Class-XII

Mathematics(041)

Section - A

1.✗ b.✗ 4

2.✗ a.✗ 1

3.✗ c.✗ $I - A$

4.✗ ~~a.✗ ±1~~ c.✗ 1

5.✗ d.✗ 4

$$\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = \alpha(2-4) - 3(1-1) + 4(4-2)$$

$$= -2\alpha + 8 = 0$$

$$2\alpha = 8$$

$$\boxed{\alpha = 4}$$

6.✗ c.✗ $2x^{2x}(1 + \log x)$

$y = x^{2x} \Rightarrow \log y = 2x \log x \quad \frac{1}{y}\frac{dy}{dx} = 2(\log x + x \cdot \frac{1}{x})$

$= 2x^{2x}(\log x + 1)$

4

7.) b.) $x = 1.5$

8.) ~~D.)~~ d.) $-16x$

$$x = A\cos 4t + B\sin 4t$$

$$\frac{dx}{dt} = -4A\sin 4t + 4B\cos 4t$$

$$\frac{d^2x}{dt^2} = -16A\cos 4t - 16B\sin 4t$$

$$= -16(x)$$

9.) b.) $(-2, -1)$

$$f'(x) = 6x^2 + 18x + 12 = 0$$

$$6(x^2 + 3x + 2) = 0$$

$$6(x+2)(x+1) = 0$$

$+ \quad - \quad +$

$-2 \qquad -1$

$\therefore [-2, -1]$

$x = A\cos 4t + B\sin 4t$

$\frac{dx}{dt} = -4A$

$x^2 + 3x + 2$

$x^2 + 2x + x + 2$

$(x+2)(x+1)$

$\int \frac{\sec^2 x}{\cancel{\sec^2} 1 + \tan^2 x} dx$ $= \sec\tan$

$\int \frac{\frac{1}{\cos}}{\frac{1}{\cos} - \frac{\sin^2}{\cos}}$

$\int \frac{1}{1-\sin x} \cdot \frac{1+\sin}{1+\sin x}$

$\int \frac{1+\sin x}{\cos^2 x}$

$\int \sec^2 x + \int \sec x \tan x$

$= \tan x + \sec x + c$

10.ꟙ b.ꟙ $\sec x + \tan x + c$

11.ꟙ d.ꟙ -2

12.ꟙ b.ꟙ 3

$3\left(\frac{dy}{dx}\right)^2 \left(\frac{d^2y}{dx^2}\right)$

13.ꟙ b.ꟙ $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

14.ꟙ c.ꟙ 7

15.ꟙ a.ꟙ $0, \frac{-1}{\sqrt{2}}, \frac{1}{\cancel{4}}$

16.ꟙ d.ꟙ 90°

$-(x-2) \cdot -\frac{(x-2)}{x-2}$

$\int_{-1}^{1} -1 \, dx$

$-\int_{-1}^{1} 1 \, dx = -[x]_{-1}^{1}$

$= -[1-(-1)]$

$= -[1+1] = -2$

$\sqrt{36+4+9} = 7$

$\frac{x}{\frac{1}{2}} = \frac{y}{\frac{1}{3}} = \frac{z}{-1}$

$\frac{x}{\frac{1}{6}} = \frac{y}{-1} = \frac{z}{-4}$

$= \frac{1}{2} \cdot \frac{1}{6} - \frac{1}{3} + \frac{4 \cdot 1}{4}$

$= \frac{1}{12} - \frac{1}{3} + \frac{1}{4}$

$= \frac{1 - 4 + 3}{12} = 44+1$

$= \frac{45}{12}$

17.) c.) $\frac{7}{8}$

18.) c.) $\frac{31}{32}$

19.) a.) Both are true and R is correct explanation of A

20.) d.) A is false but R is true.

20.) a.) Both ~~a~~ A & R ~~are~~ and R is correct explanation of A

Section - B

21. $f(x) = \tan^{-1} x$

Domain $= R$ {i.e. $(-\infty, \infty)$}, where, R = set of real numbers

Range $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

22. b. LHL

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin^2 \lambda x}{x^2} = \lim_{h \to 0} \frac{\sin^2 \lambda (0-h)}{(0-h)^2}$$

$$= \lim_{h \to 0} \frac{\sin^2 \lambda h}{h^2} \times \frac{\lambda^2}{\lambda^2} = \lambda^2 \lim_{\lambda h \to 0} \left(\frac{\sin \lambda h}{\lambda h}\right)^2$$

$$= \lambda^2$$

RHL

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sin^2 \lambda x}{x^2} = \lim_{h\to 0} \frac{\sin^2 \lambda(0+h)}{(0+h)^2}$$

$$= \lim_{h\to 0} \frac{\sin^2 \lambda h}{h^2} \times \frac{\lambda^2}{\lambda^2} = \lambda^2 \lim_{h\to 0} \left(\frac{\sin \lambda h}{\lambda h}\right)^2$$

$$= \lambda^2$$

Now,

Put $x = 0$, in $f(x) = 1$

$f(0) = 1$

As, $f(x)$ is continuous at $x = 0$

$\Rightarrow$ LHL = RHL = $f(0)$

$\Rightarrow \lambda^2 = 1$

$\Rightarrow \boxed{\lambda = \pm 1}$ ans $\lambda = \pm 1$

23.4) $\frac{2x}{8} + \frac{y}{8} = \frac{8}{8}$ $\quad y = 2 \quad y = 4$

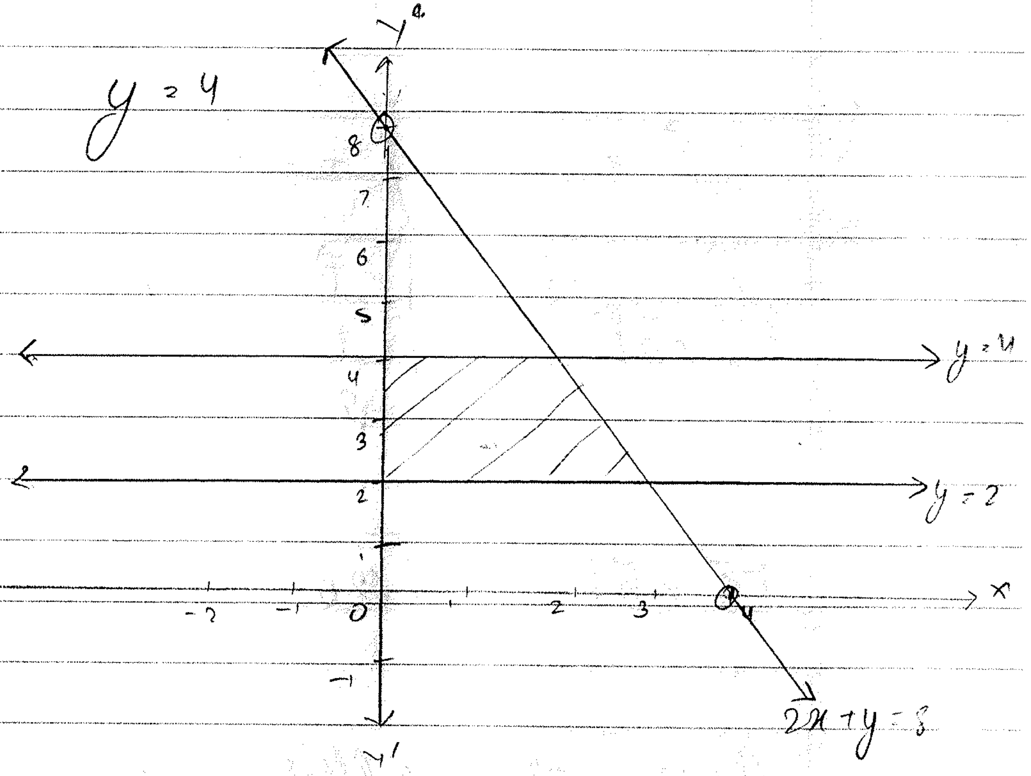
$\frac{x}{4} + \frac{y}{8} = 1$

~~Point~~ of intersection

$x = \frac{1}{2}(8 - y)$

$2x + 2 = 8$

~~x =~~



Shaded area $= \int_2^4 \frac{1}{2}(8 - y)\,dy$

$= \frac{1}{2}\left[8y - \frac{y^2}{2}\right]_2^4 = \frac{1}{2}\left[\cancel{32 - 8} - (16 - 2)\right]$

$= \frac{1}{2}\left[24 - 14\right] = 5$ sq units

ans ~~5~~ sq. unit

24. b.

