4. If a person's speed with the current is 10 km/hr and the speed of the current is 2 km/hr, then what is the person's speed against the current?
5. A boat goes downstream in half the time it takes to go upstream. What is the ratio of the speed of the boat in still water to that of the stream?
6. What is the time taken by a train 150 m long to cross a signal post if the train is travelling at 30 m/s?
7. What is the time taken by a train 200 m long travelling at 20 m/s to cross a tunnel 100 m long?
8. What is the time taken by a train 100 m long to get through two tunnels of 150 m each, if the distance between the tunnels is 400 m and the train is travelling at 36 kmph?
9. A man rows a certain distance downstream in 4 hrs with the stream and returns in 6 hrs. Find the total distance travelled if his speed in still water is 1 kmph.
10. In 3 hrs, a boat can be rowed 9 km upstream or 18 km downstream. Find the speed of the boat in still water.

Challenge Problems # 02

1. A military convoy 10 km long had to drive a distance of 24 km i.e., each member of the convoy travels 24 km only. One of the military policemen in a motorcycle started at the back of the convoy as it moved off, rode to the front of the convoy, returned to the back of the convoy just as the convoy came to a halt. Assume that the convoy's speed and the motorcyclist's speed were constant, how far did the motorcyclist ride?
   (1) 39 km (2) 36 km
   (3) 30 km (4) 32 km

2. Two guns were fired from the same place at an interval of 13 minutes but a person in a train approaching the place hears the second report 12 minutes 30 seconds after the first. Find the speed of the train, supposing that sound travels at the speed of 330 m/s.

3. I take 4 hours less to row down a 12-mile stream than I take to row up. For this 24-mile roundtrip, if I double my rowing speed, I would take half an hour less to row downstream than to row upstream. Find the speed of the stream in miles/hr.
   (1) 6 (2) 8
   (3) 0 (4) None of these

4. Newton stated that when a body falls from rest, the distance of the body from starting point at any instant is directly proportional to square of the time it takes to cover that distance. Suppose a pen is drooped from a certain height. In the 7^{th} second, it covers 169 m. Find the distance covered in the 10^{th} second and find out what percentage this distance represents of the total distance covered in ten seconds. (Ignore the length of the pen).
   (1) 273 m, 21% (2) 260 m, 20%
   (3) 247 m, 19% (4) 234 m, 18%

5. Ram and Shyam run a race between points A and B, 5 km apart. Ram starts at 9 am from A at a speed of 5 km/hr, reaches B, and returns to A at the same speed. Shyam starts at 9.45 am from A at a speed of 10 km/hr, reaches B and comes back to A at the same speed. At what time do Ram and Shyam first meet each other?
   (1) 10.00 am (2) 10.10 am
   (3) 10.20 am (4) 10.30 am

Notes / Rough Work