Text Book for A.P.

Intermediate

First Year

Physics

F

Board of Intermediate Education

Andhra Pradesh

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Sri. V. Rama Krishna, I.R.S.
Secretary

PREFACE

"I hear and I forget - I see and I remember - I do and I understand - I think and I learn"

The Board of Intermediate Education, Andhra Pradesh, Vijayawada made an attempt to provide work books for the thirst time to the Intermediate students with relevant and authentic material with an aim to engage them in academic activity and to motivate them for self learning and self assessment. These work books are tailored based on the concepts of "learning by doing" and "activity oriented approach" to sharpen the students in four core skills of learning – Understanding, Interpretation, Analysis and Application

The endeavour is to provide ample scope to the students to understand the underlying concepts in each topic. The workbook enables the student to practice more and acquire the skills to apply the learned concept in any related context with critical and creative thinking. The inner motive is that the student should shift from the existing rote learning mechanism to the conceptual learning mechanism of the core concepts

I am sure that these compendia are perfect tools in the hands of the students to face not only the Intermediate Public Examinations but also the other competitive Examinations

My due appreciation to all the course writers who put in all their efforts in bringing out these work books in the desired modus.

--- V. Rama Krishna, I.R.S.
Secretary,
B.I.E., A.P., Vijayawada.

PHYSICS - WORKBOOK - II Year

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PHYSICS – I

2. UNITS AND MEASUREMENTS

*** END ***

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2. UNITS AND MEASUREMENTS

Physical Quantity: Any quantity which can be measured directly (or) indirectly (or) in terms of which the laws of physics can be expressed is called physical quantity. There are two types of physical quantities
1) Fundamental quantities 2) Derived quantities
Fundamental Quantities: Physical quantities which cannot be expressed interms of any other physical quantities are called fundamental physical quantities.
Ex. length, mass, time, temperature etc..
Derived Quantities: Physical Quantities which are derived from fundamental quantities are called derived quantities.
Ex. Area, density, force etc...
Unit of physical quantity: A unit of measurement of a physical quantity is the standard reference of the same physical quantity which is used for comparison of the given physical quantity.
Fundamental unit : The unit used to measure the fundamental quantity is called fundamental unit.
Ex: metre for length, kilogram for mass etc..
Derived unit : The unit used to measure the derived quantity is called derived unit.
Ex: m^2 for area, gm $cm^{-}3$ for density etc...
Note : The numerical value obtained on measuring a physical quantity is inversely proportional to the magnitude of the unit chosen.

$n \alpha \frac{1}{U} \Rightarrow nU = \text{constant}$

$\Rightarrow \boldsymbol{n_1 U_1 = n_2 U_2}$

Where
n_1 and n_2 are the numerical values and
U_1 and U_2 are the units of same physical quantity in different systems

According to SI system there are three categories of physical quantities.
1)fundamental quantities
2)supplementary quantities and
3)derived quantities

S .No.	Physical Quantity	SI unit	Symbol
01	Length	Meter	M
02	Mass	Kilogram	Kg
03	Time	Second	S
04	Thermo dynamic temperature	Kelvin	K
05	Light intensity (luminous intensity)	Candela	Cd
06	Electric current	Ampere	A
07	Amount of substance	Mole	Mol
	Supplementary Quantities		
01	Plane angle	Radian	Rad
02	Solid Angle	Steridian	Sr

Measurement of length

- Light year is an astronomical unit of length. One light year is the distance travelled by light in one year in vacuum . This unit is used in astronomy.

Light year $\times =$**9.46** 10^{15} ***m***

- Par sec is also an astronomical unit of length

Parsec $=$ **3.26 light years** $\times =$**30.84** 10^{15} ***m***

- The distance between the sun and the earth is called the astronomical unit (A. U)

1 A.U. = 1.496×10^{11} m

- One Mile = 1.6 km
- Sea – mile is a unit of distance. It is the length of the arc that subtends an angle of one minute at the center of the earth.

1 Angstrom(A^o **)=10^{-10} m=10^{-8} cm**

- nanometre(nm) $= 10^{-9}$ 10 A^0
- fermi$=$ 10^{-15} m micron $= 10^{-6}$ m
- X-ray unit $= 10^{-13}$ m
- Bohr radius $\times =$0.5 10^{-10} m

Measurement of mass:

Atomic mass unit (a.m.u) =$\times =$1.67 10^{-27} kg

One Quintal = 100 kg
Metric ton = 1000 kg

Measurement of time :

A physical quantity having the same unit in all the systems of measurement is the time.

Some important conversions:

1 kmph = 5/18 ms	1KWH	=	36 X 10^5
1 Joule = 10^7 erg	1 HP	=	746 W
1 Newton = 10^5 dyne	1 degree	=	0.017 ral
1 Calorie = 4.18 J	1 Cal/gm	=	4180 J/kg
1 ev = 1.6 x 10^{-19}J	1 Kg Wt	=	9.8 N
1 gm/cm^3 = 1000 kg/m3	1 tesla	=	10^4 gauss
1 Lit = 1000 cm^3 = 10^{-3}m^3	1 A/m	=	4 x 10^{-3} oersted
1 Born = 10^{-28}m^2	1 Weber	=	10^8 maxwell

Some physical constants and their values:

1 atm pressure = pressure exerted by 76cm of Hg column

$= 1.013\ 10^5\ Pa$

Avagadro number (N) = 6.023×10^{23}

10mu = 1.67×10^{-27}kg = 931.5mcv

Permitivity of free space = 8.854×10^{-12} Fm^{-1} (or) c^2/nm^2

Permeability of free space (μ_0) $= 4\pi \times 10^{-7}$ Hm-1

Joule's constant (J) = 4.186 J cal^{-1}

Planck's constant(h) = 6.62×10^{-34} JS

Rydberg's constant(R) = $1.0974\ 10 \times 7\ m^{-1}$

Boltzmann's constant(K^B)= $1.38\ 10 \times^{-23}$ JK^{-1}

Stefan's constant (σ) $\times$ $= 5.67\ 10^{-8}$ w m^{-2} K^{-4}

Universal gas constant(R) = 8.314 *mol* $^{-1}$ K^{-1}

= 1.98 *cal* mol^{-1} K^{-1}

Wien's constant (b) = 2.93×10^{-3} *metre Kelvin*

Accuracy : The closeness of a measured value with the true value is called "accuracy"

Precision : The closeness of a measured values of repeated measurement of a physical quantity by an instrument is called "precision".

Error : The difference between the observed value and the true value is called "error".

The result of every measurement by any measuring instrument contains some uncertainty. This uncertainty in measurement is called error.

Types of errors : Errors are broadly classified into 3 types

(i) Systematic errors
(ii) Random errors
(iii) Gross Errors

Systematic Errors : The errors due to a definite cause and which follow a particular rule are called systematic errors. Ex. Least count error, instrumental error, theoretical error, parallel error.

Random Errors : The random errors can arise due to random and unpredictable fluctuations in experimental conditions.

Ex: Error due to line voltage changes, back lash errors are due to screw and nut.

<u>Gross Errors</u> : The cause for gross errors are improper recording, neglecting the sources of the error, reading the instrument incorrectly, sheer carelessness
<u>Ex</u>: In a tangent galvanometer arranged as wrongly

<u>True Value</u> : In the measurement of a physical quantity the arithmetic mean of all readings which is found to be very close to the most accurate reading is to be taken as True value of the quantities.
If $a_1, a_2, a_3, \ldots\ldots\ldots\ldots a_n$ are readings then

True value = $a_{mean} = \frac{1}{n} \sum_{i=1}^{n} a_i = \frac{a_1+a_2+\cdots\ldots\ldots+a_n}{n}$

<u>Absolute Error</u> : The magnitude of the difference between the true value of the measured physical quantity and the value of individual measurement is called absolute error of the measurement. This is denoted by $|\Delta a|$
Absolute error =|True value - measured values|
$|\Delta a_1| = |\Delta a_2| = |a_{mean} - a_2|, \ldots\ldots\ldots\ldots\Delta a_i = |a_{mean} - a_1|$

The absolute error $|\Delta a|$ is always positive.

<u>Mean absolute error</u>: The arithmetic mean of all the absolute errors is taken as the mean absolute error of the physical quantity. It is represented by Δa_{mean}

$$\Delta a_{mean} = \frac{|\Delta a_1|+|\Delta a_2|+\cdots\ldots\ldots+|\Delta a_n|}{n}$$

$$\Delta a_{mean} = \sum_{i=1}^{n} \frac{|\Delta a_i|}{n}$$

The mean absolute error is always positive.
<u>Relative error</u>: The relative error is the ratio of the mean absolute error Δa_{mean} to the mean value of the quantity measured.
Relative error = $\frac{\Delta a_{mean}}{a\ mean}$

Percentage error: When the relative error is expressed in percentage (denoted by %) it is called the percentage error of (a)

Percentage error (δa) = [$\frac{\Delta a_{mean}}{a\ mean}$] x 100

<u>Combination of errors</u> :

a) Error of a sum (or) a difference : (RULE- 1) : When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.
   (i) Error due to addition : If Z = A + B
   Z = ΔA +ΔB (Max possible error)

$$Z + \Delta Z = (A + B) \pm (\Delta A + \Delta B)$$

Relative error : $\frac{\Delta A + \Delta B}{A - B}$, Percentage of error : $\frac{\Delta A + \Delta B}{A - B}$ x 100

(ii) Error due to subtraction : If Z = A - B

Z = ΔA + ΔB (Max possible error)

$Z + \Delta Z = (A - B) \pm (\Delta A + \Delta B)$

Relative error : $\frac{\Delta A + \Delta B}{A - B}$

Percentage of error : $\frac{\Delta A + \Delta B}{A - B}$ x 100

Note : in addition or subtraction, absolute error is same. In subtraction the percentage error increases

(b) **<u>Error of a product or a quotient</u>** : (Rule -2) : Where two quantities are multiplied or divided, the relative error is the result is the sum of the relative errors in the multipliers.

(i) **<u>Error due to Multiplication</u>** : If Z = AB then $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$

$\frac{\Delta Z}{Z}$ is called fractional error or relative error.

Percentage error $\frac{\Delta Z}{Z}$ X 100 = [$\frac{\Delta A}{A}$ X 100] + [$\frac{\Delta B}{B}$ X 100]

percentage error equal to the sum of individual percentage errors.

ii) <u>Error due to division</u>: if $Z = \frac{A}{B}$

Maximum possible relative error = $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$

Max. percentage error in division= $\frac{\Delta A}{A}$ X 100 + $\frac{\Delta B}{B}$ X 100

(c)Error due to Power**:** If $Z = A^n$; $\frac{\Delta Z}{Z} = n \cdot \frac{\Delta A}{A}$

(d) Error – General form : If $Z = \frac{A^P B^Q}{C^R}$ then maximum fractional error in Z is

$$\frac{\Delta Z}{Z} = P \frac{\Delta A}{A} + q\frac{\Delta B}{B} + r \frac{\Delta C}{C}$$

Maximum Percentage error in Z is $\frac{\Delta Z}{Z}$ x 100 = P $\frac{\Delta A}{A}$ x 100 + q$\frac{\Delta B}{B}$ x 100+r $\frac{\Delta C}{C}$ x 100

<u>Significant figures</u> : The reported result of measurement is a number that includes all digits in the number that are known reliably plus the first digit that is uncertain. "The reliable digits plus the first uncertain digit are known as significant digits or significant figures".

Ex : Time period of a simple pendulum is 1.62 second. The digit 1 and 6 are reliable while the digit 2 is uncertain. The measured value has three significant figures.

Rules for determining the number of significant figures:

1) All the non-zero digits are significant.
2) All the non-zero digits in a given number are significant without any regard to the location of the decimal point if any.
   Ex: 1845.2 (or) 184.52 (or) 18.452 (or) 1.8452 (or) 18452 all have the same number of significant digits, i.e. 5.
3) If the number is less than one, all the zeros to the right of the decimal point but to the left of first non-zero digit are not significant.
   Ex: 0.002308 the underlined zeroes are not significant.
4) All zeros occurring between two non zero digits are significant without any regard to the location of decimal point if any.
   Ex: 106008 (or) 106.008 or 1.06008 has also got six significant digits
5) The terminal (or) trailing zeros in a number without a decimal point are not significant.
   Ex : 123m = 12300 cm = 123000 mm has three significant figures
6) The terminal (or) trailing zeros in a number without a decimal point are significant.
   Ex. 3.500 (or) 0.06900 have four significant figures.
7) For a number greater than 1 without any decimals, the trailing zeroes are not significant.
   Ex. 2030 has 3 significant digits.

Rounding off numbers: The result of computation with approximate numbers, which contain more than one uncertain digit, should be rounded off.

Rules : i) The preceding digit is raised by 1 if the immediate insignificant digit to be dropped is more than 5.

Ex : (1) A number 2.746 rounded off to three significant figures is 2. 75

(2) 4728 is rounded off to three significant figures as 4730.

ii) The preceding digit is to be left unchanged if the immediate insignificant digit to be dropped is less than 5.

Ex: (i) A number 2.743 rounded off to three significant figure is 2.74

(ii) A number 4723 is rounded off to three significant figures as 4720.

iii) If the immediate insignificant digit to be dropped is 5 then there will be two different cases

a) If the preceding digit is even then it is to be unchanged and 5 is dropped

Ex . A number 2.745 is rounded off to three significant figures as 2.74

(ii) A number 4.7253 is rounded off to three significant figures as 4.72

b) If the preceding digit is odd, it is to be raised by 1

Ex: (i) A number 2.735 is rounded off to three significant figures as 2.74

(iii) **A number** 4.7153 is rounded off to three significant figures as 2.72.

Rules for arithmetic operations with significant figures : -

(1) In multiplication or division, the final result should retain only as many significant figures as are there in the original number with the least number of significant figures.

Ex: 1.2 x 2.54 x 3.26 = 9.93648 . But final answer is 9.9

(2) Density should be reported to three significant figures

$$\text{Density} = \frac{4.237\ gms}{2.51\ cm^3} = 1.69\ \text{gm cm}^{-3}$$

(3) If speed of light = 3.00 x 10^8 m/s then significant figures -3 .

If one year = 365.25 days (or) 3.1557 x 10^7 sec. Then significant figures = 5.

If the light year = 9.47 x 10^{15} then significant figures – 3.

In addition or subtraction the final result should retain only that many decimal places as are there in the number with the least decimal places.

Ex. (1) 2.2 + 4.08 + 3.12 + 6.38 = 15.78. But final answer is 15.8

(2) 436.32 gm + 227.2 gm + 0.301 gm = 663.821gm. But final answer – 663.8 gms

Dimensions of physical quantities :

Dimensions: Dimensions of a physical quantity are the powers to which the fundamental quantities are to be raised to represent that quantity.

Ex : (1) Velocity = $L^1 T^{-1}$ Here dimension of length ⇒ 1 ; dimension of time ⇒(1)

(2) Force = $M^1 L^1 T^{-2}$ Here dimension of mass → 1

Dimension of length → 1

Dimension of time →(-2)

Dimensional Formula : An expression showing the powers to which the fundamental quantities are to be raised to represent the derived quantity is called dimensional formula of that quantity.

In general the dimensional formula of a quantity can be written as [$M^x L^y T^z$] Here x,y,z are dimensions.

Dimensional Equations : An equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the physical quantity.

Ex : Velocity (v) = $M^0 L^1 T^{-1}$

Force (F) = $M^1 L^1 T^{-2}$

Uses of Dimensional Equations :

(1) To convert units from one system to another system
(2) To check the validity of given physical equations
(3) To derive a relations between various physical quantities .

Limits of Dimensional Method :

(1) The proportionality constant in an equation cannot be obtained by dimensional method.
(2) The equations involving trigonometrical or exponential junction cannot be derived.
(3) The equation containing more than three physical quantities cannot be derived
(4) If the formula of a physical quantity is represented by the sum of some physical quantities dimensional method cannot be used to derive that formula.

Dimensional Quantities : Dimensionless quantities are those which do not have dimensions but have a fixed value.

(i) **Dimensionless quantities without units** : -
Ex. Pure numbers, angle, trigonometric functions ,logarthemic functions. etc.

(ii) **Dimensional quantities with units** :
Ex. Angular displacement, radian Joule's constant etc.

Dimensional Constants : The physical quantities which have dimensions and have a fixed value are called dimensional constants.

Ex. Gravitational constants (G), plank constants (h), universal gas constant (R) velocity of light in vacuum (c) etc.

ONE WORD QUESTIONS :

1. One shake is equal to ___________sec.
2. The S. I unit Luminous Intensity (light intensity) is _________
3. Young's modulus of steel is 1.9 x 10^{11} N /m^2 when expressed in CGS units of dync / cm^2 it will be ________
4. One atomic mass unit (a. m . u) is equal to ______kg.

5. If the unit of mass and velocity are increased by two times then the unit of momentum will be increased by __________.
6. How many electron volts are equal to one KWH (kilo watt hour) ____________
7. If y = $\frac{A}{B}$, the maximum percentage error in the measurement of y will be _____
8. The respective number of significant figures for the number 23.023, 0.0003 and 21 x 10^{-3} are _____
9. The diameter of a sphere is 3.54 m. Its volume with due regard to significant figures ___________m^3.
10. A body travels uniformly a distance of (13.8 ± 0.2)m in a time (4.0 ± 0.3) s, then the velocity of the body is ___________m/sec
11. The dimensional formula for the coefficient of kinematic viscosity is ________
12. The dimensional formula of "calorie" is _______________

TRUE OR FALSE QUESTIONS

1. One kilowatt hour (1 KWH) is equal to 3.6 x 10^{6} joule. -----→ T/ F
2. The physical quantity time is having the same unit in all the systems of measurement . _________ T/ F
3. If the absolute error in two physical quantities A and B are ΔA and ΔB respectively. Then the absolute error in the value of (A – B) is (ΔA – ΔB) ______T / F
4. One calorie is equal to 4.184 joule _____ T / F
5. If Y = a – b, the maximum percentage of error in the measurement of Y will be as [$\frac{\Delta a+ \Delta b}{a-b}$] ___________T / F
6. The number of significant figures in 0. 09600 is five. __________ T/ F
7. The value of $\frac{9.27}{47}$ with due regard to significant figures is 0.23.
8. When energy is released in erg the number of significant figure is four. If it is expressed in joule the no of significant figures will become four . ______T/F
9. When 58.986 is rounded off to four significant figures then it becomes 58.98
10. If F, M, T are the force, mass and time then F M^{-1} T^{2} denotes time _____T/ F

MATCH THE FOLLOWING

(I) Match the column I and column II

Column – I	Column – II
a) Distance between earth and stars	1) Micron
b) Inter atomic distance in a solid	2) angstrom
C) Size of the nucleus	3) Light year
D) Wave length of infrared laser	4) fermi

(A) a-3, b-1, c-4, d-2 (B) a-2, b-3, c-4, d-1

(C) a-3, b-2, c-4, d-1 (D) a-2, b-1, c-3, d-4

(II) Match the column I and column II

Column – I	**Column – II**
a) Zero error	1) Due to improper recording
b) Backlash error	2) Due to personal error
C) Parallax error	3) Due to (+ve) or (-ve)
D) Gross error	4) Due to loose fitting of screw or nut

(A) a-3, b-4, c-2, d-1 (B) a-4, b-3, c-1, d-2

(C) a-1, b-2, c-3, d-4 (D) a-2, b-1, c-4, d-3

(III) Match the given units with physical quantities.

Column – I	Column – II
a) Boltzmann constant	1) Jouls - sec
b) Universal gas constant	2) Nm^{-1}
C) Force constant (or) Spring Constant	3) Joule, mol^{-1} k^{-1}
D) Plank's constant	4) Joule k^{-1}

(A) a-4, b-3, c-1, d-2 (B) a-2, b-1, c-4, d-3

(C) a-4, b-3, c-2, d-1 (D) a-1, b-2, c-3, d-4

(IV) Match the following by using significant figures

Column – I (numbers)	**Column – II (number of significant figures)**
a) 0.12345	1) 1
b) 0.1210	2) 3
c) $\frac{47.23}{2.3}$	3) 2
d) 3×10^8	4) 5

(A) a-4, b-2, c-3, d-1 (B) a-1, b-2, c-3, d-4

(C) a-2, b-4, c-1, d-3 (D) a-3, b-1, c-4, d-2

(V) Match the given dimensional formulas with physical quantities.

Column – I (physical quantities)	**Column – II (Dimensional formula)**
a) Angular momentum	1) $M^{-1} L^3 T^{-2}$
b) Surface tension	2) $M L^2 T^{-1}$

C) Gravitational constant 3) $M L^2 T^0$
D) Moment of inertia 4) $M L^0 T^{-2}$

(A) a-2, b-3, c-1, d-4 (B) a-2, b-4, c-1, d-3
(C) a-3, b-2, c-4, d-1 (D) a-3, b-4, c-2, d-1

(VI) Match the Column-I and Column-II

Column – I (physical quantities)	Column – II (Dimensional formula)
a) Permeability (μ)	1) $M^{-1} L^{-3} T^4 A^2$
b) Permitivity (ε)	2) $M L^2 T^{-2} A^{-2}$
C) Magnetic Flux ($\emptyset$)	3) $M L^2 A^{-1}$
D) Magnetic Induction (B)	4) $M L^0 T^{-2} A^{-1}$

(A) a-1, b-2, c-4, d-3 (B) a-4, b-2, c-1, d-3
(C) a-3, b-4, c-1, d-2 (D) a-2, b-1, c-4, d-3

MULTIPLE CHOICE QUESTIONS :

1. Electron volt is the unit of ________________
   a) Power b) potential difference c) charge d) energy
2. The reliability of a measurement depends on ___________
   a) Precision b) accuracy c) systematic error d) random error
3. The one which is not the unit of length is ___________
   a) Angstrom Unit b) Micron c) par- sec d) steridian
4. If μ is the permeability and ε is the permittivity then $\frac{1}{\sqrt{\mu\varepsilon}}$ is equal to ____
   a) Speed of sound in vacuum b) speed of light in vacuum
   c)Speed of sound in medium d) speed of light in medium
5. The relation between the unit "μ" and the magnitude " K", of a physical quantity is __________
   a) $\mu \propto k$ b) $\mu \propto \frac{1}{K}$ c) $\mu \propto K^2$ d) $\mu \propto \sqrt{K}$
6. The time period of seconds pendulum is measured repeatedly for three times by two stop watches A, B . If the readings are as follows then

Stop Watch (A)	Stop Watch (B)
2.01 sec	2.56 sec
2.10 sec	2.55 sec
1.98 sec	2.57 sec

(A) A is more accurate but B is more precise
(B) B is more accurate but A is more precise
(C) A, B are equally precise
(D) A, B are equally accurate

7. The diameter of a wire as measured by a screw guage was found to be 1.002 cm ,

1.00 cm , 1.006 cm, the absolute error in the first reading is _____

a) 0.001 cm b) 0.004 cm c) 0.006 cm d) 0.003cm

8. The least count of a stop watch is ( ) sec; the time of 20 oscillations of a pendulum is measured to be 25 sec. The maximum percentage error in this measurement is ______

a) 8% b) 1 % c) 0.8% d) 16%

9. When the number 0.046508 is reduced to 4 (four) significant figures, then its becomes ____________

a) 0.0465 b) 4650.8 x 10^{-5} c) 4.651 x 10^{-2} d) 4.650 x 10^{-2}

10. The numbers 3.945 and 3.935 on rounding off to three significant figures will give ____ & ____

a) 3.95 and 3.94 b) 3.94 and 3.93 c) 4.651 x 10^{-2} d) 4.650 x 10^{-2}

11.The sum of the numbers 436.32, 227.2 amd 0.501 in appropriate significant figures is _________

a) 663.821 b) 664 c) 663.8 d) 663.82

12. In the given following the dimensionless error is _________

a) systematic error b) gross error c) random error d) relative error

13. The physical quantity that has no dimension is _____________

a) Angular velocity B) Linear Momentum c) Angular Momentum d) Strain

14. The fundamental physical quantities that have same dimension in the dimensional formula of torque and angular momentum are ____________

a) Mass, Time b) Time, Length c) Mass, Length d) None of those

15. Dimension of capacity x Resistance (C x R) is _____________

a) Frequency b) energy C) Time period d) Current

16. If m is mass, Q is charge and B is magnetic induction then [$\frac{m}{BQ}$] has the same dimensions as ________

a) Frequency b) $\frac{1}{Frequency}$ c) Velocity d) Acceleration

17. The equation which is dimensionally correct among the following is __________

a) V = μ + $\frac{1}{2}$ at b) V = μt + at c) S = μt + at^3 d) t =S + μV

18. The velocity of sound in air (V), pressure (P) and density of air (d) are related as V α $P^x D^y$. The value of x and y are ____________

a) 1, $\frac{1}{2}$ b) - $\frac{1}{2}$, - $\frac{1}{2}$ c) $\frac{1}{2}$, $\frac{1}{2}$ d) $\frac{1}{2}$, - $\frac{1}{2}$

19. The velocity of an object varies with time as V = At ^{3} + Bt ^{2} +ct + D , then the dimensions of C are ________

a) LT^{-1} b) LT^{-2} c) LT^{-3} d) LT^{-4}

20. The velocity of the waves on the surface of water is proportional to , where wave length, p – density and g- acceleration due to gravity, which of the following relation is correct

a) $\propto = \beta \neq \Upsilon$ b) $\beta = \Upsilon \neq \propto$ c) $\Upsilon = \propto \neq \beta$ d) $\propto \neq \beta \neq \Upsilon$

ANSWERS

ONE WORD QUESTIONS :

1. 10^{-8}	2. Candela	3. 1.9×10^{12}	4. 1.66×10^{-27}
5. Four	6. 2.25×10^{25}	7. $[\frac{\Delta A}{A} + \frac{\Delta B}{B}] \times 100$	8. 5, 1, 2
9. 19. 5	10. [3. 45 ± 0.3]	11. $M^0 L^2 T^{-1}$	12. $ML^2 T^{-2}$

TRUE OR FALSE QUESTIONS :

1. TRUE	2. TRUE	3. FALSE	4. TRUE	5. TRUE	6. FALSE
7. TRUE	8. TRUE	9. FALSE	10. FALSE.		

MATCH THE FOLLOWING QUESTIONS :

1. C	2. A	3. C	4. A	5. B	6. D

2.

MULTIPLE CHOICE QUESTIONS :

1. D	2. B	3. D.	4. D	5. B	6. A	7. A	8. C	9. C	10. D
11. B	12. D	13. D	14. C	15. C	16. B	17. A	18. D	19. B	20. C

LEVEL – 1 QUESTIONS

1. The radius of a sphere is measured as (10±0.02%) cm the error in the measurement of its volume is ____________ **Ans : 2.52 cc**
2. If measure two quantities as A = 1.0 m ± 0.2m, B = 2.0m ± 0.2m , then should report the correct value for $\sqrt{AB}$ as ___________**Ans : 1.4m ± 0.2m**
3. If the ratio of fundamental units in two systems are 2 : 3, the ratio of force in these two systems is ______________ **Ans : 1 : 1**
4. In an experiment the values of refractive indices of glass where found to be 1.54, 1.53, 1.44, 1.54, 1.56 and 1. 45 in successive measurement . Find
   a) Mean absolute error b) relative error c) percentage of relative error
   Ans : (a)0.04 (b)0.03 (c)3%
5. The pressure on a square plate is measured by measuring the force on the plate and the length of the sides of the plate. If the maximum error in measurement of force and length are respectively 4% and 2% then the maximum error in measurement of pressure is __________
   Ans : 8 %
6. When a current of (2.5 ± 0.5) ampere flows through a wire, it develops a potential difference of (20±1) volt. The resistance of the wire is ________________. **Ans : (8 ±2)Ω**

7. The velocity of a body is given by V= $At^2 + Bt+C$. If V and t are expressed in S. I. units. What are the units of A, B C ?

Ans : $(\frac{m}{sec^3}, \frac{m}{sec^2}, \frac{m}{sec})$

8. The dimensional formula for the product of two physical quantities P and Q is [$M L^2 T^{-2}$]the dimensional formula of P/ Q is [MT^{-2}]then P and Q respectively are _____________.

Ans : Force & displacement

9. The physical quantity which has the dimensional formula of [$\frac{energy}{mass\ x\ length}$] is _______________

Ans : Acceleration

10. Find the dimensional formula of $\frac{a}{b}$ in the equation P $\frac{a-ct^2}{b\ x}$ where P = pressure, x = displacement, and t = time **Ans : MT^{-2}**

LEVEL – II QUESTIONS

1. The distance covered by a body in time (5.0 ± 0.6) sec is (4.0 ± 0.4)m. Calculate the speed of the body and also determine the percentage error in the speed. **Ans : V= (8.0 ±1.04) m/3, 13%)**
2. Let (ϵ_0) denote the dimensional formula of permittivity of vaccum. If M is mass, L is length, T is time, and A is electric current.

   Ans : $M^{-1} L^{-3} T^4 A^2$

3. Velocity of wave is directly proportional to modulus of elasticity (E) and density "d" of medium. Find the expression of V using dimensional analysis. **Ans : V = $\sqrt{\frac{E}{d}}$**
4. If the determination of Young's modulus of a given wire the force, length, radius and extension in the wire are measured as (100 ± 0.01) N, (1.25±0.002) m (0.001 ± 0.00002) m and (1.01±0.00002) m respectively. Find the percentage error in the measurement of Young Modulus.

   Ans : 4.37

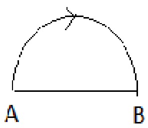
5. If C is the velocity of light, h is the Plank's constant and G is gravitational constant are taken as fundamental quantities, then find the dimensional formula for mass.

   Ans : $h^{½} G^{-1/2} C^{1/2}$

3. MOTION IN A STRAIGHT LINE

SYNOPSIS

- Mechanics is the branch of physics that deals with the motion of objects and the forces that cause the motion to change.
- Newtonian Mechanics proposed by Issac Newton. It gives correct results
- When speeds involved are far less than the speed of light in vacuum and masses involved are much greater than the mass of an atom. Mass and time are absolute.
- Relativistic Mechanics proposed by Einstein. It gives correct results when speeds involved are comparable with the speed of light in vacuum and masses are much greater than the mass of an atom. Mass and time are not absolute. They are relative.
- Kinematics is branch of mechanics which deals with the study of motion without going into the cause of motion.
- Dynamics is branch of mechanics which is concerned about the causes that cause motion.
- The study of motion of objects along a straight line is known as **Rectilinear motion** (or) motion in a straight line. 

- The study of motion of objects along a curved line is known as Curvilinear motion. 

- In Kinematics, we shall treat the objects in motion as point objects.
- Rest and motion are relative, not absolute.
- **Rest :** If a body does not change its position as time passes, it is at rest.
- **Motion :** It is change in position of an object with time .
- Motion is a combined property of the object under study and the observer. There is no meaning of rest (or) motion without the viewer.

  Ex : A book is placed on the table

  (a) It is at rest if it is viewed from the room.

  (b) It is moving if it is viewed from the moon.

Frame of reference : To specify position of a object, we need a reference point and a set of axes. It is convenient to choose a rectangular coordinate system consisting of three mutually perpendicular axes, labelled X, Y and Z axes. The point of intersection of these three axes is called origin and serves as the reference point. To measure time, we position a clock in this system. This coordinate system along with a clock constitutes a frame of reference.

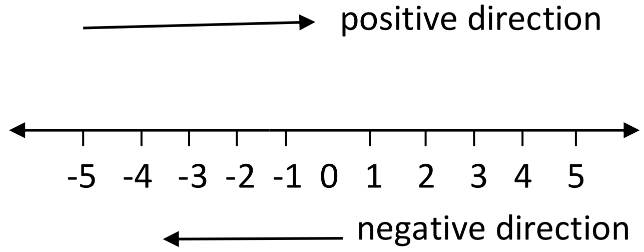
- There is no rule or restriction on the choice of frame. We can choose a frame of reference according to our convenience to describe the situation under study.

- There are different frames of references.
  Some of them are
  (1)Cartesian (rectangular) coordinate system.
  (2)Polar coordinate system
  (3)Cyclindrical coordinate system
  (4) Spherical coordinate system.

- If just one coordinate is sufficient to specify the position of a particle completely, then its motion is called one dimensional motion (ID) (or) motion in a straight line.
- If two/three coordinates are required to specify the position of particle completely, then its motion is called the dimensional (2D)/three dimensional (3D) motion.
- **Translatory motion :** Motion of an object from one position to another position as a whole.
- **Path (or) trajectory :** It is the line joining the successive positions of a particle in motion.
- **Distance (or) path length :** The actual length of the path covered by a moving particle in between two points.
- It is a scalar. SI unit : m (metre), Dimensional [$M^0 L T^0$]
- There are infinite distances between any two points.
- **Displacement :** The shortest distance covered by a body in between two points.
  Displacement = $\Delta x = x_2 - x_1$ = change in position.

It is a vector quantity SI = m [metre] Dimensional formula : [$M^0 L T^0$]

- In one – dimensional there are only two directions



Distance ≥ |displacement |

- If a particle starts from a point and reaches the same point at the end of hs journey, then displacement is zero, but distance covered is not zero.
- Motion along a straight line, in the same direction,
  Distance = |displacement |
- Speed (v) : The distance covered by a body in unit time.
- Speed (v) = $\frac{distance}{time} = \frac{s}{t}$

  It is a scalar. SI unit : ms^{-1} D. F : [$M^0 L T^{-1}$]

  It is always positive.

(a)Average speed (V_{av}) : The total distance travelled by a particle divided by the total time interval.

$$V_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{S}{t_2 - t_1}$$

(b)Instantaneous speed (v) or speed : The speed of a moving particle at an instant of time.

$$V = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{d s}{d t}$$

It is magnitude of velocity.

(c) Uniform speed : If an object travels equal distances in equal intervals of time.

$$V = \frac{S}{t}$$

(d) Non- uniform speed : If an object travels equal distances in unequal time intervals.

- Speedometer of a vehicle gives instantaneous speed.
- Average speed (V_{av}) = $\frac{S_1+S_2+S_3+\cdots\ldots\ldots}{t_1+t_2+t_3+\cdots\ldots\ldots}$

- <u>When bodies moving with different distances with different speeds :</u>
- Average speed (V_{av}) = $\frac{S_1+S_2+S_3+\cdots\ldots\ldots}{\frac{S_1}{V_1}+\frac{S_2}{V_2}+\frac{S_3}{V_3}+\ldots\ldots}$

If $S_1 = S_2 = S$, then $V_{av} = \frac{2V_1V_2}{V_1+V_2}$ (or) $\frac{2}{V_{av}} = \frac{1}{V_1} + \frac{1}{V_2}$

- **<u>When bodies moving with different speeds in different time intervals :</u>**

Average speed (V_{av}) = $\frac{V_1t_1+V_2t_2+V_3t_3+\cdots\ldots\ldots}{t_1+t_2+t_3+\cdots\ldots\ldots}$

If $t_1 = t_2 = t$, then $V_{av} = \frac{V_1+V_2}{2}$

- **Uniform motion :** An object is said to be in uniform motion in a straight line, if its displacements is equal in equal intervals of time. Otherwise, the motion is said to be non-uniform.
- **Velocity (V) :** The displacement covered by a body in unit time (or) rate of change of displacement with respect to time.

  Velocity $= \frac{\text{change in displacement}}{\text{time}}$ ⇨ $V = \frac{\vec{S}}{t}$.

  It is a vector quantity . SI unit ms^{-1} D. F = $[M^0 L T^{-1}]$

(a)Average velocity (V_{av}) : The total displacement divided by the total time interval.

$$\overline{\mathbf{V}}_{av} = \frac{\mu_2-\mu_1}{t_2-t} = \frac{\Delta\,\bar{x}}{\Delta\,t}$$

(b)**Instantaneous velocity :** The velocity at an instant of time.

$$V_{av} = \lim_{\Delta t \to 0} \frac{\Delta\,\bar{x}}{\Delta\,t} = \frac{d\,\bar{x}}{d\,t}$$

- It is the rate of change of position with respect to time at that instant.
- Slope of the tangent to the position –time graph at that instant gives instantaneous velocity.

(c)Uniform velocity : When a particle covers equal displacements in equal intervals of time, however small these intervals may be.

(d) Non-uniform (or) variable velocity : When a body covers equal displacements in unequal intervals of time.

- The direction of the velocity is simply same as the direction along which an object is moving.
- Distance, displacement, speed, velocity ⟶ relative.

Change in velocity : $\Delta \vec{v} = \vec{V}_f - \vec{V}_i$

- Velocity of a particle changes, when its magnitude, (or) direction (or) both changed.
- Motion in a straight line, in the same direction,
  $|\text{average velocity}|$ = average speed.
- Average speed ≥ $|\text{average velocity}|$ over a given time interval.
- For a moving body, average speed > 0
  Average velocity > (or) < (or) = 0
- In uniform motion, average velocity = instantaneous velocity.
- Area under the velocity – time curve represents the displacement over a given interval of time.
  Average velocity tells us how fast an object has been moving over a given total time interval.
- Instantaneous velocity tells us how fast an object moves at an instant of time.
- If after motion the body comes back to its initial position,
- Average velocity = 0, Average speed > 0 and finite.

- Acceleration ($\vec{a}$) = The change in velocity in unit time (or) rate of change of velocity with respect to time.

  $$\vec{a} = \frac{\Delta V}{t} = \frac{\text{change in velocity}}{\text{time}}$$

  It is vector quantity. Its direction is same as that of change in velocity.

  SI unit : ms^{-2} , D. F = [$M^0 L T^{-2}$]

  (a) **Average acceleration ($\vec{a}_{av}$) :** The change in velocity divided by the time interval.

  $$\vec{a}_{av} = \frac{\bar{V}_2 - \bar{V}_1}{t_2 - t_1} = \frac{\Delta \bar{V}}{\Delta t}$$

(b) **Instantaneous acceleration ($\vec{a}$) :** The acceleration at an instant of time.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$$

$$\vec{a} = \frac{d\bar{v}}{dt} = \frac{d\bar{v}}{dt} \times \frac{dx}{dt} = V\frac{dv}{dx}$$

(c) **Uniform (or) constant acceleration :** If the velocity of the body changes by equal amounts in equal intervals of time.

$$a = \frac{v-u}{t} \rightarrow V = u + at$$

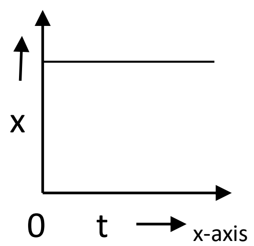
(d) **Non-uniform acceleration :** If the velocity of the body changes by unequal amounts in equal intervals of time.

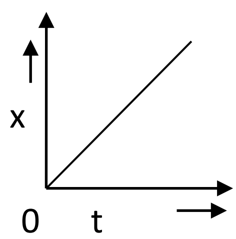
(e) **Deceleration (or) Retardation :** If the speed of a particle decreases with time, we say that it is decelerating (or) retardation. The acceleration of the body is opposite to that of the velocity and then the body decelerates.

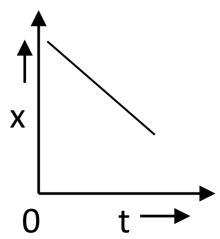
- The slope of tangent to the velocity- time curve at that instant gives instantaneous acceleration.
- Acceleration and velocity cannot change values abruptly at an instant. Changes are always continuous.
- Area between the acceleration –time graph gives change in velocity.

Velocity V	**Acceleration A**	**Motion**
+ -	+ -	Speeding up Speeding up
+ -	- +	Slowing down Slowing down
- (or) +	0	Constant velocity
0	- (or) +	Speeding up from rest
0	0	Remaining at rest

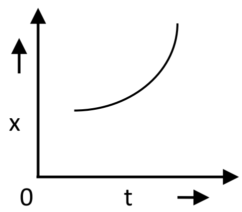
Position – time graphs :

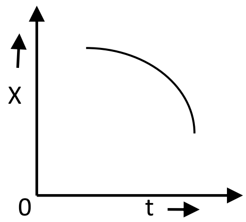


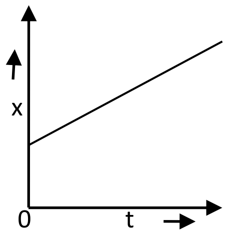




(1) Stationary object (Rest) 2) uniform positive velocity 3) uniform negative velocity

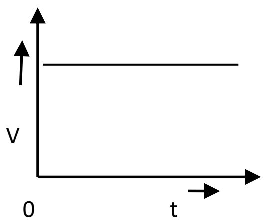




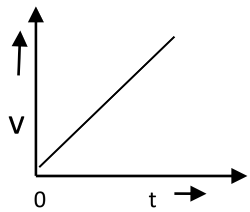


(4) positive acceleration (5) negative acceleration (6)uniform motion (a=0)

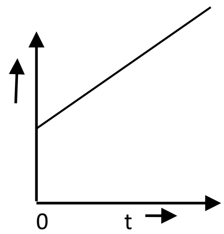
Velocity – time graphs



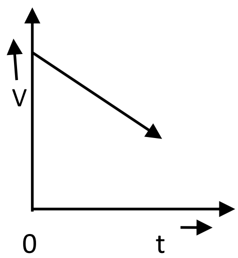
(1)constant velocity (a=0)



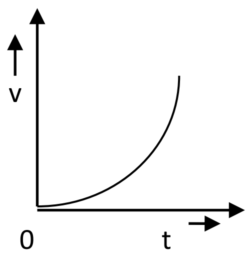
2) constant acceleration
With initial velocity =0



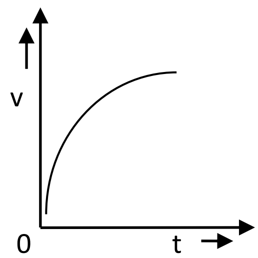
3. Constant acceleration with
with initial velocity



4) constant retardation ($V_0 \neq 0$)



5) Increasing acceleration



6) Decreasing acceleration

Kinematic equations for uniformly accelerated motion :

1. $v = V_0 + at$
2. $x = V_0 t + \frac{1}{2} at^2$
3. $V^2 = V_0^2 + 2ax$
4. $S_n = V_0 + a\left(n - \frac{1}{2}\right)$
5. $X = \left(\frac{V + V_0}{2}\right) t = \vec{v}\, t$
6. $\vec{v} = \frac{V + V_0}{2}$ = average velocity

If the position of the object at time t = 0 is 0.

If the particle starts at $x = x_0$, x is above equations is replaced by $(x – x_0)$

Free fall : An object falling under the influence of gravity g = 9.8 ms^{-2} neglecting air resistance.

It is a case of motion with uniform acceleration.

Free falling velocity	Vertically projected body
Initial velocity (V_0) = 0	Initial velocity (V_0) = V_0
Acceleration (a) = g	Acceleration (a) = - g
Distance travelled (x) = height = h	At maximum height , (h) V = 0
(i) V = gt	(i) $V = V_0 - gt$
(ii) $x = \frac{1}{2} g t^2$	(iii) $X = V_0 t - \frac{1}{2} g t^2$
(ii) $V^2 = 2 gh$	(iii) $V^2 = V_0^2 - 2gx$
(iv) $S_n = g (n - \frac{1}{2})$	(v) $S_n = V_0 - g (n - \frac{1}{2})$

For a body projected vertically upwards with intial velocity (V_0) :

(f) Maximum height h = $\frac{V_0^2}{2\,g}$

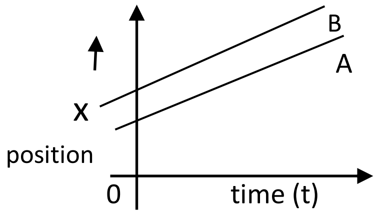
(g) Time of ascent = Time of descent = $\frac{V_0}{g}$ (neglect air resistance)

(h) Total time of flight to come back to the point of projection = $\frac{2\,V_0}{g}$

(i) Velocity of fall at the point of projection = V_0

(j) Velocity of a body dropped from height h, V = $\sqrt{2gh}$

Relative velocity : Let us take the two objects A and B moving with average velocity V_A and V_B.



If x_A (0) and x_B (0) are positions of objects A and B respectively at t= 0, their position x_A (t) and x_B(t) at time t are given by

$x_A (t) = x_A (0) + v_A t$

$x_B (t) = x_B (0) + v_B t$

Then the displacement from object A to object B is given by

$X_{BA} (t) = x_B (t) - x_A (t)$

$= [x_B(0) - x_A(0)] + (V_B - V_A)\,t$

By this we say velocity of object B relative to object A is $V_B - V_A$.

$\therefore V_{BA} = V_B - V_A$.