$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ $\qquad\qquad$ $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = \hat{i}(-1+21) - \hat{j}(1-6) + \hat{k}(-7+2)$$

$$= 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(20)^2 + (5)^2 + (-5)^2} = \sqrt{400+25+25} = \sqrt{450}$$

$$= 15\sqrt{2}$$

Area of parallelogram $= |\vec{a} \times \vec{b}| = 15\sqrt{2}$ sq. units

25. A$(1, 2, -1)$

$$5x - 25 = 14 - 7y = 35z$$

$$\frac{x-5}{\left(\frac{1}{5}\right)} = \frac{y-2}{\left(\frac{-1}{7}\right)} = \frac{z}{\left(\frac{1}{35}\right)}$$

Required line is

$$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda\left(\frac{1}{5}\hat{i} - \frac{1}{7}\hat{j} + \frac{1}{35}\hat{k}\right)$$

Or $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + 1\hat{k})$

Cartesian equation:

$$\frac{x-1}{7} = \frac{y-2}{-5} = \frac{z+1}{1}$$

Section - C

26(b)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$^2 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+6+12 & 2-4+6 & 3+2+3 \\ 3-6+4 & 6+4+2 & 9-2+1 \\ 4+6+4 & 8-4+2 & 12+2+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$A^3 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix} = \begin{bmatrix} 19+2+42 & 4+24+18 & 8+16+45 \\ 57-2+14 & 12-24+6 & 24-16+15 \\ 76+2+14 & 16+24+6 & 32+16+15 \end{bmatrix}$$

$$\cancel{A^3 = \begin{bmatrix} 63 & 46 & 69 \\ 69 & 0 & 23 \\ 92 & 54 & 63 \end{bmatrix}} \quad A^3 = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

LHS

$$= A^3 - 23A - 40I = \begin{bmatrix} 63 & 48 & 69 \\ 69 & 0 & 23 \\ 92 & 54 & 63 \end{bmatrix} - 23\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 48 & 69 \\ 69 & 0 & 23 \\ 92 & 54 & 63 \end{bmatrix} - \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix}$$

LHS

$$= A^3 - 23A - 40I = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix} - \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} 63-23-40 & 46-46 & 69-69 \\ 69-69 & -6+46-40 & 23-23 \\ 92-92 & 46-46 & 63-23-40 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= O =$ RHS

Hence proved

~~27.b~~

~~Q.2~~

$\cancel{y = \tan x + \sec x}$

$\frac{dy}{dx} = \sec^2 x + \sec x \tan x = \sec x(\sec x + \tan x)$

$\frac{d^2y}{dx^2} = \sec x \tan x (\sec x + \tan x) + \sec x(\sec x \tan x + \sec^2 x)$

$= \sec^2 x \tan x + \sec x \tan^2 x + \sec^2 x \tan x + \sec^3 x$

27.b Q.2

Let, $y = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

Let, $x = \sin\theta$

$y = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$

$y = \sec^{-1}(\sec\theta)$

$t = \sin^{-1}(2x\sqrt{1-x^2})$

Let $x = \sin\theta$

$t = \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$t = \sin^{-1}(\sin 2\theta)$

$y = \theta$ $\qquad$ $t = 2\theta$

$y = \sin^{-1} x$ $\qquad$ $t = 2\sin^{-1} x$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{dt}{dx} = \frac{2}{\sqrt{1-x^2}}$

$$\frac{dy}{dt} = \frac{dy/dx}{dt/dx} = \frac{1}{\sqrt{1-x^2}} \cdot \frac{\sqrt{1-x^2}}{2} = \frac{1}{2}$$

ans $\frac{1}{2}$

28 (b) (ii)

$$I = \int_0^{2\pi} \frac{1}{1+e^{\sin x}}\,dx \quad -(1)$$

$$I = \int_0^{2\pi} \frac{1}{1+e^{\sin(2\pi-x)}}\,dx \qquad \left\{ \because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx \right\}$$

$$I = \int_0^{2\pi} \frac{1}{1+e^{-\sin x}}\,dx = \int_0^{2\pi} \frac{e^{\sin x}}{1+e^{\sin x}}\,dx \quad \text{— (ii)}$$

(i) + (ii)

$$2I = \int_0^{2\pi} \frac{\cancel{1+e^{\sin x}}}{\cancel{1+e^{\sin x}}}\,dx = \int_0^{2\pi} 1\,dx$$

$$2I = [x]_0^{2\pi}$$

$$2I = 2\pi$$

$$I = \pi$$

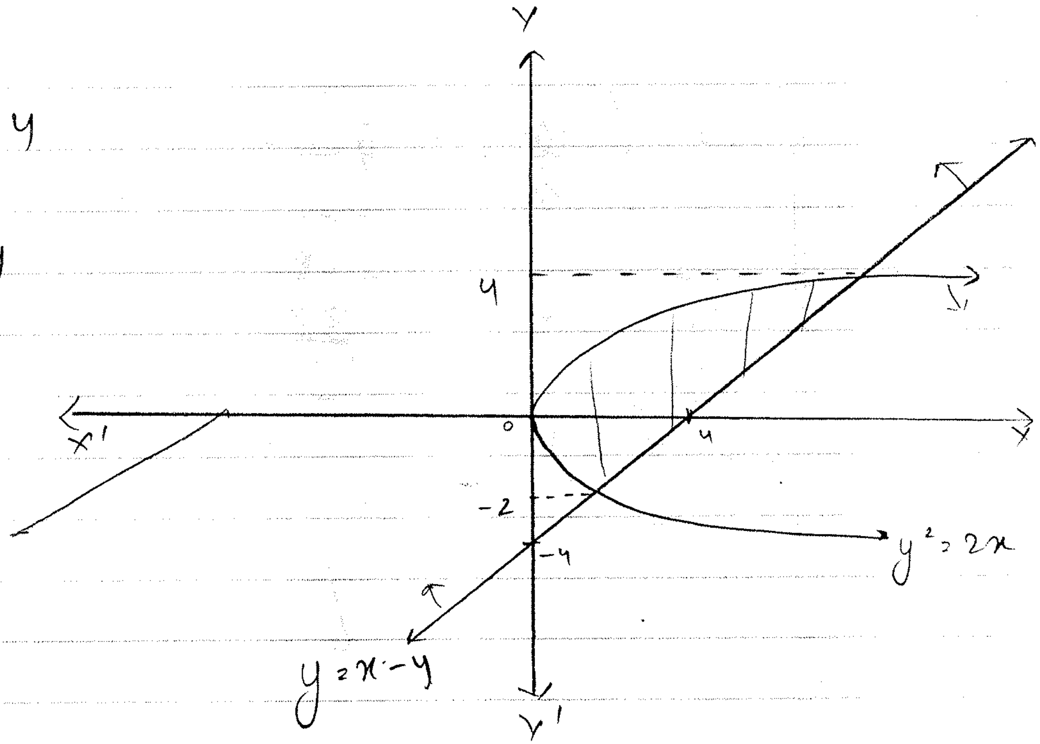
ans π

29.4 $y^2 \le 2x$ $\quad y \geq x - 4$

$y^2 = 2x$ $\quad y = x - 4$

$x - y = 4$

$\frac{x}{4} + \frac{y}{(-4)} = 1$



Point of intersection

$y^2 = 2x$ $\quad y = x - 4$

$\Rightarrow x = y + 4$

$y^2 = 2(y+4)$

$y^2 = 2y + 8 \Rightarrow y^2 - 2y - 8 = 0$

$\Rightarrow (y-4)(y+2) = 0$

$y = -2, 4$

Required area $= \int_{-2}^{4} (y+4)\,dy - \int_{-2}^{4} \frac{y^2}{2}\,dy$

$$= \left[\frac{y^2}{2} - 4y - \frac{y^3}{6} \right]_{-2}^{4}$$

$$= \left[8 + 16 - \frac{32}{3} - \left(2 - 8 + \frac{4}{3} \right) \right]$$

$$= \left[24 - \frac{32}{3} + 6 - \frac{4}{3} \right] = \left[30 - \frac{36}{3} \right]$$

