JEE-Main-28-01-2025 (Memory Based)
[MORNING SHIFT]
Maths

Question: If $\int_{-\pi/2}^{\frac{\pi}{2}} \frac{96(x^2.cosx)}{1+e^x} dx$ **=** $\mathbf{a}\pi^3 + \beta$ **(where** $\alpha + \beta$ **are positive integers), then** $\alpha + \beta$ **equal to**

Options:

(a) 144

(b) 100

(c) 64

(d) 196

Answer: (b)

$$I = 96 \int_{-\pi/2}^{\pi/2} \frac{x^2 + \cos x}{1+e^x} dx$$

$$I = 96 \int_{-\pi/2}^{\pi/2} \frac{x^2 + \cos x}{1+e^{-x}} dx$$

$$2I = 96 \int_{-\pi/2}^{\pi/2} (x^2 + \cos x) dx$$

$$I = 96\left(\tfrac{x^3}{3} + \sin x\right)_0^{\pi/2} = 96\left(\tfrac{\pi 3}{24} + 1\right)$$

$$= 4\pi^3 + 96$$

$$\alpha + \beta = 100$$

Question: Number of ways to form 5 digit numbers greater than 50000 with the use of digits 0,1,2,3,4,5,6,7 such that sum of first and last digit is not more than 8, equal to

Options:

(a) 5119

(b) 5120

(c) 4607

(d) 4068

Answer: (c)

Available digit 0, 1, 2, 3, 4, 5, 6, 7

Middle three spaces can be filled in

1st place 5, then last place can be 0, 1, 2, 3 as sum is less than or equal to 8

So total no's = 1 × 512 × 4 = 2048

1st place 6, then last place can be 0, 1, 2

So total no. of ways = 1 × 512 × 3 = 1536

1st place 7, then last place can be 0, 1

So total no. of ways = 1 × 512 × 2 = 1024

So total no. of ways = 2048 + 1536 + 1024 = 4608

Excluding 50000, 4608 - 1 = 4607

Question: If $f(x)=\dfrac{2^x}{2^x+\sqrt{2}}, x\in R,$ then $\sum_{k=1}^{81} f\left(\dfrac{k}{82}\right)$ **is equal to**

Options:

(a) $81\sqrt{2}$

(b) 82

(c) $\frac{81}{2}$

(d) 42

Answer: (c)

$$f(x) = \frac{2^x}{2^x+\sqrt{2}} \Rightarrow f(x) + f(1-x) = 1$$

$$s = \sum_{n=1}^{81} f\left(\frac{k}{82}\right)$$

$$s = \sum_{n=1}^{81} f\left(\frac{82-x}{82}\right) = \sum_{n=1}^{81} f\left(1-\frac{4}{82}\right)$$

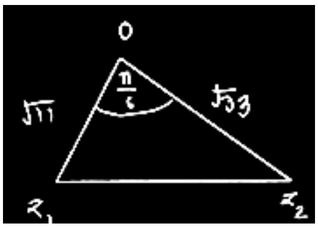
$$2s = \sum_{n=1}^{81} f\left(\frac{k}{82}\right) + f\left(1-\frac{k}{82}\right)$$

$$= \sum_{n=1}^{81} 1 = 81$$

$$s = \frac{81}{2}$$

Question: $z_1 = \sqrt{3} + 2\sqrt{2}i$ & $\sqrt{3}\left|Z_1\right| = \left|Z_2\right|$ **and arg(z_2) =arg(z_1) +** $\frac{\pi}{6}$ **then area, of triangle with vertices z_1, z_2 and origin**

Solution:



$$\text{Area} = \frac{1}{2}\sqrt{11}\,.\,\sqrt{33}\,\sin\frac{\pi}{6}$$

$$= \frac{1}{2}\,.11\sqrt{3}.\,\frac{1}{2}$$

$$= \frac{11\sqrt{3}}{4}$$

Question: The relation R={(x,y)| x, y $\in$ z, x + y = even} then R is

Options:

(a) Equivalence

(b) Reflexive & Transitive but - not Symmetric

(c) Symmetric & Transitive but not reflexive

(d) Reflexive & symmetric but not transitive

Answer: (a)

$$R = \{(x, y) : x + y = \text{even}, x, y \in z\}$$

$$\text{Reflexive as } x + x = \text{even}$$

$$\text{symmetric as } x + y = y + x$$

$$\text{Transitive as } x + y = \text{even } y + z = \text{even}$$

$$x + 2y + z = \text{even}$$

$$x + z = \text{even}$$

$$Equivalence.$$

Question: $\cos\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{33}{65}\right)$ **is equal to**

Options:

(a) 0

(b) 1

(c) $\frac{32}{65}$

(d) $\frac{33}{65}$

Answer: (a)

$$\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{33}{65}\right)$$

$$\cos\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{5}{12} + \sin^{-1}\frac{33}{65}\right)$$

$$\cos\left(\tan^{-1}\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}} + \sin^{-1}\frac{33}{65}\right)$$

$$\cos\left(\tan^{-1}\frac{14\times4}{33} + \cot^{-1}\frac{56}{33}\right)$$

$$\cos\frac{\pi}{2} = 0$$

Question: $\int_0^x tf(t)dt = x^2 f(x) f(2)3, f(6) = ?$

Solution :

$$\int_0^x tf(x)dt = x^2 f(x)$$

$$xf(x) = 2xf(x) + x^2 f'(x)$$

$$\Rightarrow xf'(x) = -f(x)$$

$$\Rightarrow \frac{f'(x)}{f(x)} = \frac{-1}{x}$$

$$\Rightarrow \ln f(x) + \ln x = \ln c$$

$$\Rightarrow xf(x) = c$$

$$2f(2) = 6f(6)$$

$$\Rightarrow f(6) = 1$$

Question: Area of region $\{(x, y) : 0 \le y \le 2|x| + 1, 0 \le y \le x^2 + 1, |x| \le 3\}$

Options:

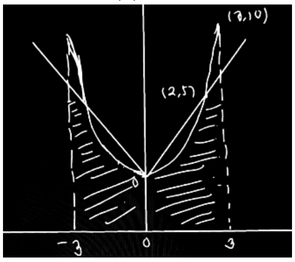
(a) $\frac{17}{3}$

(b) $\frac{32}{3}$

(c) $\frac{64}{3}$

(d) $\frac{80}{3}$

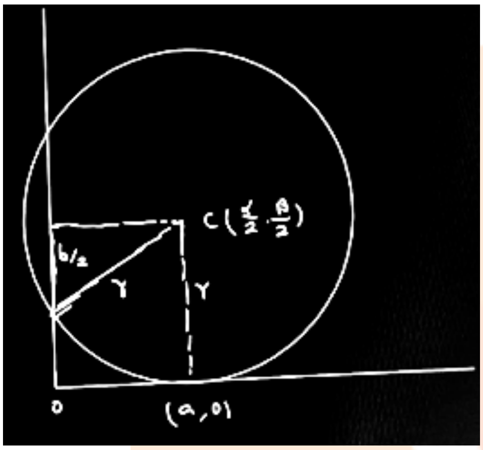
Answer: (c)



$$A = 2\left[\int_0^2 (x^2+1)dx + \int_2^3 (2x+1)dx\right]$$
$$= 2\left[\left(\frac{x^3}{3}+x\right)_0^2 + \left(x^2+x\right)_2^3\right]$$
$$= 2\left(\frac{8}{3}+2+9+3-4-2\right) = 2\left(8+\frac{8}{3}\right)$$
$$= 2 \times \frac{32}{3} = \frac{64}{3}$$

Question: $x^2 - \alpha x + \beta y - \gamma + y^2 = 0$ if touches x axis at a point A(a, 0)& cut intercept of bv. find value of a and b on y axis

