# 5. VECTOR AND THREE DIMENSIONAL GEOMETRY

## I. MCQ (2 marks each )

1) If $|\bar{a}| = 3$ , $|\bar{b}| = 4$ , then the value of $\lambda$ for which $\bar{a} + \lambda\,\bar{b}$ is perpendicular to

$\bar{a} - \lambda\,\bar{b}$ is …….

A ) $\frac{9}{16}$ B) $\frac{3}{4}$ C) $\frac{3}{2}$ D) $\frac{4}{3}$

2) $(\hat{\imath} + \hat{\jmath} - \hat{k}).(\hat{\imath} - \hat{\jmath} + \hat{k})$ =__________

A) $\hat{\imath} - \hat{\jmath} - \hat{k}$ B) 1 C) -1 D) $-\hat{\jmath} + \hat{k}$

3) The angle $\theta$ between two non-zero vectors $\bar{a}$ & $\bar{b}$ is given by $\cos\theta = \cdots$

A) $\frac{\bar{a}.\bar{b}}{|\bar{a}||\bar{b}|}$ B) $\bar{a}.\bar{b}$ C) $|\bar{a}||\bar{b}|$ D) $\frac{|\bar{a}||\bar{b}|}{\bar{a}.\bar{b}}$

4) If sum of two unit vectors is itself a unit vector, then the magnitude of their difference is...

A ) $\sqrt{2}$ B) $\sqrt{3}$ C) 1 D) 2

5) If $\alpha, \beta, \gamma$ are direction angles of a line and $\alpha = 60^{o}$ , $\beta = 45^{o}$, then $\gamma$ = _____

A ) $30^0$ or $90^0$ B) $45^0$ or $60^0$ C) $90^0$ or $30^0$ D) $60^0$ or $120^0$

6 ) The distance of the point $(3, 4, 5)$ from Y- axis is _____

A ) 3 B) 5 C) $\sqrt{34}$ D) $\sqrt{41}$

7) If $\cos\alpha$ , $\cos\beta$ , $\cos\gamma$ are the direction cosines of a line then the value of

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is ____

A ) 1 B) 2 C) 3 D) 4

8) If $|\bar{a}| = 2, |\bar{b}| = 5, and\ \ \bar{a}.\bar{b} = 8$ then $|\bar{a} - \bar{b}| =$ __

A) 13 B) 12 C) $\sqrt{13}$ D) $\sqrt{21}$

9) If $\overline{AB} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$, and A( 1, 2 ,-1 ) is given point then coordinates of B are____

A) (3, 3, -4) B) ( -3, 3 -2) C) ( 3, 3, 2) D) (-3, 3, 4)

10) If l, m, n are direction cosines of a line then $l\hat{\imath} + m\hat{\jmath} + n\hat{k}$ is ___

A ) Null vector B) the unit vector along the line.

C) Any vector along the line D) a vector perpendicular to the line.

11) The values of $c$ that satisfy $|c\,\bar{u}| = 3$, $\bar{u} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ **is** ___

A) $\sqrt{14}$ B) $3\sqrt{14}$ C) $\frac{3}{\sqrt{14}}$ D) 3

12. The value of $\hat{\imath}.\left(\hat{\jmath} \times \hat{k}\right) + \hat{\jmath}.\left(\hat{\imath} \times \hat{k}\right) + \hat{k}.\left(\hat{\imath} \times \hat{\jmath}\right)$

A ) 0 B) $-1$ C) 1 D) 3

13. The two vectors $\hat{\jmath} + \hat{k}$ & $3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ represents the two sides AB and AC, respectively of a ΔABC. The length of the median through A is