Similarly velocity of object A relative to object B is

$\therefore V_{AB} = V_A - V_B$.

In relative velocity we take
In the same direction difference and in opposite direction addition of two individual velocities.

When v_A and v_B are inclined to each other at angle

$V_{AB} = \sqrt{v_A^2 + v_B^2 - 2\,v_A v_B\,cos\theta}$

If V_{AB} makes angle β with V_A then

$$\tan\beta = \frac{V_{B\sin\theta}}{V_A - V_B\cos\theta}$$

FILL UP THE BLANKS

1. Rest and motion are _____________ (absolute / relative)
2. We can tell the speed of the car at any instant by looking at its _______ (odometer / speedometer)
3. To study the motion of objects without going into the causes of motion is __________ (kinematics/ Dynamics)
4. The length of the path transverse by an object is called ________. (Distance/ Displacement)
5. The velocity – time graph is a straight line parallel to time axis, when object is in ___________ (rest/ motion).
6. The rate of change of displacement with time is known as _____.
7. Area under velocity –time curve over a given time interval gives ______.
8. The slope of the tangent drawn on a position-time graph at any instant is equal to _____________.

9. If a body covers equal distances in equal intervals of time however small they may be, the change in speed in time 't' is equal to _______.
10. A particle moves in a circle of radius R. In half of the period of revolution its displacement is ___________ and distance covered is __________.
11. The slope of tangent to the velocity time curve at that instant gives ________.
12. A hunter aims a gun at a monkey hanging from a tree some distance away. The monkey drops from the branch at the moment he fires the gun hoping to avoid the bullet. Can monkey escapes from the bullet. _____ (yes /no).
13. An object accelerated downward under the influence of force of gravity. The motion of object is said to be ______(uniform motion / uniform acceleration).
14. Free fall of an object in vacuum is a case of motion with ___(uniform motion/uniform acceleration).
15. An object in free fall undergoes an increase in _____(speed/ acceleration).
16. If gravity between the sun and the earth suddenly vanished, earth would continue moving in ____________ (a curve / a straight line).

II. TRUE / FALSE

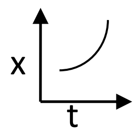
1. A particle in one-dimensional motion with zero speed, may have non-zero velocity.
2. The magnitude of displacement may (or) may not be equal to the path length traversed by an object.
3. In general, magnitude of velocity is equal (or) greater than the speed.
4. Straight line motion involves change in direction, then the magnitude of average velocity is equal to the average speed.
5. When the magnitude of displacement is equal to distance only when a body moves in a straight line.
6. When a body projected vertically upwards at its highest point, its velocity and acceleration are zero.
7. Displacement is a scalar, distance is a vector.
8. In Kinematics, big bodies are also considered as particles, but in dynamics, the mass of objects can be considered.

9. Two balls of different masses are thrown vertically upwards with the same speed. They pass through the point of projection in their downward motion with the same speed.
10. For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.
11. If the observer and the object are moving at velocities $\vec{V}_1$ and $\vec{V}_2$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\vec{V}_2 - \vec{V}_1$.
12. A particle in one –dimensional motion with zero speed at an instant may have non-zero acceleration at that instant.
13. A particle in one-dimensional motion with constant speed must have zero acceleration.
14. A particle in one-dimensional motion with positive value of acceleration must be speeding up.
15. A railway carriage moving without jerks between two stations can be considered as a point object .

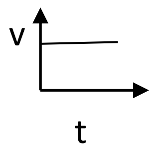
III. MATCHINGS

1. 1. Velocity — A. m/s
   2. SI unit of speed — B. Time dependent
   3. Distance — C. path independent
   4. Displacement — D. path dependent

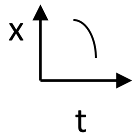
2. 1. Stationary object — A.



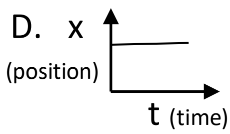
   2. uniform motion — B.



   3. Accelerated body — C.



   4. Decelerated body — D.

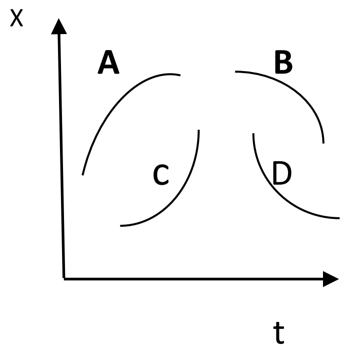


3.

Physical Quantity	Dimensional Formula
1. Distance	A. [$M^0 L T^{-1}$]
2. Speed	B. [$M^0 L T^{-2}$]
3. Acceleration	C. [$M^0 L T^0$]
4. Force	D. [$M L T^{-2}$]

4.Figure shows the position –time graph of particles moving along a straight line. Match the entries of column –I with the entries of Column – II.

Column – I	Column – II
1.The particle A is	P. Accelerating
2.The particle B is	Q. Decelerating
3.The particle C is	R. Speeding up
4.The particle D is	S. Slowing down



5. A balloon rises up (initial velocity is zero) with constant net acceleration of 10 m/s^2. After 2 seconds a body drops from the balloon after further 2 seconds match the column – I with column –II (g = 10 ms $^{-2}$).

Column – I	Column – II
1. Height of the body from the ground	P. Zero
2. Speed of the body	Q. 10 SI units
3. Displacement of the body in 2 sec After it drops from the balloon	R. 40 SI unit
4. Acceleration of the body	S. 20 SI unit

IV. VERY SHORT ANSWER QUESTIONS

1. The states of rest and motion are relative. Explain.
2. Can the velocity of an object be in a direction other than the direction of acceleration of the object ? If so, give example.
3. How is average velocity different from instantaneous velocity ?
4. When the magnitude of average velocity is equal to the average speed ?
5. Can a particle have a constant velocity and varying speed ?

6. Can an object accelerate (a) if its velocity is constant ? (b) if its speed is constant ?
7. Can an object have a constant speed even though the velocity is changing ? If so give example ?
8. Give any two examples of a case where the velocity of an object is zero, but its acceleration is not zero ?
9. A vehicle travels half the distance with speed V_1 and the other half within speed V_2. What is the average speed ?
10. A bird holds a fruit in its beak and flies parallel to the ground. It lets go of the fruit at some height. Describe the trajectory of the fruit as it falls to the ground as seen by (a) the bird (b) a person on the ground.

IV. MULTIPLE CHOICE QUESTIONS :

1. A particle starts from origin and moves along x-axis to a point 25m and to -25m. Then the ratio of distance to displacement is

   a) 1 : 3　　b) 1 : 2　　c) 2 : 1　　d) 3 : 1

2. A carom board (40 cm x 40 cm) has the queen at the centre. When the queen is hit by the striker, it moves to the front edge, rebounds and goes into the hole behind the striking line. The displacement of the queen is

   a) 40 cm　　b) $20\sqrt{2}$ cm　　c) 20 cm　　d) $40\sqrt{2}$ cm

3. A train is 100 m long and is moving with a speed of 72 km ph. The time taken by the train to cross a bridge of length 0.5km is

   a) 25 S　　b) 35 S　　c) 30 S　　d) 28 S

4. A car travels from the place A to B with a speed V_1. During its journey from B to A its speed is V2. Then the average speed in complete journey is

   A) $\frac{2V_1V_2}{V_1+V_2}$　　b) $\frac{V_1+V_2}{2}$　　c) $\frac{2\,(V_1+V_2)}{V_1V_2}$　　d) $\sqrt{\frac{V_1+V_2}{2}}$

5. A body is moving along a straight line path with constant velocity. At an instant of time the distance travelled by it is S and its displacement is D, then

   a) D < S　　b) D > S　　c) D = S　　d) D ≤ S

6. If distance covered by a particle is zero, what can you say about its displacement ?

   a) It may (or) may not be zero　　b) It cannot be zero

   c)It is negative　　d) It must be zero

7. The slope of velocity- time graph of motion with uniform velocity is equal to
   a) acceleration b) displacement c) zero d) none of these
8. What determines the nature of the path followed by the particle ?
   a) Speed b) Velocity c) acceleration d) both (b) and (c)
9. A bus starts moving with acceleration 2 m/s^2. A cyclist 96m behind the bus, starts simultaneously towards the bus at 20 m/s. After what time will he be able to overtake the bus ?
   a) 4 S b) 8 S c) 18 S d) 16 S
10. A bullet fired into a wooden block loses half of its velocity after penetrating 40 cm. It comes to rest after penetrating a further distance of
   a) $\frac{22}{3}$ cm b) $\frac{40}{3}$ cm c) $\frac{20}{3}$ cm d) $\frac{22}{5}$ cm
11. A motor car is going due north at a speed of 50 km/h . It makes a 90^0 left turn without changing the speed. The change in the velocity of the car is about
   a) 50 kmph towards west b) 70 km/h towards south west
   c) 70 km/h towards north west d) zero
12. A person travelling on a straight line moves with a uniform velocity V_1, for some time and with uniform velocity V_2 for the next equal time. The average velocity V is given by
   a) $V = \frac{V_1 + V_2}{2}$ b) $V = \sqrt{V_1 V_2}$ c) $\frac{2}{V} = \frac{1}{V_1} + \frac{1}{V_2}$ d) $\frac{1}{V} = \frac{1}{V_1} + \frac{1}{V_2}$
13. A stone is released from an elevator going up with a acceleration a. The acceleration of the stone after the release is
   a) a upward b) (g-a) upward
   c) (g – a) downward d) g downward
14. A particle experiences constant acceleration for 20 S after starting from rest. If it travels a distance S_1, in the 10 seconds and distance S_2 in the next 10 seconds, then
   a) $S_2 = S_1$ b) $S_2 = 2S_1$ c) $S_2 = 3S_1$ d) $S_2 = 4 S_1$
15. The distance –time graph of a particle at time t makes angle 45^0 with time axis. After one second, it makes angle 60^0 with the time axis. What is the acceleration of the particle ?
   a) $\sqrt{3}$ - 1 b) $\sqrt{3}$ + 1 c) $\sqrt{3}$ d) 1

16. The dependence of velocity of a body with time is given by the equation $V = 20 + 0.1\ t^2$. The body is in

a) uniform retardation b) uniform acceleration

c) non-uniform acceleration d) zero acceleration

17. A particle starting with certain initial velocity and uniform acceleration covers a distance of 12m in first 3 seconds and a distance of 30m in next 3 seconds. The initial – velocity of the particle is

a) 3 ms^{-1} b) 2.5 ms^{-1} c) 2 ms^{-1} d) 1 ms^{-1}

18. A train of 150m length is going towards North direction at a speed of 10 ms^{-1}. A parrot flies at a speed of 5 ms^{-1} towards south direction parallel to the railway track. The time taken by the parrot to cross the train is equal to

a) 12 S b0 8 S c) 15 s d) 10 s

19. A ship A is moving westwards with a speed of 10 km h^{-1} and a ship B 100km south of A, is moving northwards with a speed of 10 km h^{-1}. The time after which the distance between them becomes shortest is

a) 5h b) $5\sqrt{2}$ h c) $10\sqrt{2}$ h d) 10 h

20. Velocity – time curve for a body projected vertically upwards is

a) parabola b) ellipse c) hyperbola d) straight line

21. A rocket is fired upward from the earth's surface such that it creates an acceleration of 19.6 ms^{-2}. If after 5s, its engine is switched off, the maximum height of the rocket from earth's surface would be

a) 980 m b) 735m c) 490m d) 245m

22. A man throws balls with same speed vertically upwards one after the other at an interval of 2 sec. What should be the speed of throw so that more than two balls are in air at any time ?

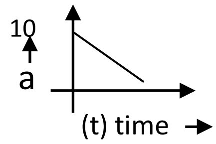
a) only with speed 19.6 ms^{-1} b) more than 19.6 ms^{-1}

c) at least 9.8 ms^{-1} d) any speed less than 19.6 ms^{-1}

23. A stone is dropped into a well in which the level of water is h below the top of the well. If V is velocity of sound, the time T after which the splash is heard is

a) $T = \frac{2h}{V}$ b) $T = \sqrt{\frac{2h}{g}} + \frac{h}{V}$ c) $T = \sqrt{\frac{2h}{v}} + \frac{h}{g}$ d) $T = \sqrt{\frac{h}{2g}} + \frac{2h}{V}$

24. A body starts from rest at time t = 0, the acceleration –time graph is shown in figure . The maximum velocity attained by the body will be



a) 110 ms^{-1} b) 55 ms^{-1} c) 65 ms^{-1} d) 550 ms^{-1}

25. A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to $v(x) = b x^{-2n}$ where b and n are constants and x is the position of the particle. The acceleration of the particle as function of x is given by

a) $-2nb^2 x^{-4n-1}$ b) $-2b^2 x^{-2n+1}$ c) $-2nb^2 x^{-4n+1}$ d) $-2nb^2 x^{-2n-1}$

26. A jet cruising at a speed of 1000 km/h ejects hot air in the opposite direction. If the speed of hot air with respect to jet is 800 km/h, than find its speed with respect to ground.

a) 1800 km/h b) 800 km/h c) 200 km/h d) zero

VI. MORE THAN ONE CORRECT ANSWERS :

(1) Consider the motion of the tip of the minute hand of a clock having a length R. In one hour

a) The displacement is zero b) the distance covered is 2π R

b) The average speed is zero d) the average velocity is zero.

(2)A particle moves along x-axis as $x = \mu(t - 2s) + a(t - 2s)^2$

a) The initial velocity of the particle is μ

b) The acceleration of the particle is a

c) The acceleration of the particle is 2a

d) At t = 2 sec , particle is at the origin.

3) An object may have

a) varying speed without having varying velocity

b) varying velocity without having varying speed

c) non zero acceleration without having varying velocity

d) non- zero acceleration without having varying speed.

4) **Mark the incorrect statements :**

a) the magnitude of the velocity of a particle is equal to its speed.

b) the magnitude of the average velocity in an interval is equal to its average speed in that interval

c) It is possible to have a situation in which the speed of the particle is never zero but the average speed in an interval is zero.

d) It is possible to have a situation in which the speed of the particle is always zero but the average speed in an interval is not zero.

5) The accelerations of a particle as seen from two frames S_1 and S_2 have equal magnitude 4 m/s^2. Mark the incorrect statements.

(a) The frames must be of rest with respect to each other

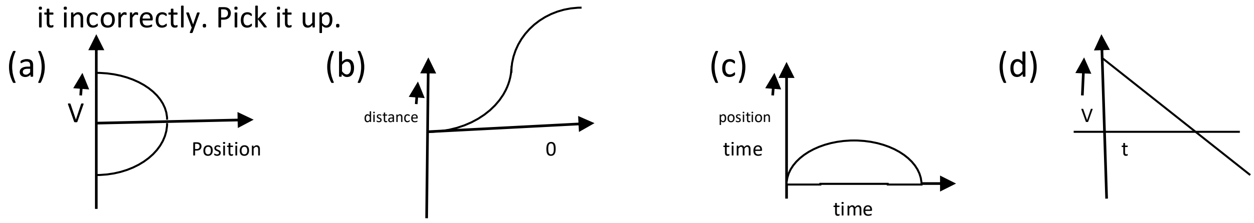
(b) The frames may be moving with respect to but neither should be accelerated with respect to the other.

(c) The acceleration of S_2 with respect to S_1 may either be zero (or) 8 m/s^2

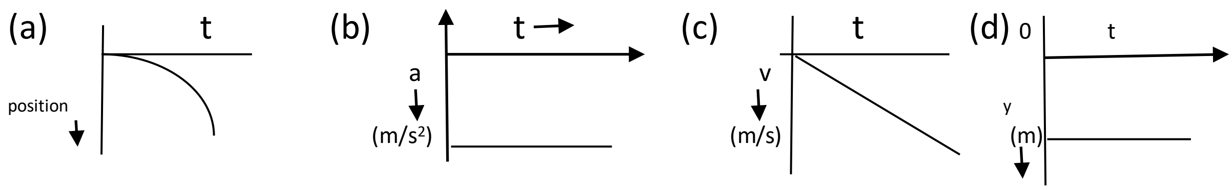
(d) The acceleration of S_2 with respect to S_1 may be anything between zero and 8 m/s^2.

VII. DIAGRAM BASED QUESTIONS

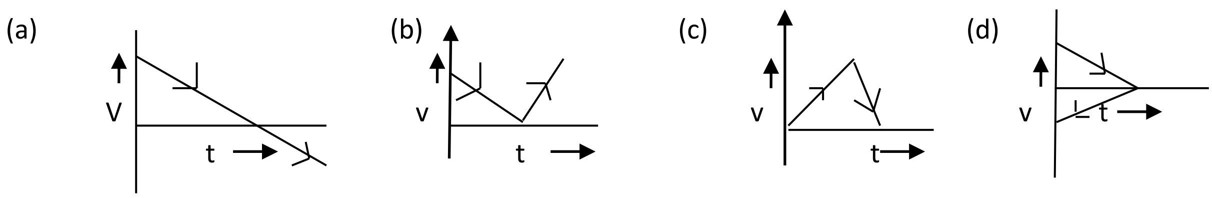
1. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.



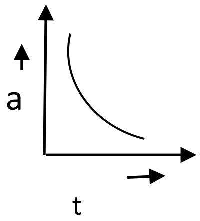
2. All the graphs below represent the same motion. One of them is incorrect. Find it.

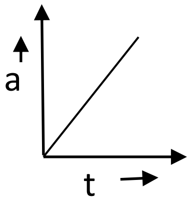


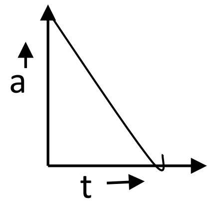
3. A body is thrown vertically upwards which one of the following graphs correctly represent the velocity versus time ?

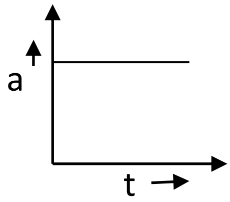


4. The distance travelled by a body moving along a line in time t is proportional to t^3. The acceleration –time graph for the motion of the body will be

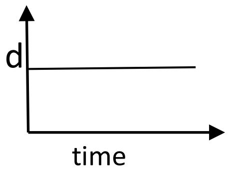
(a) 

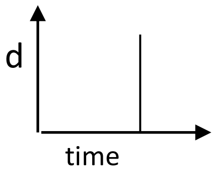
(b) 

(c) 

(d) 

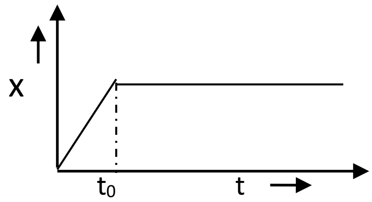
5. Which of the following is not possible for a body in uniform motion ?

(a) 

b) 

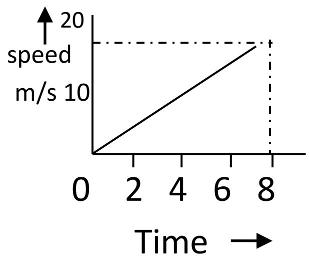
c) both (a) & (b) d) none of these

6. Figure shows the displacement-time graph of a particle moving on the x-axis.



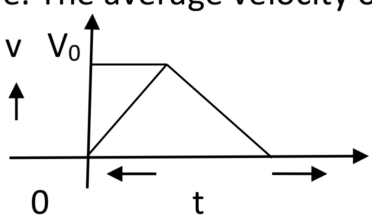
(a) The particle is continuously going in positive x direction
(b) The particle is at rest
(c) The velocity increases up to a time t_0 and then becomes constant.
(d) The particle moves at a constant velocity up to a time t_0 and then stops

7. The speed of a car as a function of time is shown in the figure. Find the distance travelled by the car in 8 seconds and its acceleration?



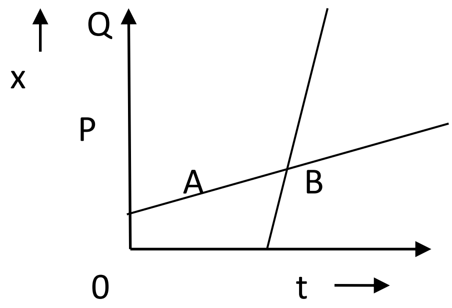
A) 80 m, 2.5 m/s^2 b) 20m, 8 m/s^2 c) 160m, 2 ms^{-2}
d) None of these.

8. A particle moves in a straight line obeying the v-t graph as shown in the figure. The average velocity of the particle over the time T is :

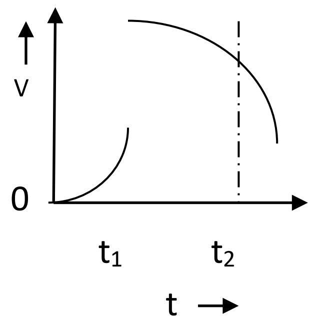


a) $\frac{V_0}{2}$ b) zero c) $\frac{2V_0}{3}$ d) $\frac{3V_0}{4}$

9. The position – time (x – t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in the figure. Choose the incorrect one :
   a) A lives closer to the school than B.
   b) B starts from the school earlier than A.
   c) B walks faster than A
   d) B overtakes A on the road once.



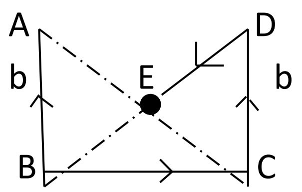
10. The velocity time graph of a particle in one-dimensional motion is shown in figure. Which of the following formulae are incorrect for describing the motion of the particle over the time-interval t_1 to t_2 :
    - $x(t_2) = x(t_1) + v_{av}(t_2 - t_1) + \frac{1}{2} a_{av}(t_2 - t_1)^2$
    - $V_{av} = [x(t_2) - x(t_1) / (t_2 - t_1)$
    - $a_{av} = [v(t_2) - v(t_1)] / (t_2 - t_1)$



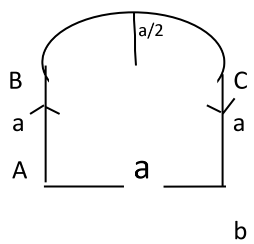
    - $x(t_2) - x(t_1)$ = area under the v- t curve bounded by the t-axis and the dotted line shown.

VIII. PROBLEMS :

1. Find the distance covered by a particle in the following figures.



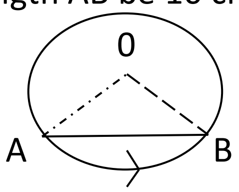
(b)



[Ans : (a) $S = l + 2b + \frac{1}{2}\sqrt{l^2 + b^2}$

(b) $S = 2a + \frac{\pi a}{2}$]

2. A particle travels from point A to B on a circular path of radius $\frac{15}{\pi}$ cm. If arc length AB be 10 cm. Find displacement.

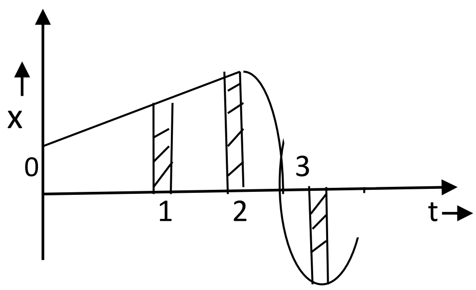


(Ans : AB = $\frac{15\sqrt{3}}{\pi}$ cm along A to B)

3. A cyclist takes two minutes to complete half revolution on a circular path of 120m radius. What is his
   a) Average speed b) Average velocity
   (Ans : (a) 3. 14 m/s
   (b) 2. 0 m/s)
4. What is the speed of the body moving with uniform acceleration at the midpoint of two points, on the straight line, where the speeds are μ and V respectively ?
   (Ans : speed = $\sqrt{\frac{V^2+\mu^2}{2}}$)
5. At a distance L = 400m away from the signal light brakes are applied to a locomotive moving with a velocity 54km/h. Determine the position of the rest of the locomotive relative to the signal light after 1 min of the application of the brakes if its acceleration is -0.3 ms^{-2}.
   (Ans : l = 25m from the signal light)
6. A car travels the first third of a distance with a speed of 10 km ph, the second third at 20 km ph and the last third at 60 km ph. What is its mean speed over the entire distance ?
   (Ans : 18 km/h (or) 5 ms^{-1})
7. A man walks on a straight road from home to a market 2.5 km away with a speed of 5 km h^{-1}. Finding the market closed due to corona effect, he instantly turns and walks back home with a speed of 7.5 km/h what is the
   a) Magnitude of average velocity and
   b) average speed of the man over the interval of time.
   (i) 0 to 30 min (ii) 0 to 50 min (iii) 0 to 40 min
   (Ans : (a) (i) 5 km/h (ii) 0 (iii) 1.875 km/h
   (b) 5 km/h (ii) 6 km/h (iii) 5.625 km/h)
8. Drops of water fall at regular intervals from the roof of a building of height 16m. The first drop strikes the ground at the same moment as the fifth drop leaves the roof. Find the distances between the successive drops ?
   Ans.[Distance between 1st and 2nd drop = 7m

Distance between 2nd and 3rd drop = 5 m
Distance between 3rd and 4th drop = 3 m
Distance between 4th and 5th drop = 1m]

9. A ball is dropped from the top of building and at the same time an identical ball B is thrown vertically upward from the ground. When the balls collide, the speed of A is twice that of B. At what fraction of the height of the building did the collision occur ?
   Ans. [fraction of height = 2/3 of height from ground]
10. A bullet moving with a speed of 150 ms^{-1} strikes a tree and penetrates 3.5 cm before stopping. What is the magnitude of its retardation in the tree and the time taken for it to stop after striking the tree ?
   Ans. [retardation a = 3.214×10^{5} ms^{-2} , and time (t) = 4.67×10^{-4} s)
11. A motorist drives north for 30 min at 85 km/h and then stops for 15 min. He continues travelling north and covers 130 km in 2 hours. What is his total displacement and average velocity ?
   Ans. [Total displacement = 172.5 km , Average velocity = 62.7 km/h]
12. A jet airplane travelling at a speed of 500 km h^{-1} ejects its products of composition at the speed of 1500 km h^{-1} relative to the jet plane. What is the speed of the latter with respect to an observer on the ground ?
   Ans. (speed = 1000 km/h)
13. A car moving along a straight highway with speed of 126 km/h is brought to a stop with in a distance of 200m. What is the retardation of the (assumed uniform) and how long does it take for the car to stop ?
   Ans. (retardation = 3.06 ms^{-2} and time = 11.4 s)
14. On a two lane road, car A is travelling with a speed of 36 km/h . Two cars B and C approach car A in opposite direction with a speed of 54 km h^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km. B decides to over take A before C does, what minimum acceleration of car B is required to avoid an accident ?
   Ans. (Acceleration of B = 1 ms^{-2})
15. Figure gives the position (x) - time (t) plot of a particle in one-dimensional motion . Three different equal intervals of time are shown. In which interval is the average speed greatest and in which is it the least ? Give the sign of average velocity for each interval ?
   Ans. (Greatest in 3, least in 2, and $v > 0$ in 1 and 2 , $v < 0$ in 3)



ANSWERS

I. FILL UP THE BLANKS

1. Relative 2. Speedometer 3. Kinematics
4. Distance 5. Motion 6. Velocity
7. Displacement 8.instantaneous velocity 9. zero
10. 2 R, π R 11. instantaneous acceleration
12. No 13. free fall
14.Uniform acceleration
15. speed 16. A straight line

II. TRUE/ FALSE

1. False 2. True 3. False 4. False 5. False
6.False 7. False 8. True 9. True 10. True
11. True 12. True 13. True 14. True 15. True

III. MATCHINGS

1. 1→B , 2 →A , 3→ D , 4→C
2. 1→ D 2→B 3→A 4 →C
3. 1→ C 2→A 3 →B 4→ D
4. 1→ Q, S 2 →Q, R 3→ P, R 4→ P, S
5. 1→R 2→ P 3→ S 4 →Q

IV. VERY SHORT ANSWER QUESTIONS :

1. The states of rest and motion are relative. They depend on frame of reference.
   Ex. When a person in a moving train, is at rest with respect to the co passenger and in motion with respect to the person on the ground (earth)

2. Yes

   For a body projected vertically upwards, the velocity of the body vertically upwards and the acceleration (g) vertically downwards.

3. **Average velocity :** $\overline{V}_{av} = \frac{\overline{x}_2 - \overline{x}_1}{t_2 - t_1} = \frac{\Delta \overline{x}}{\Delta t}$

   Average velocity tell us how fast an object has been moving over a given time interval.

   Instantaneous velocity : V = Lim $\lim_{\Delta t \to 0} \frac{\overline{\Delta} x}{\Delta t} = \frac{\overline{d} x}{d t}$

   It gives how fast an object moves at a particular instant.

   In uniform motion, both are equal, but in non-uniform motion both are different.

4. If the motion of an object is along a straight line and in the same direction, then

   $|Average\ velocity|$= average speed.

5. No

   If the velocity is constant, then the magnitude of instantaneous velocity i. e speed is constant.

6. (a) No , if the velocity is constant, that means there is no change in the velocity hence acceleration is zero.

   (b) yes

   An object moving along a curved path with constant speed has varying velocity because the direction of velocity changes from point to point along the trajectory (uniform circular motion).

7. Yes

   In uniform circular motion, an object moving along a curved path with constant speed has varying velocity because the direction of velocity changes from point to point along the trajectory.

8. (i) At the highest point of a vertically projected body, its velocity = 0, acceleration $\neq 0$, (= g)

(ii) At the extreme point of an object executing simple harmonic motion, its velocity = 0, but acceleration is not equal to zero (a $\neq$ 0)

9. Average speed (V_{av}) = $\frac{L}{\frac{L}{2V_1}+\frac{L}{2V_2}}$ = $\frac{L}{\frac{L}{2}(\frac{1}{v_1}+\frac{1}{v_2})}$

$$\therefore \boxed{V_{av} = \frac{2V_1V_2}{V_1+V_2}}$$

10. (a)straight line
(b) parabola

V. MULTIPLE CHOICE QUESTIONS

1. D	2. B	3. C	4. A	5. C
6.D	7. C	8. D	9. B	10 B
11.B	12. A	13. D	14. C	15. A
16. C	17. D	18. D	19. A	20 D
21. B	22. B	23. B	24. B	25. A
26. C				

VI. More than one correct answer

1. a, b, d	2. c, d	3. b, d	4. b, c , d	5. a, b, c

VII. Diagram based questions

1. B	2. D	3. A	4. B	5. C
6. D	7. A	8. A	9. B	10. A

VIII. PROBLEMS

1. (a) S = l + 2b + $\frac{1}{2}\sqrt{l^2+b^2}$ (b) S = 2a + $\frac{\pi a}{2}$

2. AB = $\frac{15\sqrt{3}}{\pi}$ cm along A to B

3. (a) 3. 14 m/s (b) 2.0 m /s

4. Speed = $\sqrt{\frac{V^2+\mu^2}{2}}$

5. L = 25m from the signal light

6. 18 km/h (or) 5 m s^{-1}

7. (a) (i) 5 km/h (ii) 0 (iii)1.875 km/h
   (b) (i) 5 km/h (ii) 6 km/h (iii) 5.625 km/h

8. Distance between 1st and 2nd drop = 7m
   Distance between 2nd and 3rd drop = 5m
   Distance between 3rd and 4th drop = 3m
   Distance between 4th and 5th drop = 1m

9. Fraction of height = $\frac{2}{3}$ of height from ground

10. Retardation a = 3. 214 x 10^{5} ms^{-2} and time (t) = 4.67 x 10^{-4} s

11. Total displacement = 172.5 km
    Average velocity = 62.7 km/h

12. Speed = 1000 km/h

13. Retardation = 3.06 ms^{-2} and time = 11.4 sec

14. Acceleration of B = 1 ms^{-2}

15. Greatest in 3, least in 2 and v > 0 in 1 and 2, V < 0 in 3.

----- THE END -------

Laws of Motion

Synopsis:

Force:

1. Force is an action that can change or tends to change the state of rest or motion of a body.
2. Force is also defined as interaction between two bodies. Two bodies can also exert force on each other even without being in physical contact.
   Example: Electric force between two charges, gravitational force between any two bodies in universe.
3. Force is a vector quantity having SI unit Newton (N) and dimension $MLT^{-2}MLT^{-2}$.
4. When many forces are acting on a single body the resultant force is obtained by using law of vector addition.

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \cdots + \vec{F_n}$$

5. The resultant of the two forces $\vec{F_1}\vec{F_1}$ and $\vec{F_2}\vec{F_2}$acting at an angle $\theta\theta$is given by

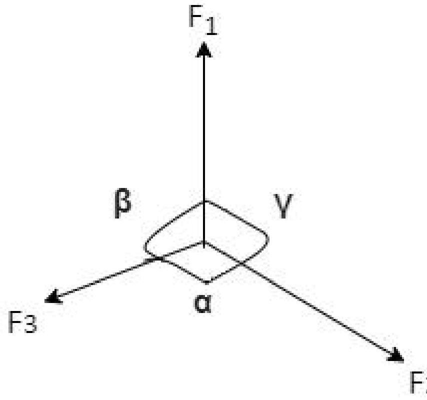
$$F = \sqrt{F_1{}^2 + F_2{}^2 + 2F_1F_2\cos\theta}$$

   the resultant force is directed at an angle $\alpha\alpha$with respect to force $F_1F_1$where

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

6. Lami's theorem: If three forces $F_1, F_2, F_3F_1, F_2, F_3$are acting simultaneously on a body and the body is in equilibrium then according to Lami's theorem

$$\frac{F_1}{\sin(\pi-\alpha)} = \frac{F_2}{\sin(\pi-\beta)} = \frac{F_3}{\sin(\pi-\gamma)}$$



where $\alpha, \beta, \gamma\alpha, \beta, \gamma$ are the angles opposite to the forces $F_1, F_2, F_3F_1, F_2, F_3$respectively.

Basic forces in nature:

1. Gravitational forces
2. Electromagnetic forces
3. Weak nuclear forces

4. Strong nuclear forces

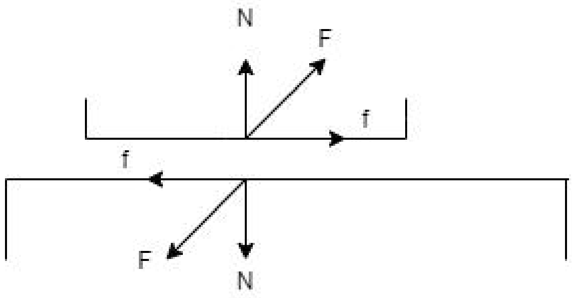
Basic forces in Mechanics:

1. Weight:Weight of an object is the force with which earth attracts it. It is also calledthe force of gravity or gravitational force.

2. Contact force:When two bodies come in contact, they exert forces on each other that are called contact forces.

   a. Normal force: It is the component of contact force normal to the surface. It measures how strongly the surfaces in contact are pressed together.

   b. Frictional force: It is the component of contact force parallel to the surface. It opposes the relative motion of two surfaces in contact.



   c. Tension: The force exerted by the end of a string, rope or chain is called tension. The direction of tension is so as to pull the body, while that of normal reaction is to push the body.

   d. Spring force: Every spring resists any attempt to change its length. The force exerted by a spring $F = -KxF = -Kx$

   Where $x = change\ in\ lengthx = change\ in\ length$

   $K = Spring\ constantK = Spring\ constant$

Newton's Laws of Motion:

First Law of Motion:

1. Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by anexternal force to change that state.

2. This law is also known as Law of Inertia.

3. Inertia: It is the property of inability of a body to change its position of rest or of uniform motion in a straight line unless some external force acts on it.

4. Mass is a measure of inertia of a body.

5. A frame of reference in which Newton's first law is valid is called Inertial Frame
i.e. If a frame of reference is at rest or in uniform motion it is called inertial frame or else it is called non inertial.

Second Law of Motion:

1. This law gives the magnitude of force.
2. The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.
3. Unit force: It is defined as the force which changes the momentum of a body by unity in unit time.

Third Law of Motion:

1. To every action there is always an equal and opposite reaction.
2. Forces always occur in pairs. Forces on a body A by B is equal and opposite to the force on body B by A.

   $$\overrightarrow{F_{AB}} = -\overrightarrow{F_{BA}}$$

3. Action and Reaction always acton different bodies.

Linear momentum:

1. The linear momentum of a body is defined as product of mass of body and its velocity.
2. $\text{Linear momentum} = \text{mass} \times \text{velocity}\text{Linear momentum} = \text{mass} \times \text{velocity}$

   $\vec{P} = m\vec{v}$

3. Linear momentum is a vector quantity.
4. SI unit of linear momentum is$kg \cdot m \cdot s^{-1} kg \cdot m \cdot s^{-1}$.
5. Its direction is same as the direction of velocity of the body.

Impulse:

1. The force which acts on a body for short time are called impulse or impulsive force.
2. Impulse is the product of force and time which equals change in momentum.
3. It is a vector quantity.
4. Its SI unit is NS.

Law of Conservation of Linear Momentum:

1. The total momentum of an isolated system of particles is conserved.
2. The law follows from second and third Law ofMotion.

Friction:

1. The force which always opposes the motion of one body over other body contact with it is calledFriction orFrictional force.
2. Frictional force is independent of the area of contact.
3. Friction is three types
   a. Static friction: The maximum frictional force present when a body just tends to slide over surface of other body is called static friction or limiting friction.
   b. Kinetic friction: The frictional force present when one body slides over another body is known as dynamic or kinetic friction.

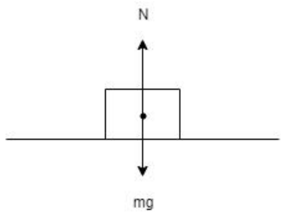
c. <u>Rolling friction</u>: When a body like wheel, cylinder or drum rolls over the surface of another body, this type of friction is called rolling friction.

$f_s > f_k > f_r$

Normal Reaction or Normal Force:

1. The force which is acting normal to the surface of body in outward direction is called normal reaction.

2. When bodyis placed on a horizontal surface then $normal\ reaction = mg$ $normal\ reaction = mg$.



3. When the body is placed on inclined surface then $normal\ reaction = mg\cos\theta$ $normal\ reaction = mg\cos\theta$

Coefficient of friction

1. The ratio between frictional force to normal reaction is called coefficient of friction.
$\mu = \frac{f}{N}$

2. It has no units and dimensions.

3. Coefficient of friction can never be greater than 1.

4. Coefficient of fiction is three types

   a. <u>Coefficient of static friction</u>: The ratio between static frictional force to normal reaction is called coefficient of static friction.
   $\mu_s = \frac{f_s}{N}$

   b. <u>Coefficient of kinetic friction</u>: The ratio between kinetic frictional force to normal reaction is called coefficient of kinetic friction.
   $\mu_k = \frac{f_k}{N}$

   c. <u>Coefficient of rolling friction</u>:The ratio between rolling frictional force to normal reaction is called coefficient of rolling friction.
   $\mu_r = \frac{f_r}{N}$

   $\mu_s,\mu_k,\mu_r\mu_s,\mu_k,\mu_r$ have no units and dimensions.

Methods to decrease friction:

1. Polishing

2. Ball Bearing

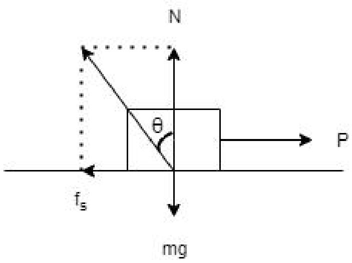
3. Lubricants

4. Streamlining
5. By proper selection of materials

Angle of friction:

The angle of friction between any two surfaces in contact is defined as angle which the resultant of the force of limiting friction $f_s f_s$ and normal reaction N makes with the direction of normal reaction N.

$\mu = \tan\theta$



Angleof Repose or Angle of Sliding:

Angle of repose or angle of sliding is defined as the minimum angle of inclination of a plane with the horizontal, such that the body placed on the plane just begins to slide down.
$\mu = \tan\alpha$

Angle of friction is equal to angle of repose.

Uniform Circular Motion:

1. Centripetal force:It is the force required to move a body uniformly in a circle. This force acts along the radius and towards the center of the circle.
   $F = \frac{mv^2}{r} = mr\omega^2$
2. Centrifugal force:It is a force that arises when a body is moving actually along a circular path by virtue of tendency of the body to regain its natural straight-line path. It acts along the radius and away from the center of the circle.

Banking of roads:

The phenomenon of raising outer edge of the curvedroad above the inner edge is called banking of roads.

$\tan\theta = \frac{v^2}{rg}$

Formula:

1. $Momentum = mass \times velocity,\ \vec{P} = m\vec{v}$$Momentum = mass \times velocity,\ \vec{P} = m\vec{v}$
2. From Newton's Second Law
   $F \propto \frac{dp}{dt} = m\frac{dv}{dt} = ma = m(\frac{v-u}{t})$
3. $\mu = \frac{f}{N}, \mu_s = \frac{f_s}{N}, \mu_k = \frac{f_k}{N}, \mu_r = \frac{f_r}{N}$$\mu = \frac{f}{N}, \mu_s = \frac{f_s}{N}, \mu_k = \frac{f_k}{N}, \mu_r = \frac{f_r}{N}$
4. Acceleration ofa body on rough horizontal plane
   $a = \frac{F}{m} - \mu_k g$
5. Smooth inclined plane
   a. For downward motion
      i. Downward acceleration$a = g \sin\theta$Downward acceleration$a = g \sin\theta$
      ii. Velocity on reaching the bottom $v = \sqrt{2gl\sin\theta} = \sqrt{2g\square}$
      Velocity on reaching the bottom $v = \sqrt{2gl\sin\theta} = \sqrt{2g\square}$
      iii. $Time\ taken\ to\ slide\ down\ the\ plane\ t = \frac{v}{g\sin\theta} = \sqrt{\frac{2l}{g\sin\theta}}$
      $Time\ taken\ to\ slide\ down\ the\ plane\ t = \frac{v}{g\sin\theta} = \sqrt{\frac{2l}{g\sin\theta}}$
   b. For upward motion
      i. Upward acceleration$a = -g \sin\theta$Upward acceleration$a = -g \sin\theta$
      ii. $Time\ taken\ to\ reach\ the\ top\ of\ surface\ t = \frac{v}{g\sin\theta} = \sqrt{\frac{2l}{g\sin\theta}}$
      $Time\ taken\ to\ reach\ the\ top\ of\ surface\ t = \frac{v}{g\sin\theta} = \sqrt{\frac{2l}{g\sin\theta}}$
6. Motion on rough inclined plane
   a. For downward motion
      i. Downward acceleration$a = g(\sin\theta - \mu_k\cos\theta)$
      Downward acceleration$a = g(\sin\theta - \mu_k\cos\theta)$
      ii. Velocity on reaching the bottom $v = \sqrt{2gl(\sin\theta - \mu_k\cos\theta)}$
      Velocity on reaching the bottom $v = \sqrt{2gl(\sin\theta - \mu_k\cos\theta)}$

iii. Time taken to slide down the plane $t = \sqrt{\frac{2l}{g\,(\sin\theta - \mu_k \cos\theta)}}$

Time taken to slide down the plane $t = \sqrt{\frac{2l}{g\,(\sin\theta - \mu_k \cos\theta)}}$

b. For upward motion

i. Upward acceleration $a = \frac{F}{m} - g(\sin\theta + \mu_k \cos\theta)$

Upward acceleration $a = \frac{F}{m} - g(\sin\theta + \mu_k \cos\theta)$

ii. Time taken to reach the top of surface $t = \sqrt{\frac{2l}{\frac{F}{m} - g\,(\sin\theta + \mu_k \cos\theta)}}$

Time taken to reach the top of surface $t = \sqrt{\frac{2l}{\frac{F}{m} - g\,(\sin\theta + \mu_k \cos\theta)}}$

7. Motion of lawn roller:

a. When pulling the lawn roller of mass M with a force F

i. Horizontal component $F_x = F\cos\theta$$F_x = F\cos\theta$

ii. Normal reaction $N = mg - F\sin\theta$$N = mg - F\sin\theta$

b. When lawn roller is pushed with a force F

i. Horizontal component $F_x = F\cos\theta$$F_x = F\cos\theta$

ii. Normal reaction $N = mg + F\sin\theta$$N = mg + F\sin\theta$

I. Very Short Question and Answers

1) What is inertia?
2) What gives the measure of inertia?
3) Define force
4) What are the basic forces in nature?
5) Can the coefficient of friction be greater than 1?
6) Why does a car with flattened tires stop sooner than the one with inflated tires?
7) What happens to the coefficient of friction if weight of the body is doubled?
8) State anyone application of impulse
9) Which physical quantity does the area between F-t graph and t-axis represent?

10) Can the coefficient of friction be 0?

