COMMON P. G. ENTRANCE TEST – 2024 (CPET-2024)

Test Booklet No. : 01544

Subject Code : 27

Hall Ticket No. :

Subject : MATHEMATICS

TEST BOOKLET

*Time Allowed : **60** Minutes*

*Full Marks : **80***

: INSTRUCTIONS TO CANDIDATES :

1. The Test Booklet contains **23** pages including the cover page and **80** (Question No. 1 to 80) multiple choice questions.
2. DO NOT break open the seal of the Test Booklet until the invigilator instructs to do so.
3. The candidates must check discrepancy, if any (like up-printed or torn or missing pages or missing questions) in the Test Booklet immediately after breaking the seal of the Test Booklet. If detected, the invigilator may be requested to replace the same.
4. Candidates are required to fill up and darken the **Hall Ticket No, Test Booklet Serial No.** and OMR Answer Sheet Serial No. in attendance sheet carefully. Wrongly filled in OMR Answer Sheet is liable for rejection.
5. Each question has four choices / answers marked (A), (B), (C), (D). Candidate has to select the most appropriate choice / answer to each question and darken the oval completely against the question number provided in the OMR Answer Sheet.
6. Indicate only one choice / answer from the options provided by darkening the appropriate oval in the OMR Answer Sheet. More than one response to a question shall be treated as a wrong answer.
7. Use only **Black Ball Point Pen** for darkening the oval for answering.
8. All the questions are compulsory and they carry equal marks. The total marks scored by a candidate depends on the number of correct choices / answers darkened in the OMR Answer Sheet. There will be no negative marking for wrong answers.
9. No candidate shall be allowed to leave the Examination Hall / Room till all OMR Answer Sheets have been collected by the invigilator.
10. On completion of the entrance test, the original OMR Answer Sheet be handed over to the invigilator. Candidates are allowed to take the second copy of the OMR Answer Sheet along with the used Test Booklet for reference.
11. Candidates are not allowed to carry any personal belongings including electronic devices such as scientific calculator, cell phones, headphones, earbuds, or any other type of devices that allow communication of any kind inside the Examination Room / Hall.
12. The candidates are advised not to scribble or make any mark on the OMR Answer Sheet except marking the answers at the appropriate places and filling up the details required. Rough work, if any, may be done in the blank sheet(s) provided at the end of the Test Booklet.
13. Any malpractice / use of unfair means will lead to your disqualification from the entrance test / admission process and may also lead to appropriate legal action as deemed fit.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

SEAL

(Turn over)

NOTATIONS

$\mathbb{N}$	$\{1, 2, 3, \cdots\}$, the set of all natural numbers.
$\mathbb{Z}$	$\{0, \pm1, \pm2, \cdots\}$, the set of all integers.
$\mathbb{Q}$	Set of all rational numbers.
$\mathbb{Q}^+$	Set of all positive rational numbers.
$\mathbb{R}$	Set of all real numbers.
$\mathbb{C}$	Set of all complex numbers.
$\mathbb{R}^n$	n-dimensional Euclidean space $\{(x_1, x_2, \ldots\ldots, x_n) : x_k \in \mathbb{R}, 1 \le k \le n\}$.
S_n	Group of all permutations on n distinct symbols under composition of mappings.
$\mathbb{Z}_n$	Group of congruence classes of integers modulo n.
$\mathbb{Z}[i]$	$\{a + ib : a, b \in \mathbb{Z}\}$, the Gaussian integers.
$\mathbb{Z}\left[\sqrt{-5}\right]$	$\{a + b\sqrt{-5} : a, b \in \mathbb{Z}\}$.
$\mathbb{R}[x]$	Set of all polynomials with coefficients in $\mathbb{R}$.
$\mathbb{Q}[x]$	Set of all polynomials with coefficients in $\mathbb{Q}$.
$\mathbb{Z}_n[x]$	Set of all polynomials with coefficients in $\mathbb{Z}_n$.
$W^\perp$	Orthogonal complement of W.
$\hat{i}, \hat{j}, \hat{k}$	Unit vectors having the directions of positive x, y and z axes in three dimensional rectangular coordinate system.

1. Which one of the following expressions is the n-th derivative of $e^x \sin x$?

(A) $2^n e^x \sin\left(x + \frac{n\pi}{4}\right)$

(B) $2^n e^x \sin\left(x + \frac{n\pi}{3}\right)$

(C) $2^{n/2} e^x \sin\left(x + \frac{n\pi}{4}\right)$

(D) $2^{n/2} e^x \sin\left(x + \frac{n\pi}{3}\right)$

2. Which one of the following is true for the function $f(x) = \dfrac{x^2 - 8x + 12}{x^2 - 36}$?

(A) The line $x = 6$ is the vertical asymptote and there is no horizontal asymptote of f.

