PHYSICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. Match List – I with List – II:

List – I	**List – II**
A) Source of microwave frequency	I) Radioactive decay of nucleus
B) Source of infrared frequency	II) Magnetron
C) Source of Gamma rays	III) Inner shell electrons
D) Source of X-rays	IV) Vibration of atoms and molecules
	V) LASER
	VI) RC circuit

Choose the correct answer from the options given below:

1) A – VI, B – V, C – I, D – IV **2)** A – VI, B – IV, C – I; D – V

3) A – II, B – IV, C – I, D – III **4)** A – II, B – IV, C – VI, D – III

Key: 3

Solution:

Source of microwave is magnetron source of IR is vibration of atoms and molecules source of Gamma rays is radioactive decay source of X-rays is inner shell electrons.

2. Two electrons each are fixed at a distance 2d. A third charge proton placed at the midpoint is displaced slightly by a distance x(x <<d) perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency:

1) $\left(\frac{2\pi\varepsilon_0 md^3}{q^2}\right)^{\frac{1}{2}}$ **2)** $\left(\frac{\pi\varepsilon_0 md^3}{2q^2}\right)^{\frac{1}{2}}$ **3)** $\left(\frac{2q^2}{\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$ **4)** $\left(\frac{q^2}{2\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$

Key: 4

Solution:

$$ma = \frac{2kq^2}{\left(l^2 + x^2\right)^{\frac{3}{2}}}x \approx \frac{2kq^2}{l^3}x \qquad a = \left[\frac{2kq^2}{ml^2}\right]x = \omega^2 x \qquad \omega = \left(\frac{q^2}{2\pi\varepsilon_0 ml^3}\right)^{\frac{1}{2}}$$

3. A soft ferromagnetic material is placed in an external magnetic field. The magnetic domains:

1) Decrease in size and changes orientation.

2) May increase or decrease in size and change its orientation.

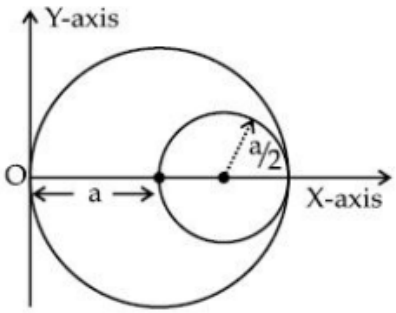
3) Have no relation with external magnetic field.

4) Increase in size but no change in orientation.

Key: 1

Solution: Magnetic domains decrease in size and change their orientation

4. A circular hole of radius $\left(\frac{a}{2}\right)$ is cut out of a circular disc of radius 'a' as shown in fig. The centroid of the remaining circular portion with respect to pint 'O' will be:



1) $\frac{2}{3}a$ **2)** $\frac{1}{6}a$ **3)** $\frac{10}{11}a$ **4)** $\frac{5}{6}a$

Key: 4

Solution:

$$x_{com} = \frac{\left(\sigma\pi a^2\right)a - \left(\sigma\frac{\pi a^2}{4}\right)\frac{3a}{2}}{\sigma\pi a^2 - \sigma\frac{\pi a^2}{4}} = \frac{5a}{6}$$

5. Zener breakdown occurs in a p – n junction having 'p' and 'n' both:

1) Lightly doped and have narrow depletion layer.

2) Heavily doped and have wide depletion layer.

3) Lightly doped and have wide depletion layer.

4) Heavily doped and have narrow depletion layer.

Key: 2

Solution:

Zener breakdown occurs when p and n both heavily doped and have wide depletion layer

6. Which of the following equation represents a travelling wave?

1) $y = Ae^{x}\cos(\omega t - \theta)$ **2)** $y = A\sin(15x - 2t)$

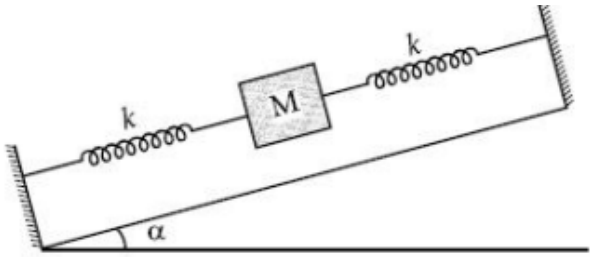
3) $y = A\sin x\cos\omega t$ **4)** $y = Ae^{-x^2}(vt + \theta)$

Key: 2

Solution:

$y = \sin(15x - 2t)$ is a travelling wave

7. In the given fig., a body of mass 'M' is held between two mass less springs, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant 'k', the frequency of oscillation of given body is:



1) $\frac{1}{2\pi}\sqrt{\frac{2k}{Mg\sin\alpha}}$ **2)** $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$ **3)** $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ **4)** $\frac{1}{2\pi}\sqrt{\frac{k}{Mg\sin\alpha}}$

Key: 3

Solution:

Effective spring constant 'k' is 2k $\quad \therefore f = \frac{1}{2\pi}\sqrt{\frac{2k}{M}}$

8. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of 'L' is 1.0 m from meter scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stop watch of 0.1s resolution. The percentage error in the determination of 'g' will be:

1) 1.33% **2)** 1.30% **3)** 1.13% **4)** 1.03%

Key: 3

Solution:

$$g = \frac{4\pi^2 l}{T^2}$$

$$\frac{\Delta g}{g}\times 100 = \left(\frac{\Delta l}{l} + \frac{2\Delta T}{T}\right)\times 100 \left(\frac{10^{-3}}{1} + \frac{2(0.01)}{1.95}\right)\times 100 = 1.13\%$$

9. An X-ray tube is operated at 1.24 million volt. The shortest wavelength of the produced photon will be:

1) 10^{-2} nm **2)** 10^{-1} nm **3)** 10^{-3} nm **4)** 10^{-4} nm

Key: 3

Solution:

$$\lambda_c = \frac{hc}{eV} = \frac{1240\ \text{nm} - \text{eV}}{e1.24\times 10^{+6}\text{V}} = 10^{-3}\text{nm}$$

10. A particle is projected with velocity v_0 along x-axis. A damping force is acting on the particle. Which is proportional to the square of the distance from the i.e. $ma = -ax^2$.? The distance at which the particle stops:

1) $\left(\frac{2v_0^2}{3\alpha}\right)^{\frac{1}{2}}$ **2)** $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{2}}$ **3)** $\left(\frac{2v_0}{3\alpha}\right)^{\frac{1}{3}}$ **4)** $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$

Key: 4

Solution:

$$ma = -\propto x^2$$

$$mv\frac{dv}{dx} = -\propto x^2$$

$$m\int_{V_0}^{0} V\,dv = -\propto \int_{0}^{x} x^2 dx \quad x = \left(\frac{3\,mv_0^2}{2\,\alpha}\right)^{\frac{1}{3}}$$

11. On the basis of kinetic theory of gases, the gas exerts pressure because its molecules:

1) Continuously stick to the walls of container

2) Suffer change in momentum when impinge on the walls of container

3) Continuously lose their energy till it reaches wall.

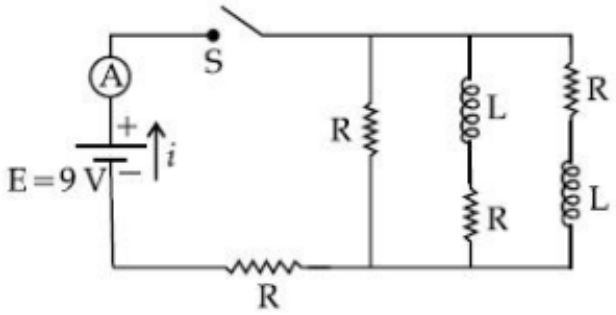
4) Are attracted by the walls of container.

Key: 2

Solution:

Molecules change their momentum due to elastic collisions with walls.

12. Fig. shows a circuit that contains four identical resistors with resistance $R = 2.0\Omega$, two identical inductors with inductance L = 2.0 mH and an ideal battery with emf E = 9 V. The current 'i' just after the switch 'S' is closed will be:



1) 3.0 A **2)** 9 A **3)** 3.37 A **4)** 2.25 A

Key: 4

Solution:

$$i = \frac{\varepsilon}{2R} = \frac{9}{2(2)} = 2.25\text{ A}$$

13. The de-Broglie wavelength of a proton and $\alpha-$particle are equal. The ratio of their velocities is:

1) 1: 4 **2)** 4: 2 **3)** 4: 3 **4)** 4: 1

Key: 4

Solution:

$$\frac{h}{m_1 v_1}=\frac{h}{m_2 v_2}\Rightarrow\frac{v_1}{v_2}=\frac{m_2}{m_1}=\frac{4}{1}$$

14. According to Bohr atom model, in which of the following transitions will the frequency be maximum?

1) n = 3 to n = 2 **2)** n = 2 to n = 1

3) n = 5 to n = 4 **4)** n = 4 to n = 3

Key: 2

Solution:

$$f=\frac{E_2-E_1}{h}\text{ is maximum}$$

15. If the source of light used in a Young's double slit experiment is changed from red to violet.

1) The fringes will become brighter

2) The intensity of minima will increase

3) Consecutive fringe lines will come closer.