$$= \left[30 - 12 \right] = 18 \text{ sq unit}$$

<u>ans</u> 18 sq unit

30. 6. Given, $|\vec{a}| = 3$ $|\vec{b}| = 4$ $|\vec{c}| = 2$

$(\vec{a} + \vec{b} + \vec{c}) = \vec{0}$

squaring,

$(\vec{a} + \vec{b} + \vec{c})^2 = 0$

$\Rightarrow (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{a} + \vec{c}\cdot\vec{b} + \vec{c}\cdot\vec{a} = 0$

$\vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{a}.\vec{c} = 0$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$9 + 16 + 4 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$2\mu = -29$

$\mu = \frac{-29}{2}$ ans $\frac{-29}{2}$

31.4 Given lines are parallel lines

$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$ $\vec{b} = (2\hat{i} + 3\hat{j} + 6\hat{k})$

$\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$d = \left|\frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|}\right|$

Ans $\vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$

$|\vec{b}| = \sqrt{4+9+36} = 7$

196
81
16
$7 \overline{)293(}$
28
13

distance between lines,

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

$\cancel{d =}$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$$= \hat{i}(6+3) - \hat{j}(12+2) + \hat{k}(6-2)$$

$$= 9\hat{i} - 14\hat{j} + 4\hat{k}$$

$$|(\vec{a}_2 - \vec{a}_1) \times \vec{b}| = \sqrt{81+196+16} = \sqrt{293}$$

$$d = \frac{\sqrt{293}}{7} \text{ unit}$$

Section - D

32) b) Let numbers be x & y

ATQ,

$x + y = 5 \Rightarrow y = (5 - x)$

then, sum of cubes,

Let, $C = x^3 + y^3$

minimise → $C = x^3 + (5-x)^3$

$$\frac{dC}{dx} = 3x^2 + 3(5-x)^2(-1)$$

$$= 3x^2 - 3(5-x)^2$$

$$= 3x^2 - 3(25 + x^2 - 10x)$$

$$\frac{dC}{dx} = 3x^2 - 75 - 3x^2 + 30x \qquad \frac{75}{30} = \frac{15}{6} = \frac{5}{2}$$

$$\frac{dC}{dx} = 30x - 75$$

Put $\frac{dC}{dx} = 0$

$$30x - 75 = 0$$

$$x = \frac{75}{30} = \frac{5}{2}$$

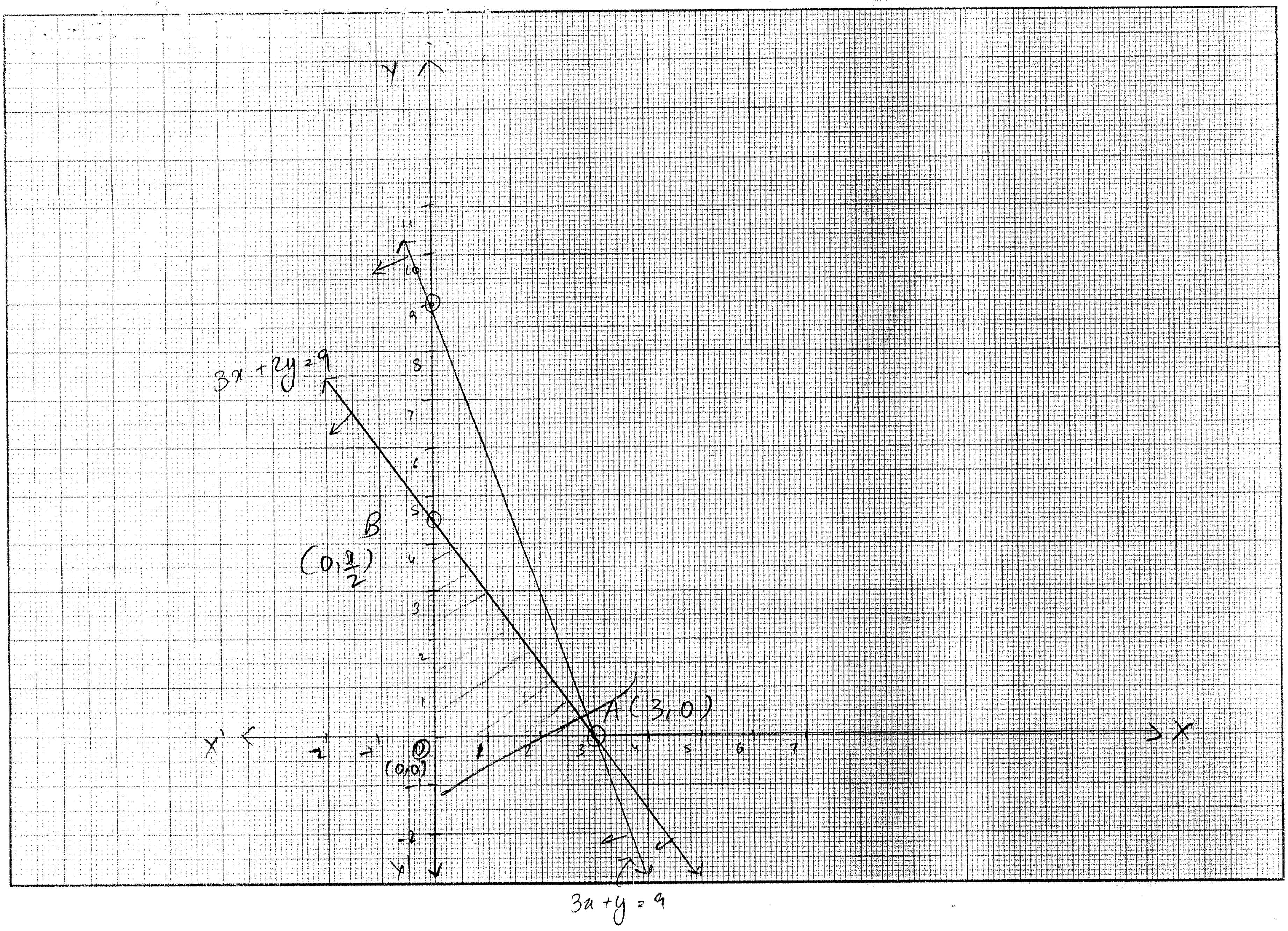
$$x = \frac{5}{2}$$

$$\frac{d^2C}{dx^2} = 30 \;\therefore\; \left.\frac{d^2C}{dx^2}\right|_{x=\frac{5}{2}} = 30 > 0$$

