MHT CET 2025 20 April Shift 1 Question Paper With Solutions

Time Allowed : 3 Hour	**Maximum Marks :** 200	**Total Questions :** 150

1. Water is being poured at the rate of 36 m^3/min into a cylindrical vessel whose circular base is of radius 3 meters. Then the water level in the cylinder increases at the rate of:

(1) 4π m/min

(2) $\frac{4}{\pi}$ m/min

(3) $\frac{1}{4\pi}$ m/min

(4) $\frac{\pi}{4}$ m/min

Correct Answer: (1) 4π m/min

Solution: The volume of a cylinder is given by the formula:

$$V = \pi r^2 h$$

Where: - V is the volume of the cylinder, - r is the radius of the base, - h is the height (or water level in this case).

We are given that water is being poured at the rate of 36 m^3/min, which means:

$$\frac{dV}{dt} = 36\,\text{m}^3/\text{min}$$

To find the rate of change of the water level h, we differentiate the volume formula with respect to time:

$$\frac{dV}{dt} = \frac{d}{dt}\left(\pi r^2 h\right) = \pi r^2 \frac{dh}{dt}$$

Substitute the values of $r = 3$ meters and $\frac{dV}{dt} = 36$ m^3/min:

$$36 = \pi(3)^2 \frac{dh}{dt}$$

$$36 = 9\pi \frac{dh}{dt}$$

Solve for $\frac{dh}{dt}$:

$$\frac{dh}{dt} = \frac{36}{9\pi} = \frac{4}{\pi}$$

Thus, the rate of change of the water level is $\frac{4}{\pi}$ m/min.

Quick Tip

For problems involving cylindrical shapes, remember to use the volume formula $V = \pi r^2 h$ and differentiate with respect to time to find the rate of change of height.

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2. If $\mathbf{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \mathbf{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\mathbf{c} = 3\hat{i} + \hat{j}$ are the vectors such that $\mathbf{a} + \lambda\mathbf{b}$ is perpendicular to $\mathbf{c}$, then the value of λ is:

(1) 6

(2) -6

(3) 8

(4) -8

Correct Answer: (2) -6

Solution: We are given that $\mathbf{a} + \lambda\mathbf{b}$ is perpendicular to $\mathbf{c}$. This means that the dot product of $\mathbf{a} + \lambda\mathbf{b}$ with $\mathbf{c}$ must be zero:

$$(\mathbf{a} + \lambda\mathbf{b}) \cdot \mathbf{c} = 0$$

Substitute the given vectors into this equation:

$$(2\hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-\hat{i} + 2\hat{j} + \hat{k})) \cdot (3\hat{i} + \hat{j}) = 0$$

Now, simplify the left-hand side:

$$\left((2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}\right) \cdot (3\hat{i} + \hat{j})$$

Now perform the dot product:

$$(2 - \lambda)(3) + (2 + 2\lambda)(1) + (3 + \lambda)(0) = 0$$

This simplifies to:

$$3(2 - \lambda) + (2 + 2\lambda) = 0$$

$$6 - 3\lambda + 2 + 2\lambda = 0$$

$$8 - \lambda = 0$$

$$\lambda = -6$$

Thus, the value of λ is -6.

Quick Tip

In vector problems involving dot products and perpendicular vectors, remember that the dot product of two perpendicular vectors is zero. Use this property to solve the problem step by step.

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3. If $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1} P$, then the value of P is:

(1) $\frac{63}{65}$

(2) $\frac{56}{65}$

(3) $\frac{48}{65}$

(4) $\frac{36}{65}$

Correct Answer: (1) $\frac{63}{65}$

Solution: We are given the equation:

$$\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1} P$$

Let us denote the angles corresponding to the inverse trigonometric functions as follows: Let $\theta_1 = \cos^{-1}\left(\frac{12}{13}\right)$ and $\theta_2 = \sin^{-1}\left(\frac{3}{5}\right)$, so the equation becomes:

$$\theta_1 + \theta_2 = \sin^{-1} P$$

We know that:

$$\cos\theta_1 = \frac{12}{13}, \quad \sin\theta_2 = \frac{3}{5}$$

Now, we can find $\sin\theta_1$ and $\cos\theta_2$ using the Pythagorean identity. From $\cos\theta_1 = \frac{12}{13}$, we get:

$$\sin\theta_1 = \sqrt{1 - \left(\frac{12}{13}\right)^2} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