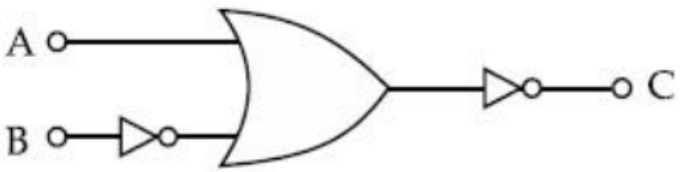
4) The central bright fringe will become a dark fringe.

Key: 3

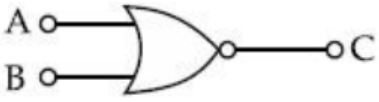
Solution:

Fringe width decreases as wavelength decreases

16.



The logic circuit shown above is equivalent to:

1)


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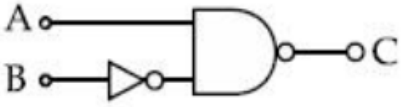

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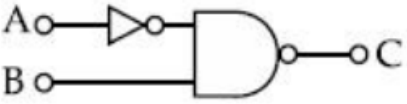

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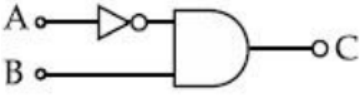


Key: 4

Solution:

$$y=\overline{A+\overline{B}}=\bar{A}B$$

17. When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is:

1) Elliptical **2)** parabolic **3)** straight line **4)** circular

Key: 1

Solution:

$$v = \omega\sqrt{A^2 - x^2}$$

$$\frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$$

18. Given below are two statements:

Statement – I: PN junction diodes can be used to function as transistor, simply by connecting two diodes, back to back, which acts as the base terminal.

Statement – II: In the study of transistor, the amplification factor β indicates ratio of the collector current to the base current.

In the light of the above statements, choose the correct answer from the options given below:

1) Both Statement – I and Statement – II are false

2) Both Statement – I and Statement – II are true

3) Statement – I is false but Statement – II is true

4) Statement – I is true but Statement – II is false

Key: 2

Solution:

$$\beta = \frac{I_C}{I_B}$$

19. A body weight 49 N on sprig balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator?

$$\left[\text{Use } g = \frac{GM}{R^2} = 9.8 ms^{-2} \text{ and radius of earth, } R = 6400 \text{ km}\right]$$

1) 49.17 N **2)** 49.83 N **3)** 49 N **4)** 48.83 N

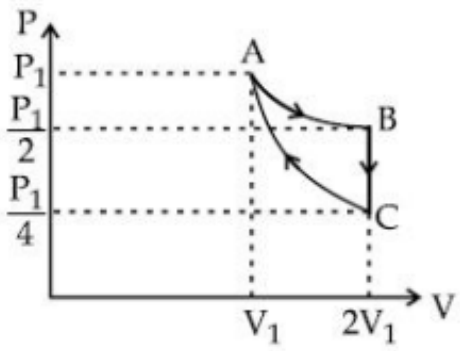
Key: 4

Solution:

$$mg = 49N$$

$$m\left(g - \omega^2 R\right) = 48.83N$$

20. If one mole of an ideal gas at (P_1, V_1) is allowed to expand reversibly and isothermally (A to B) its pressure is reduced to one-half of the original pressure (see fig.) This is followed by a constant volume cooling till its pressure is reduced to one-fourth of the initial value $(B \to C)$. Then is restored to its initial state by a reversible adiabatic compression (C to A). The network done by the gas is equal to:



1) $RT \ln 2$ **2)** $-\frac{RT}{2(\gamma-1)}$ **3)** $RT\left(\ln 2 - \frac{1}{2(\gamma-1)}\right)$ **4)** 0

Key: 3

Solution:

$$W_{BC} = 0$$

$$W_{AB} + W_{BC} + W_{CA} = RT\left[l \ln 2 - \frac{1}{2(\gamma-1)}\right]$$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. A uniform thin bar of mass 6 kg and length 2.4 meter is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is …… $\times 10^{-1}\,kg\,m^2$

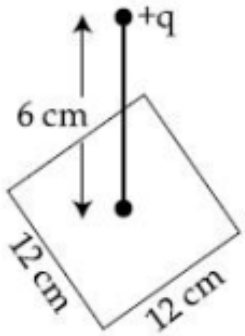
Key: 8

Solution:

$$I = 6 \times \frac{m}{6}\left[\frac{a^2}{12} + \frac{3a^2}{4}\right] = \frac{5ma^2}{6} \text{ where } a = 0.4$$

$$= \frac{5}{6} \times 6 \times (0.4)^2 = 0.8 \text{ kg} - m^2$$

22. A point charge of $+12\ \mu C$ is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in fig. The magnitude of the electric flux through the square will be …….. $\times 10^3\ Nm^2/C$:



Key: 226

Solution:

By symmetry, $\phi = \dfrac{q}{6\varepsilon_0} = \dfrac{12\times10^{-6}}{6\times8.85\times10^{-12}} = 225.98\times10^3$ SI units

23. Two cars are approaching each other at an equal speed of 7.2 km/hr. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be …… Hz. (velocity of sound in air is 340 m/s)

Key: 16

Solution:

$f = 676$ Hz

$f' = \left(\dfrac{v+u}{v-u}\right) f \approx 692$ Hz

Beat frequency = $f' - f = 16$ Hz

24. Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of their kinetic energy $(K.E)_A : (K.E)_B$ will be $\dfrac{A}{1}$, so the value of A will be ……

Key: 2

Solution:

$K_1 : K_2 = \dfrac{P_1^2}{2m_1} : \dfrac{P_2^2}{2m_2} = m_2 : m_1 \left(\because P_1 = P_2\right) = 2:1$

25. An electromagnetic wave of frequency 3 GHz enters a dielectric medium of relative electric permittivity 2.25 from vacuum. The wavelength of this wave in that medium will be ….. $\times 10^{-2}$cm

Key: 667

Solution:

$$\lambda = \frac{C}{\mu f} = \frac{C}{\sqrt{k} f} = \frac{3\times 10^8}{\sqrt{2.25}\times 3\times 10^9} = \frac{1}{15} m$$

$$= \frac{100}{15} cm = 6.666 cm$$

26. A series LCR circuit is designed to resonate at an angular frequency $\omega_0 = 10^5$ rad / s. The circuit draws 16 W power from 120 V source at resonance. The value of resistance 'R' in the circuit is …… Ω .

Key: 900

Solution:

$$P = \frac{V^2}{R}$$

$$R = \frac{120^2}{16} = 900\Omega$$

27. The root mean square speed of molecules of a given mass of a gas at 27^0C and 1 atmosphere pressure is 200 ms^{-1}. The root mean square speed of molecules of the gas at 127^0C and 2 atmosphere pressure is $\frac{x}{\sqrt{3}} ms^{-1}$. The value of 'x' will be …….

Key: 400

Solution:

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{200}{V_2} = \sqrt{\frac{300}{400}} = \frac{\sqrt{3}}{2}$$

$$V_2 = \frac{400}{\sqrt{3}} m / s$$

28. A cylindrical wire of radius 0.5 mm and conductivity 5×10^7 S / m is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3\pi$ mA. The value of 'x' is …….

Key: 5

Solution:

$$i = \sigma EA$$

$$= 5\times10^{7}\times10\times10^{-3}\times\pi\times25\times10^{-8}$$

$$= 125\pi\times10^{-3}$$

$$= 125\pi mA$$

29. A signal of 0.1 kW is transmitted in a cable. The attenuation of cable is –5 dB per km and cable length is 20 km. The power received at receiver is 10^{-x} W. The value of 'x' is ……

$$\left[\text{Given in dB} = 10\log_{10}\left(\frac{P_0}{P_i}\right)\right]$$

Key: 8

Solution:

$$100 = 10\log_{10}\left(\frac{P_i}{P_0}\right)\Rightarrow\frac{P_i}{P_0} = 10^{10}$$

$$P_0 = \frac{P_i}{10^{10}} = \frac{0.1\times10^{3}}{10^{10}} = 10^{-8}W$$

30. A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F. The elongation, if its length and diameter is doubled and subjected to the same force will be ……. cm

Key: 2

Solution:

$$e = \frac{4Fl}{\pi d^{2}Y}\Rightarrow e\propto\frac{l}{d^{2}}$$

$$\frac{e_1}{e_2} = \frac{l_1}{l_2}\left(\frac{d_2}{d_1}\right)^{2} = 2\Rightarrow e_2 = \frac{e_1}{2} = \frac{0.04}{2} = 0.02\ m = 2cm$$

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. In Polymer Buna-S: 'S' stands for:

1) Strength **2)** Styrene **3)** Sulphur **4)** Sulphonation

Key: 2

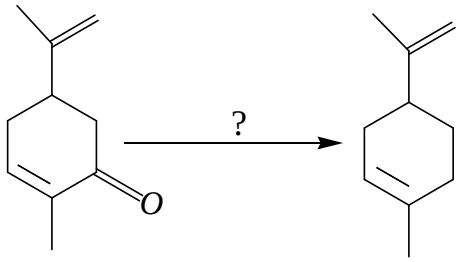
Solution:

In buna – S monomer's are

1, 3-butadine and styrene

's' stands or styrene

32.