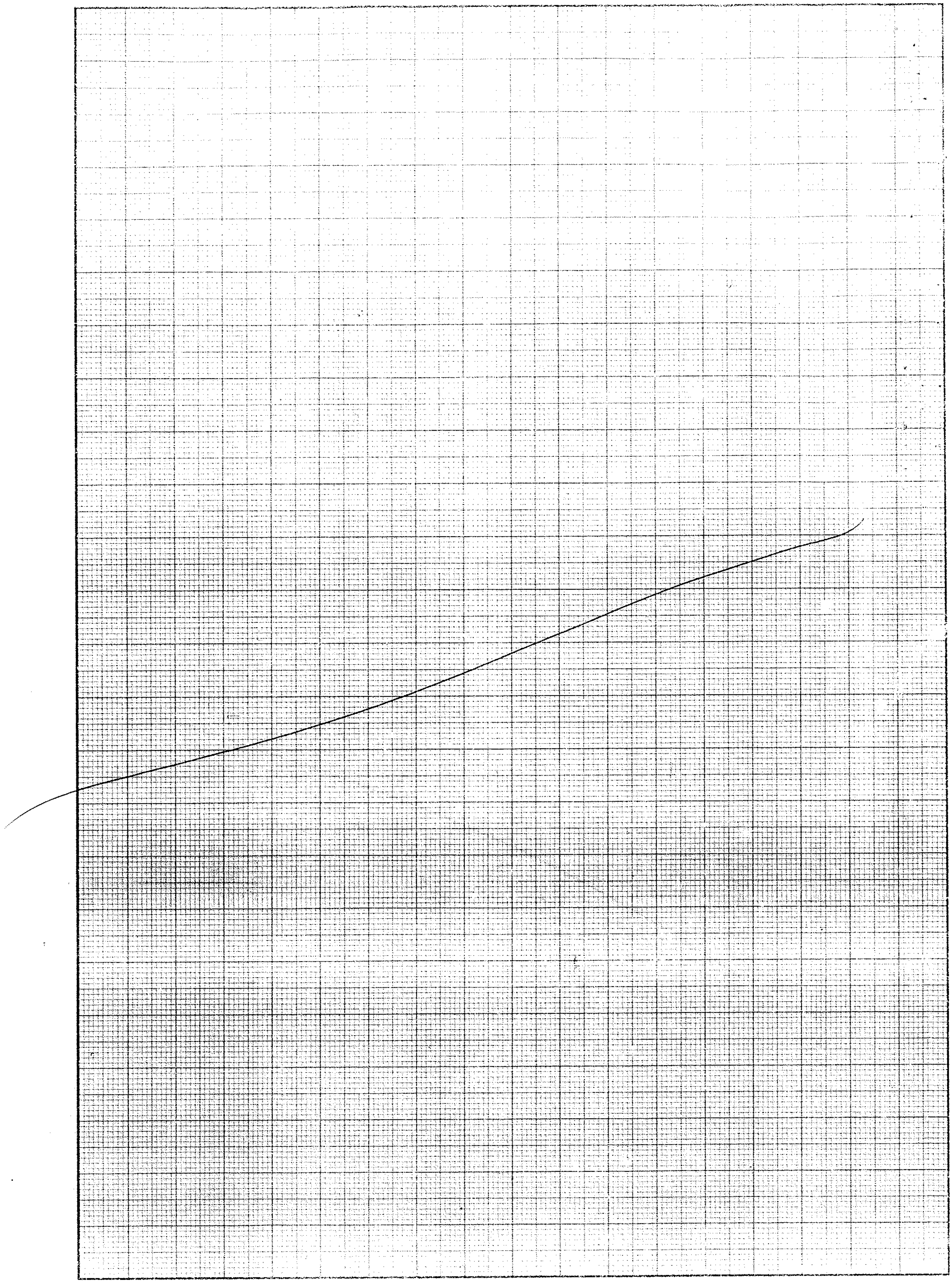
$\therefore$ Sum of cubes is least when $x = \frac{5}{2}$ & $y = \frac{5}{2}$

$\therefore$ Sum of squares $= x^2 + y^2 = \frac{25}{4} + \frac{25}{4} = \frac{25}{2}$

ans $\frac{25}{2}$



Y
X'
X
Y'
3x+2y=9
B
(0, 9/2)
O
(0,0)
A(3,0)
3x+y=9



33.

$$I = \int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x)\, dx = \int_0^{\pi/2} 2 \sin x \cos x \tan^{-1}(\sin x)\, dx$$

let, $\sin x = t \Rightarrow dt = \cos x\, dx$

$$I = 2\int_0^1 t \tan^{-1}(t)\, dt$$

let $I_1 = \int \underset{\text{II}}{t} \, \underset{\text{I}}{\tan^{-1} t}\, dt$

$$I_1 = \tan^{-1} t \cdot \frac{t^2}{2} - \frac{1}{2}\int \frac{t^2+1-1}{1+t^2}\, dt = \frac{t^2}{2}\tan^{-1} t - \frac{1}{2}\int\left(1 - \frac{1}{1+t^2}\right) dt$$

$$I_1 = \frac{t^2}{2}\tan^{-1} t - \frac{t}{2} + \frac{1}{2}\tan^{-1} t + c.$$

$$I = 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{t}{2} + \frac{1}{2}\tan^{-1} t\right]_0^1$$

$$= 2\left[\frac{1}{2}\tan^{-1}(1) - \frac{1}{2} + \frac{1}{2}\tan^{-1}(1) - \left(0 - 0 + \frac{1}{2}\tan^{-1} 0\right)\right]$$

$$I = 2\left(\frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}\right) = 2\left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$I = \frac{\pi}{2} - 1 \qquad \text{ans } \frac{\pi - 2}{2} = \cancel{\frac{1}{2}(\pi - 2)}$$

34.

$$P = 70x + 40y$$

Subject to constraints

$3x + 2y \le 9$ $\quad$ $3x + y \le 9$ $\quad$ $x \ge 0$ $\quad$ $y \ge 0$

$3x + 2y = 9$ $\quad$ $3x + y = 9$

$\frac{x}{3} + \frac{y}{\left(\frac{9}{2}\right)} = 1$ $\quad$ $\frac{x}{3} + \frac{y}{9} = 1$

Corner points are $O(0,0)$, $A(3,0)$, $B\left(0, \frac{9}{2}\right)$

Put $O(0,0)$ in $P = 70(0) + 40(0) = 0$
Put $A(3,0)$ in P : $P = 210$ ← Maximum
Put $B(0,\frac{90}{2})$ in P : $P = 180$

∴ Maximum value of P is 210 at $(3,0)$

35. a) Let,

E : He answered correctly

H_1 : He knows the answer

H_2 : He guesses the answer

$P(H_1) = \frac{3}{5}$ $P(H_2) = \frac{2}{5}$ $P(E|H_1) = 1$ $P(E|H_2) = \frac{1}{3}$

From Bayes Theorem

$$P(H_1|E) = \frac{P(H_1)\,P(E|H_1)}{P(H_1)\,P(E|H_1) + P(H_2)\,P(E|H_2)}$$

$$P(H_1/E) = \frac{\frac{3}{5} \times 1}{\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{1}{3}} = \frac{3}{3 + \frac{2}{3}}$$

$$= \frac{3}{\frac{9+2}{3}}$$

$$P(H_1/E) = \frac{9}{11}$$

ans $\frac{9}{11}$

Section - E

2 T.x Total possible relations from B to G $= 2^{3 \times 2} = 2^6 = 64$

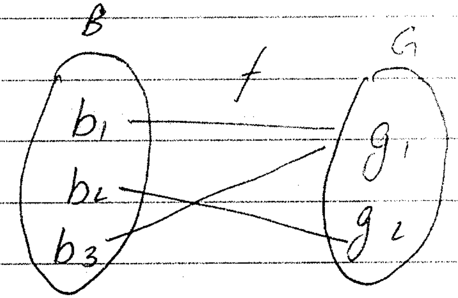
Total functions from B to G $= 2^3 = 8$

XIX

III) 2nd option

$$f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$$

It is not bijective as



$f(b_1) = g_1$ & $f(b_3) = g_1$

As $f(b_1) = f(b_3)$

though $b_1 \neq b_3$

∴ It is not one one as ~~for~~ b_1 & b_3 are related to same element g_1

As it is not one one hence it is not bijective

37.) 1.4. Let, cost of : 1 pen = ₹ x

1 bag = ₹ y

1 instrument box = ₹ z

then, ATQ,

$$\cancel{3x+}\ 5x + 3y + z = 160$$

$$2x + y + 3z = 190$$

$$x + 2y + 4z = 250$$

Given system of equations can be written as,

$$\underbrace{\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}}_{B}$$

$$AX = B$$

$$A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$$

ii) $|A| = 5(4-6) - 3(8-3) + 1(4-1)$

$= -10 - 15 + 3$

$|A| = -22 \neq 0$ $\therefore A^{-1}$ exists

iii) $A_{11} = -2$ $\quad A_{12} = -5$ $\quad A_{13} = 3$

$A_{21} = -10$ $\quad A_{22} = 19$ $\quad A_{23} = -7$

$A_{31} = 8$ $\quad A_{32} = -13$ $\quad A_{33} = -1$

$$\operatorname{adj}(A) = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}^T = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A) = -\frac{1}{22} \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{22} \begin{bmatrix} 2 & 10 & -8 \\ 5 & -19 & 13 \\ -3 & 7 & 1 \end{bmatrix}$$

