MATHEMATICS

Maximum Marks: 80
Time Allowed: Three hours

*(Candidates are allowed **additional 15 minutes** for **only** reading the paper. They must **NOT** start writing during this time.)*

This Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B** **OR** **Section C.***

Section A**: Internal choice has been provided in **two questions of two marks each, two questions of four marks each and two questions of six marks each.

Section B:** Internal choice has been provided in **one question of two marks and one question of four marks.

Section C:** Internal choice has been provided in **one question of two marks and one question of four marks.

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A - 65 MARKS

Question 1

In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

(i) A relation R on $\{1, 2, 3\}$ is given by $R = \{(1,1), (2,2), (1,2), (3,3), (2,3)\}$. **[1]**

Then the relation R is:

(a) Reflexive.

(b) Symmetric.

(c) Transitive.

(d) Symmetric and Transitive.

(ii) If A is a square matrix of order 3, then |2A| is equal to: [1]

(a) 2|A|

(b) 4|A|

(c) 8|A|

(d) 6|A|

(iii) If the following function is continuous at $x = 2$ then the value of k will be: [1]

$$f(x) = \begin{cases} 2x+1, & if\ x < 2 \\ k, & if\ x = 2 \\ 3x-1, & if\ x > 2 \end{cases}$$

(a) 2

(b) 3

(c) 5

(d) -1

(iv) An edge of a variable cube is increasing at the rate of 10 cm/sec. How fast will the volume of the cube increase if the edge is 5 cm long? [1]

(a) 75 cm^3/sec

(b) 750 cm^3/sec

(c) 7500 cm^3/sec

(d) 1250 cm^3/sec

(v) Let $f(x) = x^3$ be a function with domain $\{0, 1, 2, 3\}$. Then domain of f^{-1} is: [1]

(a) $\{3, 2, 1, 0\}$

(b) $\{0,-1,-2,-3\}$

(c) $\{0, 1, 8, 27\}$

(d) $\{0,-1,-8,-27\}$

(vi) For the curve $y^2 = 2x^3 - 7$, the slope of the normal at (2, 3) is: [1]

(a) 4

(b) $\frac{1}{4}$

(c) -4

(d) $\frac{-1}{4}$

(vii) Evaluate: $\int \frac{x}{x^2+1}\, dx$ [1]

(a) $2\log(x^2 + 1) + c$

(b) $\frac{1}{2}\log(x^2 + 1) + c$

(c) $e^{x^2+1} + c$

(d) $\log x + \frac{x^2}{2} + c$

(viii) The derivative of $\log x$ with respect to $\frac{1}{x}$ is: [1]

(a) $\frac{1}{x}$

(b) $\frac{-1}{x^3}$

(c) $\frac{-1}{x}$

(d) $-x$

(ix) The interval in which the function $f(x) = 5 + 36x - 3x^2$ increases will be: [1]

(a) $(-\infty, 6)$

(b) $(6, \infty)$

(c) $(-6, 6)$

(d) $(0, -6)$

(x) Evaluate: $\int_{-1}^{1} x^{17} \cos^4 x\, dx$ [1]

(a) ∞

(b) 1

(c) -1

(d) 0

(xi) Solve the differential equation: $\frac{dy}{dx} = cosec\, y$ [1]

(xii) For what value of k the matrix $\begin{bmatrix} 0 & k \\ -6 & 0 \end{bmatrix}$ is a skew symmetric matrix? [1]

(xiii) Evaluate: $\int_0^1 |2x+1| dx$ **[1]**

(xiv) Evaluate: $\int \frac{1+\cos x}{\sin^2 x} dx$ **[1]**

(xv) A bag contains 19 tickets, numbered from 1 to 19. Two tickets are drawn randomly in succession with replacement. Find the probability that both the tickets drawn are even numbers. **[1]**

Question 2 **[2]**

(i) If $f(x) = [4-(x-7)^3]^{\frac{1}{5}}$ is a real invertible function, then find $f^{-1}(x)$

OR

(ii) Let A = R – {2} and B = R – {1}. If f: A →B is a function defined by $f(x) = \frac{x-1}{x-2}$ then show that f is a one – one and an onto function.

Question 3 **[2]**

Evaluate the following determinant without expanding.

$$\begin{vmatrix} 5 & 5 & 5 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix}$$

Question 4 **[2]**

The probability of the event A occurring is $\frac{1}{3}$ and of the event B occurring is $\frac{1}{2}$. If A and B are independent events, then find the probability of neither A nor B occurring.

Question 5 **[2]**

Solve for x :

$5\tan^{-1} x + 3\cot^{-1} x = 2\pi$

Question 6 **[2]**

(i) Evaluate: $\int \cos^{-1}(\sin x)\, dx$

OR

(ii) If $\int x^5 \cos(x^6)dx = k \sin(x^6) + C$, find the value of k.

Question 7 **[4]**

If $\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\left(\frac{23}{36}\right)$ then prove that $24x^2 - 23x - 12 = 0$

Question 8 **[4]**

If $y = e^{ax} \cos bx$, then prove that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = 0$

Question 9 **[4]**

(i) In a company, 15% of the employees are graduates and 85% of the employees are non-graduates. As per the annual report of the company, 80% of the graduate employees and 10% of the non-graduate employees are in the Administrative positions. Find the probability that an employee selected at random from those working in administrative positions will be a graduate.

OR

(ii) A problem in Mathematics is given to three students A, B and C. Their chances of solving the problem are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability that

(a) exactly two students will solve the problem.

(b) at least two of them will solve the problem.