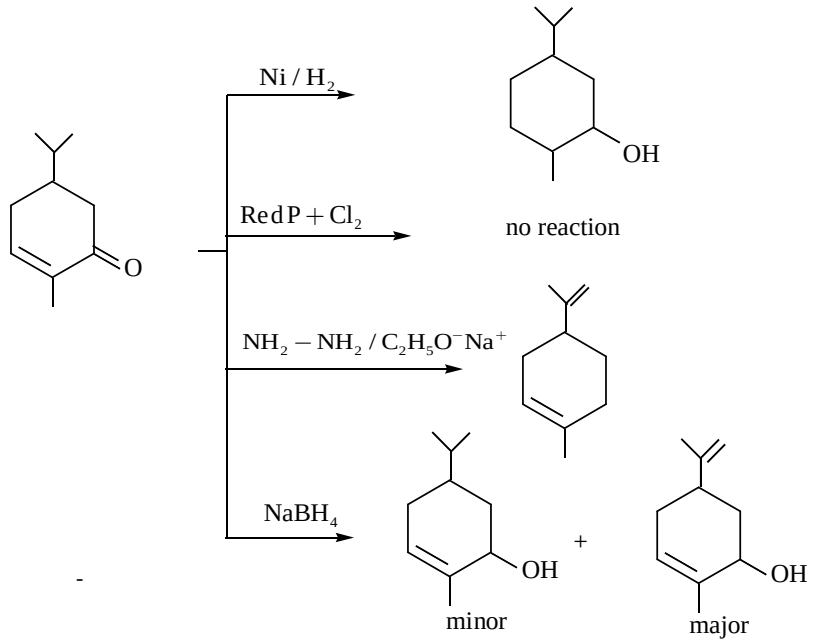
Which of the following reagent is suitable for the preparation of the product in the above reaction?

1) Ni / H_2 **2)** $\text{Red}\, P + Cl_2$

3) $NH_2 - NH_2 / C_2H_5\overset{\ominus}{O}\overset{\oplus}{N}a$ **4)** $NaBH_4$

Key: 3

Solution:



33. Match List – I and List – II.

List – I	List – II
a) Valium	i) Antifertility drug
b) Morphine	ii) Pernicious anaemia
c) Norethindrone	iii) Analgesic
d) Vitamin B_{12}	iv) Tranquilizer

1) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i) **2)** (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

3) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii) **4)** (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

Key: 3

Solution: Valium $\Rightarrow$ tranguilizer

Morphine $\Rightarrow$ analgesic

Norethidrone $\Rightarrow$ anti fertility drug

Vitamin $B_{12} \Rightarrow$ pernicious anaemia

34. The calculated magnetic moments (spin only value) for species $\left[FeCl_4\right]^{2-}$, $\left[Co\left(C_2O_4\right)_3\right]^{3-}$ and MnO_4^{2-} respectively are:

1) 4.90, 0 and 1.73 BM **2)** 5.92, 4.90 and 0 BM

3) 5.82, 0 and 0 BM **4)** 4.90, 0 and 2.83 BM

Key: 1

Solution:

	$\left(FeCl_4\right)^{2-}$,	$\left(Co\left(C_2O_4\right)_3\right)^{3-}$,	MnO_4^{-2}
No. of un paired electrons	4,	0,	1
	4.90,	0,	1.735

35. Match List – I with List – II.

List – I	List – II
a) Aluminium	i) Siderite
b) Iron	ii) Calamine
c) Copper	iii) Kaolinite
d) Zinc	iv) Malachite

Choose the correct answer from the options given below:

1) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii) **2)** (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

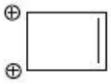
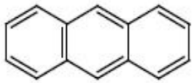
3) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv) **4)** (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

Key: 1

Solution: $Al \rightarrow$ kaolinite $Fe \rightarrow$siderite

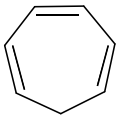
$Cu \rightarrow$ malachite $Zn \rightarrow$cala min e

36. Which one of the following compounds is non-aromatic?

1)  **2)**  **3)**  **4)** 

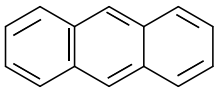
Key: 1

Solution:

 non-aromatic

 aromatic

 aromatic

 aromatic

37. Given below are two statements:

Statement I: The value of the parameter "Biochemical Oxygen Demand (BOD)" is important for survival of aquatic life.

Statement II: The optimum value of BOD is 6.5 ppm.

In the light of the above statements, choose the most appropriate answer from the options given below:

1) Both statement I and Statement II are true

2) Both statement I and Statement II are false

3) Statement I is true but Statement II is false

4) Statement I is false but Statement II is true

Key: 1

Solution: Pure water B.O.D value < 5 ppm

38. The correct shape and I – I – I bond angles respectively in I_3^- ion are:

1) Trigonal planar; 120^0 **2)** T – shaped; $180^0 and 90^0$

3) Linear; 180^0 **4)** Distorted trigonal planar; $135^0 and 90^0$

Key: 3

Solution: $I_3^{-1} \rightarrow 2b.p + 3l.p$

Linear, 180^0

39. Most suitable salt which can be used for efficient clotting of blood will be:

1) $Mg(HCO_3)_2$ **2)** $NaHCO_3$

3) $FeCl_3$ **4)** $FeSO_4$

Key: 3

Solution: $FeCl_3 \rightarrow$ coagulation of blood particles

40. According to Bohr's atomic theory:

a) Kinetic energy of electron is $\alpha \frac{Z^2}{n^2}$

b) The product of velocity (v) of electron and principal quantum number (n), $'vn' \alpha Z^2$.

c) Frequency of revolution of electron in an orbit is $\alpha \frac{Z^3}{n^3}$

d) Coulombic force of attraction on the electron is $\alpha \frac{Z^3}{n^4}$

Choose the most appropriate answer from the options given below:

1) (A) only **2)** (C) only

3) (A) and (D) only **4)** (A), (C) and (D) only

Key: 3

Solution: I) $KE \propto \frac{z^2}{n^2}$

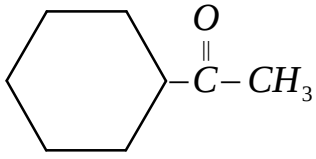
II) $V \times n = z$

III) $f = \frac{4\pi^2 z^2 e^4}{n^3 h^3} f \propto \frac{z^2}{n^3}$

IV) Coloumbic attractive forces $\propto \frac{z^3}{n^4}$

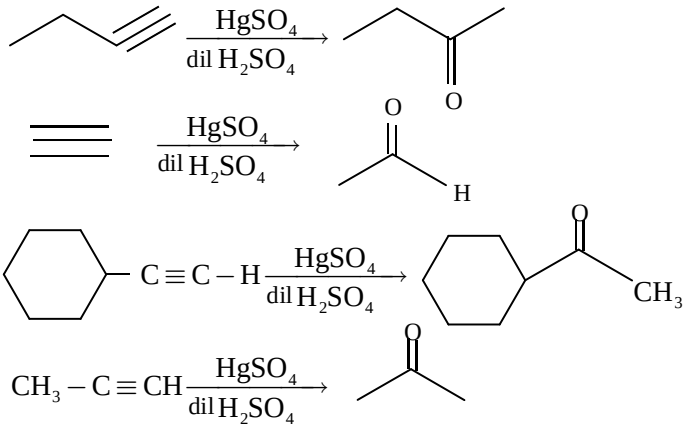
41. Which one of the following carbonyl compounds cannot be prepared by addition of water on an alkyne in the presence of $HgSO_4$ *and* H_2SO_4 ?