38.) I.

$$(x^2 - y^2)\,dx + 2xy\,dy = 0$$

$$2xy\,dy = -(x^2 - y^2)\,dx$$

$$\frac{dy}{dx} = -\frac{(x^2 - y^2)}{2xy}$$ ÷ Numerator & Denominator by x^2

$$\frac{dy}{dx} = \frac{-\left(1 - \left(\frac{y}{x}\right)^2\right)}{2\left(\frac{y}{x}\right)}$$

$$\frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2 - 1}{2\left(\frac{y}{x}\right)} = g\left(\frac{y}{x}\right)$$

II ⅰ) Let $v = \frac{y}{x} \Rightarrow y = xv$

$$\frac{dy}{dx} = x\frac{dv}{dx} + v$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v^2-1}{2v}$$

$$x\frac{dv}{dx} = \frac{v^2-1}{2v} - v = \frac{v^2-1-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{-1-v^2}{2v} = -\frac{(v^2+1)}{2v}$$

$$\frac{2v}{v^2+1}\,dv = -\frac{1}{x}\,dx$$

Integrating both sides,

$$\log|v^2+1| = -\log|x| + \log|c|$$

$$\log|v^2+1| = \log\left|\frac{c}{x}\right|$$

$$v^2 + 1 = \frac{c}{x}$$

$$\frac{y^2}{x^2} + 1 = \frac{c}{x}$$

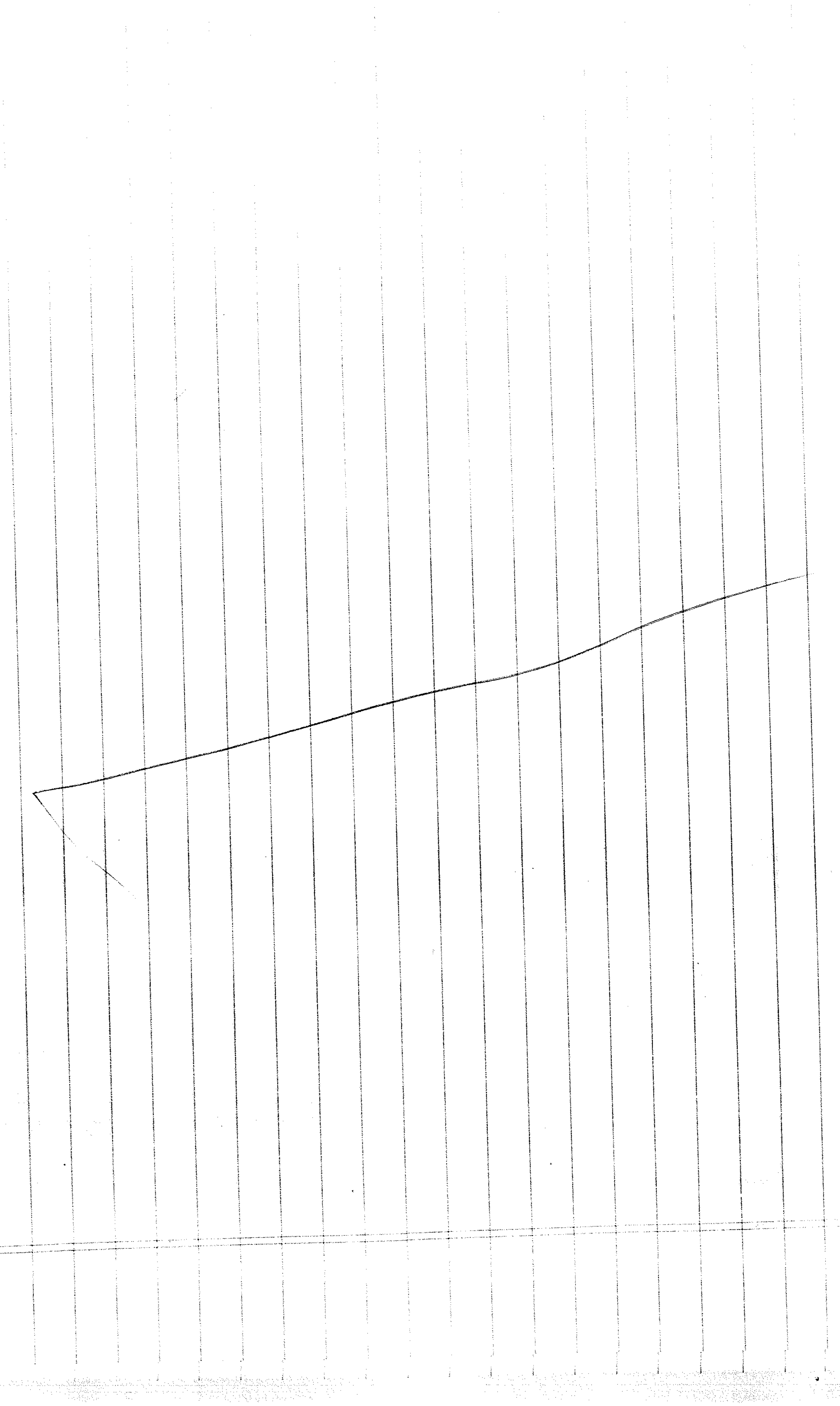
$$\frac{y^2}{x} + x = c$$

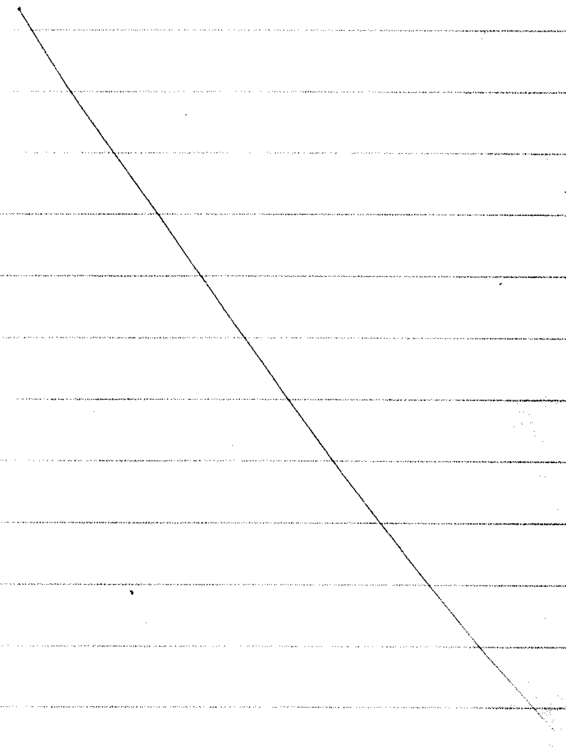
(6²)