Solution :



$$r = \sqrt{\frac{\alpha^2}{4} + \frac{\beta^2}{4} - \gamma} = \frac{\beta}{2}$$
$$\frac{\alpha^2}{4} - \gamma = 0$$
$$\alpha^2 = 4\gamma, \frac{\alpha}{2} = a$$
$$\alpha = 2a$$
$$2\sqrt{\frac{13^2}{4} - \gamma} = b$$
$$\frac{\beta^2}{4} = \gamma = \frac{b^2}{4}$$
$$b^2 = \beta^2 - 4\gamma$$
$$\left(2a, b^2\right) = \left(\alpha, \beta^2 - 4\gamma\right)$$

Question: $x^2 + |2x - 3| - 4 = 0$ has sum of squares of roots

Solution :

$x^2+|2x-3|-4=0$

If $x \geq \frac{3}{2}, x^2+2x-7=0$

$x=\frac{-2 \pm \sqrt{4+28}}{2}=\frac{-24\sqrt{2}}{2}$

$=-1 \pm 2\sqrt{2}$

$x=2\sqrt{2}-1$

If $x<\frac{3}{2}, x^2-2x-1=0$

$x=\frac{2 \pm 2\sqrt{2}}{2}=1+\sqrt{2}$

$x=1-\sqrt{2}$

sum of squares of roots $=\left(2\sqrt{2}-1\right)^2+\left(1-\sqrt{2}\right)^2$

$=12-6\sqrt{2}$

Question: The image of point (4, 4, 3) in the plane $=\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{3}$ **is (α, β, γ) find $\alpha+\beta+\gamma$.**

Solution :

Given line is x = 1 + 2t, y = 2 + t, z = 1 + 3t

Vector joining (4, 4, 3) to the general point on line

(1 + 2t, 2 + t, 1 + 3t) is $\vec{v}$ (- 3 + 2t, - 2 + t, - 2 + 3t)

Orthologous projection $\vec{v}.\vec{d}=0$ d is direction

Vector of the line as $\vec{d}=2i+j+3k$

So (-3 + 2t)2 + (-2 + t) $\cdot$ 1 + (-2 + 3t) $\cdot$ 3 = 0

-14 + 14t = 0 ⇒ t = 1

Point of projection is (3, 3, 4)

Vector joining the projection point (3, 3 4) and (4, 4, 3) - (1, 1, 1)

So image point = (3, 3, 4) + (1, 1, -1) = (2, 2, 5)

Sum of coordinates = 2 + 2 + 5 = 9

Question: $a_0=1$, $a_1=\frac{1}{2}$, $2a_n=5a_{n-1}-3a_{n-2}$ find $\sum_{i=1}^{100} a_n$.

Options:

(a) $3a_{100}+197$

(b) $2a_{100}$ - 1000
(c) $3a_{99}$ - 100
(d) $2a_{99}$ - 100
Answer: (a)

$$a_0 = 1, a_1 = \frac{1}{2}, 2a_n - 3a_{n-1} = 2a_{n-1} - 3a_{n-2}$$

$$\Rightarrow 2a_n - 3a_{n-1} = 2a_1 - 3a_0$$

$$= 1 - 3 = -2$$

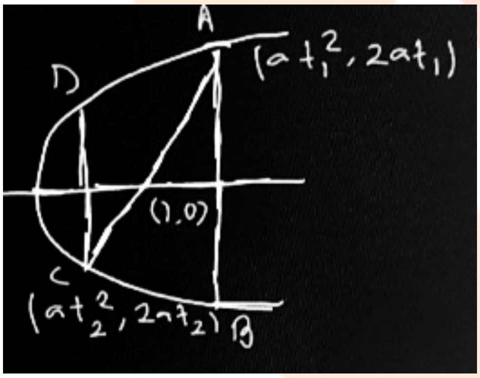
$$2\sum_{n=1}^{100} a_n - 3\sum_{n=1}^{100} a_{n-1} = -200$$