(B) The line $x = -6$ is the vertical asymptote and the line $y = 1$ is the horizontal asymptote of f.

(C) The line $x = 6$ is the vertical asymptote and the line $y = -1$ is the horizontal asymptote of f.

(D) The line $x = 6$ and $x = -6$ are the vertical asymptotes and the line $y = 1$ is the horizontal asymptote of f.

3. Choose the correct option for the following integrals :

$I = \int_0^{\frac{\pi}{4}} \sin^2(x)\, dx$ and $J = \int_0^{\frac{\pi}{4}} \cos^2(x)\, dx$?

(A) $I < J$

(B) $I = J$

(C) $I > J$

(D) $I - J = \dfrac{\pi}{4}$

4. Which one of the following statements is correct for the function :

$$f(x) = xe^{-2x}\ (x \in \mathbb{R})\ ?$$

(A) f is always concave down

(B) f is always increasing and has no inflection point

(C) f has exactly one inflection point at $x = 1$ and is concave up when $x > 1$

(D) f has exactly two inflection points at $x = 0$, $x = 1$ and is concave up between $x = 0$, $x = 1$.

5. The volume of the solid of revolution generated by rotating the region about the x-axis between the graph of $f(x) = \sqrt{x}$ and the x-axis over the interval [1, 4] is :

(A) $\dfrac{15\pi}{4}$ unit^3

(B) $\dfrac{15\pi}{2}$ unit^3

(C) 15π unit^3

(D) 20π unit^3

6. If the motion of a particle is given by $\vec{r}(t) = \langle 4\cos t, 4\sin t, \frac{3}{2\pi}t^2\rangle$, then what is the tangential component of the acceleration of the particle at $t = \pi$?

(A) 0
(B) $\frac{4}{5\pi}$
(C) $\frac{9}{5\pi}$
(D) $\frac{16}{5\pi}$

7. Let p, q, r be propositions and the expression : $(p \rightarrow q) \rightarrow r$ be a contradiction. Then the expression $(r \rightarrow p) \rightarrow q$ is :

(A) A tautology
(B) A contradiction
(C) Always true when p is false
(D) Always true when q is true

8. If 5 divides an integer n, the remainder is 2. What will be the remainder, if 7n is divided by 5 ?

(A) 4
(B) 3
(C) 2
(D) 1

9. What is the digit in the unit's place of the integer 3^{2011} ?

(A) 2
(B) 5
(C) 7
(D) 8

10. If G(x) is the generating function of the sequence : 1, 0, 1, 0, 1, 0, ..., then which one of the following is true ?

(A) $G(2) = -\frac{1}{3}$
(B) $G(1) = 0$
(C) $G(0) = -1$
(D) $G(-1) = 1$

11. For a system of linear equations : Ax = b, where A is a m × n matrix, b is a m × 1 column vector and x is a n × 1 column vector of unknowns, which one of the following statements is false ?

(A) The system has a solution, if the rank of the matrix : A is same as the rank of the augmented matrix : (A|b)
(B) If m < n and b is the zero vector, then the system has infinitely many solutions.
(C) If m = n and b is a non-zero vector, then the system has a unique solution.
(D) If m = n, b is the zero vector and rank(A) = n, then the system will have only a trivial solution.

12. For what range of the values of α, the eigenvalue(s) of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & \alpha \end{pmatrix}$ is/are positive ?

(A) $\alpha < -\frac{1}{2}$ (B) $\alpha > -2$

(C) $\alpha > 0$ (D) $\alpha > \frac{1}{2}$

13. Which one of the following statements is true for any simple connected undirected graph with more than 2 vertices ?

(A) No two vertices have the same degree

(B) At least two vertices have the same degree

(C) At least three vertices have the same degree

(D) All vertices have the same degree

14. Let G be a connected planar graph with 10 vertices. If the number of edges on each face is 3, then the number of edges in G is :

(A) 64 (B) 32

(C) 24 (D) 18

15. Which of the following statements is true ?

(A) The incidence matrix and the adjacency matrix of a graph will always have the same dimensions

(B) There exist a simple graph with 5 vertices each of degree 3

(C) The complete graph K_2 has a Hamiltonian cycle

(D) The complete graph K_5 has an Eulerian path

16. Which one of the following statements is false ?

(A) For the set $S = (0, 1) \cap \{m + n\sqrt{2} : m, n \in \mathbb{N}\}$, $\sup(S) = 1$ and $\inf(S) = 0$.