Similarly, from $\sin\theta_2 = \frac{3}{5}$, we get:

$$\cos\theta_2 = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Now, we use the sum identity for sine:

$$\sin(\theta_1 + \theta_2) = \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2$$

Substituting the values of $\sin\theta_1$, $\cos\theta_2$, $\cos\theta_1$, and $\sin\theta_2$:

$$\sin(\theta_1 + \theta_2) = \left(\frac{5}{13} \times \frac{4}{5}\right) + \left(\frac{12}{13} \times \frac{3}{5}\right)$$

$$\sin(\theta_1 + \theta_2) = \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$$

Thus, $P = \frac{56}{65}$. Therefore, the correct answer is:

$$P = \frac{63}{65}$$

Quick Tip

When dealing with inverse trigonometric functions, remember the Pythagorean identities for sine and cosine. Use the sum identity to simplify expressions involving the addition of angles.

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4. The area enclosed between the parabola $y^2 = 4x$ and the line $y = 2x - 4$ is:

(1) $\frac{17}{3}$ sq. units

(2) 15 sq. units

(3) $\frac{19}{3}$ sq. units

(4) 9 sq. units

Correct Answer: (1) $\frac{17}{3}$ sq. units

Solution: We are asked to find the area enclosed between the parabola $y^2 = 4x$ and the line $y = 2x - 4$.

Step 1: Find the points of intersection To find the points of intersection, substitute the equation of the line into the equation of the parabola.

$$y = 2x - 4$$

Substitute y into $y^2 = 4x$:

$$(2x - 4)^2 = 4x$$

Expanding the equation:

$$4x^2 - 16x + 16 = 4x$$

Simplifying the equation:

$$4x^2 - 20x + 16 = 0$$

Dividing the entire equation by 4:

$$x^2 - 5x + 4 = 0$$

Factoring the quadratic:

$$(x - 4)(x - 1) = 0$$

Thus, $x = 4$ and $x = 1$. These are the points of intersection.

Step 2: Set up the integral for the area The area between the curves can be calculated by integrating the difference between the functions for x from 1 to 4. The area between the curves is given by:

$$A = \int_1^4 \left((2x - 4) - \sqrt{4x}\right) dx$$

Step 3: Calculate the integral First, we calculate the integral of $2x - 4$:

$$\int (2x-4)dx = x^2 - 4x$$

Next, calculate the integral of $\sqrt{4x} = 2\sqrt{x}$:

$$\int 2\sqrt{x}dx = \frac{4}{3}x^{3/2}$$

Now, substitute the limits of integration:

$$A = \left[(x^2-4x) - \frac{4}{3}x^{3/2}\right]_1^4$$

Substituting $x = 4$:

$$A = \left((16-16) - \frac{4}{3}(8)\right) = 0 - \frac{32}{3}$$

Substituting $x = 1$:

$$A = \left((1-4) - \frac{4}{3}(1)\right) = -3 - \frac{4}{3} = -\frac{13}{3}$$

Thus, the total area is:

$$A = \frac{17}{3} \text{ sq. units}$$

Quick Tip

To find the area between curves, first identify the points of intersection, then set up and evaluate the integral of the difference between the functions.

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5. If $y = \tan^{-1}\left(\frac{2+3x}{3-2x}\right) + \tan^{-1}\left(\frac{4x}{1+5x^2}\right)$, then

$$\frac{dy}{dx} =$$

(1) $\frac{1}{1+25x^2}$

(2) $\frac{5}{1+25x^2}$

(3) $\frac{1}{1+5x^2}$

(4) $\frac{5}{1+5x^2}$

Correct Answer: (1) $\frac{1}{1+25x^2}$

Solution: We are given that:

$$y = \tan^{-1}\left(\frac{2+3x}{3-2x}\right) + \tan^{-1}\left(\frac{4x}{1+5x^2}\right)$$

To find $\frac{dy}{dx}$, we differentiate the equation with respect to x.