A ) $\frac{\sqrt{34}}{2}$ B) $\frac{\sqrt{48}}{2}$ C) $\sqrt{18}$ D) $\sqrt{34}$

14. If $\bar{a}$ , $\bar{b}$ and $\bar{c}$ are non coplanar unit vectors, and $\bar{a} \times \left(\bar{b} \times \bar{c}\right) = \frac{\bar{b}+\sqrt{3}\bar{c}}{2}$, then the angle between $\bar{a}$and $\bar{c}$ is ___

a) $\frac{5\pi}{3}$ b) $\frac{5\pi}{6}$ c) $\frac{\pi}{3}$ d) π

## II. Very Short Answers ( 1 mark )

1. Find the magnitude of a vector with initial point : $(1\,,-3\,,4)$ ; terminal point : $(1\,,0\,,-1)$.
2. Find the coordinates of the point which is located Three units behind the YZ-plane, four units to the right of the XZ-plane and five units above the XY-Plane.
3. $A(2{,}3)$, $B(-1{,}5)\,,C(-1{,}1)$ and $D(-7{,}5)$ are four points in the Cartesian plane, Check if , $\overline{CD}$ is parallel to $\overline{AB}$.
4. Find a unit vector in the opposite direction of $\bar{u}$.Where $\bar{u} = 8\hat{\imath} + 3\hat{\jmath} - \hat{k}$.
5. The non zero vectors $\bar{a}$ and $\bar{b}$ are not collinear find the value of $\lambda$ and $\mu$ :

   if $\bar{a} + 3\bar{b} = 2\lambda\bar{a} - \mu\bar{b}$

6. If $\bar{a} = 4\hat{\imath} + 3\hat{k}$ and $\bar{b} = -2\hat{\imath} + \hat{\jmath} + 5\hat{k}$ then find $2\bar{a} + 5\bar{b}$

7. Find the distance from $(4, -2, 6)$ to the XZ- Plane.

8. If the vectors $2\hat{\imath} - q\hat{\jmath} + 3\hat{k}$ and $4\hat{\imath} - 5\hat{\jmath} + 6\hat{k}$ are collinear then find the value of $q$.

**9.** Find $\bar{a} . \bar{b} \times \bar{c}$, if $\bar{a} = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ , $\bar{b} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ , $\bar{c} = -5\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

10. If a line makes angle $90^0, 60^0$ and $30^0$ with the positive direction of X, Y and Z axes respectively, find its direction cosines.

11. Find $\hat{k} \times (\hat{\jmath} \times \hat{\imath})$

**III. Short Answers ( 2 mark )**

1. The vector $\bar{a}$ is directed due north and $|\bar{a}| = 24$. The vector $\bar{b}$ is directed due west and $|\bar{b}| = 7$. find $|\bar{a} + \bar{b}|$.

2. Show that following points are collinear $P(4,5,2), Q(3,2,4), R(5,8,0)$

3. If a vector has direction angles $45^0$ and $60^0$ find the third direction angle.

4. If $\bar{c} = 3\bar{a} - 2\bar{b}$ then prove that $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

5. If $|\bar{a} . \bar{b}| = |\bar{a} \times \bar{b}|$ & $\bar{a} . \bar{b} < 0$ , then find the angle between $\bar{a}$ & $\bar{b}$ .

6. Find the direction ratios of a vector perpendicular to the two lines whose direction ratios are 1, 3, 2 and –1, 1,2
7. If $\bar{a}$, $\bar{b}$ and $\bar{c}$ are position vectors of the points A, B, C respectively and $5\bar{a} - 3\bar{b} - 2\bar{c} = \bar{0}$, then find the ratio in which the point C divides the line segment BA.
8. If $\bar{a}$ and $\bar{b}$ are two vectors perpendicular each other, prove that
$$(\bar{a} + \bar{b})^2 = (\bar{a} - \bar{b})^2$$
9. Find the position vector of point R which divides the line joining the points P and Q whose position vectors are $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $-5\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ in the ratio 3 : 2
(i) internally (ii) externally.
10. Find a unit vector perpendicular to the vectors $\hat{\jmath} + 2\hat{k}$ & $\hat{\imath} + \hat{\jmath}$