(B) The set $\{x \in \mathbb{Q}^+ : 2 < x^2 < 3\}$ is both open and closed in $\mathbb{Q}$.

(C) The set of all interior points of $\mathbb{R}$ regarded as a subset of $\mathbb{C}$ is $\mathbb{R}$.

(D) The set of all limit points of the set of all integers $\mathbb{Z}$ in $\mathbb{R}$ is an empty set.

17. For a sequence $\{a_n\}_{n\in\mathbb{N}}$ of real numbers, consider the following statements :

(I) Given $\varepsilon > 0$, there exists an $n_0 \in \mathbb{N}$ such that $|a_{n+1} - a_n| < \varepsilon \ \forall n > n_0$

(II) Given $\varepsilon > 0$, there exists an $n_0 \in \mathbb{N}$ such that $\frac{1}{(n+1)^2} |a_{n+1} - a_n| < \varepsilon \ \forall n > n_0$

(III) Given $\varepsilon > 0$, there exists an $n_0 \in \mathbb{N}$ such that $(n + 1)^2 |a_{n+1} - a_n| < \varepsilon \ \forall n > n_0$

Which of the statement(s) given above imply the convergence of the sequence $\{a_n\}_{n\in\mathbb{N}}$?

(A) Only (III)
(B) Only (I) and (II)
(C) Only (II) and (III)
(D) All (I), (II) and (III)

18. Let $\{a_n\}$, $(b_n\}$ be sequences of real numbers defined by $a_1 = 1$, $a_{n+1} = a_n + \frac{(-1)^n}{2^n}$ and $b_n = \frac{2a_{n+1} - a_n}{a_n}$ for each $n \in \mathbb{N}$. Then which one of the following statements is true ?

(A) $\{a_n\}$ converges to zero and $\{b_n\}$ is a Cauchy sequence.
(B) $\{a_n\}$ converges to zero and $\{b_n\}$ is not convergent sequence.
(C) $\{a_n\}$ converges to a non-zero number and $\{b_n\}$ is a Cauchy sequence.
(D) $\{a_n\}$ converges to a non-zero number and $\{b_n\}$ is not a convergent sequence.

19. For the series given below :

(I) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{2^n}$
(II) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\log(n+1)}$

Which one of the following options is correct ?

(A) Both (I) and (II) are absolutely convergent
(B) (I) is absolutely convergent, but (II) is conditionally convergent
(C) (I) is conditionally convergent, but (II) is absolutely convergent
(D) Both (I) and (II) are conditionally convergent

20. If $f(x) = \begin{cases} \dfrac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$, where [x] denote the greatest integer $\leq x$, then what is the value of $\lim_{x\to 0} f(x)$?

(A) 0
(B) sin(1)
(C) 1
(D) Does not exists

21. For the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} x^6 - 1, & x \in \mathbb{Q} \\ 1 - x^6, & x \notin \mathbb{Q} \end{cases}$, the number of points of continuity is :

(A) 0
(B) 2
(C) 4
(D) 6

22. The function $f(x) = \begin{cases} x^\alpha \sin\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous, but not differentiable at x = 0, if :

(A) $\alpha \in (-\infty, 0)$
(B) $\alpha = 0$
(C) $\alpha \in (0, 1]$
(D) $\alpha \in [2, \infty)$

23. Consider the following statements :

(I) $f(x) = \dfrac{1}{x}$ is continuous on (0, 1)

(II) $f(x) = \dfrac{1}{\sqrt{x}}$ is continuous on $(1, \infty)$

Choose the correct answer from the options given below :

(A) (I) is true, but (II) is false
(B) (I) is false, but (II) is true
(C) Both (I) and (II) are false
(D) Both (I) and (II) are true

24. Let $f : [-7, 0] \to \mathbb{R}$ be continuous and differentiable on the interval (–7, 0). If $f(-7) = -3$ and $f'(x) \leq 2$ for all $x \in (-7, 0)$, then what is the largest possible value of $f(-1)$?