11) Why aero planeand automobiles are streamlined?

12) Guess the effect of temperature on the coefficient of friction

13) What is the effect of lubrication on coefficient of friction?

14) What is the influence of normal reaction on the area of contact?

15) How can you move the body up the smooth plane without acceleration?

II. Answer True or False

1) Inertia is a force which keeps stationary objects at rest and moving objects in motion at constant velocity.

2) Inertia is a force which brings all objects to a rest position.

3) All objects haveinertia.

4) A more massive object has more inertia then a less massive object.

5) Fast moving objects how more inertia then slow-moving objects.

6) An object would not have any inertia in a gravity free environment.

7) Inertia is the tendency of all objects to resist motion and ultimately stop.

8) The mass of an object is mathematically related to weight of the object.

9) Newton's first law of motion is applicable to both moving and non-moving objects.

10) If a football is moving upward and rightwards towards the peak of its trajectory then there are both rightwards and upwards forces acting upon it.

11) Force always cause objects to move.

12) An object can experience two or more forces and not accelerate.

13) Force is a vector quantity, there is always a direction associated with it.

14) The angle made by the resultant normal reaction and frictional force with the normal reaction is known as angle of friction.

15) The angle of inclination at which the body tends to slide down is known as angle of repose.

16) Rolling friction does not depend on area of contact.

17) To every action there is always an equal and opposite reaction.

18) The pseudo force is always directed opposite to the direction of acceleration of the non-inertial frame.

19) If external force on a body is zero its acceleration is not zero.

20) The second law of motion is consistent with the first law F=0 implies a=0.

III. Fill Up the Blanks

1) Friction opposes the ______________ between the surfaces in contact with each other.

2) Friction depends on the _____________ of the surfaces.

3) Friction produces_____________.

4) Sprinkling powder on the carrom board _____________ friction.

5) Sliding friction is _____________ than the static friction.

6) _____________ provides the centripetal force needed to allow a car to turn a corner.

7) To every action there is an _____________ and _____________ reaction.

8) Momentum is a_____________ quantity.

9) Momentum SI unit is _____________.

10) Newtons second law of motion can be written as $Force = mass \times$ _________
$Force = mass \times$ _________

11) Newtons second law of motion can be written as

$Force =$ _______ $of change Force =$ _______ $of change$ of _____________.

12) Forces in newtons third law pair have equal _____________ but actin opposite_____________.

13) Newton's first law of motion _____________ force.

14) Newton's second law of motion gives us a _____________ of force.

15) A body does not change its state of rest of uniform motion unless an_____________ to change that state.

16) Newton built on __________ ideas and laid the foundation of mechanics in terms of threelaws.

17) The rate of change of momentum of a body is directly proportional to the_____________.

18) The total momentum of an isolated system of interacting particles is_____________.

19) Impulse is the product of force and time which equals _____________.

20) Law of conservation of momentum follows from __________ and _________law of motion.

IV. Multiple Choice Questions:

1) Force exerted on a body changes its

a) Direction of motion b) momentum c) kinetic energy d) all of the above

2) There are two statements
Statement A: Rate of change of momentum corresponds to force.
Statement B: Rate of change of momentum corresponds to kinetic energy.
Which one of the following is correct

a) A only b) B only c) both A and B are correct d) both A, B are wrong

3) A truck and car are moving on a planeroad with same kinetic energy.They are brought to rest by application of brakes which provide equal retarding forces.Which one of the following statements is true?

   a) distance traveled by the truck is shorter than car before coming to rest

   b) distance traveled by car is shorter than truck before coming to rest

   c) distance traveled depends on individual velocity of both vehicles

   d) both will travel same distance before coming to rest

4) A block of mass $m_1 m_1$is released from top of smooth inclined plane and it slides down the plane. Another block of mass $m_2 m_2$such that $m_2 > m_1 m_2 > m_1$is dropped from the same point and falls vertically downwards
Which one of the following statements will be true if the friction offered by air is negligible

   a) both blocks willreach ground at same time

   b) both blocks will reach ground with the same speed

   c) speed of both the blocks when they reached ground will depend on their masses

   d) block A reaches ground before block B

5) What is the time taken by the body to slide down an inclined plane if the length of inclined planeIf length of inclined plane is L, a is the retardation and $\theta\theta$ is the angle of inclination?

   a) $\sqrt{\frac{2L}{g\sin\theta}}\sqrt{\frac{2L}{g\sin\theta}}$ b) $\sqrt{\frac{2L}{a\sin\theta}}\sqrt{\frac{2L}{a\sin\theta}}$ c) $\sqrt{\frac{2L}{(g+a)\sin\theta}}\sqrt{\frac{2L}{(g+a)\sin\theta}}$ d) $\sqrt{\frac{2l}{(g-a)\sin\theta}}\sqrt{\frac{2l}{(g-a)\sin\theta}}$

6) Choose correct statement

   a) A body can be accelerated by frictional force.

   b) There can be 0 friction.

   c) Kinetic friction is greater than rolling friction.

   d) Frictional force and area of contact between two surfaces are proportional.

7) What is the force acting on the particle if the motion of particle is givenas $y = ut + \frac{1}{2}gt^2$ $y = ut + \frac{1}{2}gt^2$

   a) $F = ma F = ma$ b) $F = 0 F = 0$ c) $F = mg F = mg$ d) $F \neq 0 F \neq 0$

8) Which of the following should be constant for a body to have constant momentum?

   a) Acceleration b) force c) velocity d) all of the above

9) Two trains A and B are running in the same direction on parallel tracks such that A is faster than B. If packets of equal weight are exchanged between the two then

   a) A will be retarded but B will be accelerated.

b) A will be accelerated but B will be retarded.

c) there will not be any change in the velocity of A but B will be accelerated.

d) there will not be any change in the velocity of B but A will be accelerated.

10) Newton's third law of motion explains the two forces namely action and reaction coming into action when the two bodies are in contact with each other, these two forces

a) always act on the same body

b) always act on different bodies in opposite direction

c) have some magnitude and direction

d) actson either body at normal to each other

11) In a rocket a large volume of gases produced by the combustion of fuel is allowed to escape through its tail nozzle in downward direction with the tremendous speed and makes the rocket to move upward.Which principle is followed in this take off of therocket?

a) momentum of inertia

b) conservation of momentum

c) Newton's third law of motion

d) Newton’s law of gravitation

12) The seatbelts are provided in the cars so that if car stops suddenly due to an emergency braking, the person sitting on the front seats are not thrown forward violently and are saved from getting injured. Can you guess the law due to which person falls in forward direction on the sudden stopping of the car?

a) Newton's first law of motion

b) Newton's second law of motion

c) Newton's third law of motion

d) Newton's law of gravitation

13) A force can accelerate a lighter vehicle more easily than a heavier vehicle which are moving, Can you guess which law is present

a) Newton's first law of motion

b) Newton's second law of motion

c) Newton's third law of motion

d) Newton's law of gravitation

14) The inertia of a moving object depends on

a) momentum of the object

b) speed of the object

c) shape of the object

d) mass of the object

15) A man slides down a light rope whose breaking strength is n times it's weight.What should be the maximum acceleration so that rope does not break?

a) $g(1-n)g(1-n)$ b) $ngng$ c) $\frac{g}{1+n}\frac{g}{1+n}$
d) $\frac{g}{1-n}\frac{g}{1-n}$

16) A lift is moving down with an acceleration a. A man in the lift drops a ball inside the lift, the acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively

a) g, g b) a, a c) (g-a), g d) a, g

17) And wooden block of mass M resting on a rough horizontal surface is pulled with a force F at an angle $\emptyset\emptyset$with the horizontal. If $\mu\mu$is the coefficient of kinetic friction between the block and surface then the acceleration of block is

a) $\frac{F}{M}(\cos\emptyset + \mu\sin\emptyset) - \mu g\frac{F}{M}(\cos\emptyset + \mu\sin\emptyset) - \mu g$ b) $\frac{F}{M}\sin\emptyset$
$\frac{F}{M}\sin\emptyset$ c) $\mu\cos\emptyset\mu\cos\emptyset$ d) $\mu F\sin\emptyset\mu F\sin\emptyset$

18) A plane inclined at an angle $\theta\theta$with the horizontal. A body of mass mrests on it. If the coefficient of friction is$\mu\mu$, then the minimum force that has to be applied parallel to the inclined so as to make the body just move up the inclined plane is

a) $mg\sin\theta mg\sin\theta$ b) $\mu mg\cos\theta\mu mg\cos\theta$ c) $\mu mg\cos\theta - mg\sin\theta$
$\mu mg\cos\theta - mg\sin\theta$ d) $\mu mg\cos\theta + mg\sin\theta\mu mg\cos\theta + mg\sin\theta$

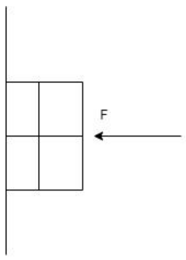
19) A block of mass m is at rest under the action of force F acting against as shown in figure. Which of the following statements is incorrect?

a) $f = mgf = mg$where f is the frictional force

b) $F = NF = N$where N is the normal force

c) F will not produce torque

d) N will not produce torque



20) A block is kept on a frictionless inclined surface with an angle of inclination$\alpha\alpha$. The inclined is given an acceleration a to keep the block stationary. Then a is equal to

a) $\frac{g}{\tan\alpha}\frac{g}{\tan\alpha}$ b) $g\ cosecag\ coseca$ c) g d) $g\tan\alpha$
$g\tan\alpha$

V. Match the following

1)

i.	Momentum	a.	J
ii.	force	b.	P
iii.	impulse	c.	F
iv.	static friction	d.	$f_r f_r$
v.	kinetic friction	e.	$f_s f_s$
vi.	rolling friction	f.	$f_k f_k$

2)

i.	$MLT^{-1}MLT^{-1}$	a.	Kinetic friction
ii.	$MLT^{-2}MLT^{-2}$	b.	impulse
iii.	Vector	c.	momentum
iv.	$mama$	d.	static friction
v.	$\mu_k N \mu_k N$	e.	force

3)

i.	Inertia	a.	product of mass and velocity
ii.	friction	b.	mass of the object
iii.	momentum	c.	rate of change of momentum
iv.	force	d.	necessary evil

4)

i.	Newton's first law of motion	a.	the acceleration produced is directly proportional to the product of mass and force applied
ii.	Newtons second law of motion	b.	when no force is exerted on an object it stays at rest or it moves in a straight line with constant speed
iii.	Newton's third law of motion	c.	to every action there is an equal and opposite reaction
iv.	Galileo's lawof inertia	d.	an object at rest or in motion will remain at rest or uniform motion when unbalanced forces applied on it

5)

i.	Force	a.	Newton
ii.	momentum	b.	acceleration
iii.	unbalanced force	c.	Kg m/s
iv.	principle on rocket works	d.	Newton's third law of motion

6)

i.	Friction	a.	when one object rolls over another
ii.	limiting friction	b.	force just sufficient to move the object
iii.	sliding friction	c.	force which opposes the motion
iv.	rolling friction	d.	for sufficient to slide one object over another

7)

i.	unbalanced force	a.	applying grease
ii.	unchanging tendency	b.	change in state
iii.	method of reducing friction	c.	Force
iv.	rate of change of momentum	d.	inertia

8)

i.	F	a.	mv

ii. p
Δp

iii. Momentum before collision

b. momentum after collision are equal

c. $\frac{p_2 - p_1 p_2 - p_1}{t \quad t}$

d. ma

9)

i. Definition of force
ii. measure of force
iii. effect of force
iv. Recoiling of gun

a. Newton's third law
b. impulse
c. Newton's second law
d. Newton's first law

10)

i. Centripetal force
ii. banking of roads
iii. Smooth inclined plane
iv. Rough inclined plane

a. $g \sin\theta g \sin\theta$
b. $mr\omega^2 mr\omega^2$
c. $g(\sin\theta - \mu_k \cos\theta) g(\sin\theta - \mu_k \cos\theta)$
d. $\frac{v^2 v^2}{rgrg}$

VI. Problems (Level – I)

$P = a + bt$

1) The linear momentum of a particle as a function of time is given by $P = a + bt$
where a and b are constants. What is the force acting on the particle?
2) Calculate the time needed for a net force of 5N to change the velocity of a 10kg mass by 2m/s.
3) A constant force acting on a body of mass 3kg changes its speed from 2m/s to 3.5m/s in 25s. What is the magnitude and direction of the force?
4) A ball of mass m is thrown vertically upward from the ground and reaches a height h before momentarily coming to rest. If g is acceleration due to gravity. What is the impulse received by the ball due to gravitational force during its flight?
5) Container of mass 200kg rests on the back of an open truck. If that truck accelerates at $ms^{-2} ms^{-2}$
1.5 . What is the minimum coefficient of static friction between the container and the bed of truck required to prevent the container from sliding off the back of the truck?
6) A fixed pulley with a smooth grow has a light string passing over it with a 4kg attached on one side and a 3kg on the other side,another 3kg is hung from the other 3kg with another light spring. If the system is released from rest find $g = 10ms^{-2}) g = 10ms^{-2})$
the common acceleration (
7) A constant retarding force of 50N is applied to a body of mass 20kg moving initially with a speed of 15m/s. How long does it take to stop?

$5ms^{-2}$

8) A rocket with a lift off mass 20,000kg is blasted upwards with an acceleration of $5ms^{-2}$
. Calculate the initial thrust(force) of the blast.
9) A bullet of mass 0.04kg moving with a speed of 90m/s enters heavy wooden block and is stopped after a distance of 60cm. What is the average resistive force exerted by the block on the bullet?

$$y = ut + \frac{1}{2}gt^2 y = ut + \frac{1}{2}gt^2$$

10) The motion of particle of mass m described by .Find the force acting on the particle

11) A batsman hits back a ball straight in the direction of bowler without changing its initial speed of 12m/s. If the mass of the ball is 0.15kg, determine the impulse imparted to the ball.

12) Determine the maximum acceleration of the train in which a box laying on its floor will remain stationary, given that the coefficient of static friction between the box and train floor is 0.15

13) If a bike with a rider having a total mass of 63kgbrakes and reduces its velocity from 8.5m/s to 0m/s in 3 seconds. What is the magnitude of braking force?

$$\vec{u} = (3\vec{i} + 4\vec{j}\vec{u} = (3\vec{i} + 4\vec{j}$$

14) A cricket ball of mass 150g has an initial velocity)m/s and a

$$\vec{v} = -(3\vec{i} + 4\vec{j}\vec{v} = -(3\vec{i} + 4\vec{j}$$

final velocity)m/s after being hit, find the change in momentum.

15) A shell of mass 0.02kg is fired by a gun of mass 100kg. If the nuzzle speed of the shell is 80m/s. What is the recoil speed of gun?

16) Force of 16N and 12N are acting on a mass of 200kg in mutually perpendicular directions. Find the magnitude of the acceleration produced.

$m_1 \: and \: m_2$

17) A light string passing over a smooth light pulley connects to blocks of masses

$m_1 \: and \: m_2$

(vertically). If the acceleration of the system is g/8, then the ratio of masses is

18) A dish of mass 10g is kept horizontal in air by firing bullets of mass 5g each, at the rate of 10 bullets per sec. If the bullets rebound with the same speed what is the velocity with which bullets are fired?

19) Two billiard balls each of mass 0.05kg moving in opposite directions with speed 6m/s collide and rebound with the same speed. What is the impulse imparted to each ball due to the other?

VII. Problems (Level – II)

1) A man in a lift feels an apparent weight W when the lift is moving up with a uniform acceleration of 1/3rd of the acceleration due to gravity. If the same man were in the same lift now moving down with a uniform acceleration that is ½ of the acceleration due to gravity then what is his apparent weight?

$30^\circ 30^\circ$

2) A block of mass of 2kg slides on an inclined plane that makes an angle of with the

$\frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2}$

horizontal. The coefficient of friction between the blocks and surface is . What force should be applied to the block so that it moves up without any acceleration?

$$y = \frac{x^2}{20} y = \frac{x^2}{20}$$

3) A block is placed on a ramp of parabolic shape given by the equation . If

$\mu_s = 0.5 \mu_s = 0.5$

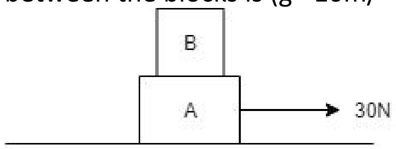
, what is the maximum height about the ground at which the block can be placed without slipping

$$\tan\theta = \mu_s = \frac{dy}{dx} \tan\theta = \mu_s = \frac{dy}{dx}$$

4) On a smooth horizontal surface, a block A of mass 10kg is kept. On this block a second block B of mass 5kg is kept. The coefficient of friction between the two blocks is 0.4. A horizontal force of 30N is applied to the lower block as shown. the force of friction

s^2s^2

between the blocks is (g= 10m/).



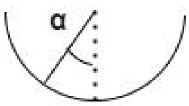
5) Two bodies of masses10kg and 20kg are respectively kept on smooth horizontal surface the two ends of light string, the horizontal force F=600N is applied to i)A ii)B along the direction of this string. What is the tension in the string in each case?

6) Two masses 8kg and 12kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration of the masses and tension in the string. when the same speed what is the impulse imparted to each ball due to the other.

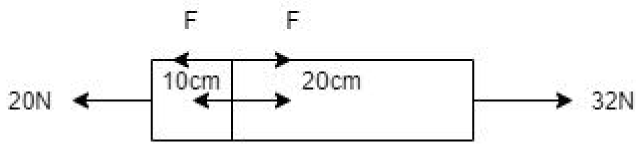
7) A stone of mass 0.25kg tiedto the end of a string is whirled round in a circle of radius 1.5m with the speed of 40 rev/min in the horizontal plane. What is the tension in the string what is the maximum speed with which the stone can be whirled around if this string can withstand a maximum tension of 200N?

8) A cyclist speeding at 18kmph on a level road takes a sharp circular turn of radius 3m without reducing the speed the coefficient of static friction between the tires and the road is 0.1 will the cyclist slip while taking the turn

9) An insect crawls up a hemispherical surface very slowly. the coefficient of friction between the insect and surface is 1/3. If the line joining the center of hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by

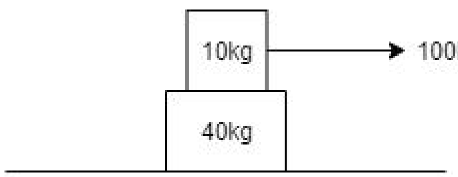


10) The figure shows a uniform rod of length 30cm having a mass 3kg. The strings shown in the figure are pulled by constant forces of 20N and 32N. Find the force exerted by the 20cm part of the road on the 10cm part. All the surfaces are smooth and the strings are light



11) Two bodies of equal masses are connected by light inextensible string passing over a smooth frictionless pulley. The amount of mass that should be transferred from one to the another so that both masses moved 50m in 5s.

12) A 40kg slab rests on a frictionless floor. A 10kg block rests on the top of the slab. The static coefficient of friction between the block and slab is 0.6. While the kinetic coefficient is 0.4. The 10kg block is acted upon by a horizontal force of 100N. The resultant acceleration of the slab will be



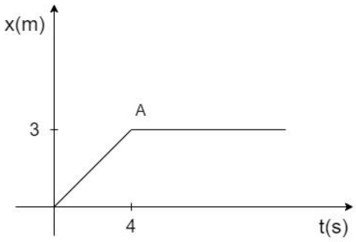
$F = 600 - 2 \times 10^5 t$

13) A bullet is fired from a gun. the force of the bullet is given by
$F = 600 - 2 \times 10^5 t$
, where F is in newton and t is in second. the force on the bullet becomes zero as soon as it leaves the barrel. what is the average impulse imparted due to the bullet?

$\beta\beta$

14) When the spring is loaded 5N its length isα. and when loaded with 4N its length is . when loaded with 9N its length will be

15) Figure shows the position time graph of a particle of mass 4kg. what is the
a) force on the particle for t<0, t>4s; 0 < t < 5
b) impulse at t=0 and t=4s



VIII. Each of these questions contains two statements, Statement 1 – Assertion and Statement 2 – Reason. Each of these questions also has four alternative choices, out of which only one is a correct answer. You have to select one of the codes 'a', 'b', 'c', 'd' given below

a – Statement 1 is True. Statement 2 is True and is the correct explanation for Statement I.
b – Statement 1 is True. Statement 2 is True, but is not the correct explanation for Statement I.
c – Statement 1 is True. Statement 2 is False.
d – Statement 1 is False. Statement 2 is True.

1) Statement I: When the car accelerates horizontally along the straight road the accelerating force is given by the push of the rare axle on the wheels.
Statement II: When the car accelerates the rear axles rotates with the greater frequency.

2) Statement I: The work done in bringing a body down from the top of a base along a frictionless inclined plane is same as work done in bringing it down the vertical side.
Statement II: The gravitational force on the body along the inclined plane is same as that along the vertical side.

3) Statement I: A bullet is fired from a rifle. If the rifle recoils freely, the kinetic energy of rifle is less than that of the bullet.
Statement II: In case of a rifle bullet system, the law of conservation of momentum is violated.

4) Statement I: A cloth covers table; some dishes are kept on it. The cloud can be pulled out without dislodging the dishes from the table.
Statement II: For every action there is an equal and opposite reaction.

5) Statement I: It is easier to pull a heavier object than to push it on a level ground.
Statement II: The magnitude of frictional forces depends on the nature of the two surfaces in contact.

ANSWERS

I. Very Short Question and Answers

1) The inability of a body to change its state of rest or state of motion by itself is known as inertia.
2) Mass is a measure of inertia.

3) The external agency which changes or tries to change the state of the body is known as force.

4) Basic forces in nature: 1) Gravitational force 2) Electromagnetic force
3) Strong Nuclear force 4) Weak Nuclear force

5) Yes, when the contact surfaces are polished too much then adhesive forces between the molecules increases and the value of coefficient of friction will be greater than unity.

6) In case of flattened tires, the rolling friction is more due to the greater deformation of tires, as friction is more it stop sooner.

7) Coefficient of friction depends only on nature of surface in contact and independent of weight of the body. So, it remains constant.

8) Shock absorbers are used in vehicles to reduce the magnitude of impulsive forces.When they travel on uneven road shock absorbers increase the time of action and reducing the force.

9) The area between F-t graph represents impulse.

10) The coefficient of friction can be negligibly small but not exactly by zero. The presence of thin film or air film can make the coefficient of friction negligibly small.

11) Aero plane and automobiles are streamlined to reduce the friction due to air.

12) The coefficient of friction may decrease due to increase of temperature.

13) Coefficient of friction decreases.

14) Area of contact increases with normal reaction.

15) By applying an external force $F = mg\sin\theta F = mg\sin\theta$ up the plane after giving an initial velocity up the plane.

II. Answer True or False

1.False inertia is not a force
2.False inertia is not a force
3.True
4.True mass is a measure of an object's inertia
5.False speed does not affect the amount of inertia
6.False
7.False inertia is the tendency to resist changes in motion
8.True weight = mg
9.True
10.False it has both an upward and rightward velocity
11.False you can be in a place but have gravity act upon you
12.True
13.True
14.True
15.True
16.False depends on area of contact
17.True
18.True
19.False acceleration is zero.
20.True

III. Fill Up the Blanks

1) motion 2) nature 3) heat 4) reduces 5) less

$kg\ ms^{-1}$ $kg\ ms^{-1}$

6) friction 7) equal, opposite 8) vector 9) 10) acceleration
11) rate, momentum 12) magnitude, direction
13) defines 14) measure 15) external force
16) Galileo's 17) applied force 18) conserved
19) changing momentum 20) second and third

IV. Multiple Choice Questions:

1) d
2) a $F = \frac{dp}{dt}, KE = \frac{p^2}{2m}$ $F = \frac{dp}{dt}, KE = \frac{p^2}{2m}$
3) d Work-Energy theorem $KE_f - KE_i = workdone = fs$
$KE_f - KE_i = workdone = fs$
KE is same and retarding force is equal, distance travelled will be same
4) b $v = \sqrt{2gh}$ $v = \sqrt{2gh}$ is independent of mass
5) c
6) c
7) c
8) c
9) a
10) b
11) b
12) a
13) b
14) d
15) a $mg - R = ma;\ mg - nmg = ma => a = g(1-n)$
$mg - R = ma;\ mg - nmg = ma => a = g(1-n)$
16) c When dropped acceleration of ball=g observed by man standing stationary on the ground. The man inside the lift is having own downward acceleration=a, therefore relative acceleration of the ball as observed by man in the lift = g - a
17) a
18) d
19) d This is the equilibrium of coplanar forces. Hence

$$\sum F_x = 0$$

$$F = N$$

$$\sum F_y = 0$$

$f = mg$
$\sum \tau_c = 0$
$\tau_N + \tau_f = 0$
$\tau_N \neq 0, \tau_f \neq 0$

20) d $ma\cos\alpha = mg sin\alpha ma\cos\alpha = mg sin\alpha$
$=> a = g\tan\alpha$

V. Match the following:

1) i – b ii – c iii – a iv – e v – f vi - d
2) i – c ii – d iii – b iv – e v - a
3) i – b ii – d iii – a iv – c
4) i – d ii – a iii – c iv – b
5) i – a ii – c iii – b iv – d
6) i – c ii – b iii – d iv – a
7) i – b ii – d iii – a iv – c
8) i – d ii – a iii – c iv – b
9) i – d ii – c iii – b iv – a
10) i – b ii – d iii – a iv – c

VI. Problems (Level – I)

1) $p = a + btp = a + bt$
$F = \frac{dp}{dt} = 0 + b(1) = b$

2) $F = \frac{m\Delta v}{t} F = \frac{m\Delta v}{t}$
$t = \frac{m\Delta v}{F} = \frac{10 \times 2}{5} = 4sec$

3) $F = \frac{m(v-u)}{t} = \frac{3(3.5-2)}{25} = 0.18NF = \frac{m(v-u)}{t} = \frac{3(3.5-2)}{25} = 0.18N$
$F = +ve => F\ is\ the\ direction\ of\ motion\ of\ the\ body$

4) $impulse\ J = \Delta p = m(v-u) impulse\ J = \Delta p = m(v-u)$
$J = m\left(\sqrt{2gh} - \left(-\sqrt{2gh}\right)\right) = \sqrt{8m^2gh}$

5) Block will not slide
$F = f$
$ma = \mu N$
$ma = \mu mg => \mu = \frac{a}{g} = \frac{1.5}{9.8} = 0.153$

6) $m_1 = 4kg, m_2 = 3+3 = 6kg$$m_1 = 4kg, m_2 = 3+3 = 6kg$

$common\ acceleration = \frac{(m_2 - m_1)}{(m_2 + m_1)} g = \frac{(6-4)}{(6+4)} 10 = 2\ m/s^2$

7) $F = \frac{m\Delta v}{t}$$F = \frac{m\Delta v}{t}$

$t = \frac{m\Delta v}{F} = \frac{20 \times 15}{50} = 6sec$

8) *Intial thrust of the blast**Intial thrust of the blast*

$F = m(g+a)$

$F = 3 \times 10^5 N$

9) $a = \frac{-v^2}{2s} = \frac{-90\times 90}{2\times 0.6} = -6750 m/s^2$$a = \frac{-v^2}{2s} = \frac{-90\times 90}{2\times 0.6} = -6750 m/s^2$

the resistive force F=ma = 0.04$\times 6750 = 270N$$\times 6750 = 270N$

10) $y = ut + \frac{1}{2}gt^2$$y = ut + \frac{1}{2}gt^2$

$v = \frac{dy}{dt} = u + gt$

$a = \frac{dv}{dt} = g$

$Force = ma = mg$

11) Change in momentum = impulse

$J = mv - mu$

$J = 0.15 \times 12 - (-0.15 \times 12)$

$J = 1.80 + 1.80$

$J = 3.6 N.s$

12) $f_s = \mu_s N$$f_s = \mu_s N$

$ma = \mu_s mg$

$a = \mu_s g = 0.15 \times 10 = 1.5 m/s^2$

13) $F = \frac{m(v-u)}{\Delta t} = \frac{63(0-8.5)}{3} = 180N$$F = \frac{m(v-u)}{\Delta t} = \frac{63(0-8.5)}{3} = 180N$

14) $Change\ in\ momentum = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v} - \vec{u})$

$Change\ in\ momentum = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v} - \vec{u})$

$\Delta\vec{p} = 0.15\ (-(3\vec{i} + 4\vec{j}) - (3\vec{i} + 4\vec{j}))$

$\Delta\vec{p} = -(0.9\vec{i} + 1.2\vec{j}) kg\ m/s$

15) Momentum before firing=0

momentum after firing = $m_b v_b - m_g v_g = 0$$m_b v_b - m_g v_g = 0$

$v_g = \frac{m_b v_b}{m_g} = \frac{0.02 \times 80}{100} = 0.016\ m/s$

16) $F = \sqrt{F_1^{\ 2} + F_2^{\ 2} + 2F_1F_2\cos\theta}$$F = \sqrt{F_1^{\ 2} + F_2^{\ 2} + 2F_1F_2\cos\theta}$

$\theta = 90^{\circ} \Rightarrow F = \sqrt{F_1^{\ 2} + F_2^{\ 2}} = \sqrt{16^2 + 12^2} = 20N$

$a = \frac{F}{m} = \frac{20}{200} = 0.1\ m/s^2$

17) $a = \frac{(m_1-m_2)}{(m_1+m_2)}g = \frac{g}{8}a = \frac{(m_1-m_2)}{(m_1+m_2)}g = \frac{g}{8}$

$\frac{(m_1 - m_2)}{(m_1 + m_2)} = \frac{1}{8}$

$8m_1 - 8m_2 = m_1 + m_2 m_1 - 8m_2 = m_1 + m_2$

$7m_1 = 9m_2$

$=> \frac{m_1}{m_2} = \frac{9}{7}$

18) Total change in momentum per second = weight of dish

$5 \times 10(2v) = 10 \times 980$

$v = 98\, cm/s$

19) Impulse on each ball $J = 2mv = 2 \times 0.05 \times 6 = 0.6Ns J = 2mv = 2 \times 0.05 \times 6 = 0.6Ns$

VII. Problems (Level – II)

1) $w_1 = mg\left(1 + \frac{a}{g}\right) = mg\left(1 + \frac{g/3}{g}\right) = \frac{4}{3}mg w_1 = mg\left(1 + \frac{a}{g}\right) = mg\left(1 + \frac{g/3}{g}\right) = \frac{4}{3}mg$

$w_2 = mg\left(1 - \frac{a}{g}\right) = mg\left(1 - \frac{g/2}{g}\right) = \frac{1}{2}mg$

$\frac{w_2}{w_1} = \frac{\frac{mg}{2}}{\frac{4}{3}mg} = \frac{3}{8}$

2) $F = mg(\sin\theta + \mu_k \cos\theta) F = mg(\sin\theta + \mu_k \cos\theta)$

$F = 2 \times 9.8(\sin 30^\circ + \frac{\sqrt{3}}{2}\cos 30^\circ)$

$F = 2 \times 9.8 \times \frac{5}{4} = 24.5N$

3) $\tan\theta = \mu_s = \frac{dy}{dx}\tan\theta = \mu_s = \frac{dy}{dx}$

$y = \frac{x^2}{20} => \frac{dy}{dx} = \frac{2x}{20}$

$=> 0.5 = \frac{2x}{20} => x = 5 => 0.5 = \frac{2x}{20} => x = 5$ m

$then\ y = \frac{x^2}{20} = 1.25m$

4) Acceleration of system

$a = \frac{F}{m_A + m_B} = 2m/s^2$

upper block is accelerated by force of friction acting upon it

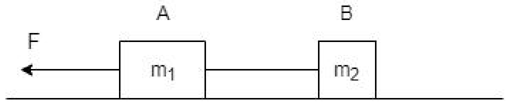
$f = m_B a = 5 \times 2 = 10N$

5) When F is applied on $m_1 m_1$, tension $T = \frac{m_2 F}{m_1+m_2} = \frac{20\times 600}{10+20} = 400N$

$T = \frac{m_2 F}{m_1+m_2} = \frac{20\times 600}{10+20} = 400N$

When F is applied on $m_2 m_2$, tension $T = \frac{m_1 F}{m_1+m_2} = \frac{10\times 600}{10+20} = 200N$

$T = \frac{m_1 F}{m_1 + m_2} = \frac{10 \times 600}{10+20} = 200N$



6) $a = \frac{(m_1 - m_2)}{(m_1+m_2)} g = (\frac{12-8}{12+8}) g = 2\ m/s^2 a = \frac{(m_1 - m_2)}{(m_1+m_2)} g = (\frac{12-8}{12+8}) g = 2\ m/s^2$

$T = \frac{2 m_1 m_2}{(m_1 + m_2)} g = \frac{(2 \times 12 \times 8)}{12+8} g = 96N$

7) Velocity $v = 2\pi \times \frac{40}{60} \times 1.5 = 2\pi \frac{m}{s} v = 2\pi \times \frac{40}{60} \times 1.5 = 2\pi \frac{m}{s}$

Tension $T = \frac{mv^2}{R} = \frac{0.25 \times 2\pi^2}{1.5} 6.6NT = \frac{mv^2}{R} = \frac{0.25 \times 2\pi^2}{1.5} 6.6N$

$maximum\ tension\ T_{max} = \frac{m{v_{max}}^2}{R}$

=> $200 = \frac{0.25\ {v_{max}}^2}{1.5} => v_{max} = 35m/s 200 = \frac{0.25\ {v_{max}}^2}{1.5} => v_{max} = 35m/s$

8) $f = \frac{mv^2}{R} f = \frac{mv^2}{R}$

$\mu_s N = \frac{mv^2}{R}$

$\mu_s mg = \frac{mv^2}{R}$

$v^2 = \mu_s Rg = 0.1 \times 3 \times 9.8 = 2.94\ m^2/s^2$

v = 18km/h=5m/s => $v^2 = 25\ m^2/s^{-2} v^2 = 25\ m^2/s^{-2}$

the condition is not obeyed; the cyclist will slip while taking the circular turn

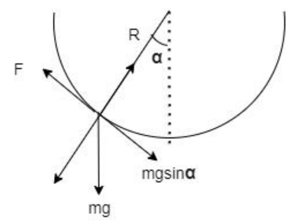
9) As clear from the figure

$F = mg \sin\alpha$

R = $mg \cos\alpha mg \cos\alpha$

=> $\frac{F}{R} = \tan\alpha \frac{F}{R} = \tan\alpha$

$\mu = \tan\alpha = \frac{1}{3} => \cot\alpha = 3$



10) Net force on rod $f = 32 - 20 = 12N f = 32 - 20 = 12N$

$acceleration\ of\ rod = \frac{f}{m} = \frac{12}{3} = \frac{4m}{s^2}$

$equaation\ of\ motion\ of\ 10cm\ part\ is$

$F - 20 = ma = 1 \times 4 => F = 24N$

$Similarly\ equation\ of\ motion\ of\ 20cm\ part\ is$

$32 - F = ma = 2 \times 4 => F = 32 - 8 = 24N$

11) $S = ut + \frac{1}{2}at^2 S = ut + \frac{1}{2}at^2$

$50 = \frac{1}{2}(a)5^2$

$=> a = 4\, m/s^2$

$a = \frac{(m_1 - m_2)}{(m_1 + m_2)} g => \frac{(m_1 - m_2)}{(m_1 + m_2)} = \frac{a}{g} = \frac{4}{10} = \frac{2}{5}$

$\%\ of\ mass\ transferred\ \frac{(m_1 - m_2)}{(m_1 + m_2)} \times 100 = 40\%$

12) Limiting force of friction of block on slab = $\mu_1 mg \mu_1 mg$ = $0.6 \times 10 \times 9.8 = 58.8N$
$0.6 \times 10 \times 9.8 = 58.8N$

acceleration of slab =a = $\frac{\mu_1 m_1 g}{m_2} = \frac{0.4 \times 10 \times 9.8}{40} = 0.98\, m/s^2$
$\frac{\mu_1 m_1 g}{m_2} = \frac{0.4 \times 10 \times 9.8}{40} = 0.98\, m/s^2$

13) $F = 600 - 2 \times 10^5 t F = 600 - 2 \times 10^5 t$

at $t = 0, F = 600N t = 0, F = 600N$

$F = 0\ on\ leaving\ the\ barrel$

$0 = 600 - 2 \times 10^5 t => t = 3 \times 10^{-3} sec$

this is the time spent by bullet in the barrel

average force = $\frac{600+0}{2} = 300N \frac{600+0}{2} = 300N$

average impulse imparted = $FtFt$= 300$\times 3 \times 10^{-3} = \times 3 \times 10^{-3} =$ 0.9Ns

14) $Let\ l\ be\ the\ natural\ length\ of\ spring Let\ l\ be\ the\ natural\ length\ of\ spring$

$then\ 5 = k(\alpha - l) and\ 4 = k(\beta - l) => k = \frac{1}{\alpha - \beta}$ and $l = 5\beta - 4\alpha$

$kx = 9 => x = \frac{9}{k} = 9(\alpha - \beta)$

$length = l + x = 5\beta - 4\alpha + 9\alpha - 9\beta = 5\alpha - 4\beta$

15) $a)\ In\ all\ three\ intervals\ acceleration\ and\ force\ are\ zero$

$a)\ In\ all\ three\ intervals\ acceleration\ and\ force\ are\ zero$

b) at t=0 sec, impulse $J = m\Delta v J = m\Delta v$

J=m$\times\ slope\ of\ x - t\ graph \times\ slope\ of\ x - t\ graph$

$J = 4 \times \frac{3}{4} J = 4 \times \frac{3}{4}$ = 3kgm/s

at t=4s, impulse = $m\Delta v m\Delta v$

$J = 4 \times \frac{-3}{4} = -3kg\, m/s$

VIII. Statement Type Questions

1) a

2) c

3) c

4) b

5) b

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Synopsis

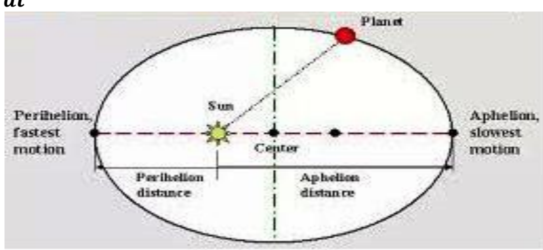
- Ptolemy proposed **geocentric** theory in his "Geography" book. According to this, all the planets along with sun revolve around the earth with earth as centre. Copernicus proposed **heliocentric** theory; all planets along with earth revolve around the sun in circular orbit with sun as the centre.

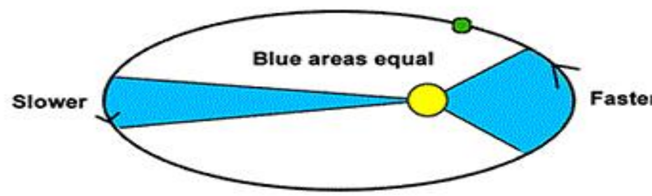
  Kepler's laws of planetary motion
- **1^St law (law of orbit):** Every planet revolves around the sun in an elliptical orbit with sun as one of its foci.

  The perihelion distance R_{min}= (1-e) a

  The Aphelion distance R_{max}= (1+e) a, for circle e=0 and for earth e=0.017.
- **II^nd law (law of area):** The radius vector joining the planet and the sun sweeps equal areas in equal intervals of time. (OR) The areal velocity of the radius vector joining the planet and the sun is constant. It is the consequence of conservation of angular momentum.

  $\frac{dA}{dt}$ = (1/2) rv = (1/2)$r^2\omega$= (L/2m)= constant.





- **III^rd Law (Law of time period):** The square of period of revolution of planet around the sun is proportional to cube of the average distance of planet around the sun. i.e. $T^2 \propto a^3$

  Newton's law of Gravitation
- The force of attraction between two masses is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. i.e. $F= Gm_1m_2/R^2$ G is the universal gravitational constant = 6.67×10^{-11} Nm^2kg^{-2}. It is a scalar quantity with dimensional formula as $M^{-1}L^3T^{-2}$.The gravitational interaction is the first force discovered in nature, is the weakest one among all forces, central and conservative force, is independent of the medium between the particles, obeys Newton's third law and principle of superposition.

  If two masses revolving around their centre of mass, then they have same angular velocity and follows $m_1r_1=m_2r_2$.
- The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the shell is concentrated at the centre of the shell.
- The force of attraction due to a hollow spherical shell of uniform density, on a point mass situated inside it is zero.
- If two identical spheres of radius r made up of same material are kept in contact with each other, then the gravitational force between them is $F \propto r^4$
- The value of G was experimentally determined by Henry Cavendish.
- The concept of gravitational field is used to overcome the difficulties encountered in universal law of gravitation. Einstein considered gravitational field as a distortion of space due to presence of matter.

Gravitational field strength

- It is defines as the gravitational force acting on a unit mass kept at a point in the gravitational field. $E_G = GM/R^2$, units- N/kg and dimensional formula is LT^{-2}. It is a vector quantity, always directed radially towards the centre of mass of the body producing the field, and follows the principle of superposition.
- In earth's gravitational field, the acceleration due to gravity may be considered as the gravitational field strength.
- A quantum of energy is associated with gravitational field called graviton, like photon in electromagnetic field.
- Gravitational field intensity due to circular ring at any point on its axis is $E_G = GMx/(R^2+x^2)^{3/2}$
  E_G is zero at the centre of the ring and maximum at $x=R/\sqrt{2}$

Gravitational potential

The amount of work done in bringing a unit mass from infinity to a certain point in the gravitational field of another massive object is called Gravitational potential.

The gravitational potential V= W/M =-GM/R, where R is the distance of the point from the centre of the given planet of mass M.

Gravitational potential is always negative, with the maximum of zero at infinity. Negative means force is opposite to displacement.

- Gravitational potential due to circular ring at any point on its axis is $V = -GM/(R^2+x^2)^{1/2}$
  Gravitational potential due to a spherical shell of radius R is $V_{inside} = V_{surface} = V_{centre} = -GM/R$
  $V_{outside} = -GM/x$, where x>R and at infinity potential become zero.
- Gravitational potential due to solid sphere $V_{inside} = -(GM/2R^3)(3R^2-x^2)$ and $V_{surface} = -GM/R$
  $V_{outside} = -GM/x$, at the centre $V_{centre} = -\frac{3GM}{2R} = V_{surface} \times \frac{3}{2}$

Gravitational potential energy

The amount of work done in bringing an object mass (m) from infinity to a certain point in the gravitational field of another massive object is called Gravitational potential energy. i.e. W=V.m

ΔU = -GMm/R

ΔU may decrease, when two bodies are moving away and is increase when two bodies are bring towards.

- The gravitational potential energy of a mass m at a point above the surface of the earth at a height h is given by U= -GMm/(R+h).The negative sign shows that if h increases, the gravitational PE decreases and becomes zero at infinity.
- Change in gravitational potential energy in lifting an object from the surface of a planet to an altitude h is given by $\Delta U = GMm(\frac{1}{R} - \frac{1}{R+h})$
  If h<<R then ΔU =mgh and ΔU = mgR when h>>R

Acceleration due to gravity

- The acceleration of a freely falling body is simply called acceleration due to gravity.
  The relation between g and G is $g = GM/R^2$, where M and R are the mass, radius of the earth.
  g depends on the mass of the planet and its radius, is independent of the mass of the body.
  Acceleration due gravity is maximum on Jupiter and minimum on Mercury. Line joining places on the earth having same values of g are called isograms. The value of the acceleration due to

gravity on the moon is about one sixth of that on the earth and on the sun is about 27 times that on the earth.

- If ρ is the mean density of the earth, g = $\frac{4}{3}\pi R\rho G$,

Effect of altitude on g

- The g value on the surface of the earth is $g = GM/R^2$ and at height h is $g_h = GM/(R+h)^2$

For very small altitudes (h<<<R) $g_h = g\,(1 - \frac{2h}{R})$

The fractional change in value of g is $\frac{\Delta g}{g} = \frac{2h}{R}$

Effect of depth on g

- The acceleration due to gravity at depth d is $g_d = g\,(1 - \frac{d}{R})$

As altitude increases g decreases, similarly as depth increases g decreases. In order to produce the same change in the value of g, then d=2h (for small values only). The value of acceleration due to gravity at the centre of the earth is zero (d=R).

Effect of rotation of earth (latitude effect)

- If g^1 is the acceleration due to gravity due to the rotation o the earth, then $g^1 = g - R\omega^2 Cos^2\emptyset$

Where ω is the angular velocity, R is the radius of the earth and Ø is the latitude of the place.