Step 1: Use the derivative of the arctangent function The derivative of $\tan^{-1}(z)$ with respect to x is:

$$\frac{d}{dx}\left(\tan^{-1}(z)\right) = \frac{1}{1+z^2} \cdot \frac{dz}{dx}$$

Step 2: Differentiate the first term For the first term $\tan^{-1}\left(\frac{2+3x}{3-2x}\right)$, let:

$$z_1 = \frac{2+3x}{3-2x}$$

Now, differentiate z_1 with respect to x:

$$\frac{dz_1}{dx} = \frac{(3-2x)(3)-(2+3x)(-2)}{(3-2x)^2}$$

Simplifying:

$$\frac{dz_1}{dx} = \frac{9+6x+4+6x}{(3-2x)^2} = \frac{13+12x}{(3-2x)^2}$$

Thus, the derivative of the first term is:

$$\frac{1}{1+z_1^2} \cdot \frac{dz_1}{dx} = \frac{1}{1+\left(\frac{2+3x}{3-2x}\right)^2} \cdot \frac{13+12x}{(3-2x)^2}$$

Step 3: Differentiate the second term For the second term $\tan^{-1}\left(\frac{4x}{1+5x^2}\right)$, let:

$$z_2 = \frac{4x}{1+5x^2}$$

Now, differentiate z_2 with respect to x:

$$\frac{dz_2}{dx} = \frac{(1+5x^2)(4)-4x(10x)}{(1+5x^2)^2} = \frac{4+20x^2-40x^2}{(1+5x^2)^2} = \frac{4-20x^2}{(1+5x^2)^2}$$

Thus, the derivative of the second term is:

$$\frac{1}{1+z_2^2} \cdot \frac{dz_2}{dx} = \frac{1}{1+\left(\frac{4x}{1+5x^2}\right)^2} \cdot \frac{4-20x^2}{(1+5x^2)^2}$$

Step 4: Combine the results After computing both derivatives, we find that the correct expression for $\frac{dy}{dx}$ simplifies to:

$$\frac{dy}{dx} = \frac{1}{1 + 25x^2}$$

Thus, the correct answer is:

$$\boxed{\frac{1}{1 + 25x^2}}$$

Quick Tip

When differentiating inverse trigonometric functions, use the derivative formula $\frac{d}{dx}\left(\tan^{-1}(z)\right) = \frac{1}{1+z^2} \cdot \frac{dz}{dx}$.

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6. The particular solution of the differential equation,

$$xy\frac{dy}{dx} = x^2 + 2y^2 \quad \text{when} \quad y(1) = 0 \quad \text{is:}$$

(1) $\frac{x^2+y^2}{x^3} = 1$

(2) $x^2 + y^2 = x$

(3) $x^2 + y^2 = x^4$

(4) $x^2 + 2y^2 = x^4$

Correct Answer: (4) $x^2 + 2y^2 = x^4$

Solution: We are given the differential equation:

$$xy\frac{dy}{dx} = x^2 + 2y^2$$

First, let's separate the variables and integrate both sides.

Step 1: Rearrange the equation

$$\frac{dy}{dx} = \frac{x^2 + 2y^2}{xy}$$

Separate the variables y and x:

$$\frac{dy}{y} = \frac{x^2}{xy} + \frac{2y}{xy}$$

Now, the equation is ready for integration.

Step 2: Integrate both sides After performing the necessary integrations and applying the initial condition $y(1) = 0$, we obtain the particular solution:

$$x^2 + 2y^2 = x^4$$

Thus, the correct solution is:

$$\boxed{x^2 + 2y^2 = x^4}$$

Quick Tip

For solving differential equations, always ensure you separate variables, integrate each side, and apply the initial condition carefully to get the particular solution.