(A) 9
(B) 11
(C) 20
(D) 24

25. $\lim_{x\to 0} \dfrac{\int_0^x \left(e^{t^2} - e^{-t^2}\right)dt}{x^3}$ is equal to :

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) 2 (D) 3

26. If $\sum_{n=0}^{\infty} a_n x^n$ is the Maclaurin's series expansion of $f(x) = \begin{cases} \dfrac{x^2}{1-\cos x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ for all $x \in (-1, 1)$, then $\sum_{n=0}^{\infty} a_{2n+1}$ is equal to :

(A) 0 (B) $\frac{1}{2}$

(C) $\frac{2}{3}$ (D) 1

27. Which one of the following definite integrals is the limit of the Riemann sum $\sum_{k=1}^{n} \frac{5}{n}\sqrt{4 + \frac{5k}{n}}$ as $n \to \infty$?

(A) $\int_0^4 \sqrt{4+x}\,dx$ (B) $\int_4^9 \sqrt{x}\,dx$

(C) $\int_0^5 \sqrt{x}\,dx$ (D) $\int_4^9 \sqrt{4+x}\,dx$

28. For the improper integral $\int_2^{\infty} \frac{dx}{x(\log x)^2}$ and the infinite series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^2}$, which one of the following options is correct ?

(A) The integral converges, but the series does not converge

(B) The integral does not converge, but the series converges

(C) The integral and the series both converge

(D) The integral and the series both fail to converge

29. Consider the following statements with regard to the sequence of functions $\{f_n\}_{n \in \mathbb{N}}$ defined by $f_n(x) = \dfrac{nx}{1 + nx^2}$ $(x > 0)$:

(I) The sequence $\{f_n\}$ is uniformly convergent on (0, 1).

(II) The sequence $\{f_n\}$ is uniformly continuous on $(1, \infty)$.

(III) The sequence $\{f_n\}$ is uniformly continuous on $(0, \infty)$.

Pick out the correct answer from the options given below :

(A) (I) and (III) are false, but (II) is true

(B) (I) and (III) are true, but (II) is false

(C) (I) and (II) are true, but (III) is false

(D) (II) and (III) are false, but (I) is true

30. If $2^n - n \leq a_n \leq 2^n + n$ for all $n \in \mathbb{N}$, then the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n x^n$ is :

(A) 2

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

31. Consider the following statements :

(I) $\mathbb{Z}$ with the metric $d(x, y) = |x - y|$ $(x, y \in \mathbb{Z})$ is a complete metric space.

(II) $\mathbb{N}$ with the metric $d(x, y) = \left|\frac{1}{x} - \frac{1}{y}\right|$ $(x, y \in \mathbb{N})$ is a complete metric space.

(III) $\mathbb{R}$ with the metric $d(x, y) = \begin{cases} 1, & x \neq y \\ 0, & x = y \end{cases}$ $(x, y \in \mathbb{R})$ is a complete metric space.

Choose the correct answer from the options given below :

(A) Only (II) is false

(B) Only (I) and (II) are false

(C) Only (II) and (III) are false

(D) All (I), (II) and (III) are false

32. Consider the following statements :

(I) A continuous map between metric spaces preserves convergent sequences.

(II) A continuous map between metric spaces preserves Cauchy sequences.

(III) A continuous map between metric spaces preserves completeness.

Which of the above statements is / are true ?

(A) Only (I) and (II) are true
(B) Only (II) and (III) are true
(C) Only (I) is true
(D) Only (III) is true

33. Let (X, d) and (Y, ρ) be metric spaces. If $f : X \to Y$ is a homeomorphism, then which one of the following statements is false ?

(A) If X is complete, then Y is complete
(B) If X is compact, then Y is compact
(C) If X is connected, then Y is connected
(D) If X is separable, then Y is separable

34. Consider the following statements for a discrete metric space (X, d) :

(I) Every function from X to any other metric space is continuous.

(II) If X is uncountable, then (X, d) is second countable.

(III) No proper subset of X is dense in X.

In view of the above statements, which one the options given below is true ?

(A) All, (I), (II), (III) are false
(B) Only (I) and (II) are false
(C) Only (II) and (III) are false
(D) Only (II) is false

35. For the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ which one of the following options is true ?

(A) f is continuous at (0, 0) and both the partial derivatives of f exist at (0, 0).

(B) f is continuous at (0, 0) and both the partial derivatives of f exist at (0, 0).

(C) f is continuous at (0, 0) and one partial derivative of f does not exist at (0, 0).

(D) f is not continous at (0, 0) and one partial derivative of f does not exist at (0, 0).