At poles Ø = 90, hence $g^1 = g$ is maximum indicates g is independent of rotation of earth at poles.

At the equator Ø = 0˚, hence $g^1 = g - R\omega^2$ is minimum.

- If the angular velocity of the earth becomes 17 times of its present value, then the acceleration due to gravity at the equator becomes zero. Hence the bodies will escape out from the equator.

The angular velocity of the earth $\omega = \sqrt{\frac{g}{R}} = 1.25\times10^{-3}$ rad/s

Effect of shape of the earth

- The shape of the earth is not exactly spherical. It is flattened at poles and bulged at equator. The equatorial radius is larger than pole radius (The polar radius is 21 km less than the equatorial radius). Hence g is minimum at equator and maximum at poles. The acceleration due to gravity changes with local conditions indicates the presence of minerals inside the earth.
- If the radius of planet decreases by n%, keeping the mass unchanged, the acceleration due to gravity on its surface increases by 2n%. i.e., $\Delta g/g$ = -2 $\Delta R/R$
- If the mass of the planet increases by m% keeping the radius constant, the acceleration due to gravity on its surface increases by m% i.e., $\Delta g/g = \Delta M/M$ where R = constant.
- If the density of planet decreases by ρ% keeping the radius constant, the acceleration due to gravity decreases by ρ%.
- If the radius of the planet decreases by r% keeping the density constant, the acceleration due to gravity decreases by r%.

Orbital velocity

- It is the velocity of object to be projected so that it revolves around the earth. In the orbit the gravitational force between the earth and the body is balanced by the centripetal force.
- The orbital velocity V_o of the body in the orbit at height h is $V_o = \sqrt{\frac{GM}{(R+h)}} = [(R+h)g_h]^{1/2}$

If body moves around the earth close to the surface of the earth, the orbital velocity $V_o = \sqrt{(gR)}$ is around 7.92km/sec. The orbital velocity doesn't depend on mass of the body, depends upon the mass and radius of the planet indicates the satellites of different masses have same orbital velocity, if they are in the same orbit.

- The orbital angular velocity $\omega_o = \frac{V_o}{(R+h)} = [\frac{g}{(R+h)}]^{1/2}$ and the time period $T = 2\pi\sqrt{[(R+h)/g_h]}$
- The frequency of revolution $n = \frac{1}{2\pi}[GM/(R+h)^3]^{1/2}$

Escape velocity

- It is the minimum velocity given to the body in order to escape from the gravitational field. It is based on conservation of energy. The KE given to the body on the surface of the planet must be equal to its gravitational potential energy at infinity.
- Escape velocity $V_e = \sqrt{\frac{2GM}{(R+h)}} = [2(R+h)g_h]^{1/2}$
- If the object is thrown from the surface of the earth then h=0, and $V_e = \sqrt{(2gR)}$ = 11.2km/sec.
- The escape velocity doesn't depend upon the mass of the body and angle of projection, depends on the mass and radius of the planet.
- The rms velocity of the molecules is greater than the escape velocity of the moon (2.31 km/sec). Hence the molecules escape from the influence of moon's gravitational field and the atmosphere is absent on the surface of the moon.
- Escape velocity = $\sqrt{2}$ ×Orbital velocity.
- The escape velocity of sun is maximum (larger than rms velocity). Hence even the lightest molecules cannot escape from there. Sufficient amount of hydrogen is present in the atmosphere of sun.
- If a body falls from the infinity, then it reaches the earth with a velocity of 11.2 km/sec.
- On doubling the energy of the satellite revolving near the earth, the satellite would escape from the earth.
- For a satellite revolving close to the earth KE= GMm/2R and PE= - GMm/R , TE= -2GMm/2R
Simply PE: KE: TE= -2:1:-1
- When the total energy of a satellite is zero (KE=PE), it escapes into the space in parabolic path.
- When a satellite is lifted from a lower orbit to higher orbit, then its PE increases and KE decreases. The amount of work done in lifting a body from the surface of the earth to a height h is given by $W = \frac{mgh}{1+\frac{h}{R}}$

Geostationary and polar satellites

- The satellite which remains stationary with respect to earth is called geostationary satellite.
- The time period of geostationary satellite is 24 hours and same direction of earth (west to east).
- The height of G-orbit is 36000 km and the radius is around 42000 km.
- Polar satellites are low altitude satellites (500km to 800km) and go round the poles of the earth in north-south direction has time period of 100 minutes. These are used to remote sensing, meteorology and environmental studies of the earth.

Weightlessness

- It is a phenomenon in which the object is in a state of free fall. Floating of astronauts in a satellite illustrate this phenomenon. In satellites the acceleration towards the centre of the earth is exactly equal to the gravity at that position.

- Then the appeared weight W_{app} = m (g-a) =0.
- This is experienced only when the effective gravitational attraction on the astronauts is negligible. It is possible when the satellite mass is very less itself. This is the reason why an astronaut does not feel weightless ness on moon, whereas he feels weightless in an artificial satellite.

❖ One word answer questions

1. According to Kepler's second law, which quantity remains constant…………………………………………..
2. The maxima and minimum velocity points in an orbit of the satellite are called ………………………………………………………………….respectively.
3. The values of acceleration due to gravity of the earth and universal gravitational constants are …………………………………………………………………
4. The value of acceleration due to gravity in the centre of the earth is……………………………………………
5. Line joining the same value of g are called……………………………………………………………………………
6. The work done to bring a mass of an object from infinity to a point in the gravitational field is called……………………………………………
7. The relation between orbital velocity and escape velocity is………………………………………………………
8. There is no atmosphere on moon due to ……………………………………………………………………………..
9. For a satellite orbiting close to the surface of the earth, PE: KE: TE=…………………………………………..
10. The relative speed and time period of revolution of geostationary satellite with respect to earth are……………………………………….
11. Feeling of weightlessness in a satellite is due to……………………………………………………………………
12. Two spheres of masses M_1 and M_2 are situated in the air and the gravitational force between them is F, If the distance is increased by 2 times, then the Gravitational force ……………………………
13. When the acceleration due to gravity at height H is equal to the acceleration due to gravity at depth D then the relation between D and H is……………………………………………………………………….
14. Escape velocity is based on the conservation of ………………………………………………………………….

❖ True/ False Questions

1. According to heliocentric theory, the earth is the centre of the universe, and all planets as well as sun revolves around the earth.
2. Find the correct statement,
   (a) Every planet revolves around the sun in circular orbit with sun is at the centre
   (b) Every planet revolves around the sun in elliptical orbit with sun is at one of its foci
   (c) Every planet revolves around the earth in circular orbit with earth is at the centre
   (d) Every planet revolves around the earth in elliptical orbit with earth is at one of its foci
3. Find the correct equation,
   (a) $T \propto R^2$ (b) $T^2 \propto R^3$ (c) $T^2 \propto R^2$ (d) $T^3 \propto R^2$
4. Find the wrong statement, Gravitational forces are
   (a) Central forces (b) Doesn't obey Newton's III law
   (c) Conservative force (d) Obeys super position principle
5. The acceleration due to gravity is maximum at equator and minimum t the poles.
6. Find the correct statement of Gravitational potential energy is
   (a) Decreasing when two bodies are moving away
   (b) Increasing when two bodies are moving away
   (c) Decreasing when two bodies are moving towards
   (d) Increasing when two bodies are moving towards

7. Find the correct statement, If the earth suddenly stops its rotation,
   (a) The acceleration due to gravity at poles increases, and at equator remains constant
   (b) The acceleration due to gravity at poles and equator remains constant
   (c) The acceleration due to gravity at poles remains constant, and at equator increases
   (d) The acceleration due to gravity at poles and equator increases.
8. If a body falls freely from infinity, then it reaches the earth with 11.2 km/sec.
9. Find the wrong statement
   (a) If the value of g is zero at equator, then the angular velocity increased by 17 times
   (b) The value of g at poles is independent of the rotation of the earth
   (c) Due to rotation of the earth, g is maximum at poles and minimum at equator
   (d) The acceleration due to gravity doesn't depend upon density and radius
10. Find the wrong statement, the orbital velocity
    (a) Doesn't depend on the mass of the body
    (b) Depends upon the mass and radius of the planet
    (c) increases with increase of height
    (d) decreases with increase of radius of the orbit
11. Find the wrong statement, the escape velocity
    (a) Doesn't depend on the mass of the body
    (b) Doesn't depend on the angle of projection
    (c) Depends upon the mass and radius of the planet
    (d) Is $\sqrt{2}$ times smaller than orbital velocity
12. The moon is the only satellite of the earth with time period around 27 days and rotational period of around 27 days about its own axis.
13. Find the wrong statement
    (a) Law of gravitation is framed base on the Newton's IIIrd law of motion
    (b) Law of gravitation cannot explain why gravity exists
    (c) Law of gravitation does not explain the presence of force even when the particle are not in physical contact
    (d) When the range is long, gravitational force become repulsive
14. Find the correct statement, If earth rotates faster than its present, then the weight
    (a) Increases at equator but same at poles
    (b) Decreases at equator but same at poles
    (c) Increases at poles but same at equator
    (d) Decreases at poles but same at equator
15. Find the correct statement, The earth is flatten at poles and bulging at equator due to
    (a) Centrifugal force is more at equator than poles
    (b) Centrifugal force is less at equator than poles
    (c) Angular velocity of spinning about its axis is more at equator
    (d) Revolution of earth around the sun in elliptical orbit

16. Find the correct statement, When height of the satellite increases from the earth, then its
    (a) PE increases, KE decreases
    (b) PE decreases, KE increases
    (c) PE increases, KE increases
    (d) PE decreases, KE decreases
17. Find the correct statement, Synchronous satellite moving
    (a) From east to west in equatorial plane
    (b) From west to east in equatorial plane
    (c) From south to north in polar plane
    (d) From north to south in polar plane
18. Find the correct statement, If gravitational force of attraction becomes zero for revolving satellite, then the satellite will
    (a) Become stationary in the orbit
    (b) Continue to move in its orbit with same velocity
    (c) Move tangentially to the original orbit with same velocity
    (d) Move towards the earth
19. Find the correct statement, If there was no atmosphere on the earth, then
    (a) The duration of day light decreases
    (b) The duration of day light increases
    (c) The duration of day light remains same
    (d) The duration of day light depends upon weather

❖ ***Multiple choice Questions Level-I***

1. The intensity of gravitational field of the earth is maximum at
   (a) Equator (b) Poles (c) Centre of the earth (d) every where
2. Which of the following quantity remains constant when earth revolves around the sun
   (a) Velocity (b) Energy (c) Linear momentum (d) Angular momentum
3. The force of attraction between two objects separated by certain distance is F, If mass of one object is doubled and of the other is halved, and the distance between them is tripled, then the new force of attraction is
   (a) F/3 (b) 3F (c) 9F (d) F/9
4. If F_1 and F_2 are the magnitudes of forces exerted by the earth on the sun and exerted by the sun on the earth respectively then
   (a) $F_1 = F_2$ (b) $F_1 < F_2$ (c) $F_1 > F_2$ (d) $F_1 \geq F_2$
5. If g is the acceleration due to gravity, R is the mean radius of the earth, then the density of the earth is
   (a) $4\pi RG/3g$ (b) $3\pi RG/4g$ (c) $3g/4\pi RG$ (d) $4g/3\pi RG$
6. If the earth stops spinning about its own axis, then the weight of the body
   (a) Decreases (b) Increases (c) remains same (d) becomes zero

7. Two planets move around the sun. The time periods and mean orbital radii of their orbits are T_1, T_2, R_1, R_2 respectively. Then the ratio of T_2/T_1 is
(a) R_1/R_2 (b) R_2/R_1 (c) $(R_1/R_2)^{3/2}$ (d) $(R_2/R_1)^{3/2}$
8. The kinetic energy of a satellite in its orbit around the earth is E. What is the kinetic energy of the satellite so as to enable it to escape from the gravitational pull of the earth?
(a) 2E (b) E^2 (c) 4E (d) $E^2/2$
9. Inside the satellite the weight of the body is determined by
(a) Spring balance (b) Pan balance (c) Digital balance (d) None of the above
10. If the area swept by the line joining the sun and the earth from Feb1 to Feb7 is A. Then the area swept by the radius vector from Feb8 to Feb28 is
(a) A (b) 2A (c) 3A (d) 9A
11. The time period of simple pendulum at the centre of the earth is
(a) Zero (b) 2 sec (c) Infinite (d) None of the above
12. When satellite loses its energy, then
(a) Radius and velocity increases (b) Radius decreases and velocity increases
(c) Radius and velocity decreases (d) Radius increases and velocity decreases
13. The escape velocity for a body projected vertically upwards from the surface of the earth is 11 km/s. If the body is projected at an angle of 45˚ with the vertical, then the escape velocity is
(a) 11km/s (b) $11/\sqrt{2}$ km/s (c) $11\sqrt{2}$ km/s (d) 22 km/s
14. When projectile attains escape velocity, then its
(a) KE = PE (b) KE > PE (c) KE < PE (d) KE = 2 PE
15. A satellite is moving with constant speed V in a circular orbit about earth. At the time of ejection the kinetic energy of the satellite is
(a) $\frac{1}{2}mv^2$ (b) mv^2 (c) $\frac{3}{2}mv^2$ (d) $2mv^2$
16. The period of revolution of an earth's satellite close to the surface of the earth is 90 minutes. The period of another earth's satellite in an orbit at a distance of three times earth's radius from its surface will be
(a) 90 minutes (b) 270 minutes (c) 720 minutes (d) $90\times\sqrt{8}$ minutes
17. If the radius of the earth is 6400km, the depth at which the acceleration due to gravity will be 5% less than the value on the surface of the earth.
(a) 320km (b) 200km (c) 480km (d) 500km
18. The kinetic energy needed to project a body of mass m from earth's surface (radius R) to infinity
(a) mgR/2 (b) mgR (c) mgR/4 (d) 2mgR
19. How much faster (around) than its normal rate should the earth rotate about its axis so that the weight of the body at equator becomes zero(R=6400km)
(a) 12 times (b) 17 times (c) 10 times (d) 14 times
20. If the mass of one planet is increased by 50% and the mass of another body is decreased by 50%, then the force between them will be
(a) Decreased by 25% (b) Increased by 25% (c) Increased by 50% (d) no change

21. If the radius of earth decreased by 10%, the mass remaining unchanged, then the acceleration due to gravity
    (a) Decreased by 19% (b) Increased by 19%
    (c) Decreased by more than 19% (d) Increased by more than 19%
22. In planetary motion the areal velocity of position vector of a planet depends on angular velocity (ω) and the distance of the planet from sun (r). If so, the correct relation for areal velocity is
    (a) $dA/dt \propto \omega r$ (b) $dA/dt \propto \omega r^2$ (c) $dA/dt \propto \omega^2 r$ (d) $dA/dt \propto \omega^2 r^2$
23. The period of moon's rotation around the earth is nearly 29 days. If moon's mass were 2 fold its present value and all other things remain unchanged, the period of moon's rotation would be nearly
    (a) 29 days (b) 48 days (c) $29\sqrt{2}$ days (d) $48\sqrt{2}$ days
24. If the distance of earth is halved from the sun, then the no. of days in a year will be
    (a) 730 days (b) 129 days (c) 182.5 days (d) 365 days
25. The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on planet B. A man jumps to a height of 2m on the surface of A. What is the height of jump by the same person on the planet B?
    (a) 28m (b) 9m (c) 18m (d) 12m
26. A uniform spherical shell gradually shrinks maintaining its shape. The gravitational potential at the centre
    (a) increases (b) decreases (c) remains same (d) Zero

❖ *Multiple choice Questions Level-II*

1. The values of acceleration of a free fall on the surface of the two planets are the same provided that the planets have same
   (a) Mass (b) Radius (c) Mass/Radius (d) Mass/ $radius^2$
2. The time period of a satellite in a circular orbit of radius R is T. The time period of another satellite revolving in a circular orbit of radius 5R will be
   (a) 5T (b) $\sqrt{5}T$ (c) $5\sqrt{5}T$ (d) T/5
3. The escape velocity from the earth surface is 11km/s. A certain planet has a radius nine times that of the earth and its mean density is one third of the same as that of the earth. The value of escape velocity from this planet would be
   (a) 11 km/s (b) 22 km/s (c) 33 km/s (d) $11\sqrt{3}$ km/s
4. If the earth shrinks such that its density becomes 8 times to the present value then the new duration of the day will be
   (a) 6 hours (b) 12 hours (c) 18 hours (d) no change
5. A particle on earth's surface is given a velocity equal to its escape velocity. Its total mechanical energy will be
   (a) Negative (b) Zero (c) Infinite (d) positive

6. A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth, then T_2/T_1 is
   (a) 1 (b) 2 (c) 3 (d) 4
7. The time period of a satellite of earth is 5 hours. If the separation is increased by 3 times the previous value, then the increased time period will be
   (a) 40 hour (b) 20 hour (c) 35 hour (d) 80 hour
8. An electron moves in a circular orbit at a distance from a proton with kinetic energy E. To escape to infinity, the additional energy must be supplied to the electron is
   (a) E (b) 2E (c) 3E (d) 0.5E
9. A satellite is moving around the earth with velocity v. To make the satellite escape, the minimum percentage increase in its velocity is
   (a) 82% (b) 41.4% (c) 2% (d) 100%
10. The escape velocities on two planets of masses M_1 and M_2 and having same radius, are V_1 and V_2 respectively, then V_1/V_2 is equal to
   (a) M_1/M_2 (b) M_2/M_1 (c) $(M_1/M_2)^2$ (d) $(M_1/M_2)^{1/2}$
11. The energy required to move a body of mass m from an orbit of radius 2R to 3R is
   (a) GMm/5R (b) GMm/6R (c) $GMm/6R^2$ (d) $GMm/5R^2$
12. A mass M is split into two parts m and (M- w), which are then separated by a certain distance. What ratio of m/M maximizes the gravitational force between the two parts?
   (a) 1/2 (b) 1/3 (c) 1/4 (d) 1/5
13. If the radius of the earth were to shrink by 1 %, with its mass remaining the same, the acceleration due to gravity on the earth's surface would be
   (a) Decreased by 1% (b) increased by 1% (c) increased by 2% (d) Decreased by 2%
14. A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is v. Due to the rotation of planet about its axis the acceleration due to gravity g at equator is 1/2 of g at poles. The escape velocity of a particle on the pole of planet in terms of v is
   (a) 2v (b) v (c) v^2 (d) $\sqrt{2v}$

❖ ***Match the fallowing***

1.
(A) law of Orbit — (P) Angular momentum constant
(B) Law of Area — (Q) Elliptical
(C) Law of time period — (R) inversely depend distance^2
(D) Gravitational law — (S) $T^2 \propto R^3$

2.
(A) Perigee — (P) Earth
(B) Apogee — (Q) Sun
(C) Ptolemy — (R) Longest distance
(D) Copernicus — (S) Shortest distance

3.
(A) $e = 1$ & $TE > 0$ — (P) Parabolic
(B) $e < 1$ & $TE < 0$ — (Q) Circular
(C) $e = 0$ & $TE < 0$ — (R) Elliptical
(D) $e = 0$ & $TE = 0$ — (S) Hyperbolic

4.
(A) $r = 0$ — (P) $-GM/R$
(B) $r = R/2$ — (Q) $-2GM/3R$
(C) $r = R$ — (R) $-11GM/8R$
(D) $r = 3R/2$ — (S) $-3GM/2R$

5.
(A) Universal gravitational constant — (P) g at equator, if earth stops rotating
(B) Increased by $R\omega^2$ — (Q) Scalar
(C) g doesn't change at poles — (R) Gravitational force
(D) Central force — (S) if earth stops rotating

6.
(A) On earth — (P) $g\left(1 - \frac{d}{R}\right)$
(B) Height — (Q) GM/R^2
(C) Depth — (R) $g\left(1 - \frac{2h}{R}\right)$
(D) Rotation — (S) $g - R\omega^2 \cos^2\phi$

7.
(A) Escape velocity — (P) 500-800km
(B) KE is doubled — (Q) 36000km
(C) Polar satellite — (R) body will escape
(D) Geostationary — (S) doesn't depend on angle of projection

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KEY

One word answer questions

Q.No	Answer
1	Areal velocity
2	Perigee and Apogee
3	$g=9.8\ m/sec^2$ $G= 6.67\times10^{-11}\ Nm^2/kg^2$
4	Zero
5	Isograms
6	Gravitational potential
7	Escape velocity $=\sqrt{2}$ Orbital velocity
8	The rms velocity is larger than escape velocity on the moon
9	PE: KE: TE= -2: 1: -1
10	Zero and infinity
11	Free fall of satellite
12	F/4
13	2H=D
14	Energy

True or False questions

Q.No	Answer
1	False
2	b
3	b
4	b
5	False
6	a
7	c
8	True
9	d
10	c
11	d
12	True
13	d
14	a
15	a
16	a
17	b
18	c
19	b

Multiple choice Level-I

Q.No	Answer
1	b
2	d
3	d
4	a
5	c
6	b
7	d
8	a
9	d
10	c
11	c
12	b
13	a
14	a
15	b
16	c
17	a
18	b
19	b
20	a
21	d
22	b
23	a
24	b
25	c
26	a

Multiple choice Level-II

Q.No	Answer
1	d
2	c
3	c
4	a
5	b
6	b
7	c
8	a
9	b
10	d
11	b
12	a
13	c
14	a

Match the followings

Q.No	Answer
1	A-Q, B-P, C-S, D-R
2	A-S, B-R, C-P, D-Q
3	A-S, B-R, C-Q, D-P
4	A-S, B-R, C-P, D-Q
5	A-Q, B-P, C-S, D-Q
6	A-Q, B-R, C-P, D-S
7	A-S, B-R, C-P, D-Q

MECHANICAL PROPERTIES OF SOLIDS

SYNOPSIS

Rigid body : When the external forces do not produce any deformation in the body is called rigid body . Ex. Diamond

Elasticity : It is the property of a material by virtue of which it resists strain when deforming forces are applied on it & recovered from strain from deforming forces removed.

Elastic Body : When the body regains its original shape and size completely after the removal of deforming forces then the body is said to be perfectly elastic . Example : Quartz.

Plastic body : If the body does not have any tendency to recover its original shape and size, it is said to be perfectly plastic Ex. Plasticine.

Stress : The restoring force per unit area set up inside the body is called stress.

Stress = $\frac{FORCE}{AREA}$ Dimensional formula : M $L^{-1}T^{-2}$ units N/m^2 or pascal

Strain : The relative charge produced in the body due to the influence of external force is called strain. It is a ratio. It has no units.

Stress is of three kinds : linear stress, shearing stress, volume stress.
Strain is of three kinds : longitudinal, shearing, volume.
The ratio of stress to strain is called modulus of elasticity.
The ratio of linear stress to longitudinal strain is called Young's modulus.

Young's Modulus y = $\frac{\frac{F}{A}}{\frac{\Delta L}{L}}$

The ratio of shearing stress to shearing strain is called modulus of rigidity.

Modulus of rigidity ϒ = $\frac{\frac{F}{A}}{\emptyset} = \frac{F}{A\,\emptyset}$

- The ratio of volume stress to volume strain is called Bulk Modulus
  - Bulk modulus B = $\frac{\Delta P}{-\frac{\Delta V}{V}}$
  - Compressibility is $\frac{1}{B}$ = - $\frac{\Delta V}{V \Delta P}$
- Shearing is exhibited by solids as they have definite shape.
- Bulk modulus of gases is very low, while that of liquids & solids is very high.
- **Poisson's ratio :** The ratio of the lateral strain to the longitudinal strain is constant for a given material.

$$\sigma = \frac{lateral\ strain}{longitudinal\ strain} = -\frac{(\frac{\Delta R}{R})}{\frac{\Delta L}{L}}$$

- Has no units & dimensions
  Theoretically σ lies between -1 & + $\frac{1}{2}$
  Practically σ lies between zero and + $\frac{1}{2}$
- Moduli of elasticity is independent of the magnitude of stress & strain. It depends only on the nature of the material of the body.
- For a rigid body Δ V or Ø = 0, Y, β or η will be ∝ elasticity of a rigid body is infinite.
- Gases are least elastic ; solids are most E s > El > E g
- Temperature increases, Y, β or η decrease

  $Y = 3B(1 - 2\sigma)$

  $Y = 2n(1 + \sigma)$

  $Y = \frac{9Bn}{3B+n}$, $\sigma = \frac{3B-2n}{6B+2n}$
- If the length of a wire is doubled, the longitudinal strain will be the stress
- For a loaded wire $\Delta L = \frac{FL}{\pi r^2 y}$, $\Delta L \propto \frac{L}{r^2}$
- If a wire of length L, area of cross section is stretched by a force F, then

  $\Delta L = \frac{FL}{AY}$

- In case of elongation its own weight F – mg, length is L/2.

$$\Delta L = \frac{M\,g\,(\frac{L}{2})}{A\,y} = \frac{M\,g\,L}{2\,A\,y} = \frac{e\,g\,L^2}{2\,y} \qquad M = e\,A\,L$$

- Thermal stress y α d θ
- Work done in a stretched wire W = $\frac{1}{2}$ x load x elongation

e ' = e (1 + k Δ e)

I. Fill in the blanks :

1. The Young's modulus for a perfectly rigid solid is __________
2. Work done per unit volume in a rigid body is __________
3. Practical value of Poisson's ratio is __________
4. The breaking stress of a wire depends upon __________
5. To reduce the bending for a given load, one should use a material with large ________
6. The value of module of rigidity for liquid is _______
7. Work done in determining body will be stored in it as potential energy is called _________
8. The delay in recovering back to the original form after the removal of the deforming force is called _____________
9. The product of breaking stress and area of cross section is called ________
10. The capacity of a substance to withstand large stress without permanent set is called ____________
11. A substance that can be elastically stretched to large strain is known as ___________
12. The lag of strain behind stress is called _____________
13. The maximum load up to which the strain is directly proportional to stress is called ___________
14. The maximum load on the wire up to which the wire exhibits the elastic property is called ________________.

II. Match the following

	I	II
1.	a) Young's Modulus	a) $\frac{\Delta V}{V}$
	b) Volume strain	b) $\frac{F\,L}{A\,\Delta\,L}$
	C) modulus of rigidity	c) $\frac{\Delta\,d}{D}$ X $\frac{L}{\Delta\,L}$
	d) Poisson's ratio	d) $\frac{F}{A\emptyset}$

a) a → b, b →a, c →d, d → c b) a → c , b →b, c →d , d → a
c) a→ b, b→ c, c→d, d →a d) a → b, b → a , c → d, d→ c

	I	II
2.	A) Hooke's law	a) Tangential strain
	b) Shearing strain	b) Temporary loss of elastic property
	c) Bulk strain	c) Elastic limit
	d) Elastic fatigue	d) 3 times the linear strain

a) a → c, b → a, c → b , d → d b) a → c, b → d, c → a, d → b
c) a → a , b → a, c →b , d →d d) a → c, b → a, c→ d, d→ b

	I	II
3.	a) for a beam depression δ	a) $\frac{\pi\,\eta\,r^4}{2\,L}$
	b) Elastic restoring couple per unit twist	b) $\frac{M\,g\,L^3}{3\,Y\,I}$
	c) Angle of shear related to angle of twist	c) $\frac{3\,B-2\,n}{6\,B+2\,n}$
	d) Poison's ratio σ	d) $L\,\phi = r\,\theta$

a) a → b, b →a , c →d, d → c b) a→ b , b→c, c →d, d→ c
c) a→ b , b → c, c → d, d→ a d) a→b , b→ a, c →d, d→ c

4.

	I	II
	a) Ratio of longitudinal stress to longitudinal strain	a) Modulus of rigidity
	b) Ratio of normal stress to volumetic strain	b) Poisson's ratio
	c) Ratio of lateral strain to longitudinal strain	c) Bulk modulus
	d) Ratio of tangential stress to shear strain	d) Young's modulus

a) a→ b , b→ c, c →a , d→ b b) a → d, b → c, c→ b , d → a
b) a→ a , b → b , c→ d , d→ a d) a → c , b → d, c → a, d→ b

5.

I	II
a) Restoring force	a) $M L^2 T^{-2}$
b) stress	b) $M^0 L^0 T^0$
c) strain	c) $M L T^{-2}$
d) Energy stored	d) $M L^{-1} T^{-2}$

a) a → c, b → d , c → b , d → a b) a → a , b → c , c → d, d → b
c) a→ b , b →c, c →d, d →a d) a→ d, b →c, c →d d →a

6.

I	II
a) Longitudinal strain	a) Volume changes
c) Shear stress	b) shape changes
d) Volume stress	c) volume doesn't change
e) Tensile stress	d) shape doesn't change

a) b → d , b → c, a→ d, d → a b)a →c, b →a, c →d, d →b
c) a→a, b →c, c→ d, d→b d) a→ b , b→ b, c →a, d→c

III. Statements with True or False :

1. Which of the following statement is false .
   A) A material which stretches more is more elastic.

B) A material which stretches to a lesser extent for a given load is considered to be more elastic.
C) Moduli of elasticity is independent of the magnitude of stress & strain.
D) Moduli of elasticity depends upon the nature of the material of the body.

2. Which of the following statement is false.
   a) Elasticity of rigid body is infinity
   b) Bulk modulus of gases is very low, while that of liquids and solids is very high.
   c) When temperature increases y, β, η decreases.
   d) Steel is more elastic than rubber because of is less than that of rubber.

3. Which of the following statement is true
   a) The elastic energy stored in a wire of Young's modulus Y is
   $$\frac{(stress)^2\ x\ volume}{2\ y}$$
   b) Energy stored per unit with volume in a stretched wire is $\frac{1}{2}$ stress x strain
   c) Strain energy = $\frac{1}{2}$ x Applied force x extension
   d) All the above.

4. Which of the following statement is false.
   a) The dimension of Poisson's ratio is $M^0 L^0 T^0$
   b) Breaking stress depends upon the area of cross section of the wire.
   c) Pressure always acts normal to the surface but stress may be normal or tangential
   d) For a material y & B are Young's modulus & bulk modulus then y > 3B

5. Which of the following is false for Young's modulus
   a) Young's Modulus y = 3B (1- 2 σ)
   b) Young's modulus y = 2 n (1 + σ)
   c) Young's modulus y = $\frac{9\ B\ n}{3\ B+n}$
   d) Young's modulus y = $\frac{3B-2n}{6\ B+\ 2n}$

6. Which of the following statement is true :
   a) Basic reason of elasticity is inter atomic force and strain energy in material of a body is stored in the forces of inter atomic energy.
   b) y of elasticity of a body depends on the longitudinal stress applied to it.
   c) y of elasticity of a body depends upon its size.
   d) None of the above.

7. Which of the following is true.
   a) Force constant of a spring is synanimous to $\frac{yA}{L}$
   b) According to Hook's law force is proportional to r
   c) Impurities effect the elastic properties of the metal to which they are added.
   d) All of the above.

8. Which of the following is true :
   a) For invarsteel the elasticity remains practically unaffected by changes of temperature.
   b) Hammering and rolling decreases the elastic property of substances.
   c) Smaller change of shape for a given stress corresponds to greater elasticity.
   d) None of the above.

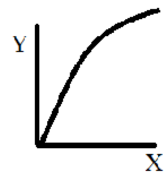
9. When a body of mass M is attached to a lower end of a wire whose upper end is fixed then the elongation of the wire is l, mark out the correct statements.
   a) Loss in gravitational PE of M is Mgl
   b) Elastic PE stored in the wire is $\frac{M\,gl}{2}$
   c) Elastic PE stored in the wire is Mgl
   d) Elastic PE stored in the wire is $\frac{M\,gl}{3}$

MULTIPLE CHOICE QUESTIONS :

1. A body subjected to strain a number of times does not obey Hook's law due to
   a) Yield point b) breaking stress c) Elastic fatigue d) Permanent set

2. The fractional change in volume per unit increase in pressure is called
   a) Pressure coefficient b) Volume coefficient c) bulk modulus
   e) Compressibility.

3. The graph shows the behaviour of a length of a steel wire in the region for which the wire obeys Hook's Law. The graph is a part of parabola the variable X & Y might represent



   a) X = stress y = strain
   b) X = strain y = stress
   c) X = strain y = elastic energy
   d) X = elastic energy y = strain

4. A wire fixed at the upper end stretched by length by applying a force F, the work done in stretching is
   a) $\frac{F}{2\,l}$ b) F l c) 2 F l d) $\frac{F\,l}{2}$

5. If stress is numerically equal to Young's modulus the elongation will be
   a) $\frac{1}{4}$ of the original length b) $\frac{1}{2}$ the original length
   c)equal to the original length d) twice the original length

6. According to Hook's law of elasticity, if stress is increased the ratio of stress to strain
   a) increases b) remains constant c) decreases d) becomes zero

7. Which of the following does not exhibit shearing
   a) twisting of wire b) wringling of washed clothes
   c) bending of beams d) curling of fingers.

8. A slightly conical wire of length L and end radii r1 & r2 is stretched by two forces F , F applied parallel to length in opposite directions and normal to end faces. If Y denotes the Young's modulus, then extension produced is
   a) $\frac{F\ L}{\pi r_1 y}$ b) $\frac{F\ L}{\pi r_2 y}$ c) $\frac{F\ L}{\pi\ r_1 r_2 y}$ d) $\frac{F\ L}{\pi\ r_1 r_2}$

9) Isothermal elasticity of a gas is equal to
   a) density b) volume c) final pressure d) specific heat

10) A stretched rubber has
   a) increased K E b) increased P E
   c) decreased K E d) decreased P E

11) For a constant hydraulic stress on an object the fractional change in the objects volume $\frac{\Delta V}{V}$ & its bulk modulus B one related as
   a) $\frac{\Delta V}{V}$ α B b) $\frac{\Delta V}{V}$ α $\frac{1}{B}$ c) $\frac{\Delta V}{V}$ α B^2 d) $\frac{\Delta V}{V}$ α B^{-2}

Level – I Multiple Choice Problems :

1. The Young's modulus is numerically equal to the stress that arises in a wire when its length changes from l to
   a) 1.25 l b) 1.5 l c) 1.75l d) 2.00 l

2. For a given material, the Young's modulus is 2.4 times that of the modulus of rigidity. Its Poisson's ratio is
   a) .22 b) .2 c) .12 d) .21

3. If the pressure of a gas is increased from 1.01×10^5 pa to 1.165×10^5 P and volume is decreased by 10% at constant temperature then the bulk modulus of the gas is
   e) 1.55×10^5 pa b) $.5 \times 10^5$ pa c) 1.55×10^4 pa d)$.5 \times 10^4$ pa

4) A force of 200 N compresses a steel rod of length 1.5m & area of cross section 3mm^2 through a distance of 4mm. Find the work done.

a) .1 J b) .2J c) .3J d) .4J

5) A force F is required to break a wire of length L and radius r. What is the force required to break a wire of the same material, twice the length and 4 times the radius.

a) 8 F b) 16 F c) 12 F d) 4 F

6) A wire 4 m long and cross section 3 x 10 $^{-6}$ m^2 is stretched by 1mm. If Y = 2 x 10 ^{11} n/m^2 . Find the energy stored in the wire.

a) . 25 J b) 0.5J c) .075J d) .75J

7) The Young's modulus of a wire of length L and radius r is Y $\frac{N}{m^2}$. If the length reduced to $\frac{L}{2}$, & radius to $\frac{r}{2}$. Its Young's modulus will be

a) $\frac{Y}{2}$ b) Y c) 2Y d) 3 Y

8) The increase in length of a wire on stretching is .025% . If the Poisson's ratio is .4, then the percentage increase in diameter is

a) .02 b) 1 C) .1 D) .01

9) The forced required to double the length of a steel wire of area of cross section 5 x 10 $^{-5}$ m^2 is Y = 20 x 10 ^{10} p a.

a) 10 ^{7} N b) 10 ^{5} N c) 10 ^{6} N d) 10 ^{8} N

10) A material has Poisson's ratio of .5 . If a uniform rod suffers a longitudinal strain of 2 x 10 $^{-3}$. What is the percentage increase in its volume ?

a) 2 % b) 1 % c) 3 % d) 0%

Level – II – Problems

1. Two wires of equal cross section but one made of steel and the other of copper, are joined end to end when the combination is kept under tension,

the elongation in the two wires are found to be equal. Find the ratio of the lengths of the two wires ,

Y_{steel} = 2×10^{11} N/m^2 Y_{copper} = 1.1×10^{11} N /m2

A) 20: 11

2. A steel wire of length 2r is stretched through 2mm, the cross- sectional area of the wire is 4mm^2. Calculate the elastic potential energy stored in the wire in the stretched condition. Young's modulus of steel is = 2×10^{11} N/m^2.

   A) .8 J

3. One end of a nylon rope of length 4. 5m & diameters 6mm is fixed to a tree limb. A monkey weighing 100N jumps to catch the free end stays there. Find the elongations of the rope and the corresponding change in the diameter.

   Y_{Nylon} = 4.8×10^{11} N / m^2 Poisson ratio of nylon = .2

   A) 8.8×10^{-9} m

4. Two blocks of masses 1 kg & 2 kg are connected in a metal wire going over a smooth pully as shown in the fig. The breaking stress of metal is 2×10^{9} N/M2. What should be the minimum radius of the wire used if it is not break, g = 10m/sec^2. **A) 4.6×10^{-5}m**

5. A load of 4kg is suspended from a ceiling through a steel wire of radius 2mm. Find the tensile stress developed in the wire when equilibrium is achieved.

   A)3.1×10^{6} n /m^2

6. A steel wire 2mm is diameter is just stretched between two rigid walls at 20^0C. If the temperature fails to 10^0 c. find the tension in the wire.

   Linear expansion of steel = 11×10^{-6} K Y = 2×10^{11} N/m^2

   A) 69.115

7. A load of 31.4 kg is suspended from a wire of radius of 10^{-3} m and density 9×10^{3} kg /m^3. Calculate the change in temperature of the wire if 75% of the work done is converted into heat

   Y = 9.8×10^{10} n/m. Heat capacity = 490 / J / kg K

   A)8.33×10^{-3} R

8. A shearing force of 10^5 N is applied on the top surface of a fixed cube of side 10 cms. If the top surface is displaced by .33 . Find rigidity modulus of the material of the cube.

   A) 3.03 x 10^{11} N/m^2.

9. A lift of mass 1000 kg is suspended by steel wire of maximum safe stress 1.4 A x 10^8 N/m^2. Find the minimum diameter of the wire, if the maximum acceleration of the lift is 1.2 m/sec.

   A).010m

KEY

I. Fill up the blanks :

1) Infinite
2) $\frac{1}{2}$ Stress x strain
3) $0 < \sigma < \frac{1}{2}$
4) The material of the wire
5) Young's modulus
6) Zero
7) Strain energy
8) elastic after effect
9) breaking force
10) resistance
11) elastomer
12) elastic hysterisis
13) proportionality limit
14) Elastic limit

II. Match the following :

1) A 2) B 3) D 4) B 5) A 6) A

III. Statements with True or False :

1) A 2) D 3) D 4) B, D 5) D 6) A
7) D 8) A, C 9) A, B

IV. Multiple choice questions :

1) A 2) B 3) D 4) D 5) D 6) C
7)D 8) A & C 9) A & B

V. Level – I Multiple choice problems :

1) $\frac{\Delta L}{L} = \frac{L_2 - L_1}{L} = \frac{2L - L}{L} = \frac{L}{L} = 1$, stress equal to y.
   A) D

2) Y = $\frac{2\,n}{1+\sigma}$, 2.4m = $\frac{2\,n}{1+\sigma}$, σ= . 2

A) B

3) B = - v $\frac{d\,P}{d\,V}$, $\frac{-100}{10}$ (1. 165 x 10 ^{5} – 1.01 x 10 ^{5}) = 1.55 x 10 ^{5}

A) A

4) Work done = $\frac{1}{2}$ load x change of length = $\frac{1}{2}$ Fx Δ L= $\frac{1}{2}$ x 200 x $\frac{4}{103}$ = .4 J

A) D

5) Breaking Stress = $\frac{F}{\pi r^2}$, in first case , it is $\frac{F\prime}{\pi\,(\,4\,r)^2}$ 16 F is required

A) B

6) PE = $\frac{1}{2}$ F. e = $\frac{1}{2}$ $\frac{A\,y\,e}{l}$, e = $\frac{A\,ye^2}{2\,l}$ = $\frac{3\,x\,10^{-6}\,x\,2\,x\,10^{11} x\,2\,x\,10^{-6}}{2\,x\,4}$ = .075J

A) C

7) y = $\frac{\frac{F}{A}}{\frac{\Delta\,l}{p}}$ = $\frac{F\,\Delta\,l}{A\,l}$ = $\frac{F\,L}{\pi r^2\,e}$ y α $\frac{L}{r^2}$; y ‘ α $\frac{\frac{L}{2}}{(\frac{r}{2})^2}$, y ’ =$\frac{2\,l}{r^2}$

∴ y’ = $\frac{y}{2}$

8) P = $\frac{\frac{\Delta\,d}{D}}{\frac{\Delta\,l}{L}}$; .4 = $\frac{\frac{\Delta\,d}{D}}{.025}$ = .01

A) D

9) L = Δ L , F = $\frac{\Delta\,L}{L}$ x A Y

= 5 X 10 $^{-5}$ x 20 x 10 ^{10} = 10 ^{7} N.

10) Volume change is $\frac{\Delta\,V}{V}$ = $\frac{(\,1-2\,x\,\sigma)\Delta L}{2}$

σ = .5, $\frac{\Delta\,V}{V}$ = 0

Volume change is 0%

A) D

Level – 2 : Problems :

1. $\frac{l}{Ls}$ = $\frac{stress}{2\,x\,10^{11} N/m^2}$ ………….(1) $\frac{l}{Lc}$ = $\frac{stress}{1.1\,x\,10^{11} N/m^2}$…………(2)

$\frac{L\,s}{L\,C}$ = $\frac{2}{1\,.1}$ = 20 : 11

2. The status in the wire $\frac{\Delta l}{l} = \frac{2\,mm}{2m}$ = 10 $^{-3}$

Stress = y x strain

= 2 x 10 ^{11} N / m^2 x 10 $^{-3}$ = 2 x 10 ^{8} N / m^2

Volume of the wire = 4 x 10 $^{-6}$ x 2m = 8 x 10 $^{-6}$ m^3

The elastic PE Stored = $\frac{1}{2}$ x stress x strain x volume = . 8 J

3. As the monkey stays in equilibrium, the tension in the rope , the weight of the monkey

Y = $\frac{stress}{strain} = \frac{T/A}{l/L}$, l $= \frac{T\,L}{A\,y}$ l = $\frac{100\ x\ 4.5}{(\pi\ x\ 9\ x\ 10^{-6})(\ 4.8\ x\ 10^{\ 11})}$ = 3.32 x 10 $^{-5}$

Poisson ratio = $\frac{\Delta\, d/d}{l/L}$, .2 = $\frac{\Delta\ d\ x\ 4.5}{(3.32\ x\ 10^{-5})(\ 6\ \ x\ 10^{\ -3})}$

Δ d = 8.8 x 10^{-9} m

4) the stress of wire = $\frac{T}{A}$, this stress should not exceed the breaking stress

T – 10 N = (1 kg) a , 20 N – T = (2kg) a

T = $\frac{40}{3}$ N

Stress = $\frac{40/3}{\pi r^2}$

If the min radius needed to avoid breaking is r.