1) $CH_3 - \overset{O}{\overset{\|}{C}} - CH_2CH_3$ **2)** $CH_3 - \overset{O}{\overset{\|}{C}} - H$

3)  **4)** $CH_3 - CH_2 - \overset{O}{\overset{\|}{C}} - H$

Key: 4

Solution:



42. Match List – I and List – II

List – I	**List – II**
a) $R-\overset{\overset{O}{\|}}{C}-Cl \rightarrow R-CHO$	i) $Br_2 / NaOH$
b) $R-CH_2-COOH \rightarrow R-\underset{\underset{Cl}{\|}}{CH}-COOH$	ii) $H_2 / Pd-BaSO_4$
c) $R-\overset{\overset{O}{\|}}{C}-NH_2 \rightarrow R-NH_2$	iii) $Zn(Hg) / Conc.HCl$
d) $R-\overset{\overset{O}{\|}}{C}-CH_3 \rightarrow R-CH_2-CH_3$	iv) $Cl_2 / \text{Re}dP, H_2O$

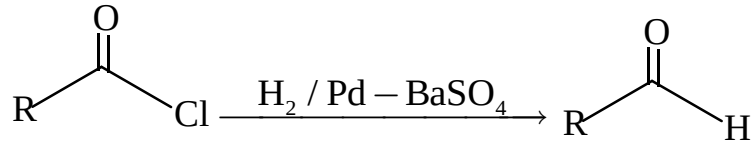
Choose the correct answer from the options given below:

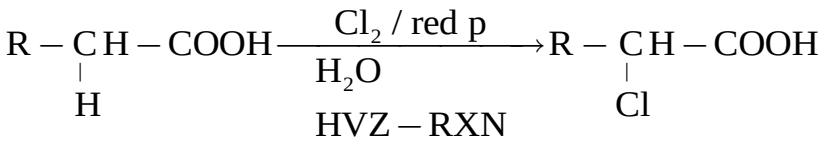
1) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii) **2)** (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

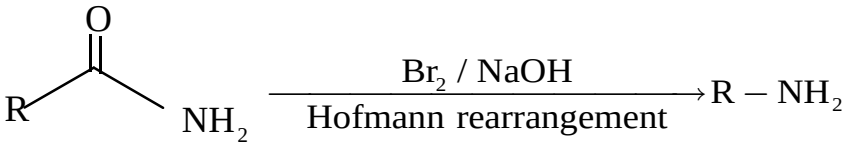
3) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii) **4)** (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

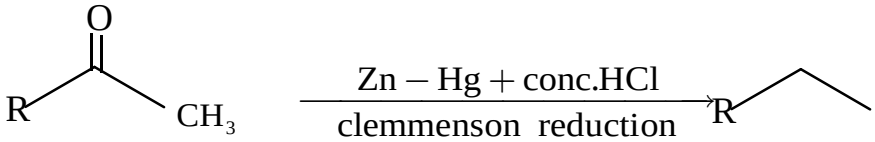
Key: 3

Solution:









43. What is the correct order of the following elements with respect to their density?

1) $Cr < Zn < Co < Cu < Fe$ **2)** $Zn < Cu < Co < Fe < Cr$

3) $Zn < Cr < Fe < Co < Cu$ **4)** $Cr < Fe < Co < Cu < Zn$

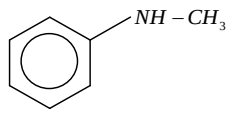
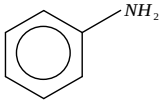
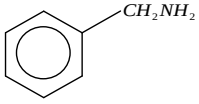
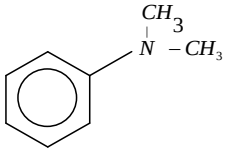
Key: 3

Solution: Order of density

Zn < Cr < Fe < Co < Cu

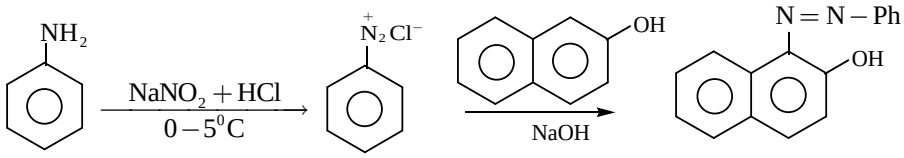
7.1 7.19 8.9

44. The diazonium salt of which of the following compounds will form a coloured dye on reaction with β – Naphthol in NaOH?

1)  **2)**  **3)**  **4)** 

Key: 2

Solution: Aromatic primary only involved in this reaction so, 1, 3, 4 are eliminated



45. Given below are two statements: one is labeled as Assertion A and the other is labeled as Reason R.

Assertion A: Hydrogen is the most abundant element in the Universe, but it is not the most abundant gas in the troposphere.

Reson R: Hydrogen is the lightest element.

In the light of the above statements, choose the correct answer from the options given below:

1) A is false but R is true

2) Both A and R are true but R is NOT the correct explanation of A

3) A is true but R is false

4) Both A and B are true and R is the correct explanation of A

Key: 2

Solution: Conceptual

46. Match List – I with List – II

List – I (Salt)	List – II (Flame colour wavelength)
a) LiCl	i) 455.5 nm
b) NaCl	ii) 670.8 nm
c) RbCl	iii) 780.0 nm
d) CsCl	iv) 589.2 nm

Choose the correct answer from the options given below:

1) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i) **2)** (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

3) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii) **4)** (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

Key: 1

Solution: $RbCl \rightarrow 780\,nm$

$CsCl \rightarrow 455.5\,nm$

$NaCl : 589.2\ nm$

$LiCl : 670.8\ nm$

47. The incorrect statement among the following is:

1) Cr_2O_3 is an amphoteric oxide

2) Red colour of ruby is due to the presence of Co^{3+}

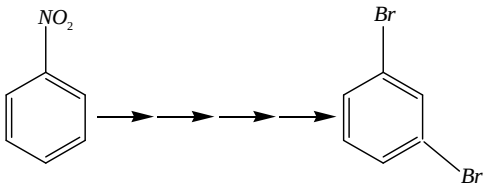
3) RuO_4 is an oxidizing agent

4) $VOSO_4$ is a reducing agent

Key: 2

Solution: Ruby red $\rightarrow Cr^{3+}$

48. What is the correct sequence of reagents used for converting nitrobenzene into m-dibromobenzene?



1) $\xrightarrow{Sn/HCL} \diagup \xrightarrow{Br_2} \diagup \xrightarrow{NaNO_2} \diagup \xrightarrow{NaBr}$

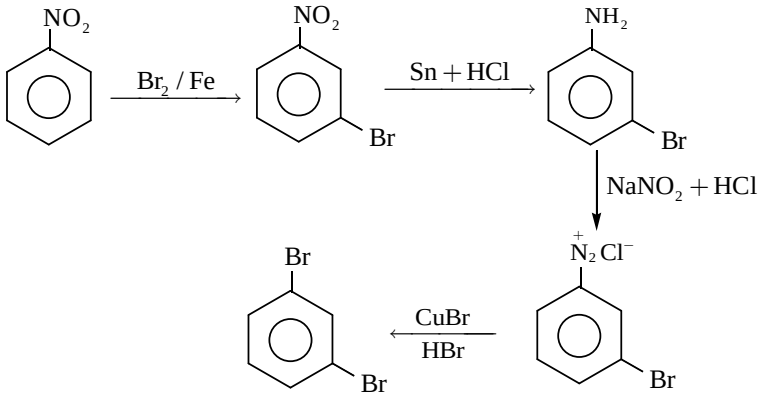
2) $\xrightarrow{Sn/HCL} \diagup \xrightarrow{KBr} \diagup \xrightarrow{Br_2} \diagup \xrightarrow{H^+}$

3) $\xrightarrow{Br_2/Fe} \diagup \xrightarrow{Sn/HCl} \diagup \xrightarrow{NaNO_2/HCl} \diagup \xrightarrow{CuBr/HBr}$

4) $\xrightarrow{NaNO_2} \diagup \xrightarrow{HCl} \diagup \xrightarrow{KBr} \diagup \xrightarrow{H^+}$

Key: 3

Solution:



sandmayer RXN

49. The correct set from the following in which both pairs are in correct order of melting point is:

1) $LiF > LiCl; MgO > NaCl$ **2)** $LiF > LiCl; NaCl > MgO$

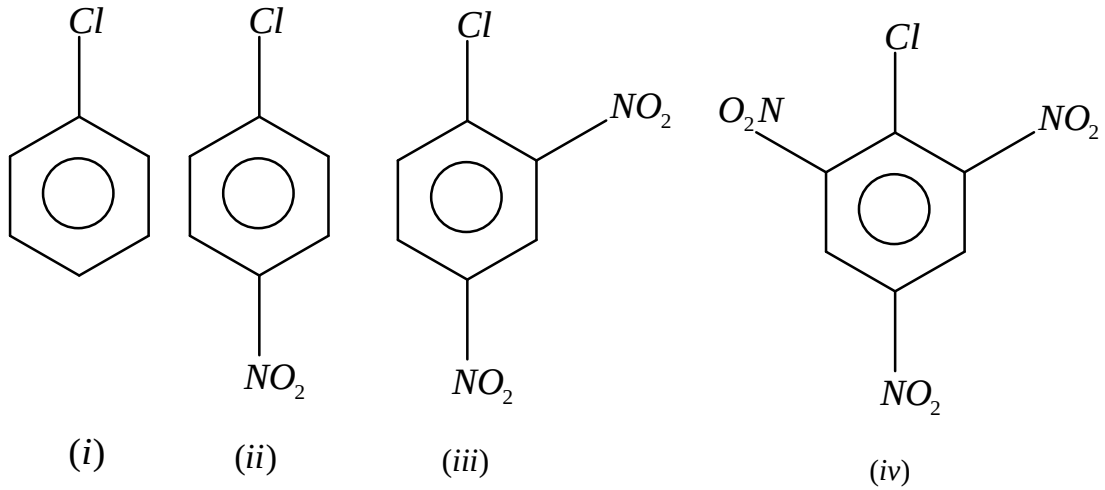
3) $LiCl > LiF >; NaCl > MgO$ **4)** $LiCl > LiF >; MgO > NaCl$