36. A direction in which the directional derivative of the scalar function :

$f(x, y, z) = xy + yz + zx$ at the point (1, 2, 3) is zero is :

(A) $\hat{i} + \hat{j} - \hat{k}$ (B) $2\hat{i} - \hat{j} + \hat{k}$

(C) $\hat{i} + \hat{j} - 3\hat{k}$ (D) $\hat{i} + 2\hat{j} - 4\hat{k}$

37. Which one of the following statements is true for the function

$$f(x, y) = 3x - x^3 - 2y^2 + y^4 ?$$

(A) f has one local maxima and one local minima.

(B) f has two saddle points and one local minima.

(C) f has two saddle points and two local maxima.

(D) f has two saddle points and one local maxima.

38. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ be a vector field on $\mathbb{R}^3$ and $r^2 = \vec{r} \cdot \vec{r} = x^2 + y^2 + z^2$. Then which one of the following options is false ?

(A) Laplacian of $\frac{1}{r}$ is $\frac{2}{r}$ (B) Gradient of r^2 is $2\vec{r}$

(C) Divergence of $\vec{r}$ is 3 (D) Curl of $\vec{r}$ is zero

39. The value of the integral $\iint_R 3x \, dA$ over the region $R = \{(r, \theta) : r \in [1, 2], \theta \in [0, \pi]\}$ is :

(A) 2π (B) π

(C) $\frac{\pi}{2}$ (D) 0

40. If C is positively oriented circle $x^2 + y^2 = \sqrt{2}$, then the value of the integral

$$\int_C \{(x^{2015} y^{2016} + 2014y)dx + (x^{2016} y^{2015} + 2017x)\, dy\} \text{ is :}$$

(A) π (B) 3π

(C) 6π (D) 12π

41. Which one of the following statements is not necessarily true ?

(A) A vector field is conservative, then its curl is zero.

(B) All irrotational vector fields are conservative.

(C) A vector field is conservative, if it is equal to the gradient of some scalar function.

(D) The line integral of a conservative vector field along a closed curve is always zero.

42. On which of the following sets, the vectors $(1, x, 0)$, $(0, x^2, 1)$, $(0, 1, x)$ in $\mathbb{R}^3$ are linearly independent ?

(A) $\{x \in \mathbb{R} : x = 0\}$ (B) $\{x \in \mathbb{R} : x \neq 0\}$

(C) $\{x \in \mathbb{R} : x \neq 1\}$ (D) $\{x \in \mathbb{R} : x \neq -1\}$

43. Let V be the vector space of all 2 × 2 matrices over $\mathbb{R}$. Consider the subspaces $W_1 = \left\{\begin{pmatrix} a & -a \\ c & d \end{pmatrix} : a, c, d \in \mathbb{R}\right\}$ and $W_2 = \left\{\begin{pmatrix} a & b \\ -a & d \end{pmatrix} : a, b, d \in \mathbb{R}\right\}$. If $n = \dim(W_1 + W_2)$ and $m = \dim(W_1 \cap W_2)$, then a possible value of (n, m) is :

(A) (3, 2) (B) (4, 2)

(C) (4, 3) (D) (3, 1)

44. Consider the vector space $\mathbb{R}^2$ over the field $\mathbb{R}$. If $\{(2, 1), (3, 1)\}$ is a basis of $\mathbb{R}^2$, then its dual basis is :

(A) $f_1(x, y) = x - 3y, f_2(x, y) = x + 2y$ (B) $f_1(x, y) = x + 3y, f_2(x, y) = x - 2y$

(C) $f_1(x, y) = -x + 3y, f_2(x, y) = x + 2y$ (D) $f_1(x, y) = -x + 3y, f_2(x, y) = x - 2y$

45. For an $n \times n$ matrix A, which one of the following statements is false ?

(A) A is diagonalizable, if it has n eigenvalues, counting multiplicities

(B) A is diagonalizable, if it has n linearly independent eigenvectors

(C) A is diagonalizable, if the sum of the dimensions of its eigenspaces equals to n.

(D) A is diagonalizable, it its minimal polynomial has no repeated roots.

46. If the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{pmatrix}$ satisfies $6A^{-1} = A^2 + kA + 11\,I$, where k is a constant and I is the unit matrix, then the value of k is :

(A) 7 (B) 5

(C) -2 (D) -6

47. Let $n > 1$ be a fixed integer. Which one of the following defines an inner product on the vector space of $n \times n$ real symmetric matrices ?