2 x 10 ^{9} = $\frac{40/3}{\pi r^2}$ r = 4.6 x 10 -5 m

5) Tension in the wire is F = 4 x 9.8 , Λ = πr^2= π (2 x 10 $^{-3}$) ^{2}

The tensile stress developed = $\frac{F}{A} = \frac{4\ X\ 9.8}{4\pi\ X\ 10^{-6}}$ = 3.1 X 10 ^{6} n /M^2

6) $\frac{\Delta\, l}{l} = \frac{e}{l}$ = α Δ t = 11 x 10 $^{-6}$ (20 -10) = 11 x 10 $^{-5}$

F = A y ($\frac{e}{l}$) = π x $\frac{2}{4}$ x 10 $^{-6}$ = 22 π= 69.115

7) $\frac{W}{V}$ = $\frac{1}{2}$ stress x strain = $\frac{1}{2}$ $\frac{(stress)^2}{y}$

W = $\frac{1}{2}$ $(\frac{M\,g}{A})^{2}$ $\frac{v}{y}$ = (BV) s Δ θ

Or Δ θ = $\frac{1}{120}$ K , = 8.33 x 10^{-3} K

8) X = l θ = 33 x 10^{-2} x 10^{-3} = 10 x 10^{-2} x θ θ = 33 x 10^{-4}

η = $\frac{F}{A\theta}$ = $\frac{10^6}{10^2\ X\ 10^{-4}\ x\ 33\ x\ 10^{-4}}$ = 3.03 x 10 11 N / m^2

9)Stress = 1. 4 x 108 N / m2 F = m (g + 9) = m (11) = 11000N

Stress = $\frac{F}{A}$, A = $\frac{F}{stress}$ = $\frac{11000}{1.4\ x\ 10^8}$

$\frac{Md^2}{4}$ = $\frac{11}{1.4\ x\ 10^5}$, d = $\sqrt{\frac{11\ x\ 4\ x\ 1}{1.4\ x\ 10^5\pi}}$ = 1010m

11. MECHANICAL PROPERTIES OF FLUIDS

Synopsis :

1) Fluid statics is the study of behaviour of fluids at rest.
2) Fluid dynamics is the study of fluids in motion.
3) Liquids and gases are fluids.
4) Fluids flow from one place to another place due to pressure difference.
5) **Pressure (Average pressure) : -**

   The normal force acting per unit area is called pressure or average pressure.
6) P = F/A . S. I. Units – pascal

   C. G. S. units – dyna/cm^2.

   P = [M L^{-1} T^{-2}]

7) **Units of Pressure :**

1 atom = 1. 01325 x 10^{J} pascal

1 bar = 760 mm of Hg (or) 760 cm of Hg

1 torr = 1 mm of Hg

8) Pressure is isotropic. The liquid pressure at any point in the liquid is equally distributed in all directions.

9) Density p = M/V units } = 1 gm /cm^2 (C. G. S.) = 10^3 kg/ m^3.

S. I unit = kg/m^3

[M L^{-3}] scalar quantity.

10) Two liquids of masses M1 , M2 densifies p_1, p_2 respectively are mixed then effective density of mixture

$$p = \frac{(M_1+M_2)\, p_1 p_2}{M_1 p_2 + M_2 p_1}$$

If $M_1 = M_2$ then $p = \frac{2p_1p_2}{p_1+p_2}$

11) If V_1, V_2 are volumes, p_1 , p_2 are densities then $p = \frac{V_1p_1+V_2p_2}{V_1+V_2}$

If $V_1 = V_2$ then $p = p_1 + p_2 /2$

12) **Variation of density with pressure :**

If pressure increases volume decreases. So density of compressible material increases.

13. **Pascal's Law :** The pressure applied to an enclosed incompressible liquid is transmitted and diminished to every point of liquid and the walls of the container.

Ex : hydrolic lift, Bhrama press.

14.Archimedis Principle : When a body is immersed partly or wholly in a fluid then it appears to looses some weight which is equal to the weight of the fluid displaced by the body.

Uses :

- To find relative density of liquids and solids.
- To find the amount of impurity in a metal.

15) Streamline flow : If the velocity of all fluid particles crossing a point remains constant both in magnitude and direction then the flow of fluid is streamline flow.

16) Turbulent flow : If the velocity of different fluid particles crossing a point does not remain constant then the flow is called turbulent flow.

17) Tube of flow : Bundle of streamlines are called tube of flow.

18) Critical velocity : Vc = $\frac{R\eta}{D\,e}$

P = Density of liquid

D = diameter of pipe

η = coefficient of viscosity

V_c = critical velocity

R = Renold number (dimensionless).

0 < R < 1000 streamline flow

0 < R < 2000 unsteady flow.

R > 2000 turbulent flow.

19. Equation of continuity : $A_1 V_1 = A_2 V_2$

A = Area

V = Velocity It is the consequence of law of conservation of mass.

20. Bernoullis theorem : When an incompressible, nonviscous, irrotational, study flow, flowing from one point to another point then at every point of its path total energy (P. E + K. E + pressure energy) per unit volume is constant.

P + ½ ev^2 + egh = constant

21. Toricillis theorem : The velocity of efflux of liquid through an orifice is equal to that of the velocity acquired by a freely falling body from a height which is equal to that of the liquid level from the orifice. V =$\sqrt{2gh}$.

22. Carburator, atomiser, sprayer etc are working on principel of Bernoulli . Bernoulli principle also explains magnus effect, reason for flying of an aeroplane and blood flow in arteries, and reason for heart- attack.

23. Coefficient of viscosity : $\eta = \frac{F}{A.\frac{dv}{dx}}$[M L^{-1} T^{-1}] increases in gases with increase in temperature.

It decreases in liquids with increase in temperature.

24. **Poisueillis Law :** F = V = $\frac{\pi \rho r^4}{8\eta\, l}$

25. **Stokes Law :** F = 6 π η ɤ v_t

26. **Terminal velocity :**V_t = $\frac{2\, r^2 g\ (e - \sigma)}{9\ \eta}$

Surface tension :

27.Cohesive forces : Attractive force between two similar type of molecules is called cohesive force.

28. Adhesive force : The attractive force between different type of molecules is called adhesive force.

29. Sphere of influence : The imaginary sphere drawn around a molecule with molecular range is sphere of influence.

30. Surface tension : The force per unit length normal to any imaginary line drawn on the surface of liquid is known as surface tension.

T =$\frac{F}{L}$ N / m

T = $\frac{W}{A}$ J / m^2 [$M L^0 T^{-2}$]

31. The work done in forming a liquid drop is the product of change in surface area and surface tension.

32. The work done in splitting a big liquid drop into n equal small drops.

W = $4 \pi R^2 T (n^{1/3} - 1)$

33. The work done in forming a big liquid drop from n equally small liquid drops

W = $4 \pi R^2 T (n - n^{2/3})$

34. Angle of contact : Angle of contact is the angle between the tangent drawn to the liquid surface at a point of contact (solid and liquid) and inside the liquid.

35. Angle of contact depends on :

1. Pair of solid and liquid
2. Temperature of liquid
3. Impurities in the liquid.

If θ> 0^0 liquid wets the solid

- θ>90^0 liquid doesn't wets the solid
- θ= 0 between pure water and glass.
- θ= 140^0 mercury and glass.

36. Capilarity : The ascent or descent of a liquid in a capillary tube is called capillarity.

37. Surface tension formula :

$$T = \frac{ɤ\,(h + \frac{ɤ}{3})\,dg}{2\cos\theta}$$

T = surface tension

R = radius of tube

H = the ascent of liquid in tube

d= density

g = acceleration due to gravity

θ= angle of contact

h does not depend upon inclination of capillary tube.

38. Generally temperature increases surface tension of liquid decreases but in a special case liquid copper and liquid cadmium surface tension increases with temperature.
39. Due to soluble impurities surface tension increases.
40. Due to insoluble impurities surface tension decreases.
41. Excess pressure in liquid drop : $\Delta p = 2T / r$
42. Excess pressure in soap bubble : $\Delta p = 4T / r$

43.Detergents (water wetting agents) :

When detergents are mixed with water they decrease the surface tension, increase wetting nature.

44. Water proofing agents : When the detergents are mixed with water they increases surface tension and decreases the wetting nature.

True / False

1. A fluid has no definite shape of its own. T/ F
2. Compressible material cannot change its volume by applying pressure on it T/ F
3. Pressure is responsible for a sharp needle pierces into skin when pressed. T/ F
4. The normal force acting per unit area is average pressure. T / F
5. Pressure is a vector quantity. T/ F
6. Dimensional formula of relative density is [ML^{-3}] T/ F
7. Pressure exerted is same in all directions in a fluid at rest is Pascals principle. T/ F
8. Toricillian vacuum is the space above the mercury column in mercury barometer. T/ F
9. A sudden fall of mercury column in a barometer indicates cyclone. T / F

10. Archimedes explains the working the hydraulic breaks. T/F
11. The tangential force opposes the relative motion between liquid layers is viscous force. T/ F
12. The units of velocity gradient is poise. T/ F
13. Bundle of streamlines is called tube of flow. T/F
14. Bernoulis principle holds good for turbulent flow. T/ F
15. The energy required to increase unit surface area of a liquid is surface tension. T/ F

Answers :

1)T	2) F	3) T	4) T	5) f
6)F	7)T	8)T	9)T	10)F
11)T	12)F	13)T	14)F	15)T

Fill up the blanks :

1. The difference between actual pressure and atmospheric pressure is ___________
2. The principle behind the carburettor of an automobile is ________
3. Flew of huts in cyclonic strom due to the principle of _________
4. The equation for speed of efflux in toricillis theorem is equal to _________
5. 1 torr = __________K. Pascal.
6. Relative density of substance is equal to the ratio of density of substance to the ______________
7. Streamlining of automobiles is to reduce________________
8. If three points PA, PB, PC are at same horizontal level in a horizontal pipe with liquid then the ratio of pressure $P_A : P_B : P_C$ = __________
9. The vertical component of force due to pressure difference on upper and lower part of the aeroplane wing is called ___________.
10. The relation between terminal velocity V_t and coefficient of viscosity η is ________
11. The phenomena which helps ants to walk on the surface of water is _____________
12. Firefighters used the equation to control the flow in the water canons is ___________
13. The equation for pressure force is ____________
14. If an ice cube melts in gravity free space then its shape is__________
15. Rain drops are spherical due to ________________

Answers :

1) Gauge pressure
2) Bernoullis principle
3) Bernoullis principle
4) V = $\sqrt{2gh}$
5) 0.133
6) Density of water at 4 ^{0} c
7) Air friction

8. 1 : 1 : 1
9. Lift
10. $V_t \propto \frac{1}{\eta}$
11. Surface tension
12. Equation of continuity
13. F = PA
14. Spherical
15. Surface tension

Objective type questions

1. If two spheres having the ratio of radii is 1 : 2 released in viscous fluid then the ratio of terminal velocity
   a) 4 : 1 b) 2 : 1 c) 1 : 2 d) 1 : 4 ( )
2. When a boat in river enters into the sea water then it
   a) Sinks little b) rises little c) remains same d)will drown completely ( )
3. Water flows through horizontal pipe of radius r at a speed V, if the radius of the pipe is doubled the speed of the flow of water is
   a) 2V b) V/2 c) V/4 d) 4V ( )
4. The dynamic lift of an aeroplane is based on
   a) Torisilli theorem b) bernoulli's theorem
   b) Conservation of angular momentum
   c) Pascals law. ( )
5. As the depth of the river increases, the velocity of flow
   a) Increases b) decreases c) remains unchanged
   d) may increase of decrease ( )
6. A good lubricant must have
   a) High viscosity b) low viscosity
   c) high density d)Low surface tension ( )
7. The paint gun works on the principle of
   a) Boyle's law b)Bernoullis law
   c) Archimedis Law d) Pascals law ( )
8. When there are no external forces shape of the liquid determined by
   a) Density of liquid b) temperature

c) surface tension d) viscosity ( )

9. The dimensional formula of velocity gradient $\frac{dv}{dx}$

a) [$L^2 T^{-2}$] b) [L T^{-1}] c) [T^{-1}] d) [L^{-1}] ( )

10) The surface tension of water when mixed with insoluble impurity

a) increases b) decreases c) increase and decrease

d) decrease and increase ( )

11. Find the excess pressure in air bubble in liquid when surface tension of liquid is equal to 72 x 10 -3 N / m diameter is 2 x 10 -3m

a) 144 b) 1.44 c) 72 d) 7.2 ( )

12. Force liquid in vessel cohesive force is less than adhesive force then

a) minuscous will be concave b) minuscous will be convex

c) a or b d) none of the above. ( )

13. A soap bubble has a radius r, the work done in increasing the radius to 3 times its original radius without any change in temperature is (surface tension M = T)

a)12 π r^2 T b) 16π r^2 T c) 64π r^2 T d) 48π r^2 d ( )

14. Pressure inside two soap bubbles are 1.01 atom, 0.02 atom then the ratio between the volumes is

a) 102 : 101 b) $102^3 : 101^3$ c) 8 : 1 d) 2 : 1 ( )

15. If pressure at half of the depth of the lake is equal to 2/3 pressure at the bottom of the lake then what is the depth of the lake

a) 10m b) 20 m c) 60 m

b) 30m ( )

Answers :

1) D	2) b	3) C	4) B	5) B	6) A
7) B	8) C	9) c	10) B	11. A	12. A
13) C	14) B	15) B.			

Matching :

1) **A** **B**

1. 1. Hydrallic breaks	a) stokes law	( )
2. Magnus effect	b) toricelli theorem	( )
3. Terminal velocity	c) Pascals Law	( )
4. Velocity of efflux	d) Bernoulis theorem	( )

2.

A		B	
1. Pressure (P)		a) [$M L^{-1} T^{-1}$]	( )
2. Coefficient of viscosity (η)		b) [$M^0 L^0 T^0$]	( )
3. Renolds number	(R_e)	c) [$M^0 L^0 T^{-1}$]	( )
4. Velocity gradient	($\frac{dv}{dx}$)	d) [$M L^{-1} T^{-2}$]	( )

3.

A		B
1. Viscosity of liquid	( )	a) decreases surface tension
2. Viscosity of gases	( )	b) decreases when η increases
3. Detergents	( )	c)decreases with increase in temperature
4. Terminal velocity of body in medium	( )	d)increases with increase in temperature

4.

A	B	
1. Cohesive forces > adhesive forces	(a) angle of contact < 90^0	( )
2. Cohesive forces < adhesive forces	(b) angle of contact > 90^0	( )
3. Cohesive forces = adhesive forces	(c)angle of contact = 0	( )
4. Water mixed with detergents	(d) angle of contact decreases	( )

	A	B
5.	1. Ants can walk on the surface of water	a) $P = \frac{2T}{r}$
	2. Roots of the plants absorb water from	

The ground	b) P = $\frac{4\,T}{r}$
3. Excess pressure in soap bubble	c) surface tension ( )
4. excess pressure in liquid drop	d) capillary raise ( )

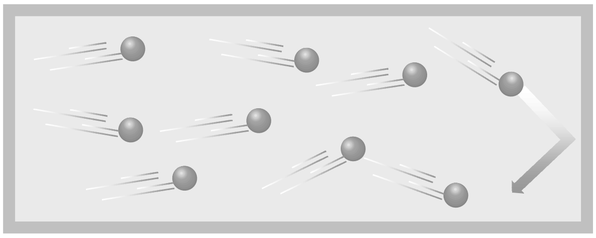
Answers :

1) 1 - c , 2- d , 3 - a , 4 - b
2) 1 - d, 2 - a, 3 - b , 4 - c
3) 1 - c, 2 - d, 3 - a, 4 - b
4) 1 - b , 2 - a, 3 - c, 4 - d
5) 1 - c, 2 - d, 3 - b, 4 - a

Kinetic Theory of Gases

Gas

In gases the intermolecular forces are very weak and its molecule may fly apart in all directions. So the gas is characterized by the following properties.

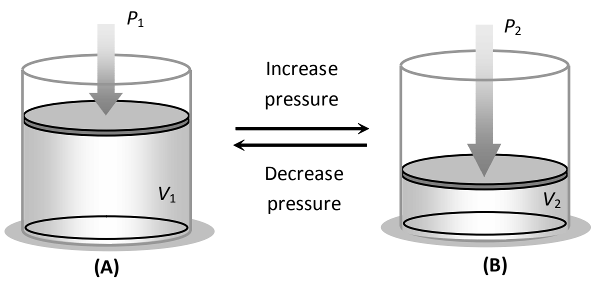


- In gases, molecules are far apart from each other and mutual attractions between them are negligible.
- They can easily compressed and expand.
- It has no shape and size and can be obtained in a vessel of any shape or size.
- It expands indefinitely and uniformly to fill the available space.
- It exerts pressure on its surroundings.

GAS LAWS :

BOYLE'S LAW:

At constant temperature, the volume of a given mass of gas is inversely proportional to its pressure.



Let us consider an ideal gas in a container with piston , it has initial volume V_1 at pressure P_1. if piston pushed inword at constant temperature , pressure increased to P_2 then its volume decreases to V_2. therefore volume is inversely proportional to pressure at constant temperature

$$i.e. \quad V \propto \frac{1}{P}$$

$$P = \frac{constant}{V}$$

$$PV = \text{constant}$$

$$P_1V_1 = P_2V_2$$

NOTE :

$$From\ Density\ \ \rho = \frac{mass}{volume} = \frac{m}{V}$$

$$\text{volume V} = \frac{m}{\rho(\text{Density of the gas})}$$

$$Then\ \ PV = \text{constant}$$

$$PV = P\left(\frac{m}{\rho}\right) = \text{constant}$$

$Here\ mass\ m = \text{constant}$

$$\therefore\ \frac{P}{\rho} = \text{constant}$$

$$\frac{P_1}{\rho_1} = \frac{P_2}{\rho_2}$$

NOTE :

♦ In PV = K, the value of the constant 'K' depends on

1. Mass of gas 2. Temperature of gas 3. System of units

♦ Boyle's law generally holds good only at low pressure and high temperatures.A gas which obey Boyle's law under all conditions of temperature and pressure is called ideal gas.

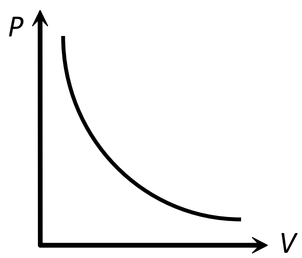
♦ Boyle's law can be experimentally verified by Quill's tube (or) Boyle's law apparatus.

♦ The graphs drawn between P & V at constant temperture of a gas are called isotherms

Graphical representation :

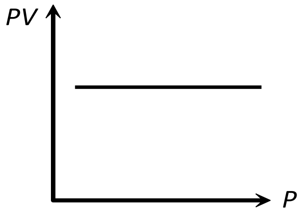
If m and T are constant

P - V Graph :

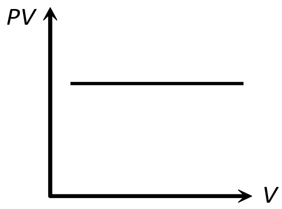


(A)

PV - P and PV - V Graph :



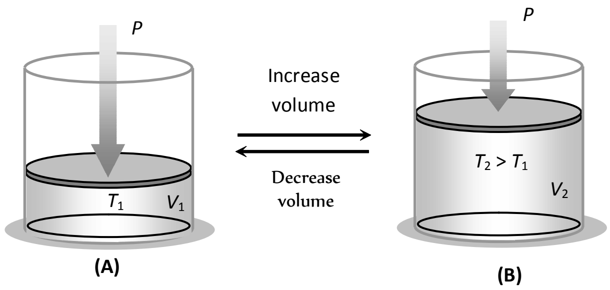
(B)



(C)

Charle's law :

If the pressure remaining constant, the volume of the given mass of a gas is directly proportional to its absolute temperature.



Let us consider an ideal gas in a container with piston , it has initial volume V_1 at absolute temperature T_1. . At constant pressure , temerature of gas increased to T_2 then its volume increases to V_2. therefore volume of gas is directly proportional to its absolute temperature at constant pressure.

$$i.e., \quad V \propto T$$

$$V = cons\tan t(T)$$

$$\frac{V}{T} = \text{constant} \qquad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

NOTE :

$$From\ Density \quad \rho = \frac{mass}{volume} = \frac{m}{V}$$

$$\text{volume V} = \frac{m}{\rho(\text{Density of the gas})}$$

$$V = \frac{m}{\rho}$$

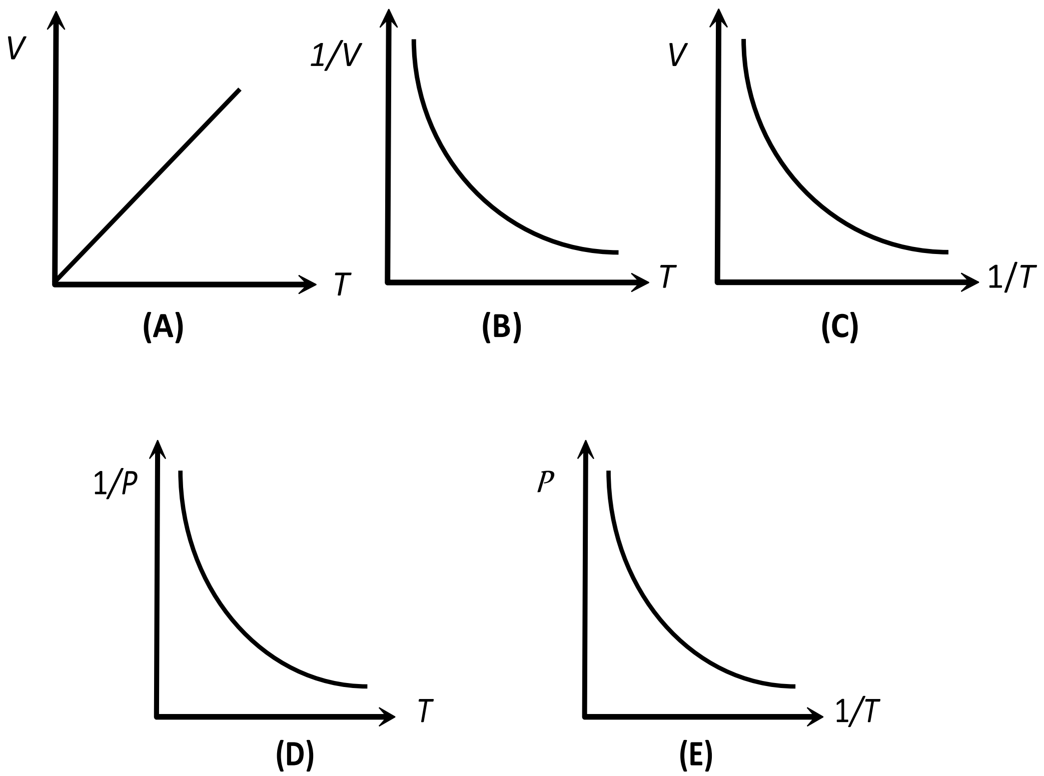
Then $\frac{V}{T} = \text{constant}$

$$\frac{V}{T} = \frac{m}{\rho T} = \text{constant}$$

$$\rho T = \text{constant}$$

$$\rho_1 T_1 = \rho_2 T_2$$

Graphical representation: If m and P are constant

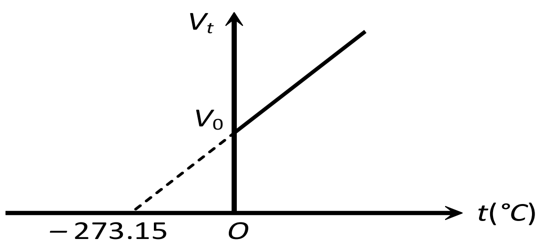


NOTE :

If the pressure remains constant, the volume of the given mass of a gas increases or decreases by $\frac{1}{273.15}$ of its volume at $0°C$ for each $1°C$ rise or fall in temperature.

$$V_t = V_0\left(1 + \frac{1}{273.15}t\right).$$

This is Charle's law for centigrade scale.



Gay-Lussac's law or pressure law :

The volume remaining constant, the pressure of a given mass of a gas is directly proportional to its absolute temperature.

Let us consider an ideal gas in a container with piston , it has initial pressure P_1 at absolute temperature T_1. At constant volume , temerature of gas increased to T_2 then its pressure increases to P_2. therefore PRESSURE of gas is directly proportional to its absolute temperature at constant volume .

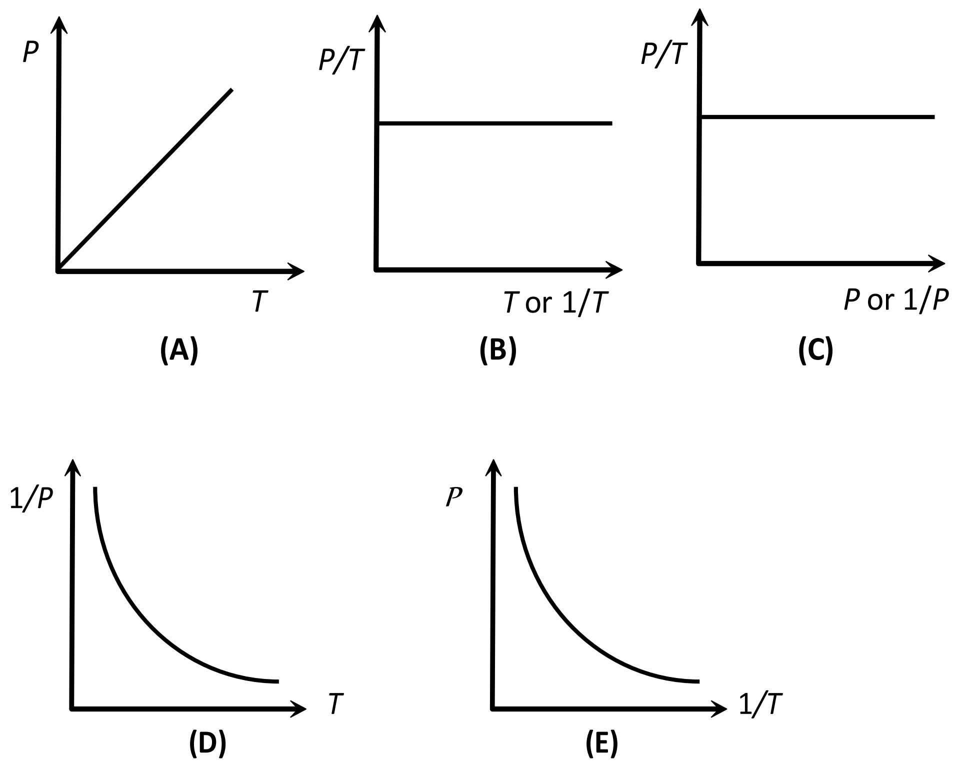
$$P \propto T$$

$$\frac{P}{T} = \text{constant}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Graphical representation :

If m and V are constants



Grahm's law of diffusion :

When two gases at the same pressure and temperature are allowed to diffuse into each other, the rate of diffusion of each gas is inversely proportional to the square root of the density of the gas

i.e. rate of diffusion $r \propto \frac{1}{\sqrt{\rho}} \propto \frac{1}{\sqrt{M}}$

$\rho \rightarrow$ is the density of the gas

M $\rightarrow$ is the molecular weight of the gas

$$\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{M_2}{M_1}}$$

If V is the volume of gas diffused in t sec then

rate of diffusion $r = \frac{V}{t}$

$$\frac{r_1}{r_2} = \frac{V_1}{V_2} \times \frac{t_2}{t_1}$$

Dalton's law of partial pressure :

The total pressure exerted by a mixture of non-reacting gases occupying a vessel is equal to the sum of the individual pressures which each gases exert if it alone occupied the same volume at a given temperature.

For n gases

$$P = P_1 + P_2 + P_3 + \ldots\ldots P_n$$

where

P = Pressure exerted by mixture

$P_1, P_2, P_3, \ldots\ldots P_n$ = Partial pressure of component gases.

Ideal Gas :

A gas which obey Boyle's law under all conditions of temperature and pressure is called ideal gas.

- Real gases obey gas laws only at low pressure and high temperatures. All Gases are real gases only.
- Attraction between the molecules of perfect gas is zero.
- Ideal or perfect gas obey gas laws at all temperatures and pressures without any limitations.
- Hydrogen or Helium behaves closely as perfect gas. Hence it is preferred in constant volume gas thermometers.

Assumption of Ideal Gases (or Kinetic Theory of Gases):

Kinetic theory of gases relates the macroscopic properties of gases (such as pressure, temperature *etc*.) to the microscopic properties of the gas molecules (such as speed, momentum, kinetic energy of molecule *etc*.)

Actually it attempts to develop a model of the molecular behaviour which should result in the observed behaviour of an ideal gas. It is based on following assumptions :

- Every gas consists of extremely small particles known as molecules. The molecules of a given gas are all identical but are different than those of another gas.
- The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses.
- Their size is negligible in comparison to intermolecular distance (10^{-9} *m*)
- The volume of molecules is negligible in comparison to the volume of gas. (The volume of molecules is only 0.014% of the volume of the gas).

- Molecules of a gas keep on moving randomly in all possible direction with all possible velocities.
- The speed of gas molecules lie between zero and infinity
- The gas molecules keep on colliding among themselves as well as with the walls of containing vessel. These collisions are perfectly elastic.
- The time spent in a collision between two molecules is negligible in comparison to time between two successive collisions.
- The number of collisions per unit volume in a gas remains constant.
- No attractive or repulsive force acts between gas molecules.
- Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.
- Molecules constantly collide with the walls of container due to which their momentum changes. The change in momentum is transferred to the walls of the container. Consequently pressure is exerted by gas molecules on the walls of container.
- The density of gas is constant at all points of the container.

Equation of State or Ideal Gas Equation :

The equation which relates the pressure (P) volume (V) and temperature (T) of the given state of an ideal gas is known as ideal gas equation or equation of state.

From Boyle's Law

$$V \propto \frac{1}{P} \quad ....(1)$$

From charles Law

$$V \propto T \quad ....(2)$$

From equation (1) and (2)

$$V \alpha \frac{T}{P}$$

$$V = K\frac{T}{P}$$

$$\frac{PV}{T} = K(cons\tan t)$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

For 1 mole of gas $K = R$ (universal gas constant.)

$$\frac{PV}{T} = R \text{ (constant)}$$

$$PV = RT$$

where R = universal gas constant.

For n moles of gas $PV = nRT$

Universal gas constant (*R*) :

A gas constant per one mole of gas is called as Universal gas constant

→ it is same for all gases

→ The value of "R" does not depend on the mass of gas or its chemical formula.

→ The fact that R is a constant for all gases is constant with Avagadro's hypothesis that "equal volumes of all gases under same conditions of temperature and pressure contains equal number of molecules".

$$R = \frac{PV}{nT} = \frac{\text{Pressure} \times \text{Volume}}{n \times \text{Temperature}} = \frac{\text{Work done}}{\text{n} \times \text{Temperature}}$$

At S.T.P. the value of universal gas constant is same for all gases

$$R = 8.31 \frac{J}{mole \times kelvin} = 1.98 \frac{cal}{mole \times kelvin} \simeq 2 \frac{cal}{mol \times kelvin} = 0.8221 \frac{litre \times atm}{mole \times kelvin}.$$

It's unit is $\dfrac{Joule}{mole \times kelvin}$

Dimension : $[ML^2T^{-2}\theta^{-1}]$

Specific gas constant (*r*) :

A gas constant per unit mass of gas is called as specific gas constant

→ it is different for different gases

It is represented by per gram gas constant

i.e., $r = \dfrac{R}{M}$.

Since the value of M is different for different gases. Hence the value of r is different for different gases.

e.g. It is maximum for hydrogen $r_{H_2} = \dfrac{R}{2}$

It's unit is $\dfrac{Joule}{kg \times kelvin}$

Dimension : $[L^2T^{-2}\theta^{-1}]$

ideal gas equation $PV = nRT$

But No of moles $n = \dfrac{m}{M}$

$$PV = \frac{m}{M}RT$$

$$PV = mrT$$

Boltzman's constant (k) :

A gas constant per one molecule of gas is called as Boltzman's constant .

$\rightarrow$ it is same for all gases

$$i.e., \ k = \frac{R}{N} = \frac{8.31}{6.023 \times 10^{23}} = 1.38 \times 10^{-23} \ J/K$$

unit is $\frac{J}{kelvin}$

dimension : $[ML^2T^{-2}\theta^{-1}]$

ideal gas equation $PV = nRT$

$$\text{But No of moles } \mu = \frac{\text{no.of molecules}}{\text{avagadro's number}} = \frac{N}{N_A}$$

$$n = \frac{N}{N_A}$$

$$PV = \frac{N}{N_A}RT$$

$$PV = Nk_BT$$

Different forms of gas equation

Quantity of gas	**Equation**	**Constant**
1 mole gas	$PV = RT$	***R = universal gas constant***
n mole gas	$PV = n\,RT$	***R = universal gas constant***
1 molecule of gas	$PV = Nk_BT$	k_B ***= Boltzmann's constant***
1 gm gas	$PV = mrT$	***r = specific gas constant***

Real Gases :

The gases actually found in nature are called real gases. They do not obeys gas Laws.

For exactly one mole of an ideal gas

$$\frac{PV}{RT} = 1 \qquad\qquad V = K\frac{T}{P}$$

$$\frac{PV}{T} = K(cons\tan t)$$

$$K = R$$

$$PV = nRT$$

Plotting the experimentally determined value of $\frac{PV}{RT}$ for exactly one mole of various real gases as a function of pressure P, shows a deviation from identity.

$\rightarrow$ The quantity $\frac{PV}{RT}$ is called the compressibility factor and should be unit for an ideal gas.

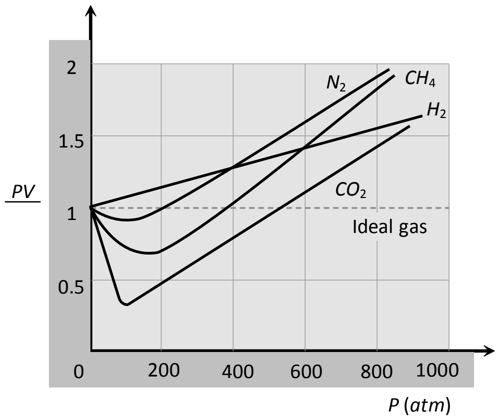


Fig. 13.7

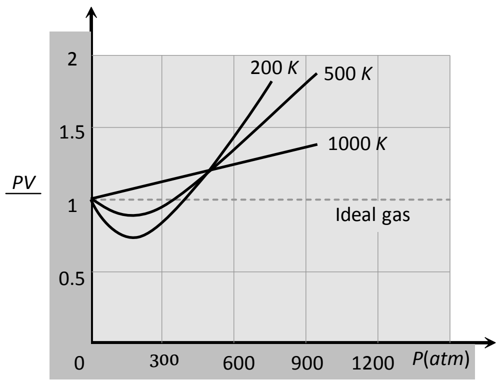


Fig. 13.8

$\rightarrow$ A real gas behaves as ideal gas most closely at low pressure and high temperature. Also can actual gas can be liquefied most easily which deviates most from ideal gas behaviour at low temperature and high pressure.

Equation of state for real gases :

It is given by Vander Waal's with two correction in ideal gas equation. The it know as Vander Waal's gas equation.

(i) **Volume correction :**

Due to finite size of molecule, a certain portion of volume of a gas is covered by the molecules themselves. Therefore the space available for the free motion of molecules of gas will be slightly less than the volume V of a gas.

Hence the effective volume becomes $(V-b)$.

(ii) **Pressure correction :**

Due to intermolecular force in real gases, molecule do not exert that force on the wall which they would have exerted in the absence of intermolecular force. Therefore the observed pressure P of the gas will be less than that present in the absence of intermolecular force.

Hence the effective pressure becomes $\left(P+\frac{a}{V^2}\right)$.

(iii) **Vander Waal's gas equations**

For 1 *mole* of gas $\left(P+\frac{a}{V^2}\right)(V-b)=RT$

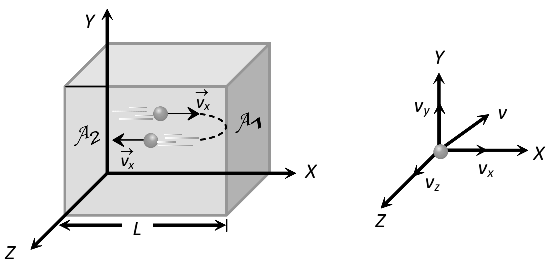
For n *moles* of gas $\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$

Here a and b are constant called Vander Waal's constant.

Dimension : $[a]=[ML^5T^{-2}]$ and $[b]=[L^3]$

Units : $a=N\times m^4$ and $b=m^3$.

Pressure of an Ideal Gas :



Consider an ideal gas (consisting of N molecules each of mass m) enclosed in a cubical box of side L.

Let Any molecule of gas moves with velocity $\vec{v}$ in any direction and collide with wall A_1 and rebounds

where $\vec{v}=v_x\hat{i}+v_y\hat{j}+v_z\hat{k}$

$$v=\sqrt{v_x^2+v_y^2+v_z^2}\ .$$

Due to random motion of molecule $v_x=v_y=v_z$

$$v^2=3v_x^2=3v_y^2=3v_z^2$$

Time between two successive collision with the wall A_1

$$\Delta t = \frac{\text{Distance travelled by molecule between two successive collision}}{\text{Velocity of molecule}}$$

$$\Delta t = \frac{2L}{v_x}$$

Then the number of collision per second. $n = \frac{1}{\Delta t} = \frac{v_x}{2L}$

This molecule collides with the shaded wall (A_1) with velocity v_x and rebounds with velocity $-v_x$.

The change in momentum of the molecule $\Delta p = (-mv_x) - (mv_x) = -2mv_x$

As the momentum remains conserved in a collision,

the change in momentum of the wall A_1 is $\Delta p = 2mv_x$ $\Delta p = 2mv_x$

After rebound this molecule travel toward opposite wall A_2 with velocity $-v_x$, collide to it and again rebound with velocity v_x towards wall A_1.

Force exerted by a single molecule on shaded wall is equal to rate at which the momentum is transferred to the wall by this molecule.

$$i.e.\ F_{\text{Single molecule}} = \frac{\Delta p}{\Delta t} = \frac{2mv_x}{(2L/v_x)} = \frac{mv_x^2}{L}$$

The total force on the wall A_1 due to all the molecules $F_x = \frac{m}{L}\sum v_x^2$

$$F_x = \frac{m}{M}(v_{x_1}^2 + v_{x_2}^2 + v_{x_3}^2 + ...) = \frac{mN}{L}\overline{v_x^2}$$

$\overline{v_x^2}$ = mean square of x component of the velocity.

Now pressure is defined as force per unit area, hence pressure on shaded wall

$$P_x = \frac{F_x}{A} = \frac{mN}{AL}\overline{v_x^2} = \frac{mN}{V}\overline{v_x^2}$$

For any molecule, the mean square velocity $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$

by symmetry $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{\overline{v^2}}{3}$$

Total pressure inside the container $P = \frac{1}{3}\frac{mN}{V}\overline{v^2} = \frac{1}{3}\frac{mN}{V}v_{rms}^2$ (where $v_{rms} = \sqrt{\overline{v^2}}$)

Relation between pressure and kinetic energy :

As we know $P = \frac{1}{3}\frac{mN}{V}v_{rms}^2$

but $M = mN$ = Total mass of the gas

$$P = \frac{1}{3}\frac{M}{V}v_{rms}^2$$

Density of gas $\rho = \frac{M}{V}$

Pressure $P = \frac{1}{3}\rho v_{rms}^2$ ... (i)

Kinetic Energy K.E = $\frac{1}{2}Mv_{rms}^2$

$\therefore$ K.E. per unit volume $\frac{K.E}{V} = E = \frac{1}{2}\left(\frac{M}{V}\right)v_{rms}^2 = \frac{1}{2}\rho v_{rms}^2$ ...(ii)

From (i) and (ii), we get $P = \frac{2}{3}E$

i.e. the pressure exerted by an ideal gas is numerically equal to the two third of the mean kinetic energy of translation per unit volume of the gas.

Effect of mass on pressure :

Pressure $P = \frac{1}{3}\frac{mN}{V}v_{rms}^2$

$P \propto \frac{(mN)T}{V}$ [As $v_{rms}^2 \propto T$]

If volume and temperature of a gas are constant

$P \propto mN$

Pressure $\propto$ (Mass of gas).

i.e. if mass of gas is increased, number of molecules and hence number of collision per second increases . *so* pressure will increase.

Effect of on pressure :

$P \propto \frac{(mN)T}{V}$

If mass and temperature of a gas are constant.

$P \propto (1/V)$,

i.e., if volume decreases, number of collisions per second will increase due to lesser effective distance between the walls resulting in greater pressure.

Effect of on pressure :

$$P \propto \frac{(mN)T}{V}$$

If mass and volume of gas are constant,

$P \propto (v_{rms})^2 \propto T$

i.e., if temperature increases, the mean square speed of gas molecules will increase and as gas molecules are moving faster, they will collide with the walls more often with greater momentum resulting in greater pressure.

Average translational kinetic energy of a gas:

Let M be the molecular mass and V be the molar volume of a gas. Let m be the mass of each molecule. Then

- Mean K.E. per mole of a gas, $E = \frac{3}{2}PV = \frac{3}{2}RT = \frac{3}{2}k_B N\, T$
- Mean K.E. per molecule of a gas $\overline{E} = \frac{3}{2}k_B T$
- K.E of 1 gram of gas $= \frac{3}{2}\frac{RT}{M}$

Note :

- Kinetic energy per molecule of gas does not depends upon the mass of the molecule but only depends upon the temperature of the gas.

  It is given as $E = \frac{3}{2}k_B T \;\; or \;\; E \propto T$
- Molecules of different gases say He, H_2 and O_2 etc. at same temperature will have same translational kinetic energy though their rms speeds are different
- Kinetic energy per mole of gas depends only upon the temperature of gas.
- Kinetic energy per gram of gas depend upon the temperature as well as molecular weight (or masso f one molecule) of the gas

  $KE_{gram} = \frac{3}{2}\frac{kT}{m} \Rightarrow KE_{gram} \propto \frac{T}{m}$
- From the above expression it is clear that higher the temperature of the gas, more will be the gas, more will be the average kinetic energy possessed by the gas molecules. At T=0, E=0, i.e. at absolute zero the molecular motion ceases.
- The kinetic interpretation of temperature gives the relation between the average kinetic energy of a gas molecule and absolute temperature.

$$i.e \quad E = (3/2)NK_B T \Rightarrow E \propto T$$

- Average kinetic energy is independent of pressure, volume and the nature of the ideal gas

PROBLEMS

1. A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100 N/m^2. During an observation time of 1 second, an atom travelling with the root–mean–square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take $R = \frac{25}{3}$ J/mol–K and $k = 1.38 \times 10^{-23}$ J/K.

   (a) Evaluate the temperature of the gas.

   (b) Evaluate the average kinetic energy per atom.

   (c) Evaluate the total mass of helium gas in the box.

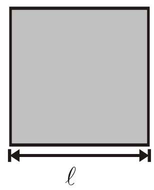
SOLUTION :

Volume of the box = $1 m^3$,

Pressure of the gas = 100 N/m^2.

Let T be the temperature of the gas

(a) Time between two consecutive collisions with one wall = $\frac{1}{500}$ sec



This time should be equal to $\frac{2I}{v_{rms}}$, where ℓ is the side of the cube.

$$2\ell v_{rms} = \frac{1}{500}$$

$$v_{rms} = 1000 \text{ m/s}$$

$$\therefore \sqrt{\frac{3RT}{M}} = 1000$$

$$T = \frac{(1000)^2 M}{3R} = \frac{(10)^6 (3 \times 10^{-3})}{3\left(\frac{25}{3}\right)} = 160K$$

(b) Average kinetic energy per atom = $\frac{3}{2}kT = \frac{3}{2}$ [(1.38×10^{-23}) 160J $= 3.312 \times^{-21}$ J

(c) From $PV = nRT = \frac{m}{M}RT$,

Mass of helium gas in the box $m = \frac{PVM}{RT}$

Substituting the values, $m = \frac{(100)(1)(4 \times 10^{-3})}{\left(\frac{25}{3}\right)(160)} = 3.0 \times 10^{-4}$ kg

2. Two ideal gases at temperature T_1 and T_2 are mixed. There is no loss of energy. If the masses of molecules of the two gases are m_1 and m_2 and number of their molecules are n_1 and n_2 respectively. Find the temperature of the mixture.

SOLUTION :

Total energy of molecules of first gas $= \frac{3}{2} n_1 k T_1$,

Total energy of molecules of second gas $= \frac{3}{2} n_2 k T_2$

Let temperature of mixture be T

then total energy of molecules of mixture $= \frac{3}{2} k(n_1 + n_2)T$

$$\frac{3}{2}(n_1 + n_2)kT = \frac{3}{2}k(n_1 T_1 + n_2 T_2)$$

$$T = \frac{(n_1 T_1 + n_2 T_2)}{(n_1 + n_2)}$$

3. 1 kg of diatomic gas is at a pressure of $8 \times 10^4 N/m^2$. The density of the gas is $4kg/m^3$. The energy of the gas due to its thermal motion is

SOLUTION :

Energy of diatomic gas due to its thermal motion is $= \frac{5}{2} PV$

$$= \frac{5}{2} P\left(\frac{m}{\rho}\right)$$

4. Gas at a pressure P_0 is contained in a vessel. If the masses of all the molecules are halved and their speeds are doubled, the resulting pressure P will be equal to

1) $4P_0$ 2) $2P_0$ 3) p_0 4) $\frac{p_0}{2}$

SOLUTION :

Pressure $P = \frac{1}{3}\rho(v_{rms})^2$

$$P \propto m(v_{rms})^2$$

$$\frac{P_2}{P_1} = \frac{m_2}{m_1}\left[\frac{v_2}{v_1}\right]^2$$

$$\frac{P_2}{P_1} = \frac{m/2}{m}\left[\frac{2v}{v}\right]^2$$

$$P_2 = 2P_0$$

Various Speeds of Gas Molecules:

The motion of molecules in a gas is characterised by any of the following three speeds.