Key: 1

Solution: LiF > LiCl

MgO > NaCl

50. The correct order of the following compounds showing increasing tendency towards nucleophilic substitution reaction is:



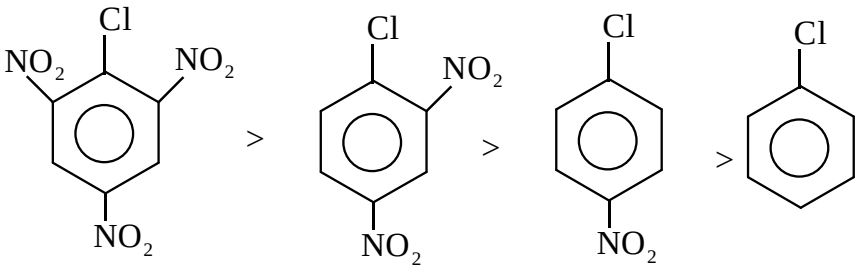
(*i*) (*ii*) (*iii*) (*iv*)

1) (iv)<(iii)<(ii)<(i) **2)** (iv)<(i)<(ii)<(iii)

3) (iv)<(i)<(iii)<(ii) **4)** (i)<(ii)<(iii)<(iv)

Key: 4

Solution:



No. of withdrawing group electrophilic character increases then reactivity order

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

51. The solubility product of PbI_2 is 8.0×10^{-6}. The solubility of lead idodide in 0.1 molar solution of lead nitrate is $x\times10^{-6}\,mol/L$. The value of x is _____(Rounded off to the nearest integer) $\left[\text{Given } \sqrt{2}=1.41\right]$

Key: 141

Solution: $(0.1+s)4s^2=8\times10^{-9}=4s^2=8\times10^{-8}$

$$S=\sqrt{2}\times10^{-4}=141.4\times10^{-6}=141$$

52. The formula of a gaseous hydrocarbon which requires 6 times of its own volume of O_2 for complete oxidation and produces 4 time its own volume of CO_2 is C_xH_y. The value of y is _____

Key: 8

Solution: $C_XH_Y + \left(X + \frac{Y}{4}\right)O_2 \rightarrow XCO_2 + \frac{Y}{2}H_2O$

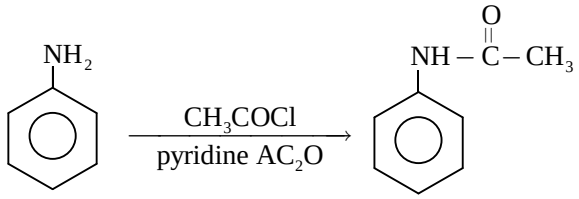
$X = 4$

$X + \frac{Y}{4} = 6 \quad , \qquad Y = 8$

53. 1.86 g of aniline completely reacts to form acetanilide, 10% of the product is lost during purification. Amount of acetanilide obtained after purification (in g) is _____ $\times 10^{-2}$.

Key: 243

Solution:



$93 \rightarrow 135$

$1.86 \rightarrow 2.70$

10% product lost 90% is there

$\frac{2.70 \times 90}{100} = 2.43 \qquad 243 \times 10^{-2}$

54. C_6H_6 freezes at 5.5^0C. The temperature at which a solution of 10 g of C_4H_{10} in 200 g of C_6H_6 freeze is $\_\_^0C$. (The molal freezing point depression constant of C_6H_6 is $5.12^0C/m$)

Key: 1.09

Solution: If $\Delta T_f = T_f^{\,0} - T_f = K_f \times m$

$5.5 - (T_f)\text{solution} = 5.12 \times \frac{10}{58} \times \frac{1000}{200}$

$5.5 - T_f \text{ of sol.} = 4.41 \quad 5.5 - 4.41 = +1.086 \simeq 1.09$

55. Among the following allotropic form of sulphur, the number of allotropic forms, which will show paramagnetism is _____

a) $\alpha-$sulphur b) $\beta-$sulphur c) S_2-form

Key: 1

Solution: S_2 - para magnetic

56. The magnitude of the change in oxidizing power of the MnO_4^- / Mn^{2+} couple is $x\times10^{-4}V$, if the H^+ concentration is decreased from 1 M to $10^{-4}M$ at 25^0C. (Assume concentration of MnO_4^- and Mn^{2+} to be same on change in H^+ concentration). The value of x is _____

(Rounded off to the nearest integer) $\left[\text{Given}:\frac{2.303RT}{F}=0.059\right]$

Key: 03776

Solution: $MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{+2} + 4H_2O$

$$E = E^0 - \frac{0.059}{5}\log_{10}\frac{\left[Mn^{+2}\right]}{\left[H^+\right]^8 MnO_4^-}$$

$$E = E^0 - \frac{0.059}{5}\log\frac{1}{1} = E^0$$

$$E = E^0 - \frac{0.059}{5}\log\left(\frac{1}{10^{-4}}\right)^8 = E^0 - \frac{0.059}{5}\log 10^{32}$$

$$E^0 - E^0 + \frac{0.059}{5}\times 32 = 0.3776V$$

57. Assuming ideal behavior, the magnitude of log K for the following reaction at 25^0C $x\times10^{-1}$. The value of x is _____(Integer answer)

$$3HC\equiv CH_{(g)} \rightleftharpoons C_6H_{6(\ell)}$$

$\left[\text{Given}:\Delta_f G^0(HC\equiv CH)=-2.04\times10^5 J\ mol^{-1};\ \Delta_f G^0(C_6H_6)=-1.24\times10^5 J\ mol^{-1}; R=8.314\ J\ K^{-1}\ mol^{-1}\right]$

Key: 150.72

Solution: $3C_2H_2 \rightleftharpoons C_6H_{6l}$

$2.4\times10^5 \quad -1.4\times10^5$

$AG_1^0 = -1.4\times10^5 - 3\times2.4\times10^5 = -8.6\times10^5$

$\Delta G_1^0 = -RT\ln k$

$\log_{10}^k = 150.72$

58. Sucrose hydrolyses in acid solution into glucose and fructose following first order rate law with a half-life of 3.33 h at 25^0C. After 9h, the fraction of sucrose remaining is f. The value of $\log_{10}\left(\frac{1}{f}\right)$ is _____$\times 10^{-2}$. (Rounded off to the nearest integer)

$[\text{Assume}: \ln 10 = 2.303, \ln 2 = 0.693]$

Key: 82

Solution: $C_{12}H_{22}O_{11} + H_2O \rightarrow C_2H_{12}O_6 + C_6H_{12}O_6$

$t_{\frac{1}{2}} = 3.33$ hours $\quad \frac{9}{3.33} = 2.7$

Fraction of sucrose remains = $\frac{1}{2^{2.7}}$

$\log\frac{1}{f} = \log 2^{2.7} = 2.7 \times 0.3010 = 0.82 \quad = 0.82 \times 100 = 82$

59. The volume occupied by 4.75 g of acetylene gas at 50^0C and 740 mmHg pressure is _____L. (Rounded off to the nearest integer)

$[\text{Given } R = 0.0826 \text{ L atm } K^{-1} \text{ mol}^{-1}]$

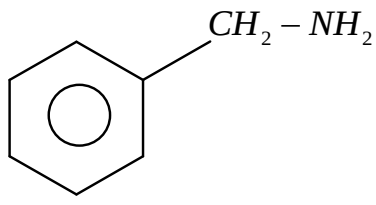
Key: 5

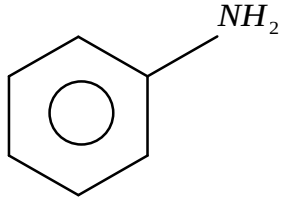
Solution: $V = \frac{nRT}{P} = \frac{4.75}{26} \times \frac{0.0826 \times 323}{\frac{740}{760}} = 5$

60. The total number of amines among the following which can be synthesized by Gabriel synthesis is _____

1) $\begin{matrix} CH_3 \\ CH_3 \end{matrix} > CH - CH_2 - NH_2$

2) $CH_3CH_2NH_2$

3) 

4) 

Key: 3

Solution: Aryl halides not involved in Gabriel synthesis (SN_2)

ABC are correct D is wrong.