Question 10 **[4]**

(i) Solve the differential equation:

$(1 + y^2)\, dx = (tan^{-1}y - x)dy$

OR

(ii) Solve the differential equation:

$(x^2 - y^2)dx + 2xydy = 0$

Question 11 **[6]**

Use matrix method to solve the following system of equations.

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$

$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

Question 12 **[6]**

(i) Prove that the semi-vertical angle of the right circular cone of given volume and least curved area is $\cot^{-1}\sqrt{2}$

OR

(ii) A running track of 440m is to be laid out enclosing a football field. The football field is in the shape of a rectangle with a semi-circle at each end. If the area of the rectangular portion is to be maximum, then find the length of its sides. Also calculate the area of the football field.

Question 13 **[6]**

(i) Evaluate: $\int \frac{3e^{2x}-2e^{x}}{e^{2x}+2e^{x}-8}\,dx$

OR

(ii) Evaluate: $\int \frac{2}{(1-x)(1+x^2)}\,dx$

Question 14 **[6]**

A box contains 30 fruits, out of which 10 are rotten. Two fruits are selected at random one by one without replacement from the box. Find the probability distribution of the number of unspoiled fruits. Also find the mean of the probability distribution.

SECTION B - 15 MARKS

Question 15 **[5]**

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) If $|\vec{a}| = 3, |\vec{b}| = \frac{\sqrt{2}}{3}$ *and* $\vec{a} \times \vec{b}$ is a unit vector then the angle between $\vec{a}$ and $\vec{b}$ will be:

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

(ii) The distance of the point $2\hat{\imath} + \hat{\jmath} - \hat{k}$ from the plane $\vec{r} \cdot (\hat{\imath} - 2\hat{\jmath} + 4\hat{k}) = 9$ will be:

(a) 13

(b) $\frac{13}{\sqrt{21}}$

(c) 21

(d) $\frac{21}{\sqrt{13}}$

(iii) Find the area of the parallelogram whose diagonals are $\hat{\imath} - 3\hat{\jmath} + \hat{k}$ *and* $\hat{\imath} + \hat{\jmath} + \hat{k}$.

(iv) Write the equation of the plane passing through the point (2, 4, 6) and making equal intercepts on the coordinate axes.

(v) If the two vectors $3\hat{\imath} + \alpha\hat{\jmath} + \hat{k}$ *and* $2\,\hat{\imath} - \hat{\jmath} + 8\hat{k}$ are perpendicular to each other, then find the value of α.

Question 16 **[2]**

(i) If A $(1, 2, -3)$ and B $(-1, -2, 1)$ are the end points of a vector $\overrightarrow{AB}$ then find the unit vector in the direction of $\overrightarrow{AB}$.

OR

(ii) If $\hat{a}$ is unit vector and $(2\vec{x} - 3\,\hat{a}) \cdot (2\vec{x} + 3\,\hat{a}) = 91$, find the value of $|\vec{x}|$

Question 17 [4]

(i) Find the equation of the plane passing through the point (1, 1, −1) and perpendicular to the planes $x + 2y + 3z = 7$ *and* $2x - 3y + 4z = 0$

OR

(ii) A line passes through the point (2, −1, 3) and is perpendicular to the lines $\vec{r} = (\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(2\hat{\imath} - 2\hat{\jmath} + \hat{k})$ and $\vec{r} = (2\hat{\imath} - \hat{\jmath} - 3\hat{k}) + \mu(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$. Obtain its equation.

Question 18 [4]

Find the area of the region bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$

SECTION C - 15 MARKS

Question 19 [5]

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) If the demand function is given by $p = 1500 - 2x - x^2$ then find the marginal revenue when $x = 10$

(a) 1160

(b) 1600

(c) 1100

(d) 1200

(ii) If the two regression coefficients are 0·8 and 0·2, then the value of coefficient of correlation r will be:

(a) ± 0·4

(b) ± 0·16

(c) 0·4

(d) 0·16

(iii) Out of the two regression lines $x + 2y - 5 = 0$ *and* $2x + 3y = 8$, find the line of regression of y on x.

(iv) The cost function $C(x) = 3x^2 - 6x + 5$. Find the average cost when $x = 2$

(v) The fixed cost of a product is ₹30,000 and its variable cost per unit is ₹800. If the demand function is $p(x) = 4500 - 100x$, find the break-even values.

Question 20 **[2]**

(i) The total cost function for x units is given by $C(x) = \sqrt{6x+5} + 2500$. Show that the marginal cost decreases as the output x increases.

OR

(ii) The average revenue function is given by $AR = 25 - \frac{x}{4}$

Find total revenue function and marginal revenue function.

Question 21 **[4]**

Solve the following Linear Programming Problem graphically.

Maximise $Z = 5x + 2y$ subject to:

$x - 2y \leq 2,$

$3x + 2y \leq 12,$

$-3x + 2y \leq 3,$

$x \geq 0, y \geq 0$

Question 22 **[4]**

(i) The following table shows the Mean, the Standard Deviation and the coefficient of correlation of two variables x and y.

Series	x	y
Mean	8	6
Standard deviation	12	4
Coefficient of correlation	0·6	

Calculate:

(a) the regression coefficient b_{xy} and b_{yx}

(b) the probable value of y when $x = 20$

OR

(ii) An analyst analysed 102 trips of a travel company. He studied the relation between travel expenses (y) and the duration (x) of these trips. He found that the relation between x and y was linear. Given the following data, find the regression equation of y on x.

$\Sigma x = 510,\ \Sigma y = 7140,\ \Sigma x^2 = 4150,\ \Sigma y^2 = 740200,\ \Sigma xy = 54900$