Root mean square speed :

It is defined as the square root of mean of squares of the speed of different molecules

$$i.e.\ v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + ...}{N}} = \sqrt{\overline{v^2}}$$

From the expression of pressure

$$P = \frac{1}{3}\rho v_{rms}^2$$

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3PV}{\text{Mass of gas}}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$$

where $\rho = \frac{\text{Mass of gas}}{V} = \text{Density of the gas}$,

$M = \mu \times$ (mass of gas),

$pV = \mu RT$,

$R = kN_A$,

$k =$ Boltzmann's constant,

$m = \frac{M}{N_A} =$ mass of each molecule.

♦ With rise in temperature *rms* speed of gas molecules increases as

$$v_{rms} \propto \sqrt{T}\ .$$

♦ With increase in molecular weight *rms* speed of gas molecule decreases as

$$v_{rms} \propto \frac{1}{\sqrt{M}}\ .$$

e.g., *rms* speed of hydrogen molecules is four times that of oxygen molecules at the same temperature.

♦ *rms* speed of gas molecules is of the order of *km/s*

e.g., at NTP for hydrogen gas

$$(v_{rms}) = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 273}{2 \times 10^{3}}} = 1840\ m/s.$$

♦ *rms* speed of gas molecules is $\sqrt{\frac{3}{\gamma}}$ times that of speed of sound in gas, as

rms speed of gas $v_{rms} = \sqrt{\frac{3RT}{M}}$

speed of sound in gas $v_s = \sqrt{\frac{\gamma RT}{M}}$

$$v_{rms} = \sqrt{\frac{3}{\gamma}} v_s$$

♦ *rms* speed of gas molecules does not depends on the pressure of gas (if temperature remains constant)

because $P \alpha\ \rho$ (Boyle's law)

if pressure is increased *n* times then density will also increases by *n* times but v_{rms} remains constant.

♦ Moon has no atmosphere because v_{rms} of gas molecules is more than escape velocity (v_{e}).

A planet or satellite will have atmosphere only if $v_{rms} < v_e$

♦ The molecules of gases will escape out from a planet. If the temperature of planet

$$T \geq \frac{Mv_e^2}{3R} \quad (\because v_{rms} \geq v_e); \quad v_e = \text{escape velocity of planet}$$

M = molecular mass of gas

♦ At $T = 0$; $v_{rms} = 0$

i.e. the *rms* speed of molecules of a gas is zero at 0 *K*. This temperature is called absolute zero.

(2) **Most probable speed :**

The particles of a gas have a range of speeds. This is defined as the speed which is possessed by maximum fraction of total number of molecules of the gas. *e.g.*, if speeds of 10 molecules of a gas are 1, 2, 2, 3, 3, 3, 4, 5, 6, 6 *km/s*, then the most probable speed is 3 *km/s*, as maximum fraction of total molecules possess this speed.

$$\text{Most probable speed } v_{mp} = \sqrt{\frac{2k_B T}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2PV}{\text{mass of gas}}} = \sqrt{\frac{2P}{\rho}}$$

(3) **Average speed :** It is the arithmetic mean of the speeds of molecules in a gas at given temperature.

$$v_{av} = \frac{v_1 + v_2 + v_3 + v_4 + ...}{N}$$

and according to kinetic theory of gases

Average speed $v_{avg} = \sqrt{\frac{8k_BT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8PV}{\pi(mass\ of\ gas)}} = \sqrt{\frac{8P}{\pi\rho}}$.

Relation between $\upsilon_{av}, \upsilon_{rms}$ and υ_{mp} :

Average speed $v_{avg} = 0.92 v_{rms}$

Most probable speed $v_{mp} = 0.816 v_{rms}$

$v_{rms} : v_{avg} : v_{mp} = 1.73 : 1.60 : 1.41$

Clearly, $v_{rms} > v_{avg} > v_{mp}$

PROBLEMS

1. The root-mean-square (rms) speed of oxygen molecules (O_2) at a certain absolute temperature is υ. If the temperature is doubled and the oxygen gas dissociates into atomic oxygen, the rms speed would be

1) υ 2) $\sqrt{2}\upsilon$ 3) 2υ 4) $2\sqrt{2}\upsilon$

SOLUTION :

rms speed of gas $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$V \alpha \sqrt{T/M}$$

$$\frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}\frac{M_1}{M_2}}$$

$$\frac{V_2}{V_1} = \sqrt{\frac{2T}{T}\frac{2M}{M}}$$

$$\frac{V_2}{V_1} = 2$$

$$V_2 = 2V$$

2. Calculate the temperature at which the oxygen moleules will have the same rm,s velocity as the hydrogen molecules at $150^0 C$. Molecular weight of oxygen is 32 and that of hydrogen is 2

SOLUTION :

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$As\ V_{1rms} = V_{2\,rms}, therefore\ T \infty M$$

$$\frac{T_1}{T_2} = \frac{M_1}{M_2}; T_2 = T_1\left(\frac{M_2}{M_1}\right)$$

$$T_2 = 423\left[\frac{32}{2}\right], T_2 = 6768K$$

$$t_2 = 6768 - 273 = 6495^0 C$$

3.Two moles of an ideal gas X occupying a volume V exerts a pressure P. The same pressure is exerted by one of another gas Y occupying a volume 2V. if the molecular weight of Y is 16 times the molecular weight of X, find the ratio of the rms speeds of the molecules of X and Y.

SOLUTION :

$$V_{rms} = \sqrt{\frac{3PV}{nM}}\left(\frac{m}{M} = n\right)$$

$$\frac{V_{1rms}}{V_{2rms}} = \sqrt{\frac{V_1}{V_2}\frac{n_2}{n_1}\frac{M_2}{M_1}}; \frac{V_{1rms}}{V_{2rms}} = \sqrt{\frac{V}{2V}\left(\frac{1}{2}\right)\frac{16}{1}}$$

$$\frac{V_{1rms}}{V_{2rms}} = 2$$

4. At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the r.m.s. speed of a helium gas atom at $-20^0 C$? Atomic mass of argon $=39.9u$ and that of helium =4.0u

SOLUTION :

Here, atomic mass of argon, $M_1 = 39.9u$

atomic mass of helium, $M_2 = 4.0u$

Suppose that the $r.m.s$ speed of argon gas atoms at temperature T is equal to the $r.m.s$. speed of Helium gas atoms at $-20^0 C$ i.e., at $T = -20 + 273 = 253K$.

Suppose that $u_{r.m.s}$ and $u_{r.m.s}$ are $u_{r.m.s}$ speeds of argon and helium gas atoms at temperatures T and T' respectively

Now. $u_{r.m.s} = \sqrt{\frac{3RT}{M}}$

$\therefore u_{r.m.s} = \sqrt{\frac{3RT}{39.9}}$ and $u_{r.m.s} = \sqrt{\frac{3R \times 253}{4.0}}$

Since $u_{r.m.s} = u_{r.m.s}$, we have

$$\sqrt{\frac{3RT}{39.9}} = \sqrt{\frac{3R \times 253}{4.0}}$$

(or) $T = \frac{253 \times 39.9}{4.0} = 2,523.7K$

5. Your are given the following group of particles, n_i represents the number of molecules with speed v_i

n_i	**2**	**4**	**8**	**6**	**3**
$v_i\left(ms^{-1}\right)$	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**

calculate (i) average speed (ii) rms speed

SOLUTION : (i) average speed

$$v_{ang} = \frac{n_1v_1+n_2v_2+n_3v_3+n_4v_4+n_5v_5}{\left(n_1+n_2+n_3+n_4+n_5\right)}$$

$$= \frac{(2\times1)+(4\times2)+(8\times3)+(6\times4)+(3\times5)}{(2+4+8+6+3)} = \frac{73}{23} = 3.17ms^{-1}$$

(ii) root mean square speed is

$$v_{rms} = \left(\frac{n_1v_1^2+n_2v_2^2+n_3v_3^2+n_4v_4^2+n_5v_5^2}{n_1+n_2+n_3+n_4+n_5}\right)$$

$$= \left(\frac{2\times1+4\times4+8\times9+6\times16+3\times25}{2+4+8+6+3}\right)^{1/2} = 3.36ms^{-1}$$

6.. If the molecular weight of two gases are M_1 and M_2 then at a temperature the ratio of root mean square velocity v_1 and v_2 will be

(a) $\sqrt{\frac{M_1}{M_2}}$ (b) $\sqrt{\frac{M_2}{M_1}}$ (c) $\sqrt{\frac{M_1+M_2}{M_1-M_2}}$ (d) $\sqrt{\frac{M_1-M_2}{M_1+M_2}}$

SOLUTION :

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V_{rms} \alpha \sqrt{\frac{1}{M}}$$

$$\frac{V_2}{V_1} = \sqrt{\frac{M_2}{M_1}}$$

7. The *r.m.s.* velocity of a gas at a certain temperature is $\sqrt{2}$ times than that of the oxygen molecules at that temperature. The gas can be

(a) H_2 (b) He (c) CH_4 (d) SO_2

SOLUTION :

$$v_{rms} \propto \frac{1}{\sqrt{M}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}}$$

$$\therefore \frac{1}{\sqrt{2}} = \sqrt{\frac{M_2}{32}} \Rightarrow M_2 = 16.$$

Hence the gas is CH_4.

8. At what temperature will the rms speed of oxygen molecule will be sufficient for escaping from earth? (Ve= 11.2km/s, m=2.76x10^{-26}kg and k=1.38x10^{-23}J/K)

1) $T = -91.2$ 2) $9.36\times10^4 K$

3) $0.36\times10^4 K$ 4) $5.36\times10^4 K$

SOLUTION :

If the temperature is T,

according to kinetic theory of gases translational $KE = \frac{3}{2}k_B T$

The oxygen molecule will escape from earth

if $\frac{3}{2}kT > \frac{1}{2}m\text{v}_e^2$

i.e. $T > \frac{m\text{v}_e^2}{3k}$

$$T > \frac{2.76\times10^{-26}\times\left(11.2\times10^3\right)^2}{3\times1.38\times10^{-23}}$$

$$T > 8.36\times10^4 K$$

9. If the density of hydrogen at STP is $0.09kgm^{-3}$, calculate the rms velocity of its molecules at 0^0C.

1) $2.84\times10^3 ms^{-1}$ 2) $1.84\times10^3 ms^{-1}$

3) $0.84\times10^3 ms^{-1}$ 4) $2\times10^3 ms^{-1}$

SOLUTION :

$$P = \frac{1}{3}\rho\left(\text{v}_{\text{rms}}\right)^2$$

$$\therefore v_{rms} = \sqrt{\frac{3P}{\rho}}$$

Here P=76cm of Hg $= 1.013 \times 10^5\, Nm^{-2}$

$$\rho = 0.09 kg\, m^{-3}$$

$$; v_{rms} = \sqrt{\frac{(3)(1.013\times10^5)}{0.09}} = 1.84\times10^3\, ms^{-1}$$

10. At what temperature is the root mean square speed of oxygen molecules equal to the r.m.s speed of carbon dioxide molecules at -23^{0}C, molecular weight of oxygen =32 and that of carbon dioxide = 44.

1) +91.2^{0}C 2) -91.2^{0}C 3) +112.2^{0}C 4) -112.2^{0}C

SOLUTION :

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V'_{rms} = \sqrt{\frac{3RT'}{M}} = \sqrt{\frac{3R\times250}{44}}$$

$$V_{rms} = V'_{rms}$$

$$\sqrt{\frac{3RT}{32}} = \sqrt{\frac{3R\times250}{44}}$$

$$T = -91.2\ ^0\text{C}$$

11. Your are given the following group of particles, n_i represents the number of molecules with speed v_i

n_i	2	4	8	6	3
$v_i\,(ms^{-1})$	1.0	2.0	3.0	4.0	5.0

most probable speed is .....

SOLUTION :

By definition,velocity belongs to more molecules is most probable speed=3.0 ms^{-1}.

12. If the density of hydrogen at STP is $0.09 kgm^{-3}$, calculate the rms velocity of its molecules at $0^0 C$.

SOLUTION :

$$P = \frac{1}{3}\rho(v_{rms})^2$$

$\therefore v_{rms} = \sqrt{\frac{3P}{\rho}}$

Here P=76cm of Hg $= 1.013 \times 10^5\, Nm^{-2}$

$\rho = 0.09 kg\, m^{-3}$;

$$v_{rms} = \sqrt{\frac{(3)(1.013\times10^5)}{0.09}}$$

$$v_{rms} = 1.84\times10^3 ms^{-1}$$

13. When the temperature of a gas is raised from $27°C$ to $90°C$, the percentage increase in the *r.m.s.* velocity of the molecules will be

(a) 10% (b) 15% (c) 20% (d) 17.5%

SOLUTION :

$$v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{(273+90)}{(273+30)}} = 1.1$$

$$\%\text{ increase} = \left(\frac{v_2}{v_1} - 1\right)\times 100 = 0.1\times100 = 10\%$$

14. The r.m.s speed of oxygen molecule (O_2) at a certain temperature T is V. If on increasing the temperature of the oxygen gas to 2T, the oxygen molecules dissociate into atomic oxygen, find the speed of the oxygen atom

1) 2V 2) V 3) V/2 4) 3V

SOLUTION :

$$V_{rms} = \sqrt{\frac{3KT}{m}}$$

$$V'_{rms} = \sqrt{\frac{3K(2T)}{m/2}}$$

$$V'_{rms} = \sqrt{\frac{3K4T}{m}}$$

$$V'_{rms} = 2V_{rms}$$

15. Calculate the temperature at which root mean square velocity of SO_2 gas molecules is same as that of O_2 **molecules at 127^0C. Molecular weights of O_2 and SO_2 are 32 and 64 respectively**

1) 527^0C 2) 800^0C 3) 500^0c 4) 627^0c

SOLUTION :

root mean square velocity of SO_2 $V_{rms} = \sqrt{\frac{3RT}{M}}$

root mean square velocity of O_2 $V_{rms} = \sqrt{\frac{3RT'}{M'}}$

$$V_{rms} = V'_{rms}$$

$$\sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT'}{M'}}$$

$$T' = 527^0C$$

16. If three molecules have velocities 0.5, 1 and 2 kms^{-1} respectively, calculate the relation between the root mean square speed and average speed

1) $V_{rms} = V_{avg}/2$ 2) $V_{rms} = V_{avg}$ 3) $V_{rms} = 1.134\ V_{avg}$ 4) $V_{rms} = 2V_{avg}$

SOLUTION :

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2}{3}} = 1.323$$

$$V_{avg} = \frac{V_1 + V_2 + V_3}{3} = 1.67 km/s$$

$$V_{rms} = 1.134\ V_{avg}$$

17. If the *r.m.s.* velocity of a gas at a given temperature (Kelvin scale) is 300 *m*/*sec*. What will be the *r.m.s.* velocity of a gas having twice the molecular weight and half the temperature on Kelvin scale =

(a) 300 *m*/*sec* (b) 600 *m*/*sec* (c) 75 *m*/*sec* (d) 150 *m*/*sec*

SOLUTION :

$$v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow v_{rms} \propto \sqrt{\frac{T}{M}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{M_1}{M_2} \times \frac{T_2}{T_1}}$$

$$= \sqrt{\frac{1}{2} \times \frac{1}{2}}$$

$$v_2 = \frac{v_1}{2} = \frac{300}{2} = 150 m/sec$$

18. At what temperature is the rms speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at $-20^0 C$? Atomic mass of argon $= 39.9u$ and that of helium =4.0u

SOLUTION :

Given that $(v_{rms})_{Ar} = (v_{rms})_{He}$ $\sqrt{\frac{3RT_{ar}}{M_{ar}}} = \sqrt{\frac{3RT_{He}}{M_{He}}}$

$$; \sqrt{\frac{3RT_{ar}}{40}} = \sqrt{\frac{3R(253)}{4}}$$

$$\frac{T}{40} = \frac{253}{4}$$

$$T = 2530K$$

$$T = 2530 - 273$$

$$T = 2257^{0}C$$

19. Two vessels having equal volume contain molecular hydrogen at one atmosphere, and helium at two atmospheres respectively. What is the ratio of rms speeds of hydrogen molecule to that of helium molecule, if both the samples are at the same temperature

SOLUTION :

According to kinetic theory of gases,

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\frac{(v_{rms})_H}{(v_{rms})_{He}} = \sqrt{\frac{M_{He}}{M_H}}$$

$$v_H = \sqrt{(4/2)}\ v_{He}$$

$$v_H = \sqrt{2}\,(v_{He})$$

20. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic) and the third contains uranium hexafluoride (polyatomic)

(a) Do the vessels contain equal number of respective molecules?

(b) Is the root mean square speed of molecules the same in the three cases ? If not, in which case is v_{rms} largest?

SOLUTION :

(a) Yes, the vessels contain equal number of respective molecules. It is because, equal volume of all the gases under same temperature and pressure contain equal number of molecules (Avogadro's hypothesis)

(b) The root mean square speed is given by $v_{rms} = \sqrt{\frac{3kT}{m}} \Rightarrow v_{rms} \propto \frac{1}{\sqrt{m}}$

Since v_{rms} .depends upon mass of the molecule of the gas, it will not be same in the three cases.

Since v_{rms} is inversely proportional to the square root of the mass of the molecules of the gas, it will be largest for the gas, whose molecules are lightest. Therefore v_{rms} .will be largest for neon gas.

DEGREE OF FREEDOM (f)

- The number of possible ways in which a system can possess internal energy is called degrees of freedom.

OR

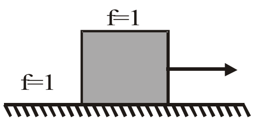
- The number of independent ways in which a molecule or an atom can exhibit motion or have energy is called it's degrees of freedom.

OR

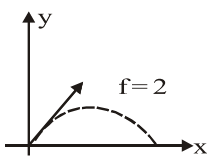
- The number of independent coordinates required to specify the dynamical state of a system is called it's degrees of freedom.

For example

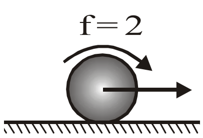
(a) Block has one degree of freedom, because it is confined to move in a straight line and has only one transistional degree of freedom.



(b) The projectile has two degrees of freedom becomes it is confined to move in a plane and so it has two translational degrees of freedom.



(c) The sphere has two degrees of freedom one rotational and another translational. Similarly a particle free to move in space will have three translational degrees of freedom.



Note : In pure rolling sphere has one degree of freedom as $KE = \frac{1}{2} mv^2 (1+\frac{K^2}{R^2}) = \frac{7}{10} mv^2$

- The degrees of freedom are of three types :

(i) Translational degrees of freedom :

The maximum number of translational degrees of freedom can be three.

These are

$$\frac{1}{2}mv_x^2, \frac{1}{2}mv_y^2, \frac{1}{2}mv_z^2.$$

(ii) Rotational degrees of freedom :

The maximum number of rotational degrees of freedom can be three. The number of degrees of freedom in this case depends on the structure of the molecule.

These are $\frac{1}{2}I_x\omega_x^2, \frac{1}{2}I_y\omega_y^2, \frac{1}{2}I_z\omega_z^2$

(iii) Vibrational degrees of freedom:

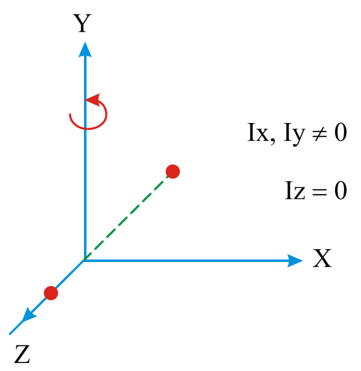
Their numbers depend on atoms in the molecule and their arrangement. These degrees of freedom are considered at a very high temperature.

Note : At room temperature only translational and rotational degrees of freedom are taken into account.

1. Monoatomic gas: The degrees of freedom of monoatomic gas molecules are due to three independent translational motions along x, y and z axis. The degrees of freedom are

$$\frac{1}{2}mv_x^2, \frac{1}{2}mv_y^2, \frac{1}{2}mv_z^2.$$

2. Diatomic (or) Linear polyatomic gas:



The molecule has three degrees of freedom of translational and two degrees of freedom of rotational these are

$$\frac{1}{2}mv_x^2, \frac{1}{2}mv_y^2, \frac{1}{2}mv_z^2, \frac{1}{2}I_x\omega_x^2, \frac{1}{2}I_y\omega_y^2.$$

Note : If vibrational degrees of freedom are taken into account, then total number of degrees of freedom of diatomic molecule becomes 7 at high temperature. These are

$$\frac{1}{2}mv_x^2, \frac{1}{2}mv_y^2, \frac{1}{2}mv_z^2, \frac{1}{2}I_x\omega_x^2, \frac{1}{2}I_y\omega_y^2, \frac{1}{2}\mu v^2, \frac{1}{2}kr^2.$$

Here $\frac{1}{2}\mu v^2$ corresponds to kinetic energy of vibration (μ is the reduced mass) and $\frac{1}{2}kr^2$ corresponds to potential energy of vibration (k is the force constant, r is the separation between the atoms)

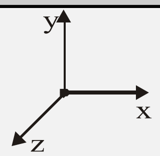
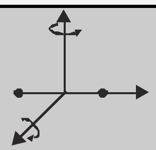
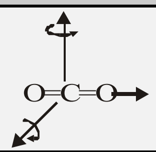
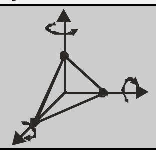
3. Triatomic or non linear polyatomic gas:

The molecule of polyatomic gases like CO_2, H_2O, NH_3, CH_4, etc. has more than two atoms.

It has 3 translational and 3 rotational degrees of freedom as shown in figure Apart from this, if such a molecule has v vibrational modes, it will have additional 2v vibrational degrees of freedom, each vibrational mode contributing 2 vibrational degrees of freedom.

Thus, total number of degrees of freedom of a polyatomic gas molecule,

i.e., $f = 3 + 3 + 2v = (6 + 2v)$

Atomicity of gas	Translational	Rotational	Total	
Monoatomic **Ex.** Ar, Ne, Ideal gas etc	3	0	3	
Diatomic **Ex.** O_2, Cl_2, N_2 etc.	3	2	5	
Triatomic (linear) **Ex.** CO_2, C_2H_2	3	2	5	
Triatomic (Non–linear) or Polyatomic **Ex.** H_2O, NH_3, CH_4	3	3	6	

At high temperatures a diatomic molecule has 7 degrees of freedom. (3 translational, 2 rotational and 2 vibrational)

Law of equipartition of energy :

According to this law, for a system in thermal equilibrium, the total energy of a dynamic system is equally distributed among its various degrees of freedom.

The energy associated with each degree of freedom is $\frac{1}{2}K_BT$ per molecule

or $\frac{1}{2}RT$ per mole.

For a molecule with f degrees of freedom

Energy related with each degree of freedom = $\frac{1}{2}\text{kT}$ Energy related with all degree of freedom = $\frac{f}{2}\text{kT}$

Energy per mole $U = f \times \frac{RT}{2} = \frac{fRT}{2}$

Molar specific heat at constant volume $C_v = \frac{\partial U}{\partial T} = \frac{fR}{2}$

Molar specific heat at constant pressure $C_p = C_v + R = \frac{fR}{2} + R = R\left(\frac{f}{2} + 1\right)$

Ratio of specific heat $\gamma = \frac{C_p}{C_v} = \left(1 + \frac{2}{f}\right)$

Note: $\gamma_{poly} = \frac{4 + f_{vib}}{3 + f_{vib}}$,

f_{vib} = number of modes of vibrations (at high temperatures)

C_v, C_p and γ for different gases:

Monoatomic gas: Degrees of freedom f = 3

Kinetic energy per mole $E = U = 3 \times \frac{RT}{2} = \frac{3RT}{2}$

Molar specific heat at constant volume $C_v = \frac{\partial U}{\partial T} = \frac{3R}{2}$

Molar specific heat at constant pressure $C_p = C_v + R = \frac{5R}{2}$

Ratio of specific heat $\gamma = \frac{C_p}{C_v} = \frac{5R/2}{3R/2} = \frac{5}{3} = 1.66$

Diatomic Gas : Degrees of freedom f = 5

Kinetic energy per mole $U = 5 \times \frac{RT}{2} = \frac{5RT}{2}$

Molar specific heat at constant volume $C_v = \frac{\partial U}{\partial T} = \frac{5R}{2}$

Molar specific heat at constant pressure $C_p = C_v + R = \frac{7R}{2}$

Ratio of specific heat $\gamma = \frac{C_p}{C_v} = \frac{7R/2}{5R/2} = \frac{7}{5} = 1.4$

Triatomic (or) polyatomic Gas:

Degrees of freedom f = 6

Kinetic energy per mole $U = 6 \times \frac{RT}{2} = 3RT$

Molar specific heat at constant volume $C_v = \frac{\partial U}{\partial T} = 3R$

Molar specific heat at constant pressure $C_p = C_v + R = 4R$

Ratio of specific heat $\gamma = \frac{C_p}{C_v} = \frac{4R}{3R} = \frac{4}{3} = 1.33$

Specific heat capacity of solids

A solid consists of a regular array of atoms in which each of the atoms has a fixed equilibrium position. As such in a solid, an atom has no translational or rotational degrees of freedom. It has only three vibrational modes along three mutually perpendicular directions. Since each vibrational mode contributes two vibrational degrees of freedom, an atom in a solid has six vibrational degrees of freedom. Further, as energy associated per degree of freedom per atom is $\frac{1}{2}k_B T$,

total inernal energy per mole (N_A atoms) of solid, i.e.,

$$U = 6\left(\frac{1}{2}k_B T\right)N_A = 3RT \quad \left(as\, k_B = \frac{R}{N_A}\right)$$

Thus, $C = \frac{dU}{dT} = \frac{d}{dT}(3RT)$ or

$C = 3R \approx 6cal/mol\,C^o \approx 25J/mol\ K$

The specific heat capacity at high temperatures (usually above 300K) is the same for all solids and is approximately equal to 3R or 6 cal/mol C° or 25 J/mol K.

Specific heat capacity of Water

A water molecule (H_2O) consists of 2 hydrogen atoms and 1 oxygen atom. Treating water like a solid, each atom in its molecule has 6 degrees of freedom and as such its each molecule has $3 \times 6 = 18$ degrees of freedom. According to the law of equipartition of energy,

energy associated per molecule per degree of freedom $= \frac{1}{2}k_B T$

internal energy associated per mole of water, $U = 18\left(\frac{1}{2}k_B T\right)N_A = 9RT$

Tus, $C = \frac{dU}{dT} = \frac{d}{dT}(9RT) = 9R$

PROBLEMS

01. Each molecule of a gas has *f* degrees of freedom. The ratio $\frac{C_p}{C_V} = \gamma$ for the gas is

1) $1+\frac{f}{2}$ 2) $1+\frac{1}{f}$ 3) $1+\frac{2}{f}$ 4) $1+\frac{(f-1)}{3}$

SOLUTION :

Molar specific heat at constant volume $C_v = \frac{\partial U}{\partial T} = \frac{fR}{2}$

Molar specific heat at constant pressure $C_p = C_v + R = \frac{fR}{2} + R = R\left(\frac{f}{2}+1\right)$

Ratio of specific heat $\gamma = \frac{C_p}{C_v} = \left(1+\frac{2}{f}\right)$

2 A cylinder of fixed capacity 44.8 litres contains helium gas at standard temperature and pressure. What is the amount of heat needed to raise the temperature of the gas in the cylinder by 15.0 °C ? (R = 8.31 J mo1^{-1} K^{-1}).

SOLUTION :

1 mole of any gas occupies 22.4 litres

so 44.8 litres of helium contains 2 moles.

Since the volume of cylinder is fixed , the heat required is C_V

Here Helium is monoatomic gas $n = 3/2$

$\therefore C_V = \frac{3}{2}R$

so heat required = $nC_V dT = 2 \times 1.5R \times 15 = 45R = 3745$

3.. The ratio of two specific heats $\frac{C_P}{C_V}$ of CO is

(a) 1.33 (b) 1.40 (c) 1.29 (d) 1.66

SOLUTION :

Co is diatomic gas, for diatomic gas

$C_P = \frac{7}{2}R$ and $C_V = \frac{5}{2}R$

$\gamma_{di} = \frac{C_P}{C_V} = \frac{7R/2}{5R/2} = 1.4$

4. Considering the gases to be ideal, the value of $\gamma = \frac{C_P}{C_V}$ for a gaseous mixture consisting of = 3 moles of carbon dioxide and 2 moles of oxygen will be $(\gamma_{O_2} = 1.4, \gamma_{CO_2} = 1.3)$

(a) 1.37 (b) 1.34 (c) 1.55 (d) 1.63

SOLUTION :

$$\gamma_{\text{mix}} = \frac{\frac{\mu_1\gamma_1}{\gamma_1 - 1} + \frac{\mu_2\gamma_2}{\gamma_2 - 1}}{\frac{\mu_1}{\gamma_1 - 1} + \frac{\mu_2}{\gamma_2 - 1}} = \frac{\frac{3\times 1.3}{(1.3-1)} + \frac{2\times 1.4}{(1.4-1)}}{\frac{3}{(1.3-1)} + \frac{2}{(1.4-1)}} = 1.33$$

5. One mole of a monoatomic ideal gas is mixed with one mole of a diatomic ideal gas. The molar specific heat of the mixture at constant volume is

(a) 8 (b) $\frac{3}{2}R$ (c) $2R$ (d) $2.5\,R$

SOLUTION :

$$(C_V)_{mix} = \frac{\mu_1 C_{V_1} + \mu_2 C_{V_2}}{\mu_1 + \mu_2}$$

$$= \frac{1\times\frac{3}{2}R + 1\times\frac{5}{2}R}{1+1} = 2R$$

$$\left((C_V)_{mono} = \frac{3}{2}R,\ (C_V)_{di} = \frac{5}{2}R\right)$$

6. A gas has molar heat capacity $C = 37.55\text{J mole}^{-1}\text{K}^{-1}$, in the process PT=constant. Find the number of degrees of freedom of the molecules of the gas.

SOLUTION :

PT = K(constant) ...... (i)

But $PV = RT \Rightarrow P = \frac{RT}{V}$ $\therefore \frac{RT}{V}\times T = K$

$$\frac{dV}{dT} = \frac{2R}{P}\ (\because PT = K) \text{ ...... (ii)}$$

From first law of thermodynamics

$$dQ = dU + dW$$

$$;\ CdT = C_v dT + PdV$$

$$C = C_v + P\left(\frac{dV}{dT}\right) \text{ as, } C = C_v + P\frac{dV}{dT}$$

$$C = C_v + P\left(\frac{2R}{P}\right) \Rightarrow C - C_v = 2R$$

$\Rightarrow C_v = C - 2R$ .... (iii)

But $C_v = \frac{fR}{2}$ .... (iv) from (iii) and (iv)

$\frac{fR}{2} = C - 2R \Rightarrow f = \frac{2(C-2R)}{R}$

$= \frac{2(37.55 - 2\times 8.3)}{8.3} = 5$

7. What is the total kinetic energy of 2g of Nitrogen gas at temperature 300 K.

SOLUTION :

Given, mass of Nitrogen gas= 2 g

molecular waight $M_W = 28$, gm

Temperature $T = 300K$,

$R = 8.31\times 10^7 erg\ mol^{-1}K^{-1}$

Kinetic energy of 2g of $N_2 = 2\times\frac{3}{2}\frac{RT}{M_W} = \frac{3RT}{M_W}$

$= \frac{3\times 8.31\times 10^7\times 300}{28} erg = 267\times 10^7 erg$

8. Calculate the total kinetic energy of one kilo mole of Oxygen gas at $27^0 C$

SOLUTION :

oxygen is a diatomic gas, its molecules have 5 degrees of freedom.

Therefore, the total kinetic energy of a molecule of Oxygen is $\frac{5}{2}k_B T$.

As, 1kg-mole of Oxygen has N molecules,
the total kinetic energy of one kg-mole of Oxygen at temperature T is

$\frac{5}{2}RT = \frac{5}{2}\times 8.31\times 10^3\times 300$

$= 6.23\times 10^6 Joule/Kg - mole$

9. Estimate the average thermal energy of a helium at (i) room temperature $(27^0 C)$ **(ii) the temperature on the surface of the sun (6,000 K) and (iii) the temperature of** $10^7 K$**. Given,** $k = 1.38\times 10^{-23} J K^{-1}$

SOLUTION :

Here, $k = 1.38\times10^{-23} JK^{-1}$

(i) $T = 27 + 273 = 300K$

$\therefore \overline{K.E} = \frac{3}{2}kT = \frac{3}{2}\times1.38\times10^{-23}\times300,$

$= 6.21\times10^{-21} J$

(ii) T=6,000 K,

$\therefore \overline{K.E.} = \frac{3}{2}kT = \frac{3}{2}\times1.38\times10^{-23}\times6,000,$ $= 1.242\times10^{-19} J$

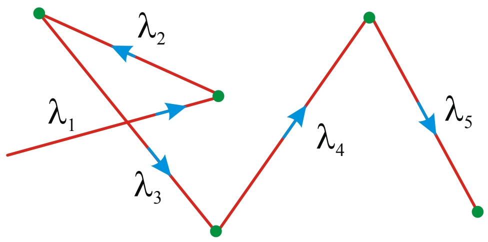
(iii) $T = 10^7 K$

$\therefore \overline{K.E.} = \frac{3}{2}kT = \frac{3}{2}\times1.38\times10^{-23}\times6,000,$

$= 2.07\times10^{-16} J$

Mean free path (λ):

→ The mean free path of a gas molecule may be defined as the average distance travelled by the molecule between two successive collisions.



Let $\lambda_1, \lambda_2, ...\lambda_n$ be the distance travelled by a gas molecule during n collisions, then the mean free path of gas molecule is given by

$$\lambda = \frac{\lambda_1 + \lambda_2 + ... + \lambda_n}{n}$$

During the collision, a molecule of a gas moves in a straight line with constant velocity. The statistical study of heat gives the mean free path as following

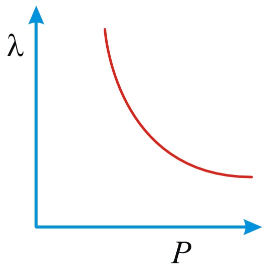
$$\lambda = \frac{1}{\sqrt{2}\pi n d^2}$$

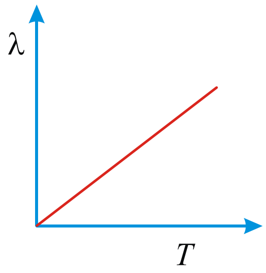
Where d is the diameter of molecule, n is the number of molecules per unit volume.

Collision frequency $f = \frac{V_{rms}}{\lambda}$

→ Mean free path depends on nature of molecule and with increase in n (number density) it decreases.

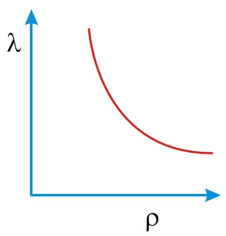
→ Here $n = \frac{N}{V} = \frac{P}{K_B T}$ Hence, $\lambda = \frac{1}{\sqrt{2}}\frac{K_B T}{\pi d^2 P}$

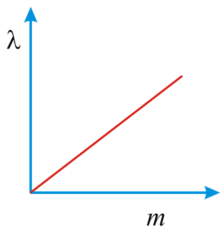




$\rightarrow \quad \lambda = \dfrac{1}{\sqrt{2}\pi n d^2} = \dfrac{m}{\sqrt{2}\pi (m.n) d^2} = \dfrac{m}{\sqrt{2}\pi d^2 \rho}$

$\rightarrow$ As $\lambda \propto \dfrac{1}{\rho}$ and $\lambda \propto m$,





Brownian motion :

It provides a direct evidence for the existence of molecules and their motion. The zig-zag motion of gas molecules is Brownian motion because it occurs due to random collision of molecules. But this motion cannot be seen. However, the zig-zag motion of pollen grains $\left(\approx 10^{-15} m\right)$ can be seen under a microscope

PROBLEMS

1. Estimate the mean free path for a water molecule in water vapour at 373K. The diameter of the molecule is $2 \times 10^{-10} m$, and at STP number of molecules per unit volume is $2.7 \times 10^{25} m^{-3}$

SOLUTION :

The number density(n) is inversely proportional to absolute temperature

$\therefore n\alpha \dfrac{1}{T}$

$\Rightarrow \dfrac{n_{373}}{n_{273}} = \dfrac{273}{373}$

$n_{373} = n_{273} \times \dfrac{273}{373}$

$n_{373} = 2.7 \times 10^{25} \times \dfrac{273}{373} = 2 \times 10^{25} m^{-3}$

Given d= $2 \times 10^{-10} m$

Hence , mean free path $\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$

$$= \frac{1}{\sqrt{2} \times 3.14 \times \left(2 \times 10^{-10}\right)^2 \times 2 \times 10^{25}} = 4 \times 10^{-7} m$$

2. Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2 atm and temperature $17^0 C$. Take the radius of nitrogen molecule to be $1 A^0$. (Molecular mass of nitrogen = 28u)

SOLUTION :

Here, $P = 2 \times 1.013 \times 10^5 = 2.026 \times 10^5 \, Nm^{-2}$;

$T = 17 + 273 = 290K; M = 28g = 28 \times 10^{-3} kg$

molecular diameter

. $d = 1 \times 2 = 2A^0 = 2 \times 10^{-10} m;$

Now , mass of a nitrogen molecule,

$$m = \frac{M}{N} = \frac{28 \times 10^{-3}}{6.02 \times 10^{23}} = 4.65 \times 10^{-26} kg$$

Also, volume occupuied by the nitrogen gas

$$V = \frac{RT}{P} = \frac{8.31 \times 290}{2.026 \times 10^5} = 1.19 \times 10^{-2} m^3$$

Therefore, density of the nitrogen gas,

$$\rho = \frac{M}{V} = \frac{28 \times 10^{-1}}{1.19 \times 10^{-2}} = 2.353 kgm^{-1}$$

Now, $\vec{\lambda} = \frac{m}{\sqrt{2}\pi d^2 \rho}$

$$= \frac{4.65 \times 10^{-26}}{\sqrt{2}\pi \times \left(2 \times 10^{-10}\right)^2 \times 2.353}$$

$$= 1.11 \times 10^{-7} m$$

Now, $v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 2.026 \times 10^5}{2.353}}$

$$= 508.24 ms^{-1}$$

Therefore, collision frequency,

$$f=\frac{v_{rms}}{\lambda}=\frac{508.24}{1.11\times10^{-7}}=4.58\times10^{9}S^{-1}$$

3. The collision frequency of nitrogen molecule in a cylinder containing nitrogen molecule in a cylinder containing at 2.0 atm pressure and temperature $17^0 C$. (Take radius of a nitrogen molecule is $1.0A^0$)

SOLUTION :

mean free path $\lambda=\frac{1}{\sqrt{2}\pi d^2\lambda}$

$$\lambda=\frac{K_BT}{\sqrt{2}\pi d^2 p}\left(p=nK_BT\right)$$

$$\lambda=\frac{\left(1.38\times10^{-23}\right)(290)}{(1.414)(3.14)\left(2\times10^{-10}\right)\left(2.026\times10^{5}\right)}=1.1\times10-7$$

$$V_{rms}=\sqrt{\frac{3K_BT}{m}}=\sqrt{\frac{3\times1.38^{-23}\times290}{28\times1.66\times10^{-27}}}=5.1\times10^{2}m/s$$

$\therefore$ collision frequency

$$f=\frac{V_{rms}}{\lambda}=\frac{5.1\times10^{2}}{1.1\times10^{-7}}=4.6\times10^{9}s^{-1}$$

4. Estimate the mean free path for a water molecule in water vapour at 373K. The diameter of the molecule is $2\times10^{-10}m$, and at STP number of molecules per unit volume is $2.7\times10^{25}m^{-3}$

SOLUTION :

The number density(n) is inversely proportional to absolute temperature

$$\therefore n\alpha\frac{1}{T}$$

$$\Rightarrow\frac{n_{373}}{n_{273}}=\frac{273}{373}$$

$$\Rightarrow n_{373}=n_{273}\times\frac{273}{373}$$

$$n_{373}=2.7\times10^{25}\times\frac{273}{373}=2\times10^{25}m^{-3}$$

Given d= $2\times10^{-10}m$;

Hence, mean free path $\lambda=\frac{1}{\sqrt{2}\pi d^2 n}=2.81\times10^{-7}m$

5. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of system is

1) 5RT 2) 11RT 3) 3RT 4) 7RT

SOLUTION :

Total internal energy of system

$$= U_{oxygen} + U_{argon} = \mu_1 \frac{f_1}{2} RT + \mu_2 \frac{f_2}{2} RT$$

$$= 2\frac{5}{2}RT + 4\frac{3}{2}RT = 5RT + 6RT = 11RT$$

[As $f_1 = 5$ (for oxygen) and $f_2 = 3$ (for argon)]

6. A container has 1 mole of nitrogen at 27^0C. The pressure in side the container is 2 atmosphere. Assuming the molecules move with root mean square speed. Then find The number of collisions per second which the molecules make with unit area of the container wall.

SOLUTION :

a) The number of molecules n present per unit volume at pressure P and temperature T.

$$n = \frac{P}{kT} = \frac{2 \times 1.05 \times 10^5}{1.38 \times 10^{-23} \times 300} = 5.07 \times 10^{25}$$

$$\upsilon_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3N_A kT}{M}}$$

$$= \sqrt{\frac{3 \times 6.02 \times 10^{23} \times 1.38 \times 10^{-23} \times 300}{28 \times 10^{-3}}} = 516.75 ms^{-1}$$

Number of collisions/m^2 = $\frac{1}{6} \times n \times \upsilon_{rms}$

$$= \frac{1}{6} \times 5.07 \times 10^{25} \times 516.75 = 4.37 \times 10^{27}$$

7. Each molecule of nitrogen gas heated in a vessel to a temperature of 5000 K has an aver age energy E_1. Some molecules of the gas escape into atmosphere at 300 K. Due to collision with air molecules, average kinetic energy of the nitrogen molecule changes to E_2.
Find the ratio E_1/E_2.

1) 7/6 2) 3/7 3) 6/7 4) 7/3

SOLUTION :

At T = 5000 K (HIGH TEMPERATURE)

f = 7

$$E_1 = f \times \frac{1}{2} KT = 7 \times \frac{1}{2} K \times 5000$$

$= 17,500$ K

At T = 300 K (LOW TEMPERATURE)

f = 5

$$E_2 = f \times \frac{1}{2} KT == 5 \times \frac{1}{2} K \times 300$$

$= 750$ K

$E_1 / E_2 = 7/3$

8. A vessel contains two non-reactive gases neon and oxygen . The ratio of their partial pressures is 3:2. Estimate the ratio of number of molecules Atomic mass of Ne=20.2 u, molecular mass of $O_2 = 32.0u$

1) $\frac{3}{2}$ 2) $\frac{2}{3}$ 3) $\frac{3}{5}$ 4) $\frac{5}{2}$

SOLUTION :

Since V and T are CONSTANTS

we have $P_1V = \mu_1 RT$ and $P_2V = \mu_2 RT$

$\therefore \frac{P_1}{P_2} = \frac{\mu_1}{\mu_2}$ where 1 and 2 refer to neon and oxygen respectively

Given $\frac{P_1}{P_2} = \frac{3}{2}$

$$\therefore \frac{\mu_1}{\mu_2} = \frac{3}{2}$$

By definition $\mu_1 = \frac{N_1}{N_A}$ and $\mu_2 = \frac{N_2}{N_A}$

where N_1 and N_2 are the number of molecules of 1 and 2

$\therefore$ The ratio of number of molecules

$$\frac{N_1}{N_2} = \frac{\mu_1}{\mu_2} = \frac{3}{2}$$

9. A vessel contains two non-reactive gases neon and oxygen . The ratio of their partial pressures is 3:2. Find mass density of neon and oxygen in the vessel. Atomic mass of Ne=20.2 u, molecular mass of $O_2 = 32.0u$

SOLUTION :

Since V and T are CONSTANTS

we have $P_1V = \mu_1 RT \ and \ P_2V = \mu_2 RT$

$$\therefore \frac{P_1}{P_2} = \frac{\mu_1}{\mu_2}$$

where 1 and 2 refer to neon and oxygen respectively

Given $\frac{P_1}{P_2} = \frac{3}{2}$

$$\therefore \frac{\mu_1}{\mu_2} = \frac{3}{2}$$

If $\rho_1 \ and \ \rho_2$ are mass densities of 1 and 2 respectively,

we have $\frac{\rho_1}{\rho_2} = \frac{m_1/V}{m_2/V} = \frac{m_1}{m_2}$

But we can also write $\mu_1 = \frac{m_1}{M_1} \ and \ \mu_2 = \frac{m_2}{M_2}$

$$\therefore \frac{m_1}{m_2} = \frac{\mu_1}{\mu_2} \times \frac{M_1}{M_2} = \frac{3}{2} \times \frac{20.2}{32.0} = 0.947$$

10: A vessel is filled with a gas at a pressure of 76cm of mercury at a certain temperature. The mass of the gas is increased by 50% by introducing more gas in the vessel at same temperature. Find out the resultant pressure of the gas.