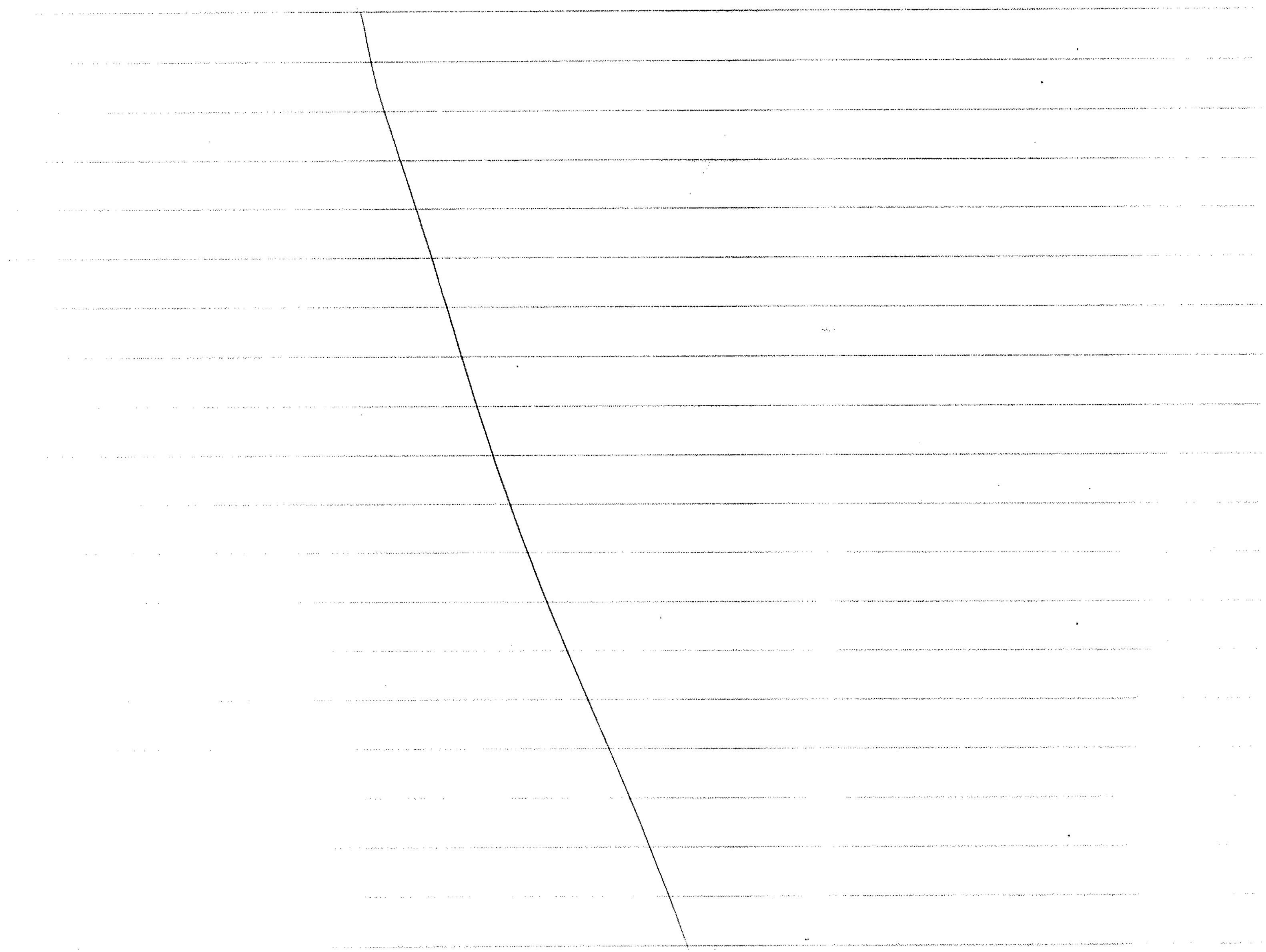
7265599

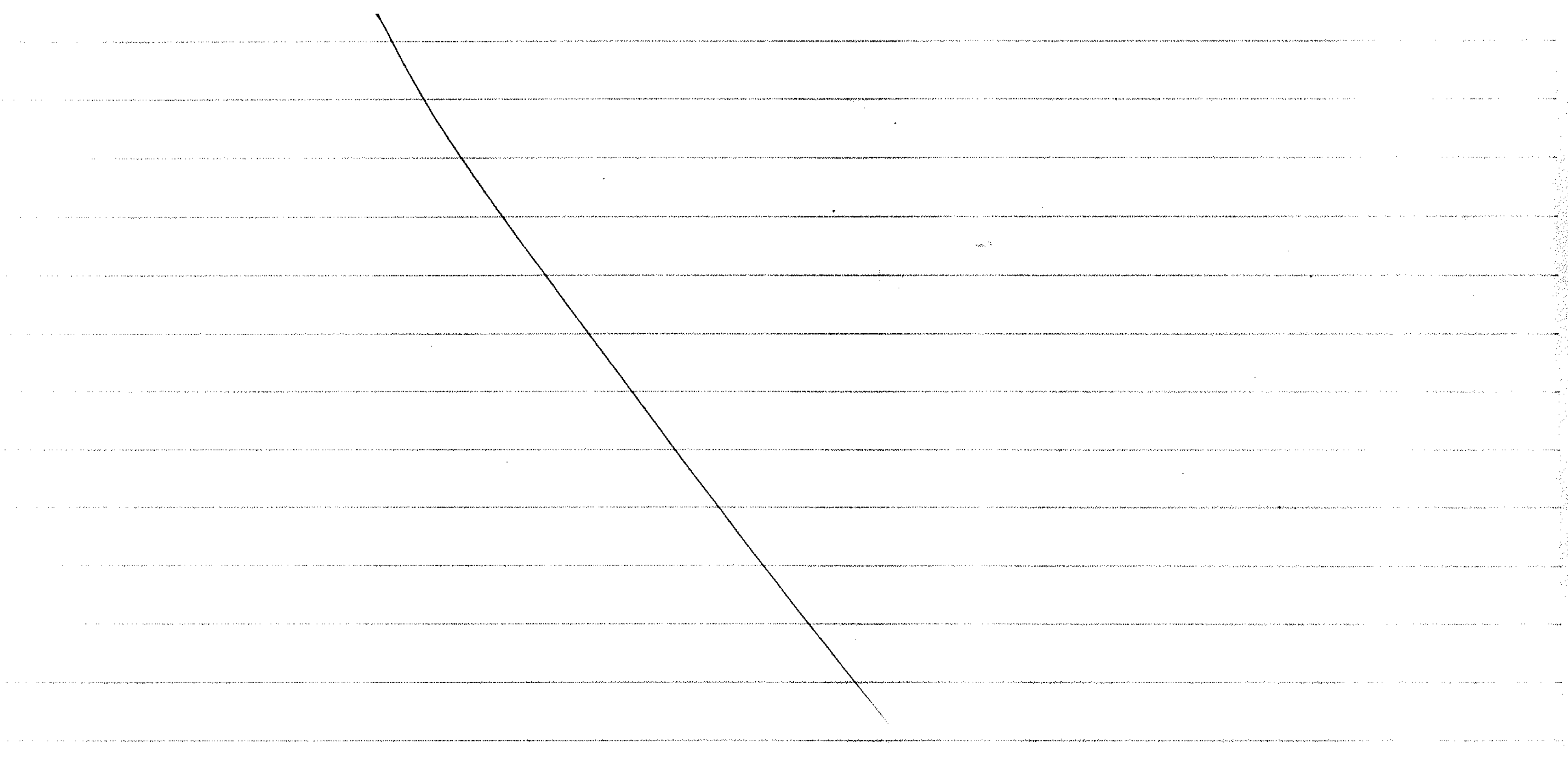
444855

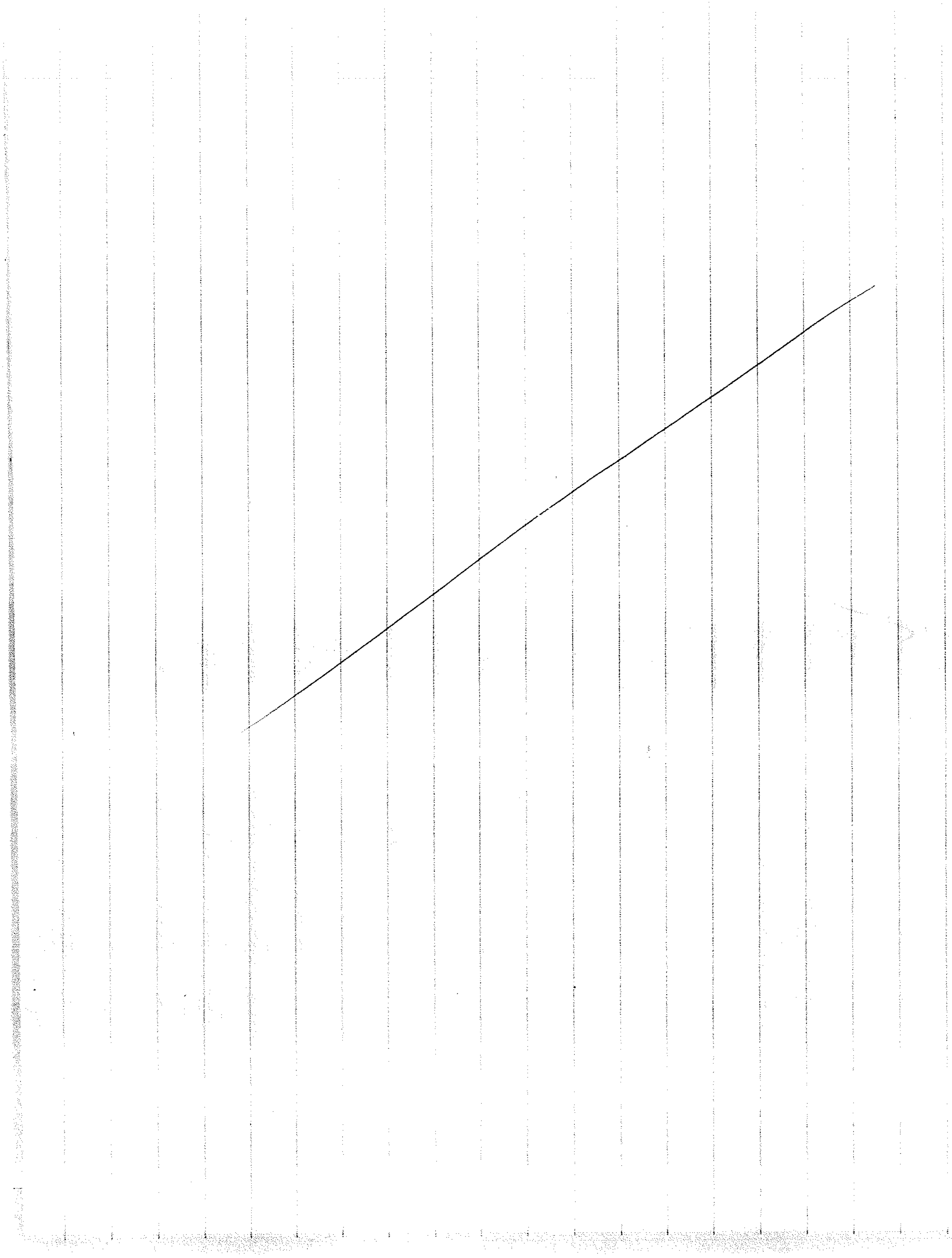
1382031

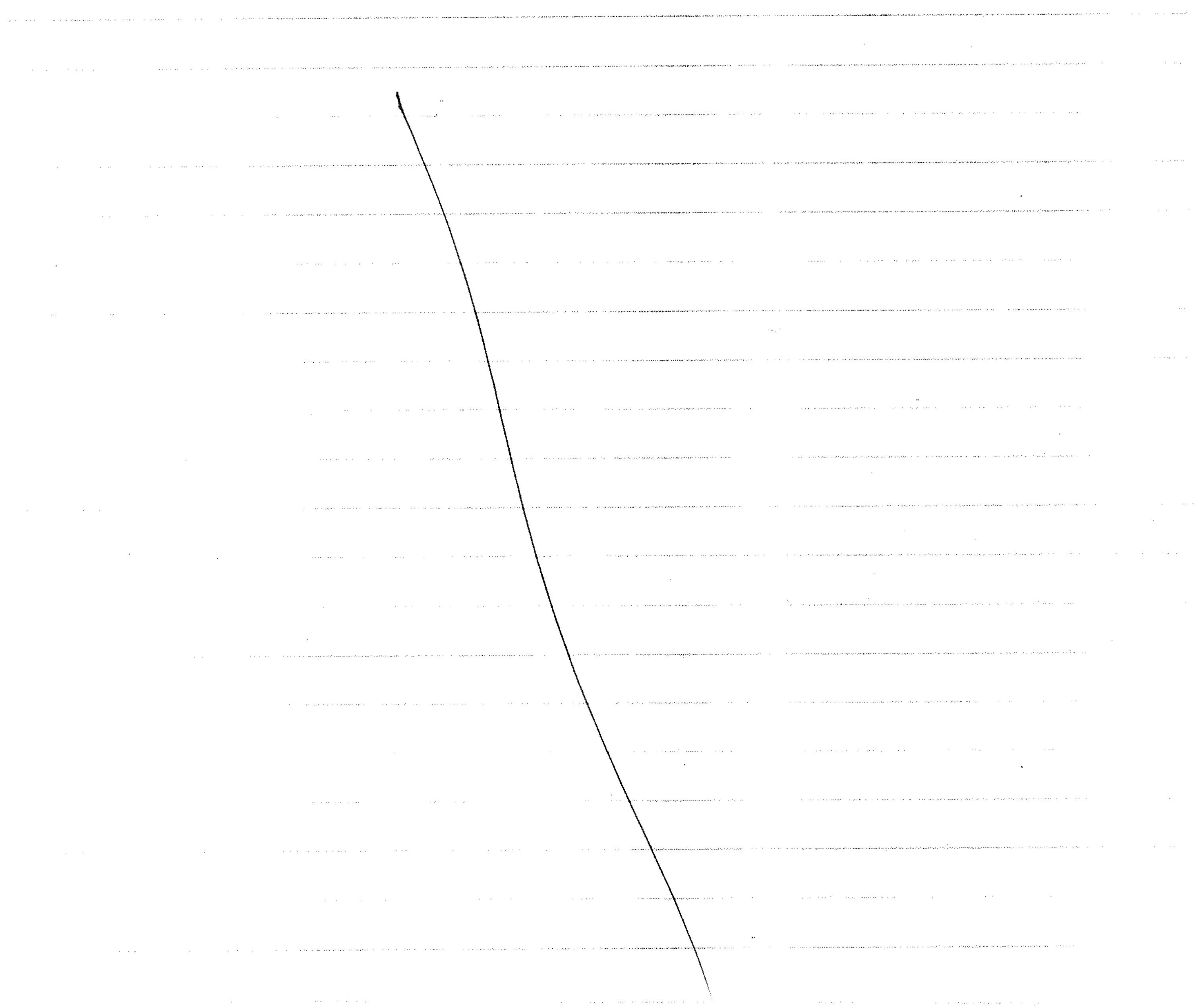


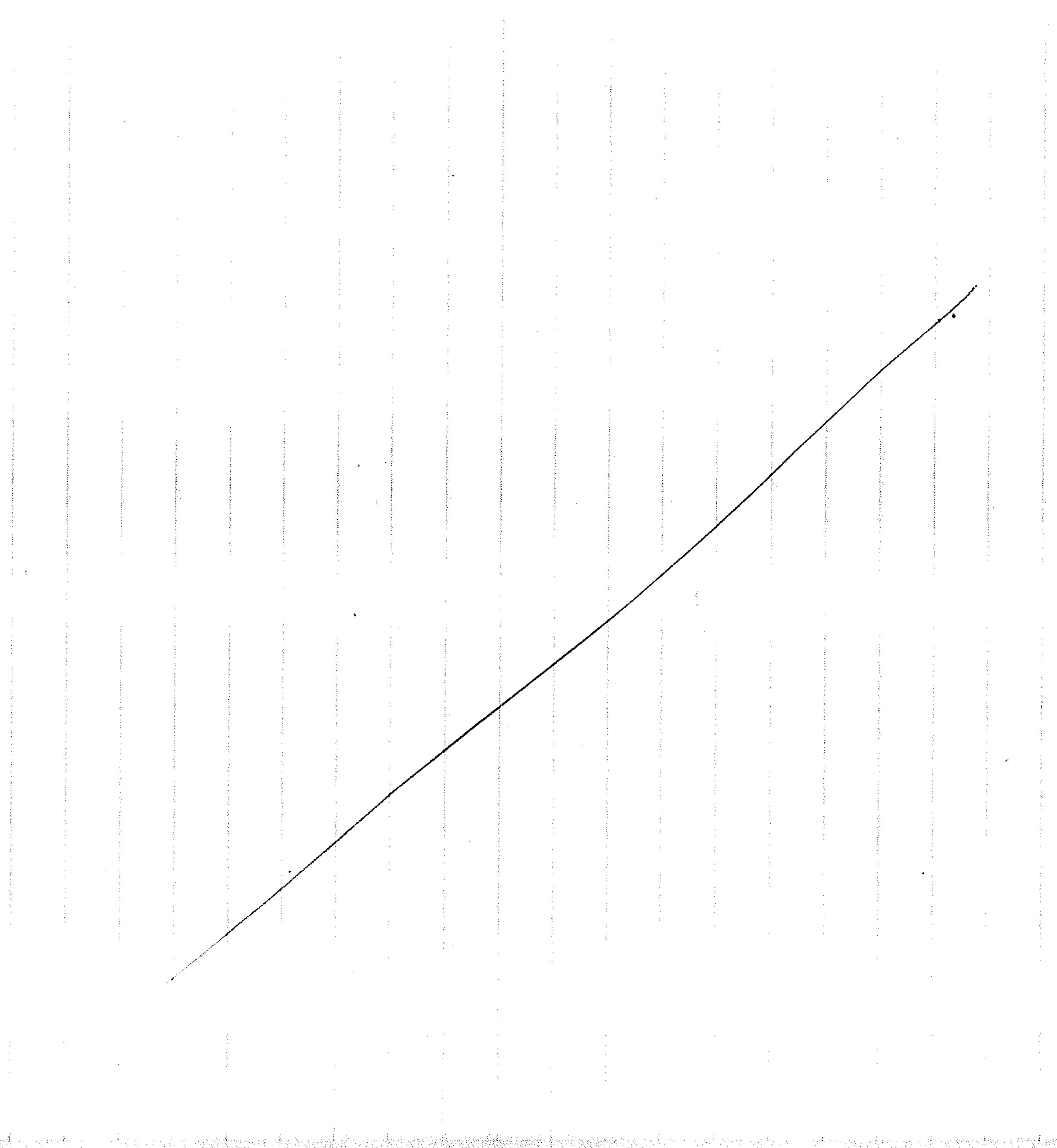


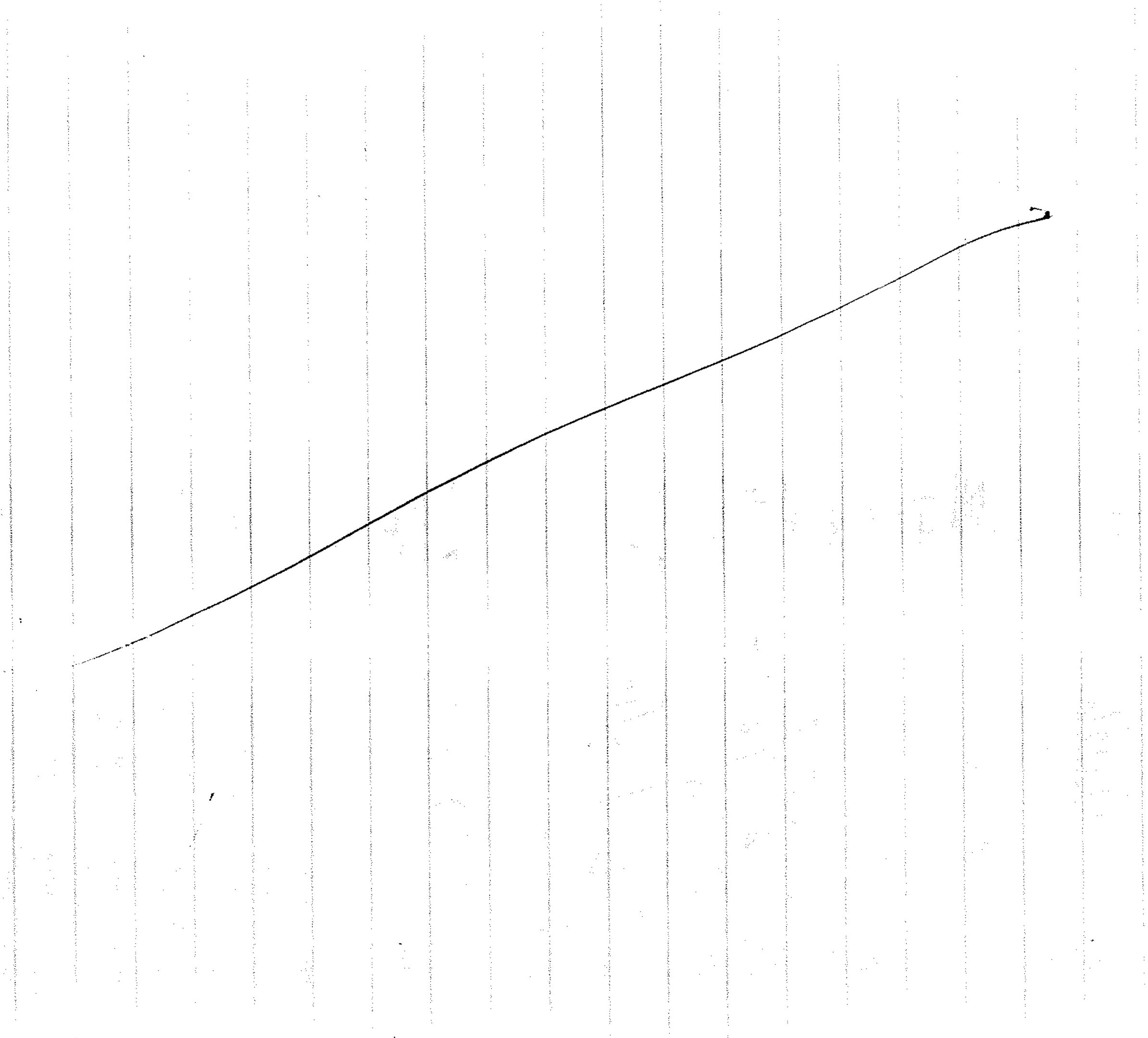












2023

Rough

$$\frac{x}{\frac{1}{2}} = \frac{y}{\frac{1}{3}} = \frac{z}{-1} \qquad \frac{x}{\frac{1}{6}} = \frac{y}{-1} = \frac{z}{-\frac{1}{4}}$$

$$= \cancel{\tfrac{1}{2}}\ \frac{1}{12} - \frac{1}{3} + \frac{1}{4} = \frac{1-4+3}{12} = 0$$

$$I = \int_2^8 \sqrt{10-(10-x)} = \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x}+\sqrt{x}}\,dx$$

$$2I = \int \frac{\sqrt{x}+\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} = \int_2^8 1\,dx = 8-2 = \frac{6}{2} = 3$$

$\tan^{-1}(\sin x)\left(-\frac{\cos 2x}{2}\right)$

$+\int \frac{\cos x}{1+\sin^2 x} \cdot \frac{\cos 2x}{2}$

$2 \sin x \cos x \tan^{-1}(\sin x)$

$2\int_0^1 \underset{II}{t} \underset{I}{\tan^{-1} t}\ dt$

$\tan^{-1} t\left(\frac{t^2}{2}\right)$

$-\frac{1}{2}\int \frac{1}{1+t^2} + t^2\ dt$