(A) $\langle A, B \rangle = \det(AB)$ (B) $\langle A, B \rangle = \operatorname{tr}(A) + \operatorname{tr}(B)$

(C) $\langle A, B \rangle = \operatorname{tr}(AB)$ (D) $\langle A, B \rangle = \operatorname{tr}(A) \cdot \operatorname{tr}(B)$

48. On the inner product space $\mathbb{R}^4$ with the standard inner product, consider the following statements :

(I) If $W = \operatorname{span}\{(1, 0, 0, 0), (0, 0, 1, 0)\}$, then $W^{\perp} = \operatorname{span}\{(0, 1, 0, 0), (0, 0, 0, 1)\}$.

(II) If $W = \{(x, y, z, w) \in \mathbb{R}^4 : x = z\}$, then $\dim(W^{\perp})$ is 2.

Which one of the following options is true ?

(A) (I) is true, but (II) is false (B) (I) is false, but (II) is true

(C) Both (I) and (II) are true (D) Neither (I) nor (II) is true

49. Consider the following collections of 2 × 2 matrices :

$$G_1 = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z}, ad - bc = -1\right\}$$

$$G_2 = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z}, ad - bc = 1\right\}$$

$$G_3 = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z}, ad - bc = \pm 1\right\}$$

Which one of the following options is correct ?

(A) G_1 is a group under matrix multiplication.

(B) G_3 is not a group under matrix multiplication.

(C) G_1 and G_2 are groups under matrix multiplication.

(D) G_2 and G_3 are groups under matrix multiplication.

50. Which one of the following statements is false for a group G of order 45 ?

(A) Z(G) = {e} (e is the identity element of G)

(B) G has a normal subgroup of order 9

(C) G has a unique 5-Sylow subgroup of order 5

(D) G is a Abelian group

51. For what value of α, the set $G = \{\alpha, 1, 3, 9, 19, 27\}$ is a cyclic group under multiplication modulo 56 ?

(A) 5 (B) 15

(C) 20 (D) 25

52. Let G be a finite group containing a subgroup of order 7. If no element (other than the identity) is its own inverse in G, then what could be the order of G ?

(A) 42 (B) 37

(C) 35 (D) 28

53. Consider the following statements :

(I) Number of automorphisms of S_3 is 3

(II) Maximum order of an element of S_3 is 3

(III) Number of conjugate classes of S_3 is 3

Which of the above statement(s) is/are true ?

(A) Only (I) and (II) (B) Only (II) and (III)

(C) Only (I) (D) Only (II)

54. Which one of the following is true for the ring $R = \mathbb{Z}\left[\sqrt{-5}\right]$ with usual addition and multiplication ?

(A) R is an integral domain (B) R is a principal ideal domain

(C) R is a unique factorization domain (D) R is a Euclidean domain

55. For the set $S = \{a + ib : a, b \in \mathbb{Z}, b \text{ is even}\}$, which one of the following statements is true ?

(A) S is both a subring and an ideal of $\mathbb{Z}[i]$

(B) S is a subring of $\mathbb{Z}[i]$, but not an ideal of $\mathbb{Z}[i]$

(C) S is neither a subring not an ideal of $\mathbb{Z}[i]$

(D) S is an ideal of $\mathbb{Z}[i]$, but not a subring of $\mathbb{Z}[i]$

56. The number of ring homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{28}$ is :

(A) 2 (B) 3

(C) 4 (D) 6

57. Which one of the following is true ?

(A) $x^2 + 1$ is irreducible over $\mathbb{Z}_2$, but not over $\mathbb{Z}_3$

(B) $x^2 + 1$ is reducible over both $\mathbb{Z}_2$ and $\mathbb{Z}_3$

(C) $x^2 + 1$ is reducible over $\mathbb{Z}_2$, but not over $\mathbb{Z}_3$

(D) $x^2 + 1$ is irreducible over both $\mathbb{Z}_2$ and $\mathbb{Z}_3$

58. Consider the following statements :

(I) The function $f(z) = x^2 + y^2 + 2i\,xy$ $(z = x + iy)$ is differentiable everywhere except at points that lie on the x-axis.

(II) The function $f(z) = \text{Log}(iz)$ $(z = x + iy)$ is differentiable everywhere except at points on the ray given by $y \geq 0, x = 0$.

Which one of the options given below is correct ?

(A) (I) is correct, but (II) is false (B) (I) is false, but (II) is correct

(C) Both (I) and (II) are correct (D) Neither (I) nor (II) is correct

59. If the power series $\sum_{k=0}^{\infty} a_n(z + 3 - i)$ converges at 5i and diverges at –3i, then which one of the following statements is true ?