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is

1) $\frac{1}{2\sqrt{2}}$ **2)** $\sqrt{7}-1$ **3)** $2\sqrt{2}-1$ **4)** $\frac{1}{\sqrt{7}}$

Key: 4

Solution: Let $\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}=\theta \Rightarrow \tan 2\theta=\frac{16\pm\sqrt{256-252}}{2\sqrt{63}}=\frac{9}{\sqrt{63}}$ or $\frac{7}{\sqrt{63}}$

$\sin 4\theta=\frac{\sqrt{63}}{8}$ $\frac{2\tan 2\theta}{1+\tan^2 2\theta}=\frac{\sqrt{63}}{8}$

$\sqrt{63}\left(1+\tan^2 2\theta\right)=16\tan 2\theta$

$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta}=\frac{9}{\sqrt{63}}$ or $\frac{7}{\sqrt{63}}$

$7\left(\tan^2\theta-1\right)+2\sqrt{63}\tan\theta=0$

$\tan\theta=\frac{-2\sqrt{63}\pm\sqrt{252+196}}{14}$

$=\frac{-2\sqrt{63}\pm\sqrt{448}}{14}$ $=\frac{-6\sqrt{7}\pm 8\sqrt{7}}{14}=\frac{1}{\sqrt{7}}$ $\Rightarrow \tan\theta=\frac{1}{\sqrt{7}}$.

62. The area of the region: $R=\left\{(x,y): 5x^2\le y\le 2x^2+9\right\}$ is

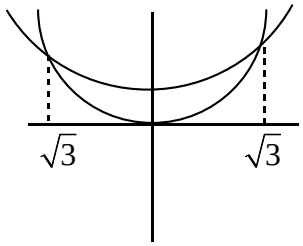
1) $11\sqrt{3}$ square units **2)** $6\sqrt{3}$ square units

3) $9\sqrt{3}$ square units **4)** $12\sqrt{3}$ square units

Key: 4

Solution:

$5x^2-y\le 0$ and $2x^2-y+9\ge 0$



$\Rightarrow A=2\int_0^{\sqrt{3}} 2x^2-5x^2+9$ $A=2\times\left(9\sqrt{3}-3\sqrt{3}\right)$

$A=12\sqrt{3}$

63. For the statements p and q consider the following compound statements:

a) $(\sim q \wedge (p \to q)) \to \sim p$ b) $((p \vee q) \wedge \sim p) \to q$

Then which of the following statements is correct?

1) (a) and (b) both are tautologies **2)** (a) is a tautology but not (b)

3) (b) is a tautology but not (a) **4)** (a) and (b) both are not tautologies

Key: 1

Solution:

p	q	$\sim q$	$p \to q$	$\sim p$	Result
T	T	F	T	F	T
T	F	T	F	F	T
F	T	F	T	T	T
F	F	T	T	T	T

p	q	$p \vee q$	$\sim p$	Result
T	T	F	F	T
T	F	T	F	T
F	T	T	T	T
F	F	T	T	T

Both a and b are tautologies

64. If the curve $y = ax^2 + bx + c, x \in R$, passes through the point (1,2) and the tangent line to this curve at origin is $y = x$, then the possible values of a,b,c are

1) $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$ **2)** $a = 1, b = 0, c = 1$

3) $a = -1, b = 1, c = 1$ **4)** $a = 1, b = 1, c = 0$

Key: 4

Solution:

$2 = a + b + c$

$\left.\frac{dy}{dx}\right|_{(0,0)} = 1 \qquad \Rightarrow b = 1$

$\Rightarrow$ possible option 4

65. Let $a, b \in R$. If the mirror image of the point $P(a,6,9)$ with respect to the line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$ is $(20, b, -a-9)$, then $|a+b|$ is equal to

1) 84 **2)** 88 **3)** 90 **4)** 86

Key: 2

Solution:

mid pt $\Rightarrow \left(\frac{a+20}{2}, \frac{b+6}{2}, \frac{-a}{2}\right)$ is locate d on line

$7(a-20) + 5(6-b) + (-9)(18+a) = 0$

$-2a-5b-272=0 \quad \Rightarrow \quad 2a+5b+272=0$

$\frac{a+20-6}{14}=\frac{b+6-4}{10}=\frac{-a+2}{18} \qquad \Rightarrow (a+14)18=14(a+2)$

$4a+14\times16=0$

$a=-56$

$b=-32$

$\Rightarrow |a+b|=88$

66. If $n\geq 2$ is a positive integer, then the sum of the series $^{n+1}C_2+2\left(^2C_2+^3C_2+^4C_2+.....^nC_2\right)$ is

1) $\frac{n(n+1)(2n+1)}{6}$ **2)** $\frac{n(2n+1)(3n+1)}{6}$

3) $\frac{n(n+1)^2(n+2)}{12}$ **4)** $\frac{n(n-1)(2n+2)}{6}$

Key: 1

Solution:

$^{n+1}C_2+2\left(^3C_3+^3C_2+^4C_2+.....+^nC_2\right)$

$^{n+1}C_2+2\left(^{n+1}C_3\right)$

$^{n+1}C_2+^{n+1}C_3+^{n+1}C_3$

$^{n+2}C_3+^{n+1}C_3=\frac{(n+2)(n+1)(n)}{6}+\frac{(n+2).n(n-1)}{6}$

$=\frac{(n+1)n}{6}(2n+1)$

67. The vector equation of the plane passing through the intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(\hat{i}-2\hat{j}\right)=-2$ and the 0point $(1,0,2)$ is

1) $\vec{r}.\left(\hat{i}+7\hat{j}+3\hat{k}\right)=\frac{7}{3}$ **2)** $\vec{r}.\left(3\hat{i}+7\hat{j}+3\hat{k}\right)=7$

3) $\vec{r}.\left(\hat{i}-7\hat{j}+3\hat{k}\right)=\frac{7}{3}$ **4)** $\vec{r}.\left(\hat{i}+7\hat{j}+3\hat{k}\right)=7$

Key: 4

Solution:

$x+y+z=1 \qquad x-2y+2=0$

$\pi_1+\lambda\pi_2=0 \qquad (x+y+z-1)+\lambda(x-2y+2)=0$

$(2)+\lambda(3)=0=\lambda=-2/3$

Reqd plane is $x+7y+3z-7=0$

$\Rightarrow \vec{r}.(i+7j+3k)=7$

68. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are

1) $(3,13)$ **2)** $(2,8)$ **3)** $(1,5)$ **4)** $(-2,8)$

Key: 2

Solution:

It's the point where slope of tgt is 4

$$\frac{dy}{dx} = 2x = 4 \quad \Rightarrow \; x = 2 \quad \Rightarrow \; pt(2,8)$$

69. The negation of the statement $\sim P \wedge (p \vee q)$ is

1) $p \vee \sim q$ **2)** $p \wedge \sim q$ **3)** $\sim p \vee q$ **4)** $\sim p \wedge q$

Key: 1

Solution:

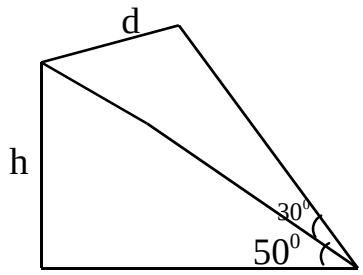
$$\sim(\sim p \wedge (p \vee q)) = p \vee (\sim p \wedge \sim q) = (p \vee \sim p) \wedge (p \vee \sim q)$$
$$= p \vee \sim q$$

70. The angle of elevation of a jet plane from a point A on the ground is 60^0. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30^0. If the jet plane is flying at a constant height, then its height is

1) $3600\sqrt{3}$m **2)** $1200\sqrt{3}$m **3)** $2400\sqrt{3}$m **4)** $1800\sqrt{3}$m

Key: 2

Solution:



$\sqrt{3}h$

$$\tan 60 = \frac{h}{\sqrt{3}h - d} \quad \Rightarrow \; h = \sqrt{3}\left(\sqrt{3}h - d\right)$$

$$2h = \sqrt{3}d$$

$$h = \frac{\sqrt{3}}{2} \times d = \frac{\sqrt{3}}{2} \times 432 \times \frac{5}{18} \times 20 = 1200\sqrt{3}$$

71. Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2 - B^2A^2)X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has

1) a unique solution **2)** infinitely many solutions

3) no solution **4)** exactly two solutions

Key: 2

Solution:

$A \to \text{Sym} \qquad B \to \text{skew} \qquad$ given system is homogenous set of eqns.