SOLUTION :

According kinetic theory of gases,

pressure $P = \frac{1}{3}\frac{m}{V}\left(v_{rms}\right)^2$.

Given, T= constant

so v_{rms} = constant.

also V=onstant

$\therefore P \propto m$

$$\frac{P_2}{P_1} = \frac{m_2}{m_1}$$

$$; \quad \frac{P_2}{76} = \frac{\left(m_1 + 50m_1\right)}{m_1} = \frac{3}{2}$$

$$P_2 = \frac{3}{2} \times 76 = 114cm \text{ of Hg}$$

11. The total number of air molecules in a room of capacity 20 m^3 at a temperature of $27^0 C$ and

1 atm pressure is.

SOLUTION :

Given ;

Volume (V) = 20 m^3

Temperature (T) = 27+273=300K

Pressure (P) = 1 atm = $1.01\times10^5\,Pa$

Total number of air molecules $N=\dfrac{PV}{K_BT}$

$$N=\frac{1.01\times10^5\times20}{1.38\times10^{-23}\times300}$$

$$N=4.87\times10^{26}$$

12. At what temperature, the mean kinetic energy of O_2 will be the same for H_2 molecules at $-73°C$

(a) $127°C$ (b) $527°C$ (c) $-73°C$ (d) $-173°C$

SOLUTION :

Mean kinetic energy of molecule depends upon temperature only. For O_2 it is same as that of H_2 at the same temperature of $-73°C$.

13. A cylinder of fixed capacity 22.4 litres contains helium gas at standard temperature and pressure. What is the amount of heat needed to raise the temperature of the gas in the cylinder by 30 °C ? (R = 8.31 J mo1^{-1} K^{-1}).

SOLUTION :

1 mole of any gas occupies 22.4 litres.

Since the volume of cylinder is fixed , the heat required $=nC_VdT$

Here Helium is monoatomic gas so $C_V=\dfrac{3}{2}R$

Heat required $=1\times1.5R\times30=45R=374.5\,J$

14. A pressure cooker contains air at 1 *atm* and 30°*C*. If the safety value of the cooler blows when the inside pressure ≥ 3 *atm*, then the maximum temperature of the air, inside the cooker can be

(a) $90°C$ (b) $636°C$ (c) $909°C$ (d) $363°C$

SOLUTION :

Since volume is constant,Hence $\dfrac{P_1}{P_2}=\dfrac{T_1}{T_2}$

$$\frac{1}{3}=\frac{(273+30)}{T_2}$$

$$T_2=909\,K=636\,°C$$

15. A container has 1 mole of nitrogen at 27⁰C. The pressure in side the container is 2 atmosphere. Assuming the molecules move with root mean square speed. If the container is made thermally insulated and moves with constant speed υ_0. If it stops suddenly, the process results in the rise of the temperature of the gas by 1⁰C. Calculate the speed υ_0.

SOLUTION :

Kinetic energy $= \frac{1}{2}m\upsilon_0^2$

Heat energy gained $= C_v \Delta T = C_v \times 1 = C_v$ and $C_P - C_v = R$

$$\frac{C_P}{V_v} - 1 = \frac{R}{C_v}$$

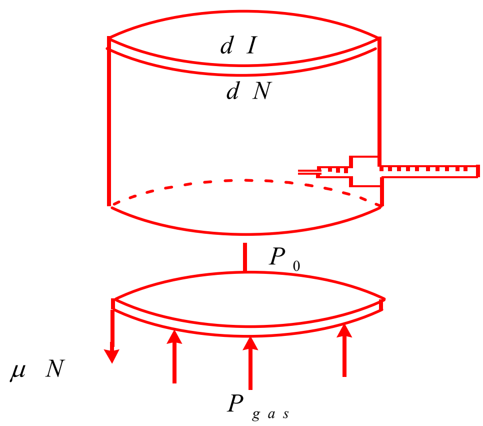
$$\gamma - 1 = \frac{R}{C_v} \quad i.e.\ C_v = \frac{R}{\gamma - 1}$$

$$C_v = \frac{1}{2}mv_0^2 \qquad \frac{R}{\gamma-1} = \frac{1}{2}mv_0^2$$

which gives $V_0 = \sqrt{\frac{2R}{m(\gamma-1)}}$

$$= \sqrt{\frac{2\times 8.31}{28\times10^{-3}\times(1.4-1)}} = 38.5ms^{-1}$$

16. Figure shows a cylindrical tube of radius r and length l, fitted with a cork. The friction coefficient between the cork and the tube is μ. The tube contains an ideal gas at temperature T, and atmosphereic pressure P_0. The tube is slowly heated; the cork pipe out when temperature is doubled. What is normal force per unit length exerted by the cork on the periphery of tube ? Assume uniform temperature throughout gas at any instant.



SOLUTION :

Since volume of the gas is constant

$$\frac{P_i}{T_i} = \frac{P_f}{T_f}$$

$$P_f = P_i\left(\frac{T_f}{T_i}\right) = 2P_i = 2P_0$$

The forces acting on the cork are shown in the figure, in equilibrium.

$$P_0 \times A + \mu N = 2P_0 A$$

$$N = \frac{P_0 A}{\mu}$$

N is the total normal force exerted by the tube on the cork; hence contact force per unit length is

$$\frac{dN}{dl} = \frac{N}{2\pi r} = \frac{P_0 A}{2\pi \mu r}$$

17. One mole of an ideal monatomic gas requires 210 J heat to raise the temperature by 10 K, when heated at constant temperature. If the same gas is heated at constant volume to raise the temperature by 10 K then heat required is

(a) 238 J (b) 126 J (c) 210 J (d) 350 J

SOLUTION :

$(\Delta Q)_P = \mu C_P \Delta T$ and $(\Delta Q)_V = \mu C_V \Delta T$

$$\frac{(\Delta Q)_V}{(\Delta Q)_P} = \frac{C_V}{C_P} = \frac{\frac{3}{2}R}{\frac{5}{2}R} = \frac{3}{5}$$

$$\left[\because (C_V)_{mono} = \frac{3}{2}R, (C_P)_{mono} = \frac{5}{2}R\right]$$

$$(\Delta Q)_V = \frac{3}{5} \times (\Delta Q)_P = \frac{3}{5} \times 210 = 126\ J$$

18. A nitrogen molecule at the surface of earth happens to have rms speed for that gas at 0°C. If it were to go straight up without colliding with other molecules, how high would it rise? (Mass of nitrogen molecule ,m = 4.65 $\times$ 10^{-26} kg,R = 8.3 J/mol/K)

SOLUTION :

The molecule goes to a height h till its entire K.E is converted into P.E

$$\therefore mgh = \frac{1}{2}m\left(v_{rms}\right)^2 ;$$

$$h = \frac{\left(v_{rms}\right)^2}{2g} = \frac{3RT}{M.2g}$$

$$h = \frac{3 \times 8.31 \times 273}{28 \times 10^{-3} \times 2 \times 9.8}$$

19. Certain amount of an ideal gas are contained in a closed vessel. The vessel is moving with a constant velocity *v*. The molecular mass of gas is *M*. The rise in temperature of the gas when the vessel is suddenly stopped is $(\gamma = C_P / C_V)$

(a) $\frac{Mv^2}{2R(\gamma+1)}$ (b) $\frac{Mv^2(\gamma-1)}{2R}$ (c) $\frac{Mv^2}{2R(\gamma+1)}$ (d) $\frac{Mv^2}{2R(\gamma+1)}$

SOLUTION :

If *m* is the total mass of the gas then its kinetic energy $= \frac{1}{2}mv^2$

When the vessel is suddenly stopped then total kinetic energy will increase the temperature of the gas.

Hence $\frac{1}{2}mv^2 = \mu C_v \Delta T = \frac{m}{M} C_v \Delta T$ [As $C_v = \frac{R}{\gamma - 1}$]

$$\frac{m}{M}\frac{R}{\gamma-1}\Delta T = \frac{1}{2}mv^2$$

$$\Delta T = \frac{Mv^2(\gamma-1)}{2R}.$$

20. The energy of a *gas/litre* is 300 joules, then its pressure will be

(a) $3 \times 10^5\ N/m^2$ (b) $6 \times 10^5\ N/m^2$

(c) $10^5\ N/m^2$ (d) $2 \times 10^5\ N/m^2$

SOLUTION :

Energy $= 300\ J/litre = 300 \times 10^3\ J/m^3$

$$P = \frac{2}{3}E = \frac{2 \times 300 \times 10^3}{3} = 2 \times 10^5\ N/m^2$$

21. Two thermally insulated bulbs, filled with air and connected by a short tube containing a valve, initially closed. The pressures, volumes and temperatures in the two vessels are P_1,V_1,T_1 and P_2,V_2,T_2 respectively. Find the P, T values after opening the valve.

SOLUTION :

When the valve is opened, the air flows from the bulb at higher pressure to the bulb at lower pressure.

In equilibrium both the vessels have the same pressure.=P

Total volume of the air, $V = V_1 + V_2$

After mixing of air, total number of moles, $n = n_1 + n_2$

After mixing of air, total number of moles, $n = n_1 + n_2$ Let the common temperature attained be T.

Hence $P(V_1+V_2)=(n_1+n_2)RT$

$$P = \frac{(n_1+n_2)RT}{(V_1+V_2)} \quad ....(1)$$

The combined system is thermally insulated; hence Q = 0;

system does no mechanical work, since dV = 0, W = pdV = 0.

From first law of thermodynamics, Q = dU + W

Hence dU = 0.

There is no change in internal energy.

The internal energy U of an ideal gas is given by

$$U = nC_\upsilon T = \frac{nRT}{(\gamma-1)} = \frac{PV}{(\gamma-1)}$$

$$U_{initial} = \frac{n_1RT_1}{(\gamma-1)} + \frac{n_2RT_2}{(\gamma-1)}$$

$$= \left(\frac{R}{\gamma-1}\right)(n_1T_1 + n_2T_2) \text{.........(2)}$$

$$U_{final} = \frac{(n_1+n_2)RT}{(\gamma-1)} \text{..........(3)}$$

$$U_{initial} = U_{final}$$

$$n_1T_1 + n_2T_2 = (n_1+n_2)T\,]$$

$$T = \left(\frac{n_1T_1 + n_2T_2}{n_1+n_2}\right) \text{.......(4)}$$

$$= \frac{P_1V_1 + P_2V_2}{\left[\left(\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right)\right]} = \frac{T_1T_2(P_1V_1 + P_2V_2)}{(P_1V_1T_2 + P_2V_2T_1)}$$

From eqn. (1)

$$P = \frac{(n_1+n_2)RT}{(V_1+V_2)} = \frac{R}{V_1+V_2}(n_1+n_2)T$$

From eqn. (4)

$$(n_1+n_2)T = n_1T_1 + n_2T_2$$

$$P = \frac{R}{(V_1+V_2)}(n_1T_1 + n_2T_2) = \frac{(P_1V_1 + P_2V_2)}{(V_1+V_2)}$$

as $P_1V_1 = n_1RT_1$

and $P_2V_2 = n_2RT_2$

Maxwell's Law (The distribution of molecular speeds).

a. The v_{rms} gives us a general idea of molecular speeds in a gas at a given temperature. This doesn't mean that the speed of each molecule is v_{rms}.

b. Maxwell derived an equation giving the distribution of molecules in different speeds as follows:

$$dN = 4\pi N\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-\frac{mv^2}{2k_B T}} dv$$

Where dN = Number of molecules with speeds between v and v+dv,

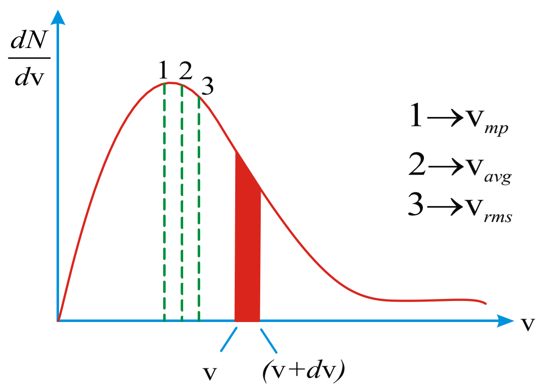
m = mass of the gas molecule

k_B = Boltzmann's constant   N = total number of molecules

c. Graph between $\frac{dN}{dv}$ (number of molecules at particular speed) and v (speed of these molecules).

From the graph it is seen that $\frac{dN}{dv}$ is maximum at most probable speed.

$$\Rightarrow \sqrt{\frac{3RT}{M}} > \sqrt{\frac{8RT}{\pi M}} > \sqrt{\frac{2RT}{M}}$$



From the graph

1) The area under the graph represents total number of molecules.

2) The shape of the curve is such that area (shown shaded enclosed by its portion on right side of v_{mp} is more than the area on the left side of v_{mp}. Thus, the number of molecules having speeds less than v_{mp} is less than the number of molecules having speeds more than v_{mp}.

Note: 1.If μ_1 moles of C_{V_1}, μ_2 moles of C_{V_2} ..... are mixed, then by conservation of energy

$$U_1 + U_2 + \ldots\ldots = U$$

$$\mu_1 C_{V_1}\Delta T + \mu_2 C_{V_2}\Delta T + ... = (\mu_1 + \mu_2 + ...)C_{V_{mix}}\Delta T$$

$$\therefore C_{V_{mix}} = \frac{n_1 C_{V_1} + n_2 C_{V_2} + ...}{n_1 + n_2 + ...} \text{ and } C_{P_{mix}} = C_{V_{mix}} + R$$

2. If μ_1 moles of γ_1 and μ_2 moles of γ_2 .... are mixed then by conservation of energy we have

$$U_1 + U_2 + ... = U$$

$$\mu_1 C_{V_1}\Delta T + \mu_2 C_{V_2}\Delta T + ... = (\mu_1 + \mu_2 + ...)C_{V_{mix}}\Delta T$$

$$\mu_1\left(\frac{R}{\gamma_1 - 1}\right) + \mu_2\left(\frac{R}{\gamma_2 - 1}\right) + .. = (\mu_1 + \mu_2 + ..)\frac{R}{\gamma_{mix} - 1}$$

$$\frac{\mu_1}{\gamma_1 - 1} + \frac{\mu_2}{\gamma_2 - 1} + ... = \frac{(\mu_1 + \mu_2 + ...)}{\gamma_{mix} - 1}$$

For two gases $\frac{\mu_1}{\gamma_1 - 1} + \frac{\mu_2}{\gamma_2 - 1} = \frac{\mu_1 + \mu_2}{\gamma_{mix} - 1}$

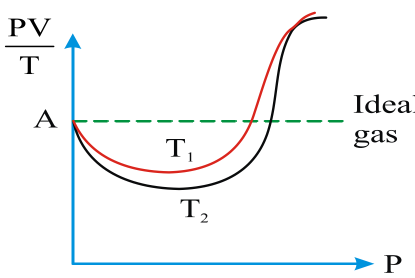
PRACTICE BITS

1. Three closed vessels A, B and C are at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and . If the average speed of the molecules in vessel A is , that of the molecules in vessel B is , the average speed of the molecules in vessel C is

(a) $(V_1 + V_2)/2$ (b) V_1 (c) $(V_1 V_2)^{1/2}$ (d) $\sqrt{3kT/M}$

KEY:b

2. **Given graph gives variation of PV/T with P for 1gm of oxygen at two different temperatures T_1 and T_2. If density of oxygen is $1.427\,kg/m^3$. The value of PV/T at point A and relation b/w T_1 & T_2 is**



1) 0.256J/K and $T_1 < T_2$ 2) 8.314 J/K and $T_1 > T_2$

3) 8.314 J/K and $T_1 < T_2$ 4) 0.256J/K and $T_1 > T_2$

KEY:4

HINT : $PV = \mu RT = \frac{m}{M}RT \Rightarrow \frac{PV}{T} = \frac{m}{M}R$

Real gas behave as ideal gases at low pressure and high temperature, so, $T_1 > T_2$

03. A gas has volume V and pressure p. The total translational kinetic energy of all the molecules of the gas is

1) $\frac{3}{2}pV$ only if the gas is monoatomic 2) $\frac{3}{2}pV$ only if the gas is diatomic

3) $>\frac{3}{2}pV$ if the gas is diatomic 4) $\frac{3}{2}pV$ **in all cases**

KEY:4

04. Three closed vessels A, B are C are at the same temperature. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of O_2 molecules in vessel A is υ_1, that of N_2 molecules in vessel B is υ_2, the average speed of O_2 molecules is vessel C is

1) $\frac{1}{2}(\upsilon_1 + \upsilon_2)$ 2) υ_1 3) $\sqrt{\upsilon_1\upsilon_2}$ 4) $\sqrt{\frac{3kT}{M}}$

KEY:2

05. If the pressure in a closed vessel is reduced by drawing out some of the gas, the mean free path of the two molecules

1) Increases 2) decreases 3) Remains unchanged

4) Increases of decrease according to the nature of the gas

KEY:2

6. The temperature of the gas consisting of rigid diatomic molecules is T = 300K. Calculate the angular root mean square velocity of a rotating molecule if its moment of inertia is equal to $I = 2.1\times10^{-39}\ g\,cm^2$.

1) $6.3\times10^{12}\ rad/\sec$ 2) $6.8\times10^{12}\ rad/\sec$

3) $3.6\times10^{12}\ rad/\sec$ 4) $3.2\times10^{12}\ rad/\sec$

KEY:1

HINT : $\frac{1}{2}I\omega^2 = 2.\frac{1}{2}kT$

KEY:1

07. The temperature of a gas is -68°c. To what temp should it be heated so that the root mean square velocity of the molecules is doubled.

1) 547°c 2) 820°c 3) 1092 K 4) 547 K

KEY:1

8. N Molecules each of mass m of gas A and 2N molecules each of mass 2m of gas B are contained in the same vessel, which are maintained at a temperature T. The mean square velocity of molecules of B type is denoted by v^2 and the mean square of the 'X' component of the velocity of A type is denoted by ω^2, then $\frac{\omega^2}{v^2}$ is

1) 2 2) 1 3) $1/3$ 4) $2/3$

KEY:4

HINT :

Mean kinetic energy of two types of molecules should be equal.

$$\text{so, } \frac{1}{2}m\left(3\omega^2\right)=\frac{1}{2}(2m)\text{v}^2 \Rightarrow \frac{\omega^2}{\text{v}^2}=\frac{2}{3}$$

09. The pressure exerted on the walls of the container by a gas is due to the fact that gas molecules are

1) Losing their kinetic energy at walls 2) Stricking to the energy

3) Changing their moment due to collision with the walls

4) Getting accelerated towards the wall

KEY:3

10. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is :

1) 4 RT 2) 15 RT 3) 9 RT 4) 11 RT

KEY:4

11. The figure given below shows the plot of versus P for oxygen gas at two different temperatures. Read the following statements concerning the curves given below

i) The dotted line corresponds to the 'ideal' gas behaviour
ii) $T_1 > T_2$
iii) The value of at the point, where the curves meet on the y-axis is the same for all gases.
Which of the above statement is true ?
1) i only 2) i and ii 3) all of these 4) none of these

KEY:3

12. Three closed vessels A, B and C at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only contains only contains a

mixture of equal quantities of. If the average speed of the molecules in vessel A is and that of the molecules in vessel B is, the average speed of the molecules in vessel C is
1) 2) 3) 4)

KEY:2

13.1 mole of an ideal gas is contained in a cubical volume V, ABCDEFGH at 300 K in figure. One face of the cube (EFGH) is made up of a material which totally absorbs any gas molecule incident on it. At any given time

1) The pressure on EFGH would be zero
2) The pressure on all the faces will be equal
3) The pressure of EFGH would be double the pressure on ABCD
4) The pressure on EFGH would be half that on ABCD.

KEY:4

14. A cubic vessel (with faces horizontal + vertical) contains an ideal gas at NTP. The vessel is being carried by a rocket which is moving at a speed of 500 in vertical direction. The pressure of the gas inside the vessel as observed by us on the ground

1) remains the same because is very much smaller than of the gas
2) remains the same because motion of the vessel as a whole does not affect the relative motion of the gas molecules and the walls
3) will increase by a factor equal to where was the original mean square velocity of the gas
4) will be different on the top wall and bottom wall of the vessel

15. A vessel of volume V = 5.0 litre contains 1.4 gm of nitrogen at temperature T=1800 K. Find the pressure of the gas if 30% of its molecule are dissociated into atom at this temperature.

1) 0.54×10^5 N/m^2 2) 1.94×10^5 N/m^2
3) 2.62×10^5 N/m^2 4) 3.75×10^5 N/m^2

KEY:2

HINT : $P = \frac{\mu}{M}\frac{RT}{V}$, $\frac{P_1}{P_2} = \frac{\mu_1}{\mu_1 + \mu_2}$

16. The pressure of an ideal gas varies according to the law $P = P_0 - AV^2$ where P_0 and A are positive constants. What is the highest temperature that can be attained by the gas.

1) $\frac{P_0}{nR}\sqrt{\frac{P_0}{A}}$ 2) $\frac{P_0}{nR}\sqrt{\frac{P_0}{2A}}$

3) $\frac{2P_0}{nR}\sqrt{\frac{P_0}{2A}}$ 4) $\frac{2P_0}{3nR}\sqrt{\frac{P_0}{3A}}$

KEY:4

HINT : $P = P_0 - AV^2$

$\Rightarrow \quad \frac{nRT}{V} = P_0 - AV^2$ at max

temperature $\frac{dT}{dV} = 0 \Rightarrow P_0 = 3AV^2$

17. The mass 15 gram of Nitrogen is enclosed in a vessel at 300K. What heat must be supplied to it to double the rms velocity of its molecules

1) 10J 2) 10KJ 3) 10^3J 4) 10^2J

KEY:2

HINT : since $v^2 \propto T \Rightarrow T_2 = 1200K$

$$Heat(H) = nc_v dT = \frac{m}{M}\frac{R}{(\gamma-1)}(T_2 - T_1)$$

18 At what absolute temperature T, is rms speed of a hydrogen molecule equal to its escape velocity from the surface of the moon? The radius of moon is R, g is the acceleration due to gravity on moon's surface, m is the mass of a hydrogen molecule and k is the Boltzmann constant.

1) $\frac{mgR}{2k}$ 2) $\frac{2mgR}{k}$ 3) $\frac{3mgR}{2k}$ 4) $\frac{2mgR}{3k}$

KEY:4

HINT : $v_{rms} = \sqrt{\frac{3KT}{m}}$ $v_e = \sqrt{2gR}$

19 Find the number of degrees of freedom of molecules in a gas whose molar heat capacity at constant pressure is equal to C_p=29 J/(mol.K)

1) 3 2) 4 3) 5 4) 6

KEY:3

HINT : $C_p = \frac{\gamma R}{\gamma - 1}; \gamma = \left(1 + \frac{2}{f}\right)$

20 Determine the gas temperature at which the root mean square velocity of hydrogen molecules exceeds their most probable velocity by $\Delta v = 400\,m/s$

1) 384K 2) 342K 3) 300K 4) 280K

KEY:1

HINT : $v_{rms} - v_p = \left(\sqrt{3} - \sqrt{2}\right)\sqrt{\frac{RT}{M}} = \Delta v$

$$\therefore T = \frac{M}{R}\left(\frac{\Delta v}{\sqrt{3} - \sqrt{2}}\right)^2 = 384k$$

21 Two chambers one containing m_1 *gram* of a gas at pressure P_1 and other containing m_2 *gram* of same gas at pressure P_2 are put in communication with each other. If temperature remains constant, the common pressure reached will be,

1) $\frac{P_1P_2(m_1+m_2)}{P_1m_2+P_2m_1}$ 2) $\frac{m_1m_2(P_1+P_2)}{P_1m_2+P_2m_1}$

3) $\frac{P_1P_2m_1}{P_1m_2+P_2m_1}$ 4) $\frac{m_1m_2P_2}{P_1m_2+P_2m_1}$

KEY:1

HINT : From Boyle's law, $\frac{P}{\rho} = Const = K$

$\rho_1 = \frac{P_1}{K}$, (or) $V_1 = \frac{m_1}{\rho_1} = \frac{m_1K}{P_1}$

llly, $V_2 = \frac{m_2K}{P_2}$

$V = V_1 + V_2 = K\left[\frac{m_1}{P_1} + \frac{m_2}{P_2}\right]$; $\rho = \frac{m_1+m_2}{V_1+V_2}$

$P = K\rho = \frac{P_1P_2(m_1+m_2)}{P_1m_2+P_2m_1}$

22. How many degrees of freedom does a gas molecule have under standard conditions if gas has density 1.3 kg/m³ and the velocity of sound propagation in the gas is v=330m/s?

1) 2 2) 3 3) 4 4) 5

KEY:4

HINT : $v = \sqrt{\frac{\gamma p}{\rho}}$

$\gamma = 1 + \frac{2}{f}$ *where f is* degrees of freedom

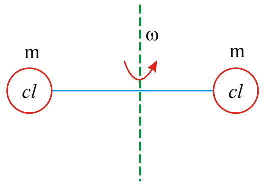
23 In crude model of a rotating diatomic molecule of chlorine (Cl_2), the two Cl atoms are $2.0\times10^{-10}\,m$ apart and rotate about their centre of mass with angular speed $\omega = 2.0\times10^{12}\,rad/s$. What is the rotational kinetic energy of one molecule of Cl_2, which has a molar mass of 70.0g/mol?

1) $2.32\times10^{-21}\,J$ 2) $2.32\times10^{21}\,J$

3) $2.32\times10^{-21}\,erg$ 4) $2.32\times10^{21}\,erg$

KEY:1

HINT :



$m = \frac{70\times10^{-3}}{2\times6.02\times10^{23}} = 5.81\times10^{-26}kg$ and

$r = \frac{2.0\times10^{-10}}{2} = 10^{-10}m$;

$M.I = 2(mr^2) = 2mr^2$

$I = 2\left(5.81\times10^{-26}\right)\left(10^{-10}\right)^2$

$\therefore K_R = \frac{1}{2}I\omega^2 = 2.32\times10^{-21}J$

24. An ideal gas undergoes a process in which PV^{-a} = constant, where V is the volume occupied by the gas initially at pressure P. At the end of the process, rms speed of gas molecules has become $a^{1/2}$ times of its initial value. What will be the value of C_V so that energy transferred in the form of heat to the gas is 'a' times of the initial energy.

1) $\frac{(a^2+1)R}{a^2-1}$ 2) $\frac{(a^2+1)R}{(a^2+1)}$ 3) $\frac{(a+1)R}{(a-1)}$ 4) $\frac{(a-1)R}{(a+1)}$

KEY:4

HINT : $\Delta Q = aU_1$

$nC(T_2 - T_1) = a\frac{f}{2}nRT_1$ -----(1)

$but\ v_{rms}\,\alpha\sqrt{T} \Rightarrow T_2 = aT_1$

(as rms speed became $\sqrt{a}$ times).

From (1) $\therefore C(a-1)T_1 = \frac{af}{2}RT_1$

$C = \frac{afR}{2(a-1)}\left(here, f = \frac{2C_v}{R}\right)$ $C = C_v + \frac{R}{1-n}$ (for polytrophic process)

where n = -a

THEORY BITS

1. Which of the following statements are true regarding the kinetic theory of gases ?

1) The pressure of the gas is directly proportional to the average speed of the molecules

2) The root mean square speed of the molecules is directly proportional to the pressure

3) The rate of diffusion is directly proportional to average speed of the molecules

4) The average kinetic energy per molecule is inversely proportional to the absolute temperature

KEY:3

2. At a given temperature if V_{rms} is the root mean square velocity of the molecules of a gas and V_s be the velocity of sound in it, then these are related as $\left(\gamma = \frac{C_p}{C_v}\right)$

1) $v_{rms} = v_s$ 2) $v_{rms} = \sqrt{\frac{3}{\gamma}} \times v_s$ 3) $v_{rms} = \sqrt{\frac{\gamma}{3}} \times v_s$ 4) $v_{rms} = \left(\frac{3}{\gamma}\right) \times v_s$

KEY:2

3.The temperature of an ideal gas is increased from 120K to 480K. If at 120K, the root mean square velocity of the gas molecules is v then at 480K, it becomes

1) 4v 2) 2v 3) v/2 4) v/4

KEY:2

4. At a given volume and temperature the pressure of a gas:

1) varies inversely as its mass 2) varies inversely as the square of its mass

3) varies linearly as its mass 4) is independent of its mass

KEY:3

5. The root mean square speed of the molecules of a gas at absolute temperature T is proportional to

1) $1/T$ (2) $\sqrt{T}$ (3) T (4) T^2

KEY:2

6. If gas molecules undergo inelastic collision with the wall of the container:

1) the temperature of the gas will decrease 2) the pressure of the gas will increase

3) neither the temperature nor the pressure will change 4) the temperature of the gas will increase

KEY:3

7. Which of the following methods will enable the volume of an ideal gas to be made four times greater? Consider absolute temperature

1) Quarter the pressure at constant temperature

2) Quarter the temperature at constant pressure

3) Half the temperature, double the pressure

4) Double the temperature, double the pressure

KEY:1

8. If k is the Boltzmann constant, the average kinetic energy of a gas molecule at absolute temperature T is

1) $kT/2$ (2) $3kT/4$ (3) kT 4) $3kT/2$

KEY: 4

9. A gas has volume V and pressure P. The total translational kinetic energy of all the molecules of the gas is

1) 3/2 PV only if the gas is monoatomic 2) 3/2 PV only if the gas is diatomic

3) > 3/2 PV if the gas is diatomic 4) 3/2 PV in all cases

KEY:4

10. The root mean square speed of the molecules of an enclosed gas is v. what will be the root mean square speed if the pressure is doubled, the temperature remaining the same?

1) v/2 (2) v (3) 2v (4) 4v

KEY:2

11. A vessel contains a mixture of 1 mole of oxygen and two moles of nitrogen at 300K. The ratio of the rotational kinetic energy per O_2 **molecule to that of per N_2 molecule is**

1) 1:1 2) 1:2 3) 2:1

4) depends on the moment of inertia of the two molecules

KEY:1

12. On any planet, the presence of atmosphere implies (C_{rms} = root mean square velocity of molecules and v_e = escape velocity)

1) $C_{rms} < v_e$ 2) $C_{rms} > v_e$ 3) $C_{rms} = v_e$ 4) $C_{rms} = 0$

KEY:1

13. Choose the correct statement. When the temperature of a gas is increased

1) the kinetic energy of its molecules increases

2) the potential energy of its molecules increases

3) the potential energy decreases and the kinetic energy increases; the total energy remaining unchanged

4) the potential energy increases and the kinetic energy decreases; the total energy remaining unchanged

KEY:1

14. The number of molecules per unit volume (n) of a gas is given by

1) $\frac{P}{kT}$ 2) $\frac{kT}{P}$ 3) $\frac{P}{RT}$ 4) $\frac{RT}{P}$

KEY:1

15. The number of molecules of N_2 and O_2 in a vessel are same. If a fine hole is made in the vessel then which gas escapes out more rapidly?

1) N_2 2) O_2 3) both 4) sometimes N_2 and sometimes O_2

KEY:1

16. The following four gases are at the same temperature . In which gas do the molecules have the maximum root mean square speed?

1) Hydrogen 2) Oxygen 3) Nitrogen 4) Carbon dioxide

KEY:1

17. Two vessels having equal volume contain molecular hydrogen at one atmosphere and helium at two atmospheres respectively. If both samples are at the same temperature, the rms velocity of hydrogen molecules is:

1)equal to that of helium 2) twice that of helium 3) half that of helium 4) $\sqrt{2}$ times that of helium

KEY:4

18. E_0 *and* E_h **respectively represent the average kinetic energy of a molecule of oxygen and hydrogen. If the two gases are at the same temperature, which of the following statements is true?**

1) $E_0 > E_h$ (2) $E_0 = E_h$ (3) $E_0 < E_h$

4) Nothing can be said about the magnitude of E_0 and E_h as the information given is insufficient.

KEY:2

19. If the Avogadro's number was to tend to infinity; the phenomenon of Brownian motion would:

1) remain completely unaffected

2) become more vigorous than that observed with present finite values of Avogadro's number, for all sizes of the Brownian particles

3) become more vigorous than that observed with the present finite value of Avogadro's number, only for relatively large Brownian particles

4) become practically unobservable as the molecular impact would tend to balance one another, for practically all sizes of Brownian particles

KEY:4

20. The root mean square speed of a group of N gas molecules, having speeds $v_1, v_2, \ldots\ldots, v_N$ **is:**

1) $\frac{1}{N}\sqrt{\left(v_1+v_2+\ldots\ldots+v_N\right)^2}$ b) $\frac{1}{N}\sqrt{\left(v_1^2+v_2^2+\ldots\ldots+v_N^2\right)}$

3) $\sqrt{\frac{1}{N}\left(v_1^2+v_2^2+\ldots\ldots+v_N^2\right)}$ 4) $\sqrt{\left(v_1+v_2+\ldots\ldots+v_N\right)^2}$

KEY:3

21. The average kinetic energy of a molecule of a gas at absolute temperature T is proportional to

1) $1/T$ (2) $\sqrt{T}$ (3) T (4) T^2

KEY:3

22. The relation between rms velocity, v_{rms} and the most probable velocity, v_{mp}, of a gas is:

1) $v_{rms} = v_{mp}$ 2) $v_{rms} = \sqrt{\frac{3}{2}} v_{mp}$ 3) $v_{rms} = \sqrt{\frac{2}{3}} v_{mp}$ 4) $v_{rms} = \frac{2}{3} v_{mp}$

KEY:2

23. Some gas at 300K is enclosed in a container. Now the container is placed on a fast moving train. While the train is in motion, the temperature of the gas:

1) rises above 300K 2) falls below 300K
3) remains unchanged 4) becomes unsteady

KEY:3

24. The average energy for molecules in one degree of freedom is :

1) $\frac{3}{2}kT$ (2) $\frac{kT}{2}$ (3) $\frac{3}{4}kT$ (4) kT

KEY:2

25. Two balloons are filled, one with pure He gas and other by air, respectively. If the pressure and temperature of these balloons are same, then the number of molecules per unit volume is:

1) more in the He filled balloon 2) same in both balloons
3) more in air filled balloon (4) in the ratio of 1:4

KEY:2

26. Which of the following phenomena gives evidence of the molecular motion?

1) Brownian movement 2) Diffusion 3) Evaporation 4) All the above

KEY:4

27. On colliding in a closed container the gas molecules:

1) transfer momentum to the walls 2) momentum becomes zero
3) move in opposite directions 4) perform Brownian motion

KEY:4

28. The root mean square velocity, v_{rms}, the average velocity v_{av} and the most probable velocity, v_{mp} of the molecules of the gas are in the order:

1) $v_{mp} > v_{avg} > v_{rms}$ 2) $v_{rms} > v_{avg} > v_{mp}$
3) $v_{avg} > v_{mp} > v_{rms}$ 4) $v_{mp} > v_{rms} > v_{avg}$

KEY:2

29. The temperature of a gas is raised while its volume remains constant, the pressure exerted by the gas on the walls of the container increases because its molecules.

(1) Lose more kinetic energy to the wall

(2) Are in contact with the wall for a shorter time

(3)Strike the wall more often with higher velocities

(4)Collide with each other with less frequency.

KEY:3

30. At upper atmosphere, an astronaut feels:

1) extremely hot 2) slightly hotter

3) extremely cool 4) slightly cooler

KEY:4

31. The average distance travelled by a molecule of gas at temperature T between two successive collisions is called its mean free path which can be expressed by (P is pressure of gas, K is Boltzmann constant, d diameter of molecule)

(1) $\frac{1}{\sqrt{2}\pi d^2 P}$ (2) $\frac{P}{\sqrt{2}\pi d^2 T}$ (3) $\frac{KT}{\sqrt{2}\pi d^2 P}$ (4) $\frac{KP}{\sqrt{2}\pi d^2 T}$

KEY:3

32. Which of the following statements about kinetic theory of gases is wrong

1)The molecules of a gas are in continuous random motion

2) The molecule continuously undergo inelastic collisions

3) The molecules do not interact with each other except during collisions

4) The collisions amongst the molecules are of short duration

KEY:2

33. The root mean square speed of gas molecules

1) is same for all gases at the same temperature

2) depends on the mass of the gas molecule and its temperature

3) is independent of the density and pressure of the gas

4) depends only on the temperature and volume of the gas

KEY:2

34. Consider a gas with density ρ and $\bar{c}$ as the root mean square velocity of its molecules contained in a volume. If the system moves as whole with velocity v, then the pressure exerted by the gas is

1) $\frac{1}{3}\rho \bar{c}^2$ 2) $\frac{1}{3}\rho\left(\bar{c}+\text{v}\right)^2$ 3) $\frac{1}{3}\rho\left(\bar{c}-\text{v}\right)^2$ 4) $\frac{1}{3}\rho\left(\bar{c}^2-\text{v}\right)$

KEY:1

35. Choose the only correct statement from the following:

1) The pressure of a gas is equal to the total kinetic energy of the molecules in a unit volume of the gas.

2) The product of pressure and volume of a gas is always constant.

3) The average kinetic energy of molecules of a gas is proportional to its absolute temperature.

4) The average kinetic energy of molecules of a gas is proportional to the square root of its absolute temperature.

KEY:3

36. If the pressure in a closed vessel is reduced by drawing out some gas, the mean free path of the molecules

(1) Decreases (2) increases (3) Remains unchanged

(4) Increases or decreases according to the nature of the gas

KEY:2

37. If P is the pressure of the gas then the KE per unit volume of the gas is

1) $\frac{P}{2}$ 2) P 3) $\frac{3P}{2}$ 4) $2P$

KEY:3

38. At absolute zero temperature, the kinetic energy of the molecules:

1) becomes zero (2) becomes maximum 3) becomes minimum (4) remains constant

KEY:1

39. Choose the correct statement from the following:

1) The average kinetic energy of a molecule of any gas is the same at the same temperature.

2) The average kinetic energy of a molecule of a gas is independent of its temperature.

3) The average kinetic energy of 1g of any gas is the same at the same temperature.

4) The average kinetic energy of 1g of a gas is independent of its temperature.

KEY:1

40. Two gases of equal mass are in thermal equilibrium. If P_a, P_b and V_a and V_b are their respective pressures and volumes, then which relation is true

(a) $P_a \neq P_b ; V_a = V_b$ (b) $P_a = P_b ; V_a \neq V_b$

(c) $\frac{P_a}{V_a} = \frac{P_b}{V_b}$ (d) $P_a V_a = P_b V_b$ KEY:d

PREVIOUS JEE MAINS QUESTIONS AND SOLUTIONS

1. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2. The ratio P_2/P_1 is –. [NA Sep. 06, 2020 (I)]

Sol : (5) Using ideal gas equation, $PV = nRT$

$$\Rightarrow P_1V_1 = nR \times 250 \; [\because T_1 = 250\text{K}] \quad \text{(i)}$$

$P_2(2V_1 = \frac{5n}{4} R \times 2000 \; [\because T_2 = 2000\text{K}]$ (ii) Dividing eq. (i) by(ii),

$$\frac{P_1}{2P_2} = \frac{4 \times 250}{5 \times 2000} \Rightarrow \frac{P_1}{P_2} = \frac{1}{5}$$

$$\frac{P_2}{P_1} = 5.$$

2. The change in the magnitude ofthe volume ofan ideal gas when a small additional pressure ΔP is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure ofthe gas were 300 K and 2 atm. respectively. If $|\Delta T| = C|\Delta P|$, then value of C in (K/atm.) is[NA Sep. 04, 2020(II)]

Sol : (150) In first case,

From ideal gas equation

$$PV = nRT$$

$P\Delta V + y\Delta P = 0$ (As temperature is constant) $\Delta V = -\frac{\Delta P}{P} y$ (i)

In second case, using ideal gas equation again

$$P\Delta V = -nR\Delta T$$

$\Delta V = -\frac{nR\Delta T}{P}$ (ii)

Equating (i) and (ii), we get

$$\frac{nR\Delta T}{P} = -\frac{\Delta P}{P} V \Rightarrow \Delta T = \Delta P \frac{V}{nR}$$

Comparing the above equation with $|\Delta T| = C|\Delta P|$, we have

$$C = \frac{V}{nR} = \frac{\Delta T}{\Delta P} = \frac{300\text{K}}{2\text{atm}} = 150\text{K/atm}$$

3. The number density of molecules of a gas depends on their distance r from the origin as , $n(r) = n_0 e^{-\alpha r^4}$. Then the total number of molecules is proportional to:[12 April 2019 II]

(a) $n_0\alpha^{-3/4}$ (b) $\sqrt{n_0}\alpha^{1/2}$ (c) $n_0\alpha^{1/4}$ (d) $n_0\alpha^{-3}$

Sol :. (a) $N = \int$ p (dv)

$$= \int_0^r n_0 e^{-\alpha r^4} \times 4\pi r^2 dr_{=4\pi n_0} \int_0^r r^2 (e^{-\alpha r^4}) dr$$

$$\propto n_0 a^{-3/4}$$

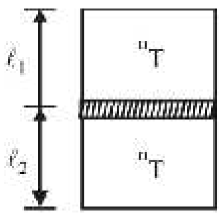
4. A vertical closed cylinder is separated into two parts by a frictionless piston ofmass m and ofnegligible thickness. The piston is free to move along the length ofthe cylinder. The length ofthe cylinder above the piston is l_1 , and that below the piston is l_2, such that $l_1 > l_2$. Each part of the cylinder contains n moles ofan ideal gas at equal temperature T. Ifthe piston is stationary, its mass, m, will be given by: (R is universal gas constant and g is the acceleration due to gravity) [12 Jan. 2019 II]

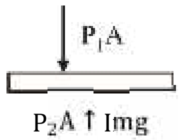
(a) $\frac{RT}{ng}\left[\frac{l_1-3l_2}{l_1 l_2}\right]$ (b) $\frac{RT}{g}\left[\frac{2l_1+l_2}{l_1 l_2}\right]$ (c) $\frac{nRT}{g}\left[\frac{1}{l_2}+\frac{1}{l_1}\right]$ (d) $\frac{nRT}{g}\left[\frac{l_1-l_2}{l_1 l_2}\right]$

Sol : (d) Clearly from figure,

$$P_2A = P_1A + mg$$

$$\text{or, } \frac{nRT \cdot A}{A\ell_2} = \frac{nRT \cdot A}{A\ell_1} + mg$$





$$\Rightarrow nRT\left(\frac{1}{l_2}-\frac{1}{\ell_1}\right) = mg$$

$$m = \frac{nRT}{g}\left(\frac{l_1 - p_2}{\ell_1 \cdot \ell_2}\right)$$

5. Thetemperamre ofan open room ofvolume 30m^3 increases from 17°C to 27°C due to sunshine. The ahnospheric pressure in the room remains 1×10^5 Pa. Ifn_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be : [2017]

(a) 2.5×10^{25} (b) -2.5×10^{25} (c) -1.61×10^{23} (d) 1.38×10^{23}

Sol : (b) Given: Temperature $T_i = 17 + 273 = 290K$

Temperature $T_f = 27 + 273 = 300K$

Atmospheric pressure, $P_0 = 1 \times 10^5\ Pa$

Volume of room, $V_0 = 30m^3$

Difference in number of molecules, $n_f - n_i$ =?