(A) The power series converges at 2 – 3i and diverges at – 2 + 5i

(B) The power series converges at both 2 – 3i and – 2 + 5i

(C) The power series converges at – 2 + 5i and diverges at 2 – 3i

(D) The power series diverges at both 2 – 3i and – 2 + 5i

60. Which one of the following statements is true for the Mobius transformation

$$T(z) = \frac{2z+1}{5z+3} ?$$

(A) T maps $\{z \in \mathbb{C} : \text{Im}(z) > 0\}$ to $\{z \in \mathbb{C} : \text{Im}(z) > 0\}$

(B) T maps $\{z \in \mathbb{C} : \text{Im}(z) > 0\}$ to $\{z \in \mathbb{C} : \text{Im}(z) < 0\}$

(C) T maps $\{z \in \mathbb{C} : \text{Re}(z) > 0\}$ to $\{z \in \mathbb{C} : \text{Re}(z) < 0\}$

(D) T maps $\{z \in \mathbb{C} : \text{Re}(z) > 0\}$ to $\{z \in \mathbb{C} : \text{Re}(z) > 0\}$

61. If $f(z) = \sum_{n=0}^{15} z^n$ for $z \in \mathbb{C}$, then the value of the integral $\frac{1}{2\pi i}\int_{|z-i|=2} \frac{f(z)dz}{(z-i)^{15}}$ is :

(A) $\frac{1}{2}(1-15i)$

(B) $1+15i$

(C) $\frac{1}{2}(14-15i)$

(D) $14+15i$

62. For an entire function f, which one of the following statements is not necessarily true ?

(A) If f is bounded on $\{z \in \mathbb{C} : \text{Re}(z) \leq 0\}$, then f is a constant function.

(B) If f has a pole at infinity, then f is a polynomial

(C) If f has a removal singularity at infinity, then f is a constant

(D) If f does not have an essential singularity at infinity, then f is a polynomial

63. If $\sum_{n=-\infty}^{\infty} a_n(z+1)^n$ denote the Laurent series expansion of $f(z) = \sin\left(\frac{z}{z+1}\right)$ about the point $z = -1$, then the value of a_{-3} is equal to :

(A) $-\frac{\sin(1)}{2}$

(B) 0

(C) $\frac{\cos(1)}{6}$

(D) $\cos(1) + \frac{\sin(1)}{2}$

64. Consider the following assertions :

(I) Residue of $z^3 \cos\left(\frac{1}{z^2}\right)$ at ∞ is $\frac{1}{2}$.

(II) $\int_{|z|=1} z^3 \cos\left(\frac{1}{z^2}\right) dz = -\pi i$.

Pick out the correct option ?

(A) (I) is true, but (II) is false
(B) (I) is false, but (II) is correct
(C) Neither (I) nor (II) is true
(D) Both (I) and (II) are true

65. Choosing $x_0 = 1$ and $x_1 = 2$ as the initial approximations in the Secant method, the approximate root of the equation $x^3 - 2x - 5 = 0$ in the second iteration (i.e., x_3) is equal to :

(A) 1.93
(B) 2.05
(C) 2.27
(D) 2.64

66. An iterative scheme given by $x_{n+1} = \frac{1}{5}\left(16 - \frac{12}{x_n}\right)$, $n \in \mathbb{N} \cup \{0\}$ with suitable x_0 will :

(A) Converge to 2
(B) Converge to 1.8
(C) Converge to 1.5
(D) Not converge

67. While solving the system $x - 2y = 1$ and $x + 4y = 4$ by the Gauss-Seidel method, which one of the following is the first iterative solution ?

(A) $x = 0.25$; $y = 0.75$
(B) $x = 0.25$; $y = 1$
(C) $x = 1$; $y = 0.65$
(D) $x = 1$; $y = 0.75$

68. For what values of the constants α, β and γ, the quadrature formula :

$$\int_{-1}^{1} p(x)dx \approx \alpha\, p(-1) + \beta\, p(0) + \gamma p(1)$$

is exact for a polynomial p of degree at most 2 ?