Deteminant of coeff matrix $= \left|A^2B^2 - B^2A^2\right| = 0$

Since $A^2B^2 - B^2A^2$ is a skew symmetric matrix of odd order

Eqn have

Non-trivial soln

$\therefore$ no of soln is ∞

72. Let f be a twice differentiable function defined on R such that $f(0)=1, f'(0)=2$ and $f^1(x) \neq 0$ for all $x \in R$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x \in R$, then the value of $f(1)$ lies in the interval

1) $(6,9)$ **2)** $(0,3)$ **3)** $(3,6)$ **4)** $(9,12)$

Key: 1

Solution:

$$f(x)f''(x) = \left(f^1(x)\right)^2$$

$$\Rightarrow \frac{\left(f^1(x)\right)^2 - f(x)f^{11}(x)}{\left(f^1(x)\right)^2} = 0 \qquad \Rightarrow \frac{d}{dx}\left(\frac{f(x)}{f^1(x)}\right) = 0$$

$$\Rightarrow \frac{f(x)}{f^1(x)} = k \Rightarrow k = \frac{1}{2}$$

$$f(x) = \pm ke^{2x} \quad f(0) = 1 \Rightarrow f(1) = e^2$$

73. Let $f: R \to R$ be defined as $f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4, \end{cases}$

Let $A = \{x \in R : f \text{ is increasing }\}$. Then A is equal to

1) $(-\infty,-5) \cup (4,\infty)$ **2)** $(-\infty,-5) \cup (-4,\infty)$

3) $(-5,\infty)$ **4)** $(-5,-4) \cup (4,\infty)$

Key: 4

Solution:

$f^1(x) = -55 \qquad n < -5$

$= 6x^2 - 6x - 120 - 5 \le x \le 4 \qquad \Rightarrow \quad f^1(x) > 0 \qquad \forall \ x \in (-5,-4)$

$= 6x^2 - 6x - 36 \qquad x > 4 \qquad \Rightarrow f^1(x) > 0 \qquad \forall \ x \in (4,\infty)$

74. If a curve $y = f(x)$ passes through the point $(1,2)$ and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)dx = \frac{62}{5}$?

1) 5 **2)** 10 **3)** $\frac{31}{5}$ **4)** $\frac{62}{5}$

Key: 2

Solution:

$x\frac{dy}{dx} + y = bx^4 \Rightarrow \frac{dy}{dx} + y\left(\frac{1}{x}\right) = x^3.b \quad \text{if} = e^{\int \frac{1}{x}dx} = x$

$xy = \int bx^4 dx \Rightarrow xy = \frac{bx^5}{5} + c$

passing through $(1, 2)\ 2 = \frac{b}{5} + c \Rightarrow c = 2 - \frac{b}{5}$

$\Rightarrow f(x) = \frac{bx^4}{5} + \frac{c}{x}$ $\frac{62}{5} = \frac{b}{25} \times 31 + C \ln x$ $\Rightarrow b = 10$

75. The probability that two randomly selected subsets of the set $\{1,2,3,4,5\}$ have exactly two elements in their intersection, is

1) $\frac{65}{2^7}$ **2)** $\frac{65}{2^8}$ **3)** $\frac{35}{2^7}$ **4)** $\frac{135}{2^9}$

Key: 4

Solution:

S = {1, 2, 3, 4, 5}

A = { } B = { }

Let two elements be selected in ${}^5C_2$ ways remaining elements can be distributed in 3 ways

$\therefore P(E) = \frac{{}^5C_2 \times 3^3}{4^5} = \frac{27 \times 5}{2^7} = \frac{135}{2^9}$

76. For the system of linear equations $x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in R$, consider the following statements

A) The system has unique solution if $k \neq 2, k \neq -2$.

B) The system has unique solution if $k = -2$.

C) The system has unique solution if k = 2.

D) The system has no-solutions if k = 2.

E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are correct ?

1) (A) and (B) only **2)** (A) and (D) only

3) (C) and (D) only **4)** (B) and (E) only

Key: 2

Solution:

$$\Delta=\begin{vmatrix}1 & -2 & 0\\ 1 & -1 & k\\ 0 & k & 4\end{vmatrix}=1\left(-4-k^2\right)+2=-\left(k^2-4\right)\Rightarrow \text{(A) statement is correct}$$

$$\Delta_1=\begin{vmatrix}1 & -2 & 0\\ -2 & -1 & k\\ 6 & k & 4\end{vmatrix}=1\left(-4-k^2\right)+2\left(-8-6k\right)=-\left(k^2+12k+20\right)$$

$$\Delta_2=\begin{vmatrix}1 & 1 & 0\\ 1 & -2 & k\\ 0 & 6 & 4\end{vmatrix}=1\left(-8-6k\right)-1\left(4\right)=-\left(6k+12\right)$$

$$\Delta_3=\begin{vmatrix}1 & -2 & 1\\ 1 & -1 & -2\\ 0 & k & 6\end{vmatrix}=1\left(-6+2k\right)+2\left(6\right)+1\left(k\right)=\left(3k+6\right)$$

$\Rightarrow$ statement D is correct

77. Let a,b,c be in arithmetic progression. Let the centroid of the triangle with vertices $(a,c),(2,b)$ and (a,b) be $\left(\frac{10}{3},\frac{7}{3}\right)$. If α,β are the roots of the equation $ax^2+bx+1=0$, then the value of $\alpha^2+\beta^2-\alpha\beta$ is

1) $\frac{69}{256}$ **2)** $-\frac{71}{256}$ **3)** $-\frac{69}{256}$ **4)** $\frac{71}{256}$

Key: 2

Solution:

$2a+2=10$ $2b+c=7$

$$\left(\alpha+\beta\right)^2-3\alpha\beta=\frac{b^2}{a^2}-\frac{3}{a}=\frac{b^2-3a}{a^2}$$

$a+1=5$ $\quad 4+2c=7$

$a=4$ $\quad 2c=3$

$\quad c=3/2$

78. Let $f(x)$ be a differentiable function defined on $[0,2]$ such that $f'(x)=f'(2-x)$ for all $x\in(0,2), f(0)=1$ and $f(2)=e^2$. Then the value of $\int_0^2 f(x)\,dx$ is

1) $1-e^2$ **2)** $1+e^2$ **3)** $2\left(1-e^2\right)$ **4)** $2\left(1+e^2\right)$

Key: 2

Solution:

$f'(x)=f'(2-x)$

$\forall x \in (0, 2)$

$f(0) = 1, f(2) = e^2$

$\int_0^2 f(x)dx = I$

$I = x f(x)\Big|_0^2 - \int_0^2 f'(x) - x$

$I = 2e^2 - \int_0^2 (2-x) f'(2-x)$

$= 2e^2 - \int_0^2 (2-x) f'(x) = 2e^2 - 2(e^2 - 1) + \int_0^2 x f'(x)$

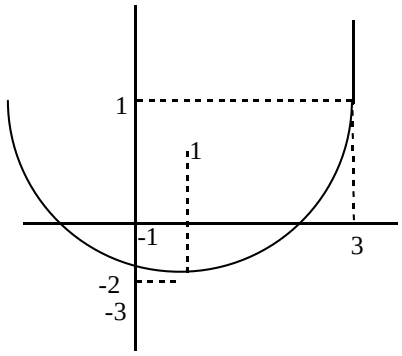
$2I = 2e^2 + 2 \Rightarrow I = 1 + e^2$

79. The value of the integral, $\int_1^3 \left[x^2 - 2x - 2\right]dx$, where $[x]$ denotes the greatest integer less or equal to x, is

1) $-\sqrt{2} - \sqrt{3} + 1$ **2)** $-\sqrt{2} - \sqrt{3} - 1$ **3)** -4 **4)** -5

Key: 2

Solution:



$x^2 - 2x - 2 = -3 \quad x = 1$

$= -2 \qquad x = 2$

$= -1 \qquad x = \dfrac{2 + 2\sqrt{2}}{2}$

$= 0 \qquad x = \dfrac{2 + 2\sqrt{3}}{2}$

$= 1 \qquad x = 3$

$\int_1^2 -3dx + \int_2^{1+\sqrt{2}} -2 + \int_{1+\sqrt{2}}^{1+\sqrt{3}} -1 + \int_{1+\sqrt{3}}^{3} 0 = -\sqrt{2} - \sqrt{3} - 1$

80. For which of the following curves, the line $x+\sqrt{3}y=2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2},\frac{1}{2}\right)$?