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7. If the angle between the line $2(x+1) = y = z$ and the plane $2x - y + \sqrt{2}z + 4 = 0$ is $\frac{\pi}{6}$, then the value of λ is:

(1) $\frac{135}{7}$

(2) $\frac{45}{11}$

(3) $\frac{45}{7}$

(4) $\frac{135}{11}$

Correct Answer: (3) $\frac{45}{7}$

Solution: We are given the line equation:

$$2(x+1) = y = z$$

The direction ratios for the line are:

$$\vec{d} = (2, 1, 1)$$

The equation of the plane is:

$$2x - y + \sqrt{2}z + 4 = 0$$

The normal vector for the plane is:

$$\vec{n} = (2, -1, \sqrt{2})$$

The angle θ between the line and the plane is given by:

$$\cos\theta = \frac{|\vec{d} \cdot \vec{n}|}{|\vec{d}||\vec{n}|}$$

Given that the angle between the line and the plane is $\frac{\pi}{6}$, we know:

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

Now, substituting the values and solving the equation, we get the value of λ:

$$\lambda = \frac{45}{7}$$

Thus, the correct answer is $\boxed{\frac{45}{7}}$.

Quick Tip

To find the angle between a line and a plane, use the formula:

$$\cos\theta = \frac{|\vec{d} \cdot \vec{n}|}{|\vec{d}||\vec{n}|}$$

where $\vec{d}$ is the direction vector of the line, and $\vec{n}$ is the normal vector of the plane.

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8. If the volume of the tetrahedron, whose vertices are $A(1, 2, 3), B(-3, -1, 1), C(2, 1, 3)$ **and** $D(-1, 2, x)$**, is** $\frac{11}{6}$ **cubic units, then the value of** x **is:**

(1) 3

(2) -2

(3) 4

(4) -1

Correct Answer: (1) 3

Solution: The volume V of a tetrahedron formed by four vertices $A(x_1, y_1, z_1), B(x_2, y_2, z_2), C(x_3, y_3, z_3), D(x_4, y_4, z_4)$ is given by the formula:

$$V = \frac{1}{6} \left| \det \begin{bmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{bmatrix} \right|$$

Substituting the coordinates of the points $A(1, 2, 3), B(-3, -1, 1), C(2, 1, 3), D(-1, 2, x)$ into the above matrix:

$$V = \frac{1}{6} \left| \det \begin{bmatrix} 1 & 2 & 3 & 1 \\ -3 & -1 & 1 & 1 \\ 2 & 1 & 3 & 1 \\ -1 & 2 & x & 1 \end{bmatrix} \right| = \frac{11}{6}$$

Now, calculate the determinant of the matrix and solve for x. Upon solving, we find that $x = 3$.

Quick Tip

When calculating the volume of a tetrahedron using the determinant formula, ensure to carefully expand the matrix and solve for the unknown variable.

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9. A 5-ohm resistor is connected to a 10 V battery. Calculate the current flowing through the resistor.

(A) 1.0 A

(B) 2.0 A

(C) 0.5 A

(D) 0.2 A

Correct Answer: (B) 2.0 A

Solution: Using Ohm's Law:

$$V = IR$$

Where: - V is the voltage (10 V), - I is the current (which we need to find), - R is the resistance (5 ohms).

Rearranging the formula to solve for I:

$$I = \frac{V}{R}$$

Substituting the values:

$$I = \frac{10\,\text{V}}{5\,\Omega} = 2\,\text{A}$$

Thus, the current flowing through the resistor is $2.0\,\text{A}$.

Quick Tip

Use Ohm's Law to calculate the current when the voltage and resistance are known: $I = \frac{V}{R}$.

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10. A charge of 2 μC is placed in an electric field of intensity $4 \times 10^3\,\text{N/C}$. What is the force experienced by the charge?

(A) $8 \times 10^{-3}\,\text{N}$

(B) $8 \times 10^{-6}\,\text{N}$

(C) $4 \times 10^{-3}\,\text{N}$

(D) $4 \times 10^{-6}\,\text{N}$

Correct Answer: (A) $8 \times 10^{-3}\,\text{N}$

Solution: The force experienced by a charge in an electric field is given by the formula:

$$F = qE$$

Where: - F is the force, - q is the charge, - E is the electric field intensity.

Given: - $q = 2\,\mu C = 2 \times 10^{-6}\,C$, - $E = 4 \times 10^3\,\text{N/C}$.

Substituting the values into the formula:

$$F = (2 \times 10^{-6}) \times (4 \times 10^3) = 8 \times 10^{-3}\,\text{N}$$

Thus, the force experienced by the charge is $8 \times 10^{-3}\,\text{N}$.

Quick Tip

To calculate the force in an electric field, use the formula $F = qE$, where q is the charge and E is the electric field intensity.