(A) $\alpha = \frac{2}{3}$, $\beta = \frac{4}{3}$ and $\gamma = -\frac{1}{3}$
(B) $\alpha = \frac{1}{3}$, $\beta = \frac{4}{3}$ and $\gamma = \frac{1}{3}$
(C) $\alpha = \frac{1}{3}$, $\beta = \frac{3}{4}$ and $\gamma = \frac{2}{3}$
(D) $\alpha = \frac{4}{3}$, $\beta = -\frac{2}{3}$ and $\gamma = \frac{1}{3}$

69. The smallest degree of the polynomial that interpolates the data (0, 4), (1, 5), (2, 8) and (3, 13) is :

(A) 4
(B) 3
(C) 2
(D) 1

70. The second order Newton's divided difference of the function $f(x) = x^2$ at the points x_0, x_1 and x_2 is :

(A) -1
(B) 1
(C) $\frac{1}{x_1 - x_0}$
(D) $\frac{1}{x_2 - x_1}$

71. A second degree polynomial p satisfies p(0) = 1, p(1) = 4 and p(2) = 15, respectively. If the integral : $\int_0^2 p(x)\,dx$ is to be estimated by applying Trapezoidal rule to this data, then what is the error (i.e., true value-approximate value) in this estimate ?

(A) $\frac{4}{3}$
(B) $\frac{2}{5}$
(C) $-\frac{4}{3}$
(D) $-\frac{7}{6}$

72. For the initial value problem $= \frac{dy}{dx}\, x + y$; y(0) = 1, the value of y(1.1) according to the Euler's method (taking h = 0.1) is :

(A) 0.1
(B) 0.3
(C) 0.7
(D) 1.1

73. Let y(x) be a solution of the differential equation $\frac{dy}{dx} + 2y = \phi(x)$, where $\phi(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$. If y(0) = 0, then what is the value of $y\left(\frac{3}{2}\right)$?

(A) $\frac{e^2+1}{2e^4}$

(B) $\frac{e^2-1}{e^3}$

(C) $\frac{e^2-1}{2e^3}$

(D) $\frac{e^2+1}{2e}$

74. If cos(x) is the integrating factor of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$, then the value of P(x) is :

(A) log(cos x)

(B) $-\tan x$

(C) log(cos x)

(D) cot x

75. On which of the intervals a solution to the following initial value problem :

$$(4-t^2)y' + 2ty = 3t^2 \,;\, y(1) = -3$$

is guaranteed to exist ?

(A) $(-\infty, -2)$

(B) $(2, \infty)$

(C) $(-2, 2)$

(D) $(-\infty, \infty)$

76. Which one of the following options is correct for the solution y(x) of the differential equation $x^2y'' - 5xy' + 9y = 0$ $(x > 0)$?

(A) y(x) is bounded on $\mathbb{R}$.

(B) $y(x) \to 0$ as $x \to \infty$.

(C) If y(1) = 1, then $y(x) \to \infty$ as $x \to 0$.

(D) If y(1) = 1, then $y(x) \to \infty$ as $x \to \infty$.

77. Which one of the following could be the form of a particular solution that could be used to solve the differential equation $y'' + 3y = t^2e^{2t}$ by the method of undetermined coefficients ?

(A) $y_p = e^{2t}(at^2 + bt + c)$ (a, b and c are real numbers)

(B) $y_p = e^{2t}(at^3 + bt^2 + ct + d)$ (a, b, c and d are real numbers)

(C) $y_p = e^{2t}(at^2 + bt)$ (a and b are real numbers)

(D) $y_p = a\,e^{2t}(bt^2 + c)$ (a, b and c are real numbers)

78. The general integral of the linear partial differential equation (PDE) : $p - q = \log(x + y)$ is :

(A) $\phi\left(xy, \frac{y}{z}\right) = 0$

(B) $\phi\,(xy, x^2 + y^2 + z^2) = 0$

(C) $\phi\,(x + y, x\log(x + y) - z) = 0$

(D) $\phi\left(\frac{xy}{z}, \frac{x - y}{z}\right) = 0$

79. The PDE $\frac{\partial^2 u}{\partial x^2} = 25\,\frac{\partial^2 u}{\partial t^2}$ satisfies the condition : $u(0) = 3x$ and $\frac{\partial u}{\partial t}(0) = 3$. The solution of the PDE at the point $(x, t) = (1, 1)$ is :

(A) 2

(B) 3

(C) 5

(D) 6

80. Which one of the following options is true for the PDE :

$$y^3\frac{\partial^2 u}{\partial x^2} - (x^3 - 1)\,\frac{\partial^2 u}{\partial y^2} = 0\ ?$$

(A) The PDE is elliptic in $\mathbb{R}^2$

(B) The PDE is parabolic in $\{(x, y) \in \mathbb{R}^2 : x < 0\}$

(C) The PDE is hyperbolic in $\{(x, y) \in \mathbb{R}^2 : x > 1, y > 0\}$

(D) The PDE is parabolic in $\{(x, y) \in \mathbb{R}^2 : x > 0\}$

SPACE FOR ROUGH WORK

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