1) $x^2+y^2=7$ **2)** $x^2+9y^2=9$ **3)** $y^2=\frac{1}{6\sqrt{3}}x$ **4)** $2x^2-18y^2=9$

Key: 2

Solution: $\frac{dy}{dx}=-\frac{1}{\sqrt{3}}$ and passes through $\left(\frac{3\sqrt{3}}{2},\frac{1}{2}\right)$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

81. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2+(y-3)^2=25$ at the point $(5,7)$ is A, then 24A is equal to _____

Key: 6

Solution:

Normal is $(y-3)=\frac{4}{3}(x-5)$

$\Rightarrow 4x-3y-11=0 \Rightarrow$ intersection with x-axis (11/4, 0)

fgf : $(4-3)=-\frac{3}{4}(x-5)$

$\Rightarrow 3x+4y-27=0$

Intersection with x-axis $\left(\frac{27}{4},0\right)$

$\therefore \text{area}=\frac{1}{2}\times 4\times 3=6$

82. let $i=\sqrt{-1}$. If $\frac{\left(-1+i\sqrt{3}\right)^{21}}{(1-i)^{24}}+\frac{\left(1+i\sqrt{3}\right)^{21}}{(1+i)^{24}}=k$, and $n=\left[\left|k\right|\right]$ be the greatest integral part of $|k|$. Then $\sum_{j=0}^{n+5}(j+5)^2-\sum_{j=0}^{n+5}(j+5)$ is equal to _____

Key: 310

Solution:

$$\frac{\left(-1+i\sqrt{3}\right)^{21}}{(1-i)^{24}}+\frac{\left(1+i\sqrt{3}\right)^{21}}{(1+i)^{24}}=k$$

$$\Rightarrow \frac{(2\omega)^{21}}{\left(\sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)\right)^{24}}+\frac{(-2\omega)^{21}}{\left(\sqrt{2}\,\text{cis}\,\frac{\pi}{4}\right)^{24}}=\frac{(2\omega)^{21}}{\left(\sqrt{2}\,\text{cis}(-6\pi)\right)^{24}}+\frac{\left(-2\omega^2\right)^{21}}{\left(\sqrt{2}\,\text{cis}\,6\pi\right)^{24}}$$

$$=\frac{(2\omega)^{21}}{\left(\sqrt{2}\right)^{24}}+\frac{\left(-2\omega^2\right)^{21}}{\left(\sqrt{2}\right)^{24}}$$

$$=\sum_{j=0}^{5} j^2+9j+20=\frac{5\times6\times11}{6}+\frac{9\times5\times6}{2}+20\times6$$

$$=55+135+120=310$$

83. The students $S_1,S_2,.....,S_{10}$ are to be divided into 3 group A,B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is_____

Key: 39449

Solution:

$$={}^{10}C_3\left(2^7-2\right)+{}^{10}C_4\left(2^6-2\right)+.....+{}^{10}C_{10}$$

$$=\sum_{r=3}^{10}{}^{10}C_r\ 2^{10-r}-2\sum_{r=3}^{10}{}^{10}C_r$$

$$=\left[(1+2)^{10}-\left({}^{10}C_0 2^{10}+{}^{10}C_1 2^9+{}^{10}C_2 2^8\right)\right]$$

$$-2\left[2^{10}-\left({}^{10}C_0+{}^{10}C_4+{}^{10}C_2\right)\right]=39449$$

84. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is_____

Key: 3

Solution:

$$a+ar+ar^2+ar^3=\frac{65}{12}\ \frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+\frac{1}{ar^3}=\frac{65}{18}$$

$$a^3r^3=1\Rightarrow ar=1$$

$$\frac{1+r+r^2+r^3}{r}=\frac{65}{12}\ \ \frac{r^3+r^2+r+1}{r^2}=\frac{65}{18}$$

$$r=\frac{18}{12}=\frac{3}{2}\Rightarrow 3\text{rd term}=r=\frac{3}{2}$$

$$2\alpha=3$$

85. For integers n and r, let $\binom{n}{r}=\begin{cases}{}^{n}C_{r}, & \text{if } n\geq r\geq 0\\ 0, & \text{otherwise}\end{cases}$

The maximum value of k for which the sum $\sum_{i=0}^{k+1}\binom{10}{i}\binom{15}{k-i}+\sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to _____

Key: 25

Solution:

$$\sum_{i=0}^{k}{}^{10}C_{i}.{}^{15}C_{k-i}=\text{coeff. of } x^{k} \text{ in } (1+x)^{25}={}^{25}C_{k}$$

$$\sum_{i=0}^{k+1}{}^{12}C_{i}.{}^{13}C_{k+i-}=\text{coeff. of } x^{k+1} \text{ in } (1+x)^{25}={}^{25}C_{k+1}$$

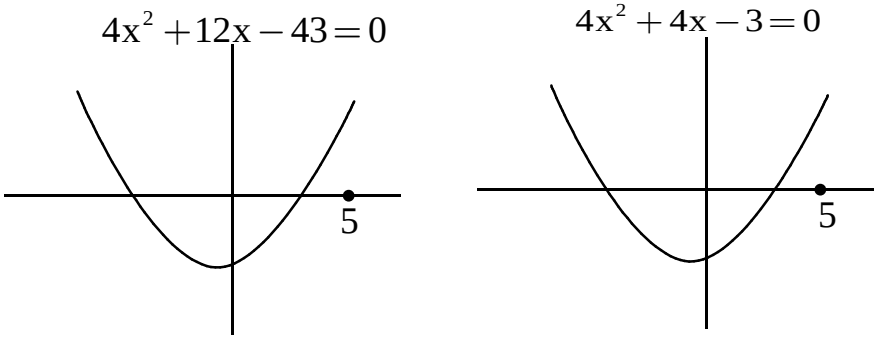
$$\Rightarrow {}^{25}C_{k}+{}^{25}C_{k+1}={}^{26}C_{k+1}$$

$\therefore$ Max. value of 'k' for which ${}^{26}C_{k+1}$ exists is k = 25

86. The number of the real roots of the equation $(x+1)^{2}+|x-5|=\frac{27}{4}$ is_____

Key: 2

Solution:



$$(x+1)^{2}+(x-5)=\frac{27}{4}\forall\ x\geq 5$$

$$(x+1)^{2}+(5-x)=\frac{27}{4}\forall\ x\leq 5$$

$$x^{2}+3x-4=\frac{27}{4}\forall x\geq 5\Rightarrow 4x^{2}+12x-43=0$$

$$x^{2}+x+6=\frac{27}{4}\forall x\leq 5\Rightarrow 4x^{2}+4x-3=0$$

$\Rightarrow$ no. of real roots is 2 from 2nd graph

87. Let a point P be such that its distance from the point $(5,0)$ is thrice the distance of P from the point $(-5,0)$. If the locus of the point P is a circle of radius r, then $4r^{2}$ is equal to _____

Key: 56.25

Solution:

$$3\sqrt{(x+5)^2+y^2}=(x-5)^2+y^2$$

$$\Rightarrow (x-5)^2+y^2=9(x+y)^2+9y^2$$

$$\Rightarrow 8x^2+8y^2+100x+200=0$$

$$x^2+y^2+\frac{25}{2}x+25=0$$

$$r=\sqrt{\frac{625}{16}-25}\Rightarrow 4r^2=\frac{225}{4}=56.25$$

88. Let λ be an integer. If the shortest distance between the lines $x-\lambda=2y-1=-2z$ and $x=y+2\lambda=z-\lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is _____

Key: 1

Solution:

$$\frac{x-\lambda}{2}=\frac{y-1/2}{1}=\frac{z}{-1}\quad \frac{x-0}{1}=\frac{y-(-2\lambda)}{1}=\frac{z-\lambda}{1}$$

Shortest distance = $\frac{\sqrt{7}}{2\sqrt{2}}$ then $|\lambda|=.....$

$A(\lambda, y_2, 0)\ B(0,-2\lambda,\lambda)$ $\qquad\qquad$ $\lambda\hat{i}+\left(\frac{1}{2}+2\lambda\right)\hat{j}-\lambda\hat{k}$

$\frac{\sqrt{7}}{2\sqrt{2}}=$ projection of $\overrightarrow{AB}$ on common normal

$$\frac{\sqrt{7}}{2\sqrt{2}}=\frac{\left|2\lambda-3\left(\frac{1}{2}+2\lambda\right)-\lambda\right|}{\sqrt{14}}$$

$$\Rightarrow\left|5\lambda+\frac{3}{2}\right|=\frac{7}{2}\Rightarrow 5\lambda=-5 \text{ or } 2$$

$\lambda=-1$ or $\frac{2}{5}$ $\qquad\qquad$ $\Rightarrow\lambda=-1$ as $\lambda\in z$ $\Rightarrow|\lambda|=1$

89. If $a+\alpha=1, b+\beta=2$ and $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}, x\neq 0$, then the value of the expression $\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is_____

Key: 2

Solution:

$a+\alpha \qquad\qquad b+\beta$

$$af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}$$

$$af\left(\frac{1}{x}\right)+\alpha f(x)=\frac{b}{x}+\beta x$$

Adding both

$$(a+\alpha)\left(f(x)+f\left(\frac{1}{x}\right)\right)=b\left(x+\frac{1}{x}\right)+\beta\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}=2$$

90. If the variance of 10 natural numbers 1,1,1,……1, k is less than 10, then the maximum possible value of k is _____

Key: 11

Solution:

Consider {1, 1, ……. 9 times} as one set and {k} as the combined variance of the 2 sets

$$=\frac{1}{n_1+n_2}\left[n_1\sigma_2^{\,2}+n_2\sigma_2^{\,2}+\frac{n_1n_2}{n_1+n_2}\left(\bar{X}_1-\bar{X}_2\right)\right]<10$$

put $n_1=9,\ n_2=1,\ \sigma_1=0,\ \sigma_2=0,\ \bar{X}_1=1,\ \bar{X}_2=k$

$$\Rightarrow (k-1)^2<\frac{1000}{9}\Rightarrow k-1<\frac{10\sqrt{10}}{3}$$

$$k<\left(\frac{10\sqrt{10}}{3}+1\right)\ \&\ K\in N$$

Max. value of k = 11