EXERCISE - 3: ADVANCED OBJECTIVE QUESTIONS

Single Choice Questions

1. Two communicating vessels contain mercury. The diameter of one vessel is four times the diameter of the other. A column of water of height h_0 = 70 cm is poured into the left hand vessel (the narrower one). How much will be mercury level rise in the right hand vessel? (Specific density of mercury = 13.6)

(a) 0.3 cm (b) 0.7 cm

(c) 0.1 cm (d) 1.0 cm

Ans. (a)

Sol. If the level in narrow tube goes down by h_1 and then in wider tube goes up by h_2.

Now

$$\pi r_1^2 h_1 = \pi r_2^2 h_2$$

$$\Rightarrow h_1 = \left(\frac{r_2}{r_1}\right)^2 h_2$$

$r_2 = nr_1$

$\Rightarrow h_1 = n^2 h_2$.

Now pressure at point A = Pressure at point B

$$h\rho g = (h_1 + h_2)\rho' g$$

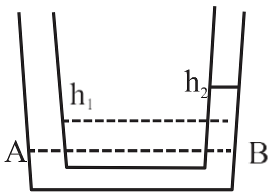
$$h = (h_1 + h_2)\frac{\rho'}{\rho}$$

$$h = (n^2 h_2 + h_2)s$$

$$h = h_2(n^2 + 1)s$$

$$h_2 = \frac{h}{(n^2+1)s}$$

$$\text{where } s = \frac{\rho'}{\rho}$$



$$h_2 = \frac{70(\text{in cm})}{(4^2+1)\times 13.6}$$

$$\Rightarrow h_2 = 0.3cm$$

2. A U-tube is partially filled with water. Oil, which does not mix with water, is next poured into one side until water rises by 25 cm on the other side. If the density of oil be 0.8, the oil level will stand higher than the water level by:

(a) 6.25 cm (b) 12.50 cm

(c) 31.25 cm (d) 20 cm

Ans. (b)

Sol. $\rho_{oil} = 0.8$

$\rho_{water} = 1$

When water goes down by x amount in one side of U tube, then it moves up by x amount in other tube.

Thus, the total difference in height of water in both side is equal to 2*x*.

Difference in hight = 2 × 25 cm = 50cm

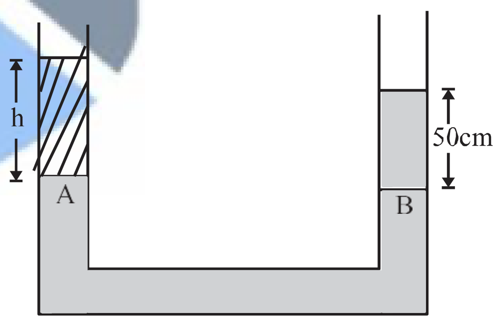
As $P_A = P_B$

$$P_0 + \rho_{oil}gh = P_0 + \rho_{water}g(50)$$

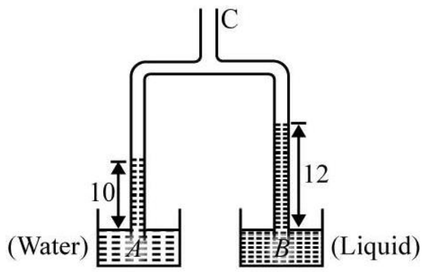
0.8 gh = g (50)

h = 62.5 cm

Thus, the oil stands higher than water by amount Δh = h - 50 = 62.5 - 50 = 12.5 cm.



3. The limbs of a glass U-tube are lowered into vessels A and B, A containing water. Some air is pumped out through the top of the tube C. The liquid in the left hand limb A and the right hand limb B rise to heights of 10 cm and 12 cm respectively. The density of liquid B is:



(a) 0.75 g/cm^3 (b) 0.83 g/cm^3

(c) 1.2 g/cm^3 (d) 0.25 g/cm^3

Ans. (b)

Sol. S → Density of liquid

Given,

$P_1 = P_0 - S_A\ g(10)$

$P_2 = P_0 - S_B\ g(12)$

$P_1 = P_2$

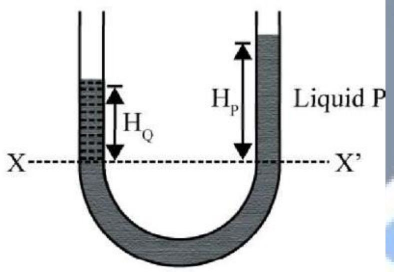
$P_0 - S_A g\ (10) = P_0 - S_B\ g(12)$

$10S_A = 12S_B$

$10 \times 1 = 12\ S_B$

$S_B = 0.83\ g/cm^3$

4. Two immiscible liquids P and Q of different densities are contained in a wide U-tube as shown in fig. The heights of the two liquids above the horizontal line XX` which cuts the boundary between the liquids are $H_P$ and $H_Q$ respectively. The U-tube is transported to a planet where the acceleration of free fall is 2/3 that on the earth, where the liquids do not evaporate and where the heights of liquid (measured relative to XX`) are h_p and h_Q respectively. Which of the given statements is correct?



(a) The liquid levels are unchanged, i.e., $h_P = H_P$ and $h_Q = H_Q$

(b) Both liquid levels rise up so that $h_P/H_P = h_Q/H_Q$

(c) Both liquid levels rise up so that $h_P - h_Q = H_P - H_Q$

(d) The liquid P falls and liquid Q rises such that $\frac{h_P}{h_Q} = \frac{2\ H_P}{3\ H_Q}$

Ans. (a)

Sol. $P_x = P_0 + H_Q p_Q g$

$P_x = P_0 + H_p p_p g$

Another planet

$P_x = P_0 + h_p p_Q \left(g \times \frac{2}{3}\right)$

$P_x = P_0 + h_p p_Q (g \times 2/3)$

The liquids levels are unchanged.

$h_p = H_p$ $\qquad h_Q = H_Q$

Only pressure dereases

The height of the liquid in the tube is therefore proportional to the pressure exerted by atmosphere.

5. A tank with a square base of area 2.0 m^2 is divided into two compartments by a vertical partition in the middle. There is a small hinged door of face area 20 cm^2 at the bottom of the partition. Water is filled in one compartment and an acid of relative density 1.5 in the other, both to a height of 4 m. If $g = 10\ ms^{-2}$, the force necessary to keep the door closed is

(a) 10 N (b) 20 N

(c) 40 N (d) 80 N

Ans. (c)