$$2s = 3[a_0 + s - a_{100}] = -200$$

$$2s = 3s - 3 + 3a_{100} = -200$$

$$s = 3a_{100} + 197$$

Question: Vertices of a trapezium lies on a parabola y2 = 4x one of its diagonal passes through (1, 0), and its length is$\frac{25}{4}$. Parallel sides are parallel to y. Find its area of trapezium.
Solution :



$$t_1 t_2 = -1$$

$$\sqrt{\left(t_1^2 + t_2\right)^2 + \left(2t_1 - 2t_2\right)^2} = \frac{25}{4}$$

$$|t_1 - t_2|\sqrt{\left(t_1 + t_2\right)^2 - 4t_1 t_2} = \frac{25}{4}$$

$$\left(t_1 - t_2\right)^2 = \frac{25}{4}$$

$$t_1 - t_2 = \frac{5}{2}$$

$$r_1 + \frac{1}{t_1} = \frac{5}{2}$$

$$t_1 = 2, t_2 = -\frac{1}{2}$$

$$A = \frac{1}{2}(8 + 2) \times \left(4 - \frac{1}{4}\right) = \frac{1}{2} \times 10 \times \frac{15}{4} = \frac{75}{4}$$

Question: A bag has 7 good and 3 defected oranges. 2 oranges are chosen randomly. Find the variance of oranges being defected.
Solution:

7 Good, 3 Defective

$$\begin{array}{c|l} x & f \\ \hline 0 & {}^{7}C_2/{}^{10}C_2 \quad = \frac{7}{15} \\ 1 & {}^{3}C_1 \cdot {}^{7}C_1/{}^{10}C_2 = \frac{7}{15} \\ 2 & {}^{3}C_2/{}^{10}C_2 \quad = \frac{1}{15} \end{array}$$

$$\bar{x} = 0 + \frac{7}{15} + \frac{2}{15} = \frac{9}{15} = \frac{3}{5}$$

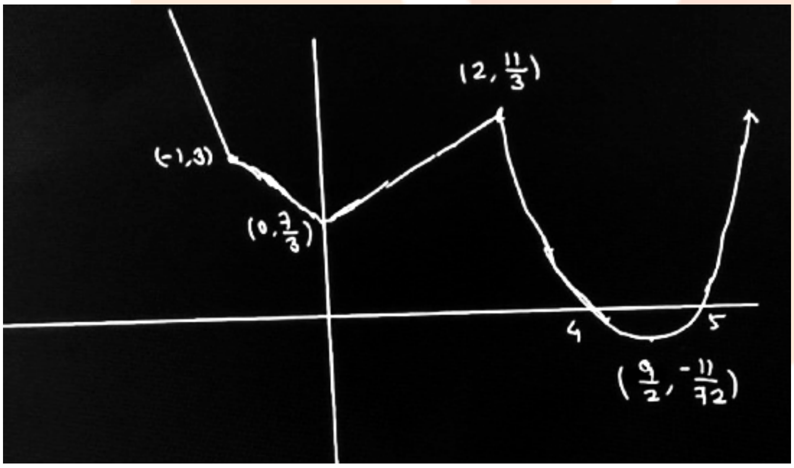
$$Var = \left(0 + \frac{7}{15} + \frac{4}{15}\right) - \left(\frac{3}{5}\right)^2 = \frac{11}{15} - \frac{9}{25}$$

$$= \frac{110-54}{150} = \frac{56}{150} = \frac{28}{75}$$

Question: The sum of all local minimum values of the function

$$\begin{cases} 1-2x & x < -1 \\ \frac{1}{3}(7+2|x|) & -1 \le x \le 2 \\ \frac{11}{18}(x-4)(x-5) & x > 2 \end{cases}$$ is

Solution :



$$\text{Sum of local minimum values} = \frac{7}{3} - \frac{11}{72}$$

$$= \frac{168-11}{72} = \frac{157}{72}$$