**IV. Short Answers ( 3 mark )**

1. If two of the vertices of the triangle are $A(3,1,4)$ and $B(-4,5,-3)$ and the centroid of a triangle is $G(-1,2,1)$, then find the co-ordinates of the third vertex C of the triangle.
2. Find the centroid of tetrahedron with vertices $\mathrm{K}(5,-7,0)$, $\mathrm{L}(1,5,3), \mathrm{M}(4,-6,3),\ \mathrm{N}(6,-4,2)$?
3. If a line has the direction ratios , $4\ , -12\ , 18$ then find its direction cosines.
4. Show that the points $A(2,-1,0)$ $B(-3,0,4), C(-1,-1,4)$ and $D(0,-5,2)$ are non coplanar.
5. Using properties of scalar triple product, prove that

$$[\bar{a}+\bar{b} \quad \bar{b}+\bar{c} \quad \bar{c}+\bar{a}] = 2\,[\bar{a} \quad \bar{b} \quad \bar{c}]$$

6. The direction ratios of $\overline{AB}$ are $-2\ ,2\ ,1$ . If $A = (4,1,5)$ and $l(AB) = 6$ units, Then find B.
7. If $G(a, 2, -1)$ is the centroid of the triangle with vertices $P(1,2,3)$ , $Q(3, b, -4)$ and $R(5,1,c)$ then find the values of $a, b$ and $c$.
8. If $A(5,1,p)$ , $B(1,q,p)$ and $C(1,-2,3)$ are vertices of triangle and $G\left(r, -\frac{4}{3}, \frac{1}{3}\right)$ is its centroid then find the values of $p\ ,q$ & $r$.
9. Prove by vector method that the angle subtended on semicircle is a right angle.
10. Prove that medians of a triangle are concurrent.
11. Prove that altitudes of a triangle are concurrent.

**V. Long Answers ( 4 mark )**

1. Express $-\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ as linear combination of the vectors $2\hat{\imath} + \hat{\jmath} - 4\hat{k}$, $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$.
2. If Q is the foot of the perpendicular from P(2,4,3) on the line joining the points A(1,2,4) and B(3,4,5), find coordinates of Q.
3. Prove that the angle bisectors of a triangle are concurrent.
4. Using vector method , find the incenter of the triangle whose vertices are $A(0,3,0)$ $B(0,0,4)$ and $C(0,3,4)$.

5. Find the angles between the lines whose direction cosines $l, m, n$ satisfy the equations $5l + m + 3n = 0$ and $5mn - 2nl + 6lm = 0$

6. Let $A(\bar{a})$ and $B(\bar{b})$ be any two points in the space and $R(\bar{r})$ be a point on the line segment $AB$ dividing it internally in the ratio $m : n$ then prove that $\bar{r} = \frac{m\bar{b}+n\bar{a}}{m+n}$.

7. D and E divides sides BC and CA of a triangle ABC in the ratio 2 : 3 respectively. Find the position vector of the point of intersection of AD and BE and the ratio in which this point divides AD and BE.

8. If $\bar{u} = \hat{i} - 2\hat{j} + \hat{k}$ , $\bar{r} = 3\hat{i} + \hat{k}$ & $\bar{w} = \hat{j}, \hat{k}$ are given vectors , then find

   $[\bar{u} + \bar{w}].[(\bar{w} \times \bar{r}) \times (\bar{r} \times \bar{w})]$

9. Find the volume of a tetrahedron whose vertices are

   $A(-1, 2, 3)$ $B(3, -2, 1)$ ,C(2, 1, 3) and D( -1, -2, 4)

10. If four points A $(\bar{a})$ , B$(\bar{b})$ ,C$(\bar{c})$ & D$(\bar{d})$ are coplanar then show that $[\bar{a} \quad \bar{b} \quad \bar{d}] + [\bar{b} \quad \bar{c} \quad \bar{d}] + [\bar{c} \quad \bar{a} \quad \bar{d}] = [\bar{a} \quad \bar{b} \quad \bar{c}]$

11. Prove that the volume of parallelopiped with coterminous edges as $\bar{a}$, $\bar{b}$ and $\bar{c}$ is $[\bar{a} \quad \bar{b} \quad \bar{c}]$.

12. Prove that the volume of tetrahedron with coterminous edges as $\bar{a}$ , $\bar{b}$ and $\bar{c}$ is $\frac{1}{6}[\bar{a} \quad \bar{b} \quad \bar{c}]$