Using ideal gas equation, $= nRT(N_0)$,

N_0 = Avogadro's number

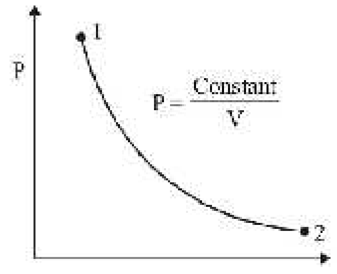
$$\Rightarrow n = \frac{Py}{RT}(N_0)$$

$$n_{f}-n_i = \frac{P_0V_0}{R}(\quad)N_0(\quad)$$

$$= \frac{1 \times 10^5 \times 30}{8.314} \times 6.023 \times 10^{23} \left(\frac{1}{300} - \frac{1}{290}\right)$$

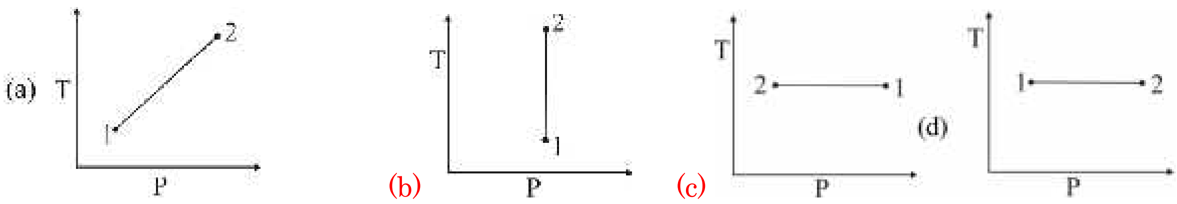
$$= -2.5 \times 10^{25}$$

6. For the P - V diagram given for an ideal gas,



v

out of the following which one correctly represents the T - P diagram? [Online April 9, 2017]



Sol : (c) From P - V graph,

$P \propto \frac{1}{V}$, T = constant and Pressure is increasing from 2 to 1 so option (3) represents correct T - P graph.

7. There are two identical chambers, completely thermally insulated from surroundings. Both chambers have a partition wall dividing the chambers in two compartments. Compartment 1 is filled with an ideal gas and Compartment 3 is filled with a real gas. Compartments 2 and 4 are vacuum. A small hole (orifice) is made in the partition walls and the gases are allowed to expand in vacuum.

Statement - 1: No change in the temperature of the gas takes place when ideal gas expands in vacuum. However, the temperature ofreal gas goes down (cooling) when it expands in vacuum.

Statement - 2: The internal energy of an ideal gas is only kinetic. The internal energy of a real gas is kinetic as well as potential. [Online April 9, 2013]

(a) Statement - 1 is false and Statement - 2 is true.

(b) Statement - 1 and Statement - 2 both are true. Statement - 2 is the correct explanation of Statement - 1.

(c) Statement - 1 is true and Statement - 2 is false.

(d) Statement - 1 and Statement - 2 both are true. Statement - 2 is not correct explanation of Statement - 1.

Sol : a) In ideal gases the molecules are considered as point particles and for point particles, there is no internal excitation, no vibration and no rotation. For an ideal gas the internal energy can only be translational kinetic energy and for real gas both kinetic as well as potential energy

8. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will [2002]

(a) increase (b) decrease (c) remain same

(d) decrease for some, while increase for others

Sol : ∴. 8. (c) The centre ofmass ofgas molecules also moves with lorry with uniform speed. As there is no relative motion of gas molecule. So, kinetic energy and hence temperature remain same

9. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to? (Given, mean kinetic energy of a molecule (at T) is 4×10^{-14} erg, $g = 980 cm/s^2$, density of mercury $= 13.6 g/cm^3$) [Sep. 05, 2020 (I)]

(a) 4.0×10^{18} (b) 4.0×10^{16} (c) 5.8×10^{16} (d) 5.8×10^{18}

Sol : (c) Given: K.E. mean $= \frac{3}{2}kT = 4 \times 10^{-14}$

$P = 2$ cm ofHg, $V = 4cm^3$

$$N = \frac{PV}{KT} = \frac{P\rho g^{y} 2 \times 13.6 \times 980 \times 4}{KT \frac{8}{3} \times 10^{-14}} = 4 \times 10^{18}$$

10. Nitrogen gas is at 300°C temperature. The temperature (in K) at which the rms speed of a H_2 molecule would be equal to the rms speed of a nitrogen molecule, is . (Molar mass of N_2 gas 28 g); [NA Sep. 05, 2020 (II)]

Sol : Room mean square speed is given by

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Here, M = Molar mass ofgas molecule T = temperature ofthe gas molecule We have given $v_{N_2} = v_{H_2}$

$$\sqrt{\frac{3RT_{N_2}}{M_{N_2}}} = \sqrt{\frac{3RT_{H_2}}{M_{H_2}}}$$

$$\square \frac{T_{H_2}}{2} = \frac{573}{28} \square T_{H_2} = 41K$$

11. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed ofthe molecules will be: [9 April 2019 I]

(a) 100m/s (b) $80\sqrt{5}$m/s (c) $100\sqrt{5}$m/s (d) 80 m/s

Sol : (c) $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \frac{(273+127)}{(273+237)} = \sqrt{\frac{400}{500}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

$$v_2 = \frac{\sqrt{5}}{2} v_1 = \frac{\sqrt{5}}{2} \times 200 = 100\sqrt{5} m/s.$$

12. If10^{22} gas molecules each ofmass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed $10^4 m/s$, the pressure exerted by the gas molecules will be of the order of: [8 April 2019 I]

(a) $10^4 N/m^2$ (b) $10^8 N/m^2$ (c) $10^3 N/m^2$ (d) $10^{16} N/m^2$

Sol : (Bouns) Rate ofchange ofmomentum during collision

$$= \frac{mv - (-mv)}{\Delta t} = \frac{2mv}{\Delta t} N$$

so pressure $p = \frac{N \times (2mv)}{\Delta t \times A}$

$$= \frac{10^{22} \times 2 \times 10^{-26} \times 10^4}{1 \times 1} = 2 N / m^2$$

13. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to: [8April 2019II] [Boltzmann Constant $k_B = 1.38 \times 10^{-23} J/K$ Avogadro Number $N_A = 6.02 \times 10^{26}/kg$ Radius ofEarth: $6.4 \times 10^6 m$ Gravitational acceleration on Earth $= 10 ms^{-2}$]

(a) 800K (b) $3 \times 10^5 K$ (c) $10^4 K$ (d) 650K

Sol : (c) $v_{rs} = v_e$

$$\sqrt{\frac{3RT}{M}} = 11.2 \times 10^3$$

or $\sqrt{\frac{3kT}{m}} = 11.2 \times 10^3$

or $\sqrt{\frac{3 \times 1.38 \times 10^{-23} T}{2 \times 10^{-3}}} = 11.2 \times 10^3$ $v = 10^4 K$

14. Amixmre of2 moles ofhelium gas (atomic mass = 4u), and 1 mole ofargon gas (atomic mass = 40u) is kept at 300 K in a container. The ratio oftheir rms speeds $\left[\frac{V_{rms}(\text{helium})}{V_{rms}(\text{argon})}\right]$ is close to: [9 Jan. 2019 I]

(a) 3.16 (b) 0.32 (c) 0.45 (d) 2.24

Sol : (a) Using $\frac{V_{1rms}}{V_{2rms}} = \sqrt{\frac{M_2}{M_1}}$

$$\frac{V_{rms}(He)}{V_{rms}(Ar)} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = 3.16$$

15. Nmoles ofa diatomic gas in a cylinder are at atemperature T. Heat is supplied to the cylinder such that the temperature remains constant but n moles ofthe diatomic gas get converted into monoatomic gas. What is the change in the total kinetic energy ofthe gas? [Online April 9, 2017]

(a) $\frac{1}{2}$nRT (b) 0 (c) $\frac{3}{2}$nRT (d) $\frac{5}{2}$nRT

Sol : (a) Energy associated with N moles ofdiatomic gas,

$U_i = N\frac{5}{2}RT$

Energy associated with n moles ofmonoatomic gas

$= n\frac{3}{2}RT$

Total energywhen n moles ofdiatomic gas converted into monoatomic $(U_f) = 2n\frac{3}{2}RT + (N - n)\frac{5}{2}RT$

$= \frac{1}{2}nRT + \frac{5}{2}NRT$

Now, change in total kinetic energy ofthe gas

$$\Delta U = Q = \frac{1}{2}nRT$$

16. In an ideal gas at temperature T, the average force that a molecule applies on the walls of a closed container depends on T as T^q. A good estimate for q is: [Online April 10, 2015]

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{1}{4}$

Sol : (c) Pressure, $P = \frac{1}{3}\frac{mN}{V}V_{rms}^2$

or, $P = \frac{(mN)T}{V}$

If the gas mass and temperature are constant then

$$P \propto (V_{rms})^2 \propto T$$

So, force □ $(V_{rms})^2 \propto T$

i. e., Value of $q = 1$

17. A gas molecule of mass M at the surface ofthe Earth has kinetic energy equivalent to 0°C. If it were to go up straight without colliding with any other molecules, how high it would rise? Assume that the height attained is much less than radius ofthe earth. (k_B is Boltzmann constant). [Online Apri119, 2014]

(a) 0 (b) $\frac{273k_B}{2Mg}$ (c) $\frac{546k_B}{3Mg}$ (d) $\frac{819k_B}{2Mg}$

Sol : d) Kinetic energy ofeach molecule,

K.E. $= \frac{3}{2}K_BT$

In the given problem,

Temperature, T = 0°C = 273K

Height attained by the gas molecule, h =?

K.E. $= \frac{3}{2}K_B(273) = \frac{819K_B}{2}$

K.E. = P. E. $\Rightarrow \frac{819K_B}{2}$ = Mgh

or h = $\frac{819K_B}{2Mg}$

18. At room temperature a diatomic gas is found to have an r.m.s. speed of1930 ms^{-1}. The gas is: [Online Apri112, 2014]

(a) H_2 (b) $C1_2$ (c) O_2 (d) F_2

Sol : (a) C = $\sqrt{\frac{3RT}{M}}$

(1930)2 = $\frac{3\times8.314\times300}{M}$

$$M = \frac{3 \times 8.314 \times 300}{1930 \times 1930} \approx 2 \times 10^{-3} kg$$

The gas is H_2.

19. In the isothermal expansion of 10g of gas fi,om volume V to 2V the work done by the gas is 575J. Mat is the root mean square speed of the molecules of the gas at that temperature? [Online April 25, 2013]

(a) 398m/s (b) 520m/s (c) 499m/s (d) 532m/s

Sol : (c) v_{rms} = $\sqrt{\frac{3pv}{\text{massofthegas}}}$

20. A perfect gas at 27°C is heated at constant pressure so as to double its volume. The final temperature of the gas will be, close to [Online May 7, 2012]

(a) 327°C (b) 2α)°C (c) 54°C (d) 3α)°C

Sol : (a) Given, $V_1 = V$

$$V_2 = 2V$$

$$T_1 = 27^\circ + 273 = 300K$$

$$T_2 = ?$$

From charle' s law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \text{ (Pressure is constant) or, } \frac{V}{300} = \frac{2V}{T_2}$$

$$T_2 = 600K = 600 - 273 = 327°C$$

21. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and it' s suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by: [2011]

(a) $\frac{(\gamma-1)}{2\gamma R} Mv^2 K$ (b) $\frac{\gamma M^2 v}{2R} K$ (c) $\frac{(\gamma-1)}{2R} Mv^2 K$ (d) $\frac{(\gamma-1)}{2(\gamma+1)R} Mv^2 K$

Sol : (c) As, work done is zero.

So, loss in kinetic energy = heat gain by the gas

$$\frac{1}{2}mv^2 = nC_v\Delta T = n\frac{R}{1'-1}\Delta T$$

$$\frac{1}{2}mv^2 = \frac{m}{M}\frac{R}{\Upsilon-1}\Delta T$$

22. Three perfect gases at absolute temperatures T_1 , T_2 and T_3 are mixed. The masses ofmolecules are m_1 , m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is : [2011]

(a) $\frac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$ (b) $\frac{n_1T_1^2+n_2T_2^2+n_3T_3^2}{n_1T_1+n_2T_2+n_3T_3}$ (c) $\frac{n_1^2T_1^2+n_2^2T_2^2+n_3^2T_3^2}{n_1T_1+n_2T_2+n_3T_3}$ (d) $\frac{(T_1+T_2+T_3)}{3}$

Sol :$P_1V_1 + P_2V_2 + P_3V_3 = PV$

$$\frac{n_1}{N_A}RT_1 + \frac{n_2}{N_A}RT_2 + \frac{n_3}{N_A}RT_3$$

$$= \frac{n_1+n_2+n_3}{N_A}RT_{mix}$$

$$T_{mix} = \frac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$$

23. One kg of a diatomic gas is at a pressure of $8 \times 10^4 N/m^2$. The density of the gas is $4 kg/m^3$. What is the energy of the gas due to its thermal motion?[2009]

(a) $5 \times 10^4 J$ (b) $6 \times 10^4 J$ (c) $7 \times 10^4 J$ (d) $3 \times 10^4 J$

Sol :. (a) Given, mass = 1 kg

Density = 4 kg m^{-3}

Volume = $\frac{\text{mass}}{\text{density}} = \frac{1}{4} \text{m}^3$

Internal energy ofthe diatomic gas

$$= \frac{5}{2}PV = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4 J$$

Alternatively:

$$\text{K.E} = \frac{5}{2}nRT = \frac{5}{2}\frac{m}{M}RT = \frac{5}{2}\frac{m}{M} \times \frac{PM}{d} [\because PM = dRT]$$

$$= \frac{5}{2}\frac{mP}{d} = \frac{5}{2} \times \frac{1 \times 8 \times 10^4}{4} = 5 \times 10^4 J$$

24. The speed ofsound in oxygen (O_2) at a certain temperature is 460 ms^{-1}. The speed ofsound in helium (He) at the same temperature will be (assume both gases to be ideal) [2008]

(a) 1421ms^{-1} (b) 500 ms^{-1} (c) 650 ms^{-1} (d) 330 ms^{-1}

Sol : (a) The speed of sound in a gas is given by $v = \sqrt{\frac{\gamma RT}{M}} v \propto \sqrt{\frac{\gamma}{M}}$ [As R and T is constant]

$$\frac{v_{O_2}}{v_{He}} = \sqrt{\frac{\gamma_{O_2}}{M_{O_2}} \times \frac{M_{He}}{\gamma_{He}}}$$

$$= \sqrt{\frac{14}{32} \times \frac{4}{167}} = 0.3237$$

$$v_{He} = \frac{v_{O_2}}{0.3237} = \frac{460}{0.3237} = 1421 m/s$$

25. At what temperature is the r.m.s velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C? [2002]

(a) 80 K (b) −73K (c) 3 K (d) 20 K

Sol : (d) RMS velocity ofa gas molecule is given by

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

Let T be the temperature at which the velocity ofhydrogen molecule is equal to the velocity ofoxygen molecule.

$$\sqrt{\frac{3RT}{2}} = \sqrt{\frac{3R \times (273 + 47)}{32}}$$

$$\therefore T = 20K$$

26. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T.

The total internal energy, U ofa mole ofthis gas, and the value of $\gamma\left(=\frac{C_p}{C_v}\right)$ are given, respectively, by: [Sep. 06, 2020 (D]

(a) U = $\frac{5}{2}$ RT and $\gamma = \frac{6}{5}$ (b) U = 5RT and $\gamma = \frac{7}{5}$

(c) U = $\frac{5}{2}$ RT and $\gamma = \frac{7}{5}$ (d) U = 5RT and $\gamma = \frac{6}{5}$

Sol :. (c) Total degree offreedom $f = 3 + 2 = 5$

Total energy, $u = \frac{nfRT}{2} = \frac{5RT}{2}$

And $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$

27. In a dilute gas at pressure P and temperature T, the mean time between successive collisions of a molecule varies with T is: [Sep. 06, 2020 (II)]

(a) T (b) $\frac{1}{\sqrt{T}}$ (c) $\frac{1}{T}$ (d) $\sqrt{T}$

Sol :. (b) Mean free path, $\lambda = \frac{1}{\sqrt{2}\pi n d^2}$

where, d = diameter ofthe molecule

n = number ofmolecules per unit volume But, mean time ofcollision, $\Gamma = \frac{\lambda}{v_{rms}}$

But $v_{rms} = \sqrt{\frac{3kT}{R}}$

$$t = \frac{\lambda}{\sqrt{\frac{3kT}{m}}} \Rightarrow t \propto \frac{1}{\sqrt{T}}$$

28. Match the C_p/C_v ratio for ideal gases with different type of molecules : [Sep. 04, 2020 (I)]

Column - I C	olumn - II
Molecule	Type C_p/C_v
(A) Monatomic	(I) 7/5
(B) Diatomic rigid molecules	(II) 9/7
(C) Diatomic non - rigid molecules	(III) 4/3
(D) Triatomic rigid molecules	(IV) 5/3

(a) (A) – (IV), (B) – (II), (C) – (I), (D) – (III)

(b) (A) – (III), (B) – (IV), (C) – (II), (D) – (I)

(c) (A) – (IV), (B) – (I), (C) – (II), (D) – (III)

(d) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Sol : (c) As we know,

$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$, where f = degree offreedom

(A) Monatomic, $f = 3$

$$\gamma = 1 + \frac{2}{3} = \frac{5}{3}$$

(B) Diatomic rigid molecules, $f = 5$

$$\gamma = 1 + \frac{2}{5} = \frac{7}{5}$$

(C) Diatomic non - rigid molecules, $f = 7$

$$\gamma = 1 + \frac{2}{7} = \frac{9}{7}$$

(D) Triatomic rigid molecules, $f = 6$

$$\gamma = 1 + \frac{2}{6} = \frac{4}{3}$$

29. A closed vessel contains 0.1 mole of a monatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to [NA Sep. 04, 2020 (I)]

Sol : (266.67) Here work done on gas and heat supplied to the gas are zero.

Let Tbe the fmal equilibrium temperature ofthe gas in the vessel.

Total internal energyofgases remain same.

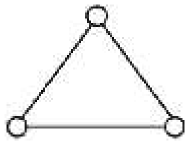
i.e., $u_1 + u_2 = u_1 + u_2$

or, $n_1 C_v \Delta T_1 + n_2 C_v \Delta T_2 = (n_1 + n_2) C_v T$

$$\square\ (0.1)C_v(200) + (0.05)C_v(400) = (0.15)C_v T$$

$$T = \frac{800}{3} = 266.67\text{K}$$

30.



Consider a gas oftriatomic molecules. The molecules are assumed to be triangular and made Of mass less rigid rods whose vertices are occupied by atoms. The internal energy ofa mole of the gas at temperature T is: [Sep. 03, 2020 (I)]

(a) $\frac{5}{2}RT$ (b) $\frac{3}{2}RT$ (c) $\frac{9}{2}RT$ (d) $3RT$

Sol : (d) Here degree of fiieedom, $f = 3 + 3 = 6$ for triatomic nonlinear molecule.

Internal energy ofa mole ofthe gas at temperature T,

$$U = \frac{f}{2}nRT = \frac{6}{2}RT = 3RT$$

31. To raise the temperature ofa certain mass ofgas by 50°C at a constant pressure, 160 calories of heat is required. When the same mass ofgas is cooled by 100°C at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)? [Sep. 03, 2020 (II)]

(a) 5 (b) 6 (c) 3 (d) 7

Sol :b) Let C_p and C_v be the specific heat capacity ofthe gas at constant pressure and volume.

At constant pressure, heat required

$$\Delta Q_1 = nC_p \Delta T$$

$\square$ 160 = $nC_p \cdot 50$ (i)

At constant volume, heat required

$$\Delta Q_2 = nC_v \Delta T$$

$\square$ 240 = $nC_v \cdot 100$ (ii)

Dividing (i) by(ii), we get

$$\frac{160}{240} = \frac{C_p}{C_v} \cdot \frac{50}{100}\ \square\ \frac{C_p}{C_v} = \frac{4}{3}$$

$\gamma = \frac{C_p}{C_v} = \frac{4}{3} = 1 + \frac{2}{f}$ (Here, f = degree of fiieedom)

$$\square\ f = 6.$$

32. A gas mixture consists of3 moles ofoxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) ofthe

mixture is: [Sep. 02, 2020]

(a) 15 (b) 13 (c) 20 (d) 11

Sol : (a) Total energy ofthe gas mixture,

$$E_{mix} = \frac{f_1 n_1 RT_1}{2} + \frac{f_2 n_2 RT_2}{2}$$

$$= 3 \times \frac{5}{2}RT + \frac{5}{2} \times 3RT = 15RT$$

33. An ideal gas in a closed container is slowly heated. As its temperature increases, which ofthe following statements are true? [Sep. 02, 2020 (II)]

(1) The mean free path of the molecules decreases

(2) The mean collision time between the molecules decreases

(3) The mean free path remains unchanged

(4) The mean collision time remains unchanged

(a) (2) and (3) (b) (1) and (2) (c) (3) and (4) (d) (1) and(4)

Sol : (a) As we know mean f : ee path

$$\lambda = \frac{1}{\sqrt{2}\left(\frac{N}{V}\right)\pi d^2}$$

Here, N = no. ofmolecule

V = volume ofcontainer

d = diameter ofmolecule

But $PV = nRT = nNKT$

$$\Rightarrow \frac{N}{V} = \frac{P}{KT} = n$$

$$\lambda = \frac{1KT}{\sqrt{2}\pi d^2 P}$$

For constant volume and hence constant number density n of gas molecules $\frac{P}{T}$ is constant.

So mean free path remains same.

As temperature increases no. of collision increases so relaxation time decreases

34. Consider two ideal diatomic gases A and B at some temperature T. Molecules ofthe gas A are rigid, and have a mass m. Molecules of the gas B have an additional vibrational mode, and have a mass $\frac{m}{4}$. The ratio of the specific heats (C_V^A and C_V^B) ofgas A and B, respectively is: [9 Jan 2020 I]

(a) 7: 9 (b) 5: 9 (c) 3: 5 (d) 5: 7

Sol : (d) Specific heat of gas at constant volume

$C_v = \frac{1}{2}fR$; f = degree of freedom

For gas A (diatomic)

f = 5(3translational +2 rotational)

$$C_V^A = \frac{5}{2}R$$

For gas B (diatomic) in addition to (3translational +2 rotational) 2 vibrational degree of freedom.

$C_v^B = \frac{7}{2}R$ Hence $\frac{C_v^A}{C_v^B} = \frac{\frac{5}{2}R}{\frac{7}{2}R} = \frac{5}{7}$

35. Two gases - argon (atomic radius 0.07nm, atomic weight 40) and xenon (atomic radius 0.1nm, atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free times is closest to: [9 Jan 2020 II]

(a) 3.67 (b) 1.83 (c) 2.3 (d) 4.67

Sol : (Bonus) Mean free path of a gas molecule is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

Here, n = number of collisions per unit volume

d = diameter of the molecule

If average speed of molecule is v then

Mean free time, $\Gamma = \frac{\lambda}{v}$

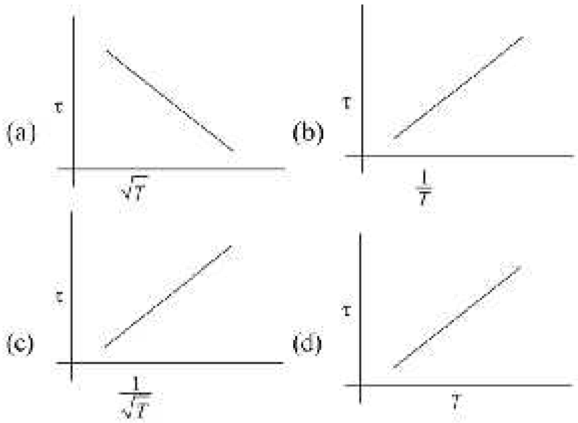
$$\square\ \Gamma = \frac{1}{\sqrt{2}\pi n d^2 v} = \frac{1}{\sqrt{2}\pi n d^2}\sqrt{\frac{M}{3RT}}$$

$$\left(\left(v = \sqrt{\frac{3RT}{M}}\right)\right)$$

$$\Gamma \propto \frac{\sqrt{M}}{d^2}\ \frac{\tau_1}{\tau_2} = \frac{\sqrt{M_1}}{d_1^2} \times \frac{d_2^2}{\sqrt{M_2}}$$

$$= \sqrt{\frac{40}{140}} \times \left(\frac{0.1}{0.07}\right)^2 = 1.09$$

36. The plot that depicts the behavior of the mean free time τ (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is: (Graphs are schematic and not drawn to scale) [8 Jan. 2020 I]



Sol :. (c) Relaxation time $(\Gamma) \propto \frac{\text{meanfreepath}}{\text{speed}} \Rightarrow \Gamma \propto \frac{1}{v}$

and, $v \propto \sqrt{T}$

$$\Gamma \propto \frac{1}{\sqrt{T}}$$

Hence graph between Γ v/s $\frac{1}{\sqrt{T}}$ is a straight line which is correctly depicted by graph shown in option (c).

37. Consider a mixture of n moles of helium gas and $2n$ moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_P/C_V value will be: [8 Jan. 2020 II]

(a) 19/13 (b) 67/45 (c) 40/27 (d) 23/15

Sol : (a) Helium is a monoatomic gas and Oxygen is a diatomic gas.

For helium, $C_{V_1} = \frac{3}{2}R$ and $C_{P_1} = \frac{5}{2}R$

For oxygen, $C_{V_2} = \frac{5}{2}R$ and $C_{P_2} = \frac{7}{2}R$

$$\gamma = \frac{N_1 C_{P_1} + N_2 C_{P_2}}{N_1 C_{V_1} + N_2 C_{V_2}}$$

$$\Rightarrow \gamma = \frac{n.\frac{5}{2}R + 2n.\frac{7}{2}R}{n.\frac{3}{2}R + 2n.\frac{5}{2}R} = \frac{19nR \times 2}{2(13nR)}$$

$\therefore (\ )(\ )\ mixture = \frac{19}{13}$

38. Two moles ofan ideal gas with $\frac{C_P}{C_V} = \frac{5}{3}$ are mixed with 3

moles of another ideal gas with $\frac{C_P}{C_V} = \frac{4}{3}$. The value of for the mixture is: [7 Jan. 2020 I]

(a) 1.45 (b) 1.50 (c) 1.47 (d) 1.42

Sol :d) Using, $\gamma_{\text{mixture}} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{v_1} + n_2 C_{v_2}}$

$$\Rightarrow \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} = \frac{n_1 + n_2}{\gamma_m - 1}$$

$$\Rightarrow \frac{3}{\frac{4}{3} - 1} + \frac{2}{\frac{5}{3} - 1} = \frac{5}{\gamma_m - 1}$$

$$\Rightarrow \frac{9}{1} + \frac{2 \times 3}{2} = \frac{5}{\gamma_m - 1} \Rightarrow \gamma_m - 1 = \frac{5}{12}$$

$$\Rightarrow \gamma_m = \frac{17}{12} = 1.42$$

39. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat ofmixture at constant volume? (R = 8.3J/mo1K) [12 April 2019 I]

(a) 19.7 J/mo1L (b) 15.7 J/mo1K (c) 17.4J/mo1K (d) 21.6J/mo1K

Sol : (c) $[C_v]_{min} = \frac{n_1[C_{v_1}]+n_2[C_{v_2}]}{n_1+n_2}$

$$= \left[\frac{2 \times \frac{3R}{2} + 3 \times \frac{5R}{2}}{2+3}\right]$$

$$= 2.1R = 2.1 \times 8.3 = 17.4 J/mol - k$$

40. A diatomic gas with rigid molecules does 10 J ofwork when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process? [12 April 2019 II]

(a) 25 J (b) 35 J (c) 30 J (d) 40 J

Sol : (b) $F = \frac{C_v}{C_p} = \frac{1}{r} = \frac{1}{(7/5)} = \frac{5}{7}$

or $\frac{W}{Q} = 1 - \frac{5}{7} = \frac{2}{7}$

or $Q = \frac{7}{2}W = \frac{7\times10}{2} = 35J$

41. A$25 \times 10^{-3} m^3$ volume cylinder is filled with 1 molofO_2 gas at room temperature (300K) . The molecular diameter of O_2, and its root mean square speed, are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule? [10 April 2019 I]

(a) -10^{12} (b) □ 10^{11} (c) □ 10^{10} (d) □ 10^{13}

Sol : (c) $V = 25 \times 10^3 m^3, N = 1$mole of$O_2$

$$T = 300K$$

$$V_r = 200 m/s$$

$$\lambda = \frac{1}{\sqrt{2} N\pi r^2}$$

Average time $\Gamma\underline{1} = \frac{\langle V \rangle}{\lambda} = 200. N\pi r^2 . \sqrt{2}$

$$= \frac{\sqrt{2} \times 200 \times 6.023 \times 10^{23}}{25 \times 10^{-3}} . \pi \times 10^{-18} \times 0.09$$

The closest value in the given option is $= 10^{10}$

42. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is: [10 April 2019 II]

(a) $\frac{2}{3}Q$ (b) $\frac{5}{3}Q$ (c) $\frac{7}{5}Q$ (d) $\frac{3}{2}Q$

Sol :. (c) Amount ofheat required (Q) to raise the temperature at constant voiume

$Q = nC_v \Delta T$ (i)

Amount ofheat required (Q_1) at constant pressure

$Q_1 = nC_P \Delta T$ (ii)

Dividing equation (ii) by(i), we get

$$\frac{Q_1}{Q} = \frac{C_p}{C_v}$$

$$\square\ Q_1 = (Q)\left(\frac{7}{5}\right)\left(\left(\because \gamma = \frac{C_p}{C_v} = \frac{7}{5}\right)\right)$$

43. An HC1 molecule has rotational, translational and vibrational motions. Ifthe rms velocity ofHCl molecules in its gaseous phase is $\overline{v}$, m is its mass and k_B is Boltzmann constant, then its temperature will be:[9 April 2019 I]

(a) $\overline{6k_B}$ (b) $\overline{3k_B}$ (c) $\frac{m\overline{v}^2}{7k_B}$ (d) $\frac{m\overline{v}^2}{5k_B}$

Sol : (a) $\frac{1}{2}m\overline{v}^2 = 3k_BT$

$$m\overline{v}^2$$

or $T = \overline{6k_B}$

44. The specific heats, C_p and C_v of a gas of diatomic molecules, A, are given (in units of$Jmo1^{-1}k^{-1}$) by29 and 22, respectively. Another gas of diatomic molecules, B, has the corresponding values 30 and 21. Ifthey are treated as ideal gases, then: [9April 2019II]

(a) A is rigid but B has a vibrational mode.

(b) A has a vibrational mode but B has none.

(c) A has one vibrational mode and B has two.

(d) Both A and B have a vibrational mode each.

Sol : (b) $\gamma_A = \frac{C_p}{C_v} = \frac{29}{22} = 1.32 < 1.4$ (diatomic) and $\gamma_B = \frac{30}{21} = \frac{10}{7} = 1.43 > 1.4$

Gas A has more than 5 - degrees of fiieedom.

45. An ideal gas occupies a volume of 2 m^3 at a pressure of3×10^6 Pa. The energy ofthe gas: [12 Jan. 2019 I]

(a) $9 \times 10^6 J$ (b) $6 \times 10^4 J$ (c) $10^8 J$ (d) $3 \times 10^2 J$

Sol : (a) Energy ofthe gas, E

$= \frac{f}{2}nRT = \frac{f}{2}PV$

$$= \frac{f}{2}(3 \times 10^6)(2) = f \times 3 \times 10^6$$

Considering gas is monoatomic i. e., f = 3

Energy, $E = 9 \times 10^6 J$

46. An ideal gas is enclosed in a cylinder at pressure of2 atm and temperature, 300 K. The mean time between two successive collisions is $6 \times 10^{-8}s$. Ifthe pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions wiil be close to: [12 Jan. 2019 II]

(a) $2 \times 10^{-7}s$ (b) $4 \times 10^{-8}s$ (c) $0.5 \times 10^{-8}s$ (d) $3 \times 10^{-6}s$

Sol : (b) Using, $\Gamma = \frac{1}{2n\pi d^2 V_{avg}}$

$$t \square \frac{\sqrt{T}}{P}\left[\because n = \frac{\text{no. of molecules}}{\text{Volume}}\right]$$

or, $\frac{t_1}{6\times10^{-8}} = \frac{\sqrt{500}}{2P} \times \frac{P}{\sqrt{300}} \approx 4 \times 10^{-8}$

47. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is : [11 Jan. 2019 I]

(a) 15 RT (b) 12 RT (c) 4 RT (d) 20 RT

Sol : (a) $U = \frac{f_1}{2} n_1 RT + \frac{f_2}{2} n_2 RT$

Considering translational and rotational modes, degrees of freedom $f_1 = 5$ and $f_2 = 3$

$$u = \frac{5}{2}(3RT) + \frac{3}{2} \times 5RT$$

$$U = 15RT$$

48. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation VT = K, where K is a constant. In this process the temperature of the gas is increased by ΔT. The amount of heat absorbed by gas is (R is gas constant): [11 Jan. 2019 II]

(a) $\frac{1}{2}R\Delta T$ (b) $\frac{1}{2}KR\Delta T$ (c) $\frac{3}{2}R\Delta T$ (d) $\frac{2K}{3}\Delta T$

Sol : (a) According to question VT = K

we also know that PV = nRT

$$\Rightarrow T = \left(\frac{PV}{nR}\right)$$

$$\square\ V\left(\frac{PV}{nR}\right) = k \Rightarrow PV^2 = K$$

$C = \frac{R}{1-x} + C_v$ (For polytropic process)

$$C = \frac{R}{1-2} + \frac{3R}{2} = \frac{R}{2}$$

$$\Delta Q = nC\Delta T$$

$= \frac{R}{2} \times \Delta T$ [here, n = 1 mole]

49. Two kg of a monoatomic gas is at a pressure of $4\times 10^4 N/m^2$ The density of the gas is $8 kg/m^3$ What is the order of energy of the gas due to its thermal motion? [10 Jan 2019 II]

(a) $10^3 J$ (b) $10^5 J$ (c) $10^4 J$ (d) $10^6 J$

Sol : (c) Thermal energy of N molecule

$$= N\left(\frac{3}{2}kT\right)$$

$= \frac{N}{N_A}\frac{3}{2}RT = \frac{3}{2}(nRT) = \frac{3}{2}PV$

$$= \frac{3}{2}P\left(\frac{m}{\rho}\right) = \frac{3}{2}P\left(\frac{2}{8}\right)$$

$$= \frac{3}{2} \times 4 \times 10^4 \times \frac{2}{8} = 1.5 \times 10^4 J$$

therefore, order = $10^4 J$

50. A 15g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about: [Take R = 8.3J/K mole] [9 Jan. 2019 II]

(a) 0.9kJ (b) 6 kJ (c) 10 kJ (d) 14 kJ

Sol :c) Heat transferred,

$Q = nC_v\Delta T$ as gas in closed vessel

To double the rms speed, temperature should be 4 times i. e., $T' = 4T$ as $v_{rms} = \sqrt{3RT/M}$

$$Q = \frac{15}{28} \times \frac{5 \times R}{2} \times (4T - T)$$

$$\left[\because \frac{C_P}{C_V} = \gamma_{diatomic} = \frac{7}{5} \& C_p - C_v = R\right]$$

or, Q = 10000J = 10kJ

51. Two moles ofan ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabaticallyto a volume 2 V. Calculate(1) the final temperature ofthe gas and(2) change in its internal energy. [2018]

(a) (1) 189 K (2) 2.7kJ

(b) (1) 195 K (2) −2.7kJ

(c) (1) 189 K (2) −2.7kJ

(d) (1) 195 K (2) 2.7kJ

Sol : (c) In an adiabatic process

$TV^{\gamma-1}$ = Constant or, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

For monoatomic gas $\gamma = \frac{5}{3}$

$(300)\, V^{2/3} = T_2(2V)^{2/3}$ □ $T_2 = \frac{300}{(2)^{2/3}}$

T_2 = 189K(final temperature)

Change in internal energy $\Delta U = n\frac{f}{2}R\Delta T$

$$= 2\left(\frac{3}{2}\right)\left(\frac{25}{3}\right)(-111) = -2.7kJ$$

52. Two moles of helium are mixed with n with moles of hydrogen. If $\frac{C_P}{C_V} = \frac{3}{2}$ for the mixture, then the value ofn is [Online Apri116, 2018]

(a) 3/2 (b) 2 (c) 1 (d) 3

Sol : (b) Using formula,

$$\gamma_{\text{mixture}} = (\ )_{\text{mix}}(\ ) = \frac{\frac{n_1\gamma_1}{\gamma_1-1} + \frac{n_2\gamma_2}{\gamma_2-1}}{\frac{n_1}{\gamma_1-1} + \frac{n_2}{\gamma_2-1}}$$

Putting the value of $n_1 = 2, n_2 = n, (\ )_{\text{mix}}(\ ) = \frac{3}{2}$

$\gamma_1 = \frac{5}{3}, \gamma_2 = \frac{7}{5}$ and solving we get, $n = 2$

53. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that $C_p - C_v = a$ for hydrogen gas $C - C_v = b$ for nitrogen gas T1i correct relation between a and b is: [2017]

(a) a = 14b (b) a = 28b (c) $a = \frac{1}{14}b$ (d) a = b

Sol : (a) As we know, $C_p - C_v = R$ where C_p and C_v are molar specific heat capacities

or, $C_p - C_v = \frac{R}{M}$

For hydrogen $(M = 2) C_p - C_v = a = \frac{R}{2}$

For nitrogen $(M = 28) C_p - C_v = b = \frac{R}{28}$

$\frac{a}{b} = 14$ or, $a = 14b$

54. An ideal gas has molecules with 5 degrees of freedom. The ratio of specific heats at constant pressure (C_p) and at constant volume (C_v) is: [Online April 8, 2017]

(a) 6 (b) $\frac{7}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{5}$

Sol : (d) The ratio of specific heats at constant pressure (C_p) and constant volume (C_v)

$$\frac{C_P}{C_V} = \gamma = \left(1 + \frac{2}{f}\right)$$

where f is degree of freedom

$$\frac{C_P}{C_V} = \left(1 + \frac{2}{5}\right) = \frac{7}{5}$$

55. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n =constant, then n is given by(Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively) : [2016]

(a) $n = \frac{C_P - C}{C - C_V}$ (b) $n = \frac{C - C_V}{C - C_P}$ (c) $n = \frac{C_P}{C_V}$ (d) $n = \frac{C - C_P}{C - C_V}$

Sol : . (d) For a polytropic process

$$C = C_v + \frac{R}{1-n} \quad C - C_v = \frac{R}{1-n}$$

$$1 - n = \frac{R}{C - C_v} \quad 1 - \frac{R}{C - C_v} = n$$

$$n = \frac{C - C_v - R}{C - C_v} = \frac{C - C_v - C_p + C_v}{C - C_v}$$

$$= \frac{C - C_p}{C - C_v}(C_p - C_{v=R})$$

56. Using equipartition of energy, the specific heat (in $Jkg^{-1}K^{-1}$) of aluminium at room temperature can be estimated to be (atomic weight of aluminium = 27) [Online April 11, 2015]

(a) 410 (b) 25 (c) 1850 (d) 925

Sol : (d) Using equipartition of energy, we have

$$\frac{6}{2}KT = mCT$$

$$C = \frac{3 \times 1.38 \times 10^{-23} \times 6.02 \times 10^{23}}{27 \times 10^{-3}}$$

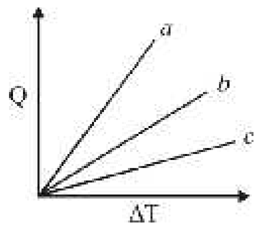
$$C = 925 J/kgK$$

57. Modern vacuum pumps can evacuate a vessel down to a pressure of 4.0×10^{-15} atm. at room temperature (300 K). Taking $R = 8.0 JK^{-1}mole^{-1}$, 1 atm = 10^5 Pa and $N_{Avogadro} = 6 \times 10^{23} mole^{-1}$, the mean distance between molecules of gas in an evacuated vessel will be of the order of. [Online April 9, 2014]

(a) 0.2μm (b) 02mm (c) 0.2 cm (d) 0.2nm

Sol :. (b)

58. Figure shows the variation in temperature (ΔT) with the amount of heat supplied (Q) in an isobaric process corresponding to a monoatomic (M), diatomic (D) and a polyatomic(P) gas. The initial state ofall the gases are the same and the scales for the two axes coincide. Ignoring vibrational degrees of freedom, the lines a, b and c respectively correspond to : [Online April 9, 2013]



(a) P, M and D (b) M, D and P (c) P, D and M (d) D, M and P

Sol : (b) On giving same amount of heat at constant pressure, there is no change in temperature for mono, dia and polyatomic.

$$(\Delta Q)_P = \mu C_p \Delta T \left(\mu = \frac{\text{No. ofmo1ecu1e. s}}{\text{Avogedro'sno}}\right)$$

or $\Delta T \propto \frac{1}{\text{no.ofmolecules}}$

59. A given ideal gas with $\gamma = \frac{C_p}{C_v} = 1.5$ at a temperature T. If the gas is compressed adiabatically to

one - fourth of its initial volume, the final temperature will be [Online May 12, 2012]

(a) $2\sqrt{2}T$ (b) $4T$ (c) $2T$ (d) $8T$

Sol : (c) $TV^{\gamma-1}$ = constant

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\Rightarrow T(V)^{1/2} = T_2\left(\frac{V}{4}\right)^{1/2}$$

[∵ $\gamma = 1.5, T_1 = T, V_1 = V$ and $V_2 = \frac{V}{4}$]

$$T_2 = \left(\frac{4V}{V}\right)^{1/2} T = 2T$$

60. If C_p and C_V denote the specific heats ofnitrogen per unit mass at constant pressure and constant volume respectively, then [2007]

(a) $C_p - C_V = 28R$ (b) $C_p - C_V = R/28$ (c) $C_p - C_V = R/14$ (d) $C_p - C_V = R$

Sol : (b) According to Mayer's relationship

$C_p - C_V = R$, as per the question $(C_p - C_V)M = R$

$$\therefore C_p - C_V = R/28$$

Here M = 28 = mass ofl unit ofN_2

61. A gaseous mixmre consists of16 g ofhelium and 16 g of oxygen. The ratio $\frac{C_p}{C_v}$ ofthe mixture is [2005]

(a) 1.62 (b) 1.59 (c) 1.54 (d) 1.4

Sol : (a) For mixture ofgas specific heat at constant volume

$$C_v = \frac{n_1C_{v_1} + n_2C_{v_2}}{n_1 + n_2}$$

62. One mole ofideal monatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$) . What is γ for the mixture? Y Denotes the ratio of specific heat at constant pressure, to that at constant volume [2004]

(a) 35/23 (b) 23/15 (c) 3/2 (d) 4/3

Sol :No. ofmoles ofhelium,

$$n_1 = \frac{m_{He}}{M_{He}} = \frac{16}{4} = 4$$

Number ofmoles ofoxygen,

$$n_2 = \frac{16}{32} = \frac{1}{2}$$

$$C_v = 4 \times \frac{3}{2}\left(4 + \frac{1}{2}\right) R + \frac{1}{2} \times \frac{5}{2} R = \frac{6R + \frac{5}{4}R}{\frac{9}{2}}$$

$= \frac{29R \times 2}{9 \times 4} = \frac{29R}{18}$ and

Specific heat at constant pressure

$$C_p = \frac{n_1 C_{p1} + n_2 C_{p2}}{(n_1 + n_2)} = -4 \times \underline{5}\left(\frac{4+\frac{1}{}}{2}\right) 2R_{+\frac{1}{2}\times\frac{7R}{2}}$$

$$= \frac{10R + \frac{7}{4}R}{} \frac{9}{2} = \frac{47R}{18}$$

$$\frac{C_p}{C_v} = \frac{47R}{18} \times \frac{18}{29R} = 1.62$$

63. During an adiabatic process, the pressure ofa gas is found to be proportional to the cube ofits absolute temperature. The ratio C_p/C_v for the gas is [2003]

(a) $\frac{4}{3}$ (b) 2 (c) $\frac{5}{3}$ (d) $\frac{3}{2}$

Sol : (d) $P \propto T^3 \Rightarrow PT^{-3}$ = constant ....(i)

But for an adiabatic process, the pressure temperature relationship is given by

$P^{1-\gamma}T^{\gamma}$ = constant

□ $PT^{\frac{\gamma}{1-\gamma}}$ =constt. (ii)

From (i) and(ii) $\frac{\gamma}{1-\gamma} = -3$ □ $\gamma = -3 + 3\gamma \Rightarrow \gamma = \frac{3}{2}$