Sol. For compartment with water,

$h = 4$ m, $p_w = 10^3\ kg/m^3$

$p_w = hp_w g = 4 \times 10^4$ pa

For compartment with acid,

$h = 4$ cm, $p_a = 1.5 \times 10^3\ kg/m^3$

$p_a = 4 \times 1.5 \times 10^3 \times 10$ pa $= 6 \times 10^4$ pa

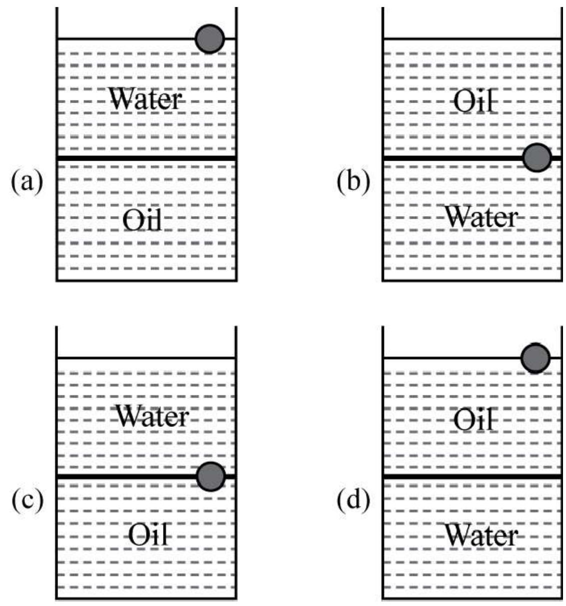
Net pressure $= p_a - p_w = 2 \times 10^4$ pa

Force on door = Area × Net pressure

$= 20 \times 10^{-4} \times 2 \times 10^4$ N

$= 40$ N

6. A ball is made of a material of density p where $p_{oil} < p < p_{water}$ with p_{oil} and p_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in mixture of this oil and water, which of the following pictures represents its equilibrium position ?

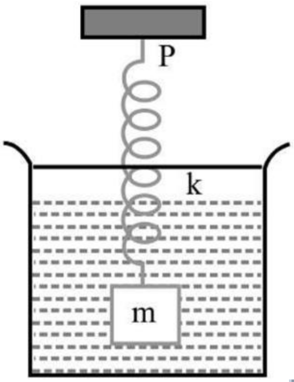


Ans. (b)

Sol. As $\rho_{oil} < \rho_{water}$ so oil should be over the water. As $\rho > \rho_{oil}$, so the ball will sink in the oil but $\rho < \rho_{water}$ so it will float in the water.

Hence option (b) is correct.

7. A cube of mass m and density D is suspended form the point P by a spring of stiffness k. The system is kept inside a beaker filled with a liquid of density d. The elongation in the spring, assuming D > d, is



(a) $\frac{mg}{k}\left(1-\frac{d}{D}\right)$ (b) $\frac{mg}{k}\left(1-\frac{D}{d}\right)$

(c) $\frac{mg}{k}\left(1+\frac{d}{D}\right)$ (d) none of these

Ans. (a)

Sol. Let's say the elongation in spring is x so by Hooke's law

$f = kx$

$kx + F_B = mg$

$\Rightarrow kx = mg - d(V_{body})g$ d is density of liquid.

$$\Rightarrow kx = mg\left(1-\frac{dV_{body}}{DV_{body}}\right)$$

$$\Rightarrow x = \frac{mg}{k}\left(1-\frac{d}{D}\right)$$

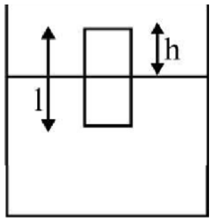
8. A cubical block of steel of each side equal to l is floating on mercury in a vessel. The densities of steel and mercury are ρ_s and ρ_m. The height of the block above the mercury level is given by

(a) $l\left(1+\frac{\rho_s}{\rho_m}\right)$ (b) $l\left(1-\frac{\rho_s}{\rho_m}\right)$

(c) $l\left(1+\frac{\rho_m}{\rho_s}\right)$ (d) $l\left(1-\frac{\rho_m}{\rho_s}\right)$

Ans. (b)

Sol.



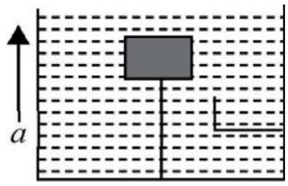
$$\rho_s l^3 g = \rho_m l^2 (l-h) g$$

$$\rho_m (l-h) = \rho_s l$$

$$l - h = \frac{\rho_s}{\rho_m} l$$

$$h = l\left(1-\frac{\rho_s}{\rho_m}\right).$$

9. The tension in a string holding a solid block below the surface of a liquid (where $\rho_{liquid} > \rho_{block}$) as in shown in the figure is T when the system is at rest.



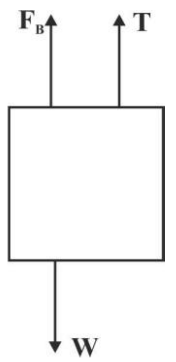
Then what will be the tension in the string if the system has upward acceleration a?

(a) $T\left(1-\frac{a}{g}\right)$ (b) $T\left(1+\frac{a}{g}\right)$

(c) $T\left(\frac{a}{g}-1\right)$ (d) $\frac{a}{g}T$

Ans. (b)

Sol. At rest



$T + F_B = W$

$\Rightarrow T + \rho_{liq} Vg = \rho_b Vg$

$\Rightarrow T = (\rho_b - \rho_{liq}) Vg$ ...(1)

When accelerating upward $g_{eff} = g + a$

Then $T' + \rho_{liq} Vg_{eff} = \rho_b Vg_{eff}$

$T'(\rho_b - \rho_{liq}) Vg_{eff}$ ...(2)

$$T' = (\rho_b - \rho_{liq}) V (g + a)$$

$= (\rho_b - \rho_{liq}) Vg + (\rho_b - \rho_{liq}) Va$

$$= T + \frac{T}{g} a = T\left(1 + \frac{a}{g}\right)$$

10. A beaker containing water is placed on the platform of a spring balance. The balance reads 1.5 kg. A stone of mass 0.5 kg and density 500 kg/m^3 is immersed in water without touching the walls of the beaker. What will be the balance reading now?

(a) 2 kg (b) 2.5 kg

(c) 1 kg (d) 3 kg

Ans. (b)

Sol. $$V_{stone} = \frac{0.5}{500} = 10^{-3} m^3$$

When it is completely submerged in water will exert Buoyaut force on it and stone wil exert same force on water in opposite direction

So new reading $= \left(1.5 + \frac{F_B}{g}\right) kg$

$$= \left(1.5 + \frac{1000 \times 10^{-3} \times 10}{10}\right)$$

$= 2.5$ kg

11. An ornament weighing 36 g in air weighs only 34 g in water. Assuming that some copper is mixed with gold to prepare the ornament, find the amount of copper in it. Specific gravity of gold is 19.3 and that of copper is 8.9.

(a) 2.2 g (b) 4.4 g

(c) 1.1 g (d) 3.6 g

Ans. (a)

Sol. Let's asume the density of copper and gold to be V_C & V_A respectively

Then $w = \rho_C V_C + \rho_A V_A = 36$.(i)

$W - F_B = 34$

$\Rightarrow F_B = 2g$

$\Rightarrow \rho_x(V_A + V_C) = 2$

$$\frac{\rho_C V_A + \rho_A V_A}{\rho_w (V_A + V_C)} = 18.$$

$\Rightarrow 9.1 V_C = 1.3 V_A$

$\Rightarrow V_A = 7V_C$

Now $8.9 V_C + 19.37 (V_C) = 36$

$$\Rightarrow V_C = \frac{36}{8.9 + 19.3 \times 7}$$

Amount of copper $= 8.9 V_C = 2.225$g.

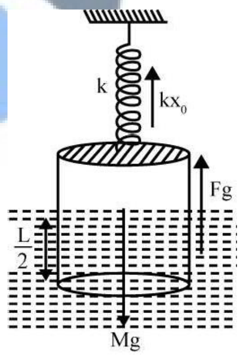
12. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is

(a) $\frac{Mg}{k}$ (b) $\frac{Mg}{k}\left(1 - \frac{LA\sigma}{M}\right)$

(c) $\frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$ (d) $\frac{Mg}{k}\left(1 + \frac{LA\sigma}{M}\right)$

Ans. (c)

Sol. Let k be the spring constant of spring and it gets extended by length x_0 in equailibrium position



In equilibrium,

$kx_0 + F_B = Mg$

$$kx_0 + \sigma \frac{L}{2} Ag = Mg$$

$$x_0 = \frac{mg - \frac{\sigma LAg}{2}}{k} = \frac{Mg}{k}\left(1 - \frac{\sigma LA}{2M}\right)$$

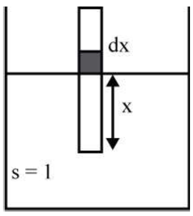
13. A long metal rod of length l and relative density σ is held vertically with its lower end just touching the surface of water. The speed of the rod when it just sinks in water is given by

(a) $\sqrt{2gl}$ (b) $\sqrt{2gl\sigma}$

(c) $\sqrt{2gl\left(1-\frac{1}{2\sigma}\right)}$ (d) $\sqrt{2gl(2\sigma-1)}$

Ans. (c)

Sol.



$$\sigma lAg - xAg = \sigma lAv\frac{dv}{dx}$$

$$g(\sigma l - x) = \sigma lv\frac{dv}{dx}$$

$$\int_0^l g(\sigma l - x)dx = \int_0^v \sigma lv\,dv$$

$$g\left(\sigma l - \frac{l}{2}\right) = \frac{\sigma v^2}{2}$$

$$\Rightarrow v = \sqrt{2gl\left(1-\frac{1}{2\sigma}\right)}$$

14. A large block of ice 5m thick has a vertical hole drilled through it and is floating in the middle of a lake. The minimum length of the rope required to scoop up bucket full of water through the hole is

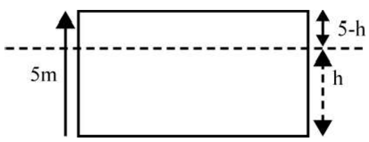
(the relative density of ice = 0.9)

(a) 1 m (b) 0.9 m

(c) 0.5 m (d) 0.45 m

Ans. (c)

Sol. Ice will be the floating in water partially with some part submerged, so water will be till height h of ice cube. So length of rope required is to be determined.



Min length of rope $= (5-h)_m$

Now by

Archimedes principle

$W_{ice} = F_{buoyant}$

$\Rightarrow \rho_{ice}A(5)g = \rho_w A(h)g$

$\Rightarrow h = 5\left(\frac{\rho_{ice}}{\rho_{water}}\right) = 5(0.9)$

$= 4.5$

$= (5 - 4.5)m = 0.5$ m

15. If a sample of metal weighs 210 g in air, 180 g in water and 120 g in a liquid:

(i) RD of metal is 3 (ii) RD of metal is 7

(iii) RD of liquid is 3 (iv) RD of liquid is (1/3)

(a) (i, ii) (b) (i, iii)

(c) (ii, iv) (d) (ii, iii)

Ans. (d)

Sol. Relative density of Metal = $\frac{\text{Weight in air}}{\text{change in weight of water}}$

$$R.D = \frac{210}{210-180} = 7$$

Density of metal = 7g/cm^3

Change in weight of liquid = upthrust in liquid

$= V_{solid} \times \rho_{liq} \times g$

$\Delta w \propto \rho_{liq}$

$$\frac{\Delta w_e}{\Delta w_w} = \frac{\rho_l}{\rho_w}$$

$$\rho_l = \frac{\Delta W_c}{\Delta W_w}\rho_w$$

$$\rho_l = \frac{210-120}{210-180}\times(1)$$

$$\rho_l = 3g/cm^3$$

16. A vessel contains oil of density 0.8 gcm^{-3} floating over mercury of density 13.6 gcm^{-3}. A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the sphere in gcm^{-3} is

(a) 3.3 (b) 6.4

(c) 7.2 (d) 12.8

Ans. (c)

Sol. Weight = Net buoyant force

$$\rho_s V_g = 0.8\left(\frac{V}{2}\right)g + 13.6\left(\frac{v}{2}\right)g$$

$$\rho_s = \frac{144}{2}g/cm^3 = 7.2g/cm^3.$$

17. A cubical block of wood of specific gravity 0.5 and a chunk of concrete of specific gravity 2.5 are fastened together. The ratio of mass of wood to the mass of concrete which makes the combination to float with its entire volume submerged in water is:

(a) 1/5 (b) 1/3

(c) 3/5 (d) 2/5

Ans. (c)

Sol. Let the volume of block = v_1

volume of concrete = v_2

Displaced volume of water = $v_1 + v_2$

Now weight of the combination = Buoyant force

$0.5 \times v_1 \times g + 2.5 \times v_2 \times g = (v_1 + v_2)g$

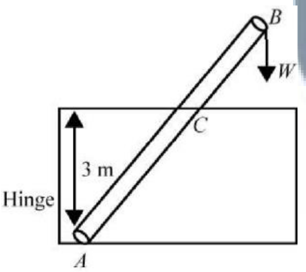
$$\frac{v_1}{v_2} = \frac{3}{1}$$

$$\frac{M_1}{M_2} = \frac{0.5 \times v_1}{2.5 \times v_2} = 3/5.$$

Passage

Using the following Passage, solve Q. 18 & 19

A rod of length 6 m has a mass 12 kg. It is hinged at one end A at a distance of 3 m below the water surface. The specific gravity of the material of rod is 0.5.



18. What weight must be attached to the other end B so that 5 m of the rod is immersed in water?

(a) 7 kgf (b) 20kgf

(c) $\frac{7}{5}$ kgf (d) $\frac{7}{2}$ kgf

Ans. (b)

Sol. given mass of rod 12 kg

Let's say Area of cross section of rod is S.

Where density of rod is 0.5×10^3 kg/m^3

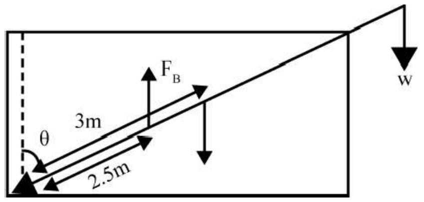
So when rod is submerged by 5m length then buoyant force

As volume $= \frac{mass}{length} \times 5m$

$$F_B = \frac{10^3}{0.5 \times 10^3} \times \frac{12}{6} \times 5 \times 10$$

= 20kg-f

Now for equation of rod, net torque about hinge should be zero.



balancing torque about hinge point we get

$w_r \times 3 \times \sin\theta + w \times 6\sin\theta = (F_B \times 2.5)\sin\theta$

$\Rightarrow w \times b = (20 \times 2.5) - (12 \times 3)g$

$$w = \left(\frac{50 - 36}{6}\right) g = \frac{7}{3} kgf$$

19. Find the magnitude and direction of the force exerted by the hinge on the rod.

(a) $\frac{17}{3}$ kgf in the downward direction

(b) 8 kgf in the downward direction

(c) 4 kgf in the downward direction

(d) 5 kgf in the downward direction

Ans. (a)

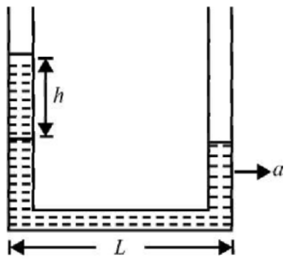
Sol. Now translational equation of rod net downward direction.

$F_H + w_r + w = F_B \Rightarrow F_H = F_B - (w_r + w)$

$$= \left[20 - \left(12 + \frac{7}{3}\right)\right] kgf$$

$$= \frac{17}{3} kgf$$

20. When at rest, a liquid stands at the same level in the tubes as shown in the figure. But as indicated, a height difference h occurs when the system is given an acceleration a towards the right. Then h is equal to

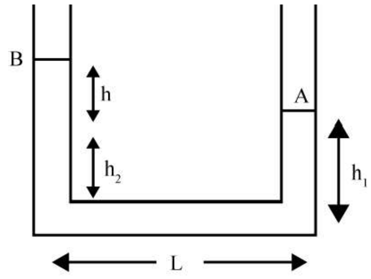


(a) $\frac{aL}{2g}$ (b) $\frac{gL}{2a}$

(c) $\frac{gL}{a}$ (d) $\frac{aL}{g}$

Ans. (d)

Sol.



$P_A + pgh_1 + paL - pg(h_1 + h) = P_B$

As $P_A = P_B = P_{atm}$

$\Rightarrow pal - pgh = 0$

$\Rightarrow h = \left(\frac{al}{g}\right).$

COMPREHENSION TYPE QUESTIONS

Passage

Using the following Passage, solve Q. 21 to Q. 26

If the container filled with liquid gets accelerated horizontally or vertically, pressure in liquids gets changed. In case of horizontally accelerated liquid (a_x), the free surface has the slope a_x/g. In case of vertically accelerated liquid (a_y) for calculation of pressure, effective g is used. A closed box with horizontal base 6 m by 6m and a height 2m is half filled with liquid. It is given constant horizontal acceleration g/2 and vertical downward acceleration g/2.

21. The angle of the free surface with the horizontal is equal to

(a) 30 (b) $\tan^{-1}(2/3)$

(c) $\tan^{-1}(1/3)$ (d) 45º

Ans. (d)

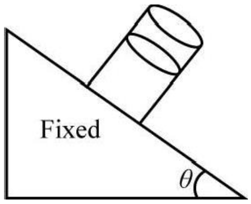
Sol. $a_y = g_{eff} = g - \frac{g}{2} = \frac{g}{2}$

$a_x = \frac{g}{2}$

$\tan\theta = \frac{a_x}{a_y} = \frac{\left(\frac{g}{2}\right)}{\left(\frac{g}{2}\right)} = 1$

$\Rightarrow \theta = 45^\circ$

22. A cylindrical vessel filled with water is released on an inclined surface of angle θ as shown in the figure. The friction coefficient of surface with vessel is m (< tan θ). Then the constant angle made by the surface of water with the incline will be



(a) $\tan^{-1}$ m (b) $\theta - \tan^{-1}$ m

(c) $\theta + \tan^{-1}$ m (d) $\cot^{-1}$ m

Ans. (a)

Sol. As pulling force = force of friction

$\Rightarrow$ m mg cosθ = mg sinθ

$\Rightarrow$ m = tanθ

$\Rightarrow \theta = \tan^{-1}$m

23. Length of exposed portion of top of box is equal to -

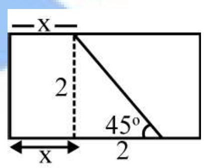
(a) 2m (b) 3m

(c) 4m (d) 2.5 m

Ans. (c)

Sol. Since container is closed so no volume of water will be spilled.

Hence the container will have liquid profile as shown



By volume conservation

$\Rightarrow x + 1 = 3 \Rightarrow x = 2$m

Hence length of exposed portion = 6 – 2 = 4m

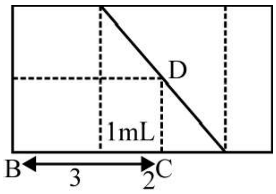
24. Water pressure at the bottom of centre of box is equal to (atmospheric pressure $= 10^{-5}\ N/m^2$, density of water $= 1000\ kg/m^3$, $g = 10\ m/sec^2$)

(a) 1.1 MPa (b) 0.11 MPa

(c) 0.101 MPa (d) 0.011 MPa

Ans. (b)

Sol. By geometry, height of water level above centre of box will be 1m (as shown)



So pressure at centre

$= 10^{5}(Pa) + 10^{3}\left(\frac{10}{2}\right)10$

$= (10^5 + \times 10^3)$ Pa

$= (0.1 + 0.01)$ MPa

$= 0.11$ MPa

25. Maximum value of water pressure in the box is equal to -

(a) 1.4 MPa (b) 0.14 MPa

(c) 0.104 MPa (d) 0.014 MPa

Ans. (b)

Sol. Maximum pressure will be at B

$P_B = P_C + 4pa_x$

$= \left(0.1 + 4 \times 10^3 \times 10\right) MPa = 0.14 MPa$

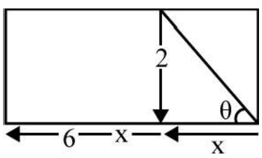
26. What is the value of vertical acceleration of box for given horizontal acceleration (g/2), so that no part of bottom of box is exposed -

(a) g/2 upward (b) g/2 downward

(c) g/4 upward (d) not possible

Ans. (a)

Sol. If no part of bottom is exposed, also volume then should be conserved then



Following case would be possible as shown

$(6 - x)6 \times 2 + 6\left(\frac{1}{2} \times 2 \times x\right) = 6 \times 6 \times 1$

$\Rightarrow 2(6 - x) + x = 6$

$\Rightarrow 12 - 2x + x = 6$

$\Rightarrow x = 6$

Hence $\tan\theta = \frac{a_x}{g_{eff}} = \frac{2}{x} = \frac{2}{6}$.

$\Rightarrow \frac{\left(\frac{g}{2}\right)}{\left(g_{eff}\right)} = \frac{1}{3}$

$\Rightarrow g_{eff} = \frac{3g}{2}$

Hence particle must have acceleration equal to $\frac{3g}{2}$ in upward direction.

27. Two bodies with volumes V and 2V are equalized on a balance. The larger body is then immersed in oil of density $d_1 = 0.9$ g/cm^3 while the smaller body is immersed in another liquid when it is found that the equilibrium of the balance is not disturbed. The density of the second liquid is then:

(a) 2.4 g/cm^3 (b) 1.8 g/cm^3

(c) 0.45 g/cm^3 (d) 2.7 g/cm^3

Ans. (b)

Sol. $V_1 = V$

$V_2 = 2V$

Both have same mass therefore, their densities are

$M_1 = M_2$

$d_2 = \frac{1}{2} \times d_1$

M_2 is immersed in oil of density 0.9g/cm^3

Buoyant force on the larger body, $M_2 = V_2 \times 0.9 \times g$

Let the density of liquid in which the smaller body M_1 is immersed be d g/cm^3

Buoyant force on smaller body, $M_1 = V_1 \times d \times g$

For the balance to remain in equilibrium

$M_2 = M_1$

$v_2 \times 0.9 \times g = v_1 \times d \times g$

$d = 0.9 \times \frac{v_2}{v_1} = 0.9 \times \frac{2v}{v} = 1.8 g/cm^3$

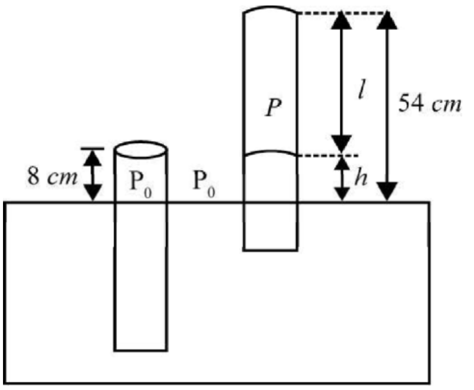
28. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure - 76 cm of Hg)

(a) 22 cm (b) 38 cm

(c) 6 cm (d) 16 cm

Ans. (d)

Sol.



$P_1V_1 = P_2P_2 \Rightarrow P_0 (A \times 8) = P(Al)$

Also, $P_0 - P = \rho gh = \rho g(54 - l)$

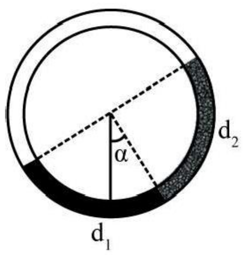
$\Rightarrow 8P_0 = l[P_0 - \rho g (54 - l)]$

$\Rightarrow 8(\rho g \times 76) = l[\rho g \times 76 - \rho g (54 - l)]$

$\Rightarrow 8 \times 76 = 22 l + l^2 \Rightarrow (l + 38)(l - 16) = 0$

$\therefore l = 16$ cm

29. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is :

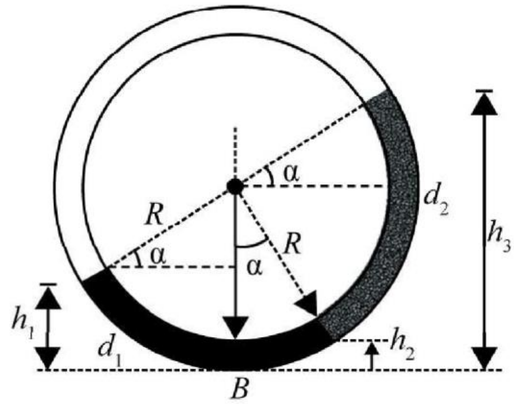


(a) $\frac{1 + \cos \alpha}{1 - \cos \alpha}$ (b) $\frac{1 + \tan \alpha}{1 - \tan \alpha}$

(c) $\frac{1 + \sin \alpha}{1 - \cos \alpha}$ (d) $\frac{1 + \sin \alpha}{1 - \sin \alpha}$

Ans. (b)

Sol.



In figure if R is the radius of tube, we have

$h_1 = R - R \sin \alpha$

$h_2 = R - R \cos \alpha$

$h_3 = R - R \sin \alpha$

Pressure due to liquid at point B is given as

$P_B = h_1 d_1 g$

and $P_B = (h_3 - h_2)d_2 g + h_2 d_1 g$

$\Rightarrow h_1 d_1 g = (h_3 - h_2)d_2 g + h_2 d_1 g$

$\Rightarrow (h_1 - h_2)d_1 = (h_3 - h_2)d_2$

$\Rightarrow \frac{d_1}{d_2} = \frac{h_3 - h_2}{h_1 - h_2} = \frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha}$

$\Rightarrow \frac{d_1}{d_2} = \frac{1 + \tan\alpha}{1 - \tan\alpha}$.

30. Two identical cylindrical vessels, each of base area A, have their bases at the same horizontal level. They contain a liquid of density ρ. In one vessel the height of the liquid is h_1 and in the other $h_2 > h_1$. When the two vessels are connected, the work done by gravity in equalizing the levels is

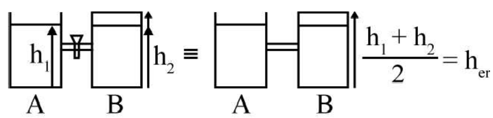
(a) $2\rho Ag (h_2 - h_1)^2$ (b) $\rho Ag (h_2 - h_1)^2$

(c) $\frac{1}{2}\rho Ag(h_2 - h_1)^2$ (d) $\frac{1}{4}\rho Ag(h_2 - h_1)^2$

Ans. (d)

Sol. On mixing volume should be conserved

So $V_f = V_i$



$\Rightarrow 2A(h_{eq}) = Ah_1 + Ah_2$

$\Rightarrow h_{eq} = \frac{h_1 + h_2}{2} = h(\text{say})$

Work done by gravity = $-\Delta U$

$$U_f = 2Ah\rho g\frac{h}{2} + Ah\rho g\frac{h}{2}$$

$$= \frac{A\rho g}{2}(h_1 + h_2)^2 = \frac{\rho Ag}{4}(h_1 + h_2)^2$$

$$U_i = (\rho Ah_1)g\left(\frac{h_1}{2}\right) + (\rho Ah_2)g\left(\frac{h_2}{2}\right)$$

$$= \frac{\rho Ag}{2}\left(h_1^2 + h_2^2\right)$$

$w_g = -\Delta U = U_i - U_f$

$$= \frac{\rho gA}{4}\left[2\left(h_1^2 + h_2^2\right) - (h_1 + h_2)^2\right]$$

$$W_g = \frac{\rho gA}{4}(h_2 - h_1)^2$$

31. Two capillary tubes A and B of radii r_a and r_b and lengths l_a and l_b respectively are held horizontally. The volume of water flowing per second through tube A is Q_a when the pressure difference across its ends is maintained at P. When the same pressure difference is maintained across tube B, the volume of water flowing per second through it is Q_b. The ratio Q_a/Q_b is

(a) $\frac{l_b}{l_a}\left(\frac{r_a}{r_b}\right)$ (b) $\frac{l_b}{l_a}\left(\frac{r_a}{r_b}\right)^2$

(c) $\frac{l_b}{l_a}\left(\frac{r_a}{r_b}\right)^3$ (d) $\frac{l_b}{l_a}\left(\frac{r_a}{r_b}\right)^4$

Ans. (d)

Sol. $V_a = \frac{\pi}{8}\frac{Pr_a^4}{\eta l_a} = Q_a$

$V_b = \frac{\pi}{8}\frac{Pr_b^4}{\eta l_b} = Q_b$

$\frac{Q_a}{Q_b} = \frac{r_a^4}{r_b^4} \times \frac{l_b}{l_a}$

$= \left(\frac{l_b}{l_a}\right) \times \left(\frac{r_a}{r_b}\right)^4$

32. Two capillary tubes A and B of equal radii $r_a = r_b = r$ and equal lengths $l_a = l_b = l$ are held horizontally. When the same pressure difference P is maintained across each tube, the rate of flow of water in each is Q. If the tubes are connected in series and the same pressure difference P is maintained across the combination, the rate of flow through the combination will be

(a) Q/2 (b) Q

(c) 2Q (d) none of these

Ans. (a)

Sol. Volume of liquid flowing per second or rate of flow is given by

$$V = \frac{\pi}{8}\frac{Pr^4}{\eta l}$$

When tubes are connected in parallel

$P_1 = P_2 = P_3$

Volume of fluid flowing per second

$V = V_1 + V_2$

$$Q = \frac{\pi}{8}\frac{Pr_a^4}{\eta l_a} + \frac{\pi}{8}\frac{Pr_b^4}{\eta l_b}$$

$Q = Q^1 + Q^1$

$\Rightarrow Q^1 = \frac{Q}{2}$

Note: When tubes are connected in series

$V_1 = V_2 = V_3$.

33. In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the lower and upper surfaces of the wing are v and $\sqrt{2}$ v respectively. If the density of air is ρ and the surface area of the wing is A, the dynamic lift on the wing is given by

(a) $\frac{1}{\sqrt{2}}\rho v^2 A$ (b) $\frac{1}{2}\rho v^2 A$

(c) $\sqrt{2}\rho v^2 A$ (d) $2\rho v^2 A$

Ans. (b)

Sol. Applying bernoulli's theorem b/w lower and upper surface.

$$p_U + \frac{1}{2}\rho v^2 = p_L + \frac{1}{2}\rho\left(\sqrt{2}v\right)^2$$

$\Rightarrow (p_L - p_U)$ = Net upward pressure.

$$\Rightarrow \Delta p = \frac{1}{2} p (v)^2$$

So, dynamic lift = $(\Delta p)A = \frac{1}{2} p v^2 A$.

34. In a cylindrical water tank there are two small holes Q and P on the wall at a depth of h_1 form upper level of water and at a height of h_2 from the lower end of the tank respectively as shown in the figure. Water coming out from both the holes strike the ground at the same point. The ratio of h_1 and h_2 is

(a) 1 (b) 2

(c) > 1 (d) < 1

Ans. (a)

Sol. For orfice A,

$$V = \sqrt{2gh_1}$$

time taken by the liquid drop to fall

$$T = \sqrt{\frac{2(h + h_2)}{g}}$$

Now,

$$Range = \sqrt{2gh} \sqrt{\frac{2(h + h_2)}{g}}$$

$$Range = \sqrt{4(h + h_2)h_1}$$

For orifice B,

$$V = \sqrt{2g(h_1 + h)}$$

Time taken by the liquid to fall T = $\sqrt{\frac{2h_2}{g}}$

Now

$$\text{Range} = \sqrt{\frac{2h_2}{g}} \left(\sqrt{2g(h_1 + h)}\right) = \sqrt{4[h_2(h_1 + h)]}$$

As range for both are same, So

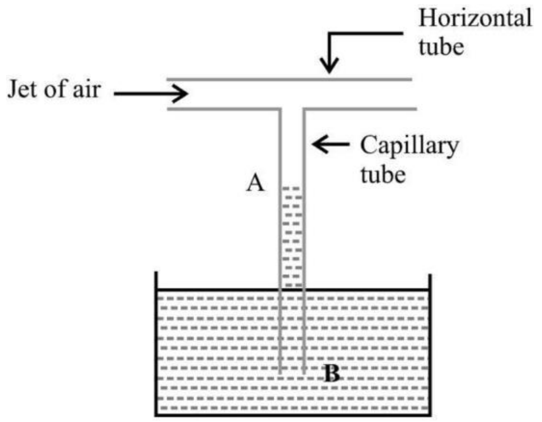
$$\sqrt{4(h + h_2)h_1} = \sqrt{4(h_1 + h)h_2}$$

$hh_1 = hh_2$

$h_1 = h_2$

$$So, \frac{h_1}{h_2} = 1$$

35. Water stands at level A in the arrangement shown in figure. What will happen if a jet of air is gently blown into the horizontal tube in the direction shown in the figure ?



(a) Water will rise above A in the capillary

(b) Water will fall below A in the capillary tube

(c) There will be no effect on the level of water in the capillary tube.

(d) Air will emerge from end B in the form of bubbles.

Ans. (a)

Sol. As a jet of air is gently blown into the horizontal tube, air particle in the tube will attain a speed. So, by Bernoulli's principle, pressure in that region will come down. So, it creates suction and hence, liquid inside the capillary tube rises above A. This is also known as Venturi Effect.

36. Tanks A and B open at the top contain two different liquids upto certain height in them. A hole is made to the wall of each tank at a depth 'h' from the surface of the liquid. The area of the hole in A is twice that of in B. If the liquid mass flux through each hole is equal, then the ratio of the densities of the liquids respectively, is

(a) 2/1 (b) 3/2

(c) 2/3 (d) 1/2

Ans. (d)

Sol. Given area of A = 2 (Area of B)

Velocity of ejection for both = $\sqrt{2gh}$

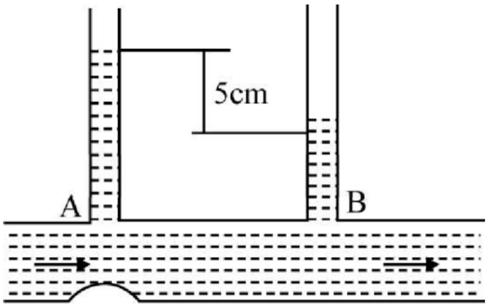
Mass flux = pAV

Since given mass flux is same for both:

$$p_1 A_1 v = p_2 A_2 v$$

$$\Rightarrow \frac{p_1}{p_2} = \frac{A_2}{A_1} = \frac{1}{2}$$

37. In the diagram shown, the difference in the two tubes of the manometer is 5 cm, the cross section of the tube at A and B is 6 mm^2 and 10 mm^2 respectively. The rate at which water flows through the tube is :
($g = 10\ ms^{-2}$)



(a) 10.0 cc/s (b)8.0 cc/s
(c) 7.5 cc/s (d) 12.5 cc/s

Ans. (c)

Sol. We apply beronoulli's equation at A and B

$$P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$$

$$\Rightarrow P_B - P_A = \rho\left(v_A^2 - v_B^2\right)$$

$$\Rightarrow h\rho g = \frac{1}{2}\rho v_A^2\left(1 - \frac{A_A^2}{A_B^2}\right)$$

$$\Rightarrow V_A = \sqrt{\frac{2gh}{\left(1 - \frac{A_A^2}{A_B^2}\right)}} = \sqrt{\frac{2\times 10\times 0.5}{1-(3/5)^2}}$$

$$\Rightarrow V_A = \sqrt{\frac{1}{16/25}} = \frac{5}{4}\ m/s$$

By continuity equation

$A_A v_B = A_B v_B$
We have $P_B - P_A = h\rho g$

From (1) $v_B = \frac{A_A v_A}{A_B}$

Fluid flow rate through tube is

$$\frac{dV}{dt} = A_A v_A = 6\times 10^{-6}\times 1.25$$

$$\Rightarrow \frac{dV}{dt} = 7.5\times 10^{-6}\ m^3/s.$$

$= 7.5$ cc/s

38. The cylindrical tube of a spray pump has a radius R, one end of which has n fine holes, each of radius r. If the speed of flow of the liquid in the tube is V, the speed of ejection of the liquid through the holes is

(a) $\frac{V}{n}\left(\frac{R}{r}\right)^{1/2}$ (b) $\frac{V}{n}\left(\frac{R}{r}\right)$

(c) $\frac{V}{n}\left(\frac{R}{r}\right)^{3/2}$ (d) $\frac{V}{n}\left(\frac{R}{r}\right)^{2}$

Ans. (d)

Sol. Using equation of continuity, AV = constant

$A_1V_2 = A_2V_2$

$$\pi R^2 V = \left(n\pi r^2\right)V'$$

where $n\pi r^2$ is the total area of n fine holes.

$$V' = \frac{R^2}{nr^2}V$$

39. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is steady. The cross-sectional area of the stream 0.15 m below the tap is (take $g = 10\ ms^{-2}$)

(a) 5.0×10^{-4} m^2 (b) 1.0×10^{-5} m^2
(c) 5.0×10^{-5} m^2 (d) 2.0×10^{-5} m^2

Ans. (c)

Sol. Volume flow rate $= 10^{-4}\ \frac{m^3}{sec}$

$$\Delta P = \rho gh = 10^3\ \frac{kg}{m^3}\times 10\ \frac{m^2}{s}\times 0.15$$

$$\Rightarrow \Delta P = 1500\ Pa$$

$$\Rightarrow \Delta P = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$

$$\Rightarrow 1500 = \frac{1}{2}\times 10^3\left(V^2 - 1\right)$$

$$\Rightarrow V = 2\ \frac{m}{s}$$

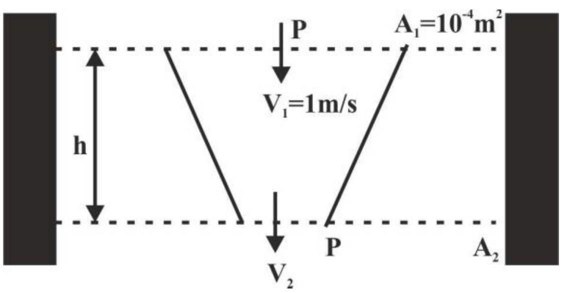
—

By equation of continuity

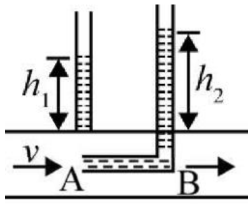
$A_1V_1 = A_2V_2$

$\Rightarrow 10^{-4} \times 1 = A_2 \times 2$

$\Rightarrow A_2 = 5\times10^{-5} m^2$



40. The figure shows a liquid of density p flowing through a tube with velocity v. The h_1 and h_2 are the heights of liquid in the straight and L-shaped tubes, respectively. Choose the correct statements.



(a) The pressure at the point A is pgh_1

(b) The pressure at the point B is pgh_2

(c) The velocity of flow is, $v = \sqrt{2gh_2}$

(d) The velocity of flow is, $v = \sqrt{2g(h_2 - h_1)}$

Ans. (d)

Sol. Acc. to Bernouli's Theorm

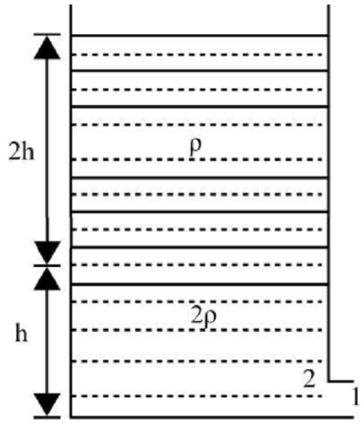
$$P_A + \frac{1}{2}PV^2 + Pgh_1 = P_B + \frac{1}{2}P(0)^2 + Pgh_2$$

$$\frac{V^2}{2} + gh_1 = gh_2$$

$$\frac{V^2}{2} = g(h_2 - h_1)$$

$$V = \sqrt{2g(h_2 - h_1)}$$

41. The velocity of the liquid coming out of a small hole of a vessel containing two different liquids of densities 2p and p as shown in the figure is



(a) $\sqrt{6gh}$ (b) $2\sqrt{gh}$

(c) $2\sqrt{2gh}$ (d) $\sqrt{gh}$

Ans. (b)

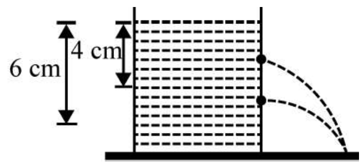
Sol. Pressure at (2), $P_1 = p_{atm} + 2hpg + 2hpg$

Applying Bernoulli's theorem between points (1) and (2) ...

$$(p_{atm} + 2hpg) + (2l)gh = p_{atm} + \frac{1}{2}(2p)v^2$$

$$\Rightarrow v = 2\sqrt{gh}$$

42. Figure shows two holes in a wide tank containing a liquid column. The water streams coming out of these holes strike the ground at the same point. The height of liquid column in the tank is



(a) 10 cm (b) 8 cm

(c) 9.8 cm (d) 980 cm

Ans. (a)

Sol. $V = \sqrt{2gh}$, h is distance from free surface $h_1 = 4$, $h_2 = 6$.

$$V_1 = \sqrt{2gh_1} = \sqrt{8g}$$

$$t = \sqrt{2h/g} = \sqrt{\frac{2(H-4)}{g}}$$, H is height of surface

$$x_1 = v_1 . t_1 = \sqrt{16(H-4)} \quad \text{... (1)}$$

$$t_2 = \sqrt{\frac{2(H-6)}{g}}$$

$$V_2 = \sqrt{12g}$$

$$x_2 = V_2 . t_2 = \sqrt{24(H-6)} \quad \ldots (2)$$

Equating (1) and (2)

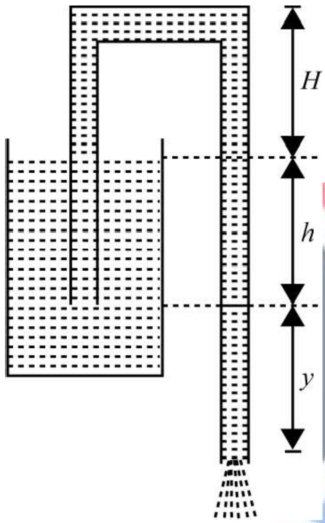
2 (H - 4) = 3 (H - 6)

2H - 8 = 3H - 18

H = -8 + 18

H = 10 cm.

43. A siphon tube is used to remove liquid from a container as shown in the figure.



If the tube is initially filled with liquid, then the speed of the liquid through the siphon is

(a) $\sqrt{2gy}$ (b) $\sqrt{2g(h+y)}$

(c) $\sqrt{2g(H+h+y)}$ (d) none of the above

Ans. (b)

Sol. Using bernull's theorem,

$$p_{atm} + \frac{1}{2}\rho V_{1A}^2 + \rho y_A .g = p_{atm} + \frac{1}{2}\rho V_D^2 + \rho . y_D .g$$

Solving by putting, $y_A = 0$, $y_D = -(h + y)$, $V_D = V$

We get: $V = \sqrt{2g(h+y)}$

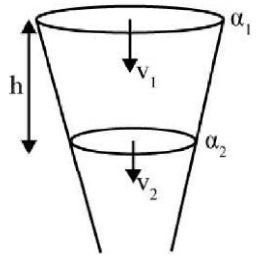
44. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The velocity as it leaves the tap is 0.4 ms^{-1}. The water diameter of the water stream at a distance 2×10^{-1} m below the tap is close to

(a) 7.5×10^{-3} m (b) 9.6×10^{-3} m

(c) 3.6×10^{-3} m (d) 5.0×10^{-3} m

Ans. (c)

Sol.



Here, $d_1 = 8 \times 10^{-3}$ m

$v_1 = 0.4$ ms^{-1}

h = 0.2m

According to equation of motion,

$$v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{(0.4)^2 + 2 \times 10 \times 0.2}$$

» 2 ms^{-1}

¥ According to equation of continuity

$a_1 v_1 = a_2 v_2$

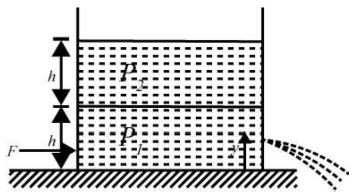
$$\pi \times \left(\frac{8 \times 10^{-3}}{2}\right)^2 \times 0.4 = \pi \times \left(\frac{d_2}{2}\right)^2 \times 2$$

$d_2 = 3.6 \times 10^{-3}$ m

Passage

Using the following Passage, solve Q. 45 to 56

A cylindrical tank having cross sectional area A = 0.5 m^2 is filled with two liquids of density $\rho_1 = 900$ kg m^{-3} and $\rho_2 = 600$ kg m^{-3}, to a height h = 60 cm each as shown in the figure. A small whole having area a = 5 cm^2 is made in right vertical wall at a height y = 20 cm from the bottom. A horizontal force F is applied on the tank to tank to keep it in static equilibrium. The tank is lying on a horizontal surface. Neglect mass of cylindrical tank in comparison to mass of liquids (take g = 10 ms^{-2}).



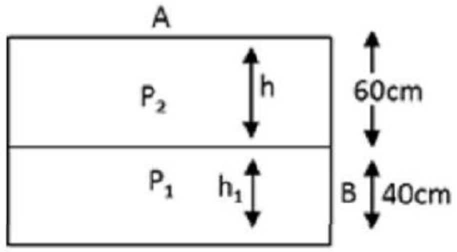
45. The velocity of efflux is

(a) 10 ms^{-1} (b) 20 ms^{-1}

(c) 4 ms^{-1} (d) 35 ms^{-1}

Ans. (c)

Sol.



Applying Bernoulli's between A and B we can write

$$P_{atm} + \rho_1 gh + \rho_2 gh = P_{atm} + \frac{1}{2}\rho_2 v^2$$

$$\Rightarrow 900 \times 10 \times \frac{40}{100} + \frac{600 \times 10 \times 60}{100} = \frac{1}{2}(900)v^2$$

$$\Rightarrow 3600 + 3600 = \frac{1}{2}(900)v^2$$

$$v^2 = \frac{2 \times 2 \times 3600}{900} \Rightarrow \sqrt{16} = 4m/s$$

46. Horizontal force F to keep the cylinder in static equilibrium, if it is placed on a smooth horizontal plane, is

(a) 7.2 N (b) 10 N

(c) 15.5 N (d) 20.4 N

Ans. (a)

Sol. $F = \left(\frac{dm}{dt}\right)v$

$= (\rho A v)v$

$= (900 \times 5 \times 10^{-4} \times 4 \times 4)$

$= 72000 \times 10^{-4} = 7.2$ N

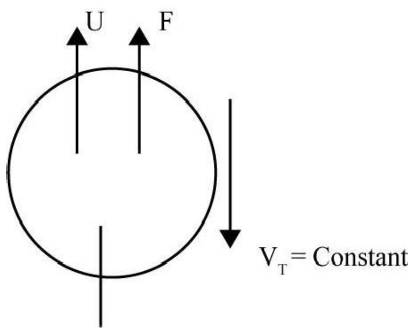
47. A tiny sphere of mass m and density x is dropped in a tall jar of glycerine of density y. When the sphere acquires terminal velocity, the magnitude of the viscious force acting on it is

(a) mgx/y (b) mgy/x

(c) mg (1 – y/x) (d) mg (1 + x/y)

Ans. (c)

Sol.



At $v = v_t$

$U + F = W$

$F = W - U$

$$= W\left[1 - \frac{U}{W}\right]$$

$$F = W\left[1 - \frac{Vyg}{Vxg}\right]$$

$$F = mg\left[1 - \frac{y}{x}\right]$$

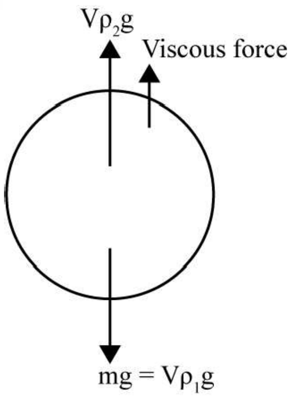
48. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2. ($\rho_2 < \rho_1$). [Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, ie, $F_{viscous} = -kv^2$ ($k > 0$). The terminal speed of the ball is

(a) $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$ (b) $\frac{Vg\rho_1}{k}$

(c) $\sqrt{\frac{Vg\rho_1}{k}}$ (d) $\frac{Vg(\rho_1 - \rho_2)}{k}$

Ans. (a)

Sol. The forces acting on the solid ball when it is falling through a liquid are mg downwards, thrust by Archimedes principle upwards and the force due to the force of friction also acting upwards. The viscous force rapidly increases with velocity, attaining a maximum when the ball reaches the terminal velocity.



$mg - V\rho_2 g - kv^2 = ma$ Where V is volume, v is the terminal velocity. When the ball is moving with terminal velcoity a = 0. Then a = 0

Therefore $V\rho_1 g - V\rho_2 g - kv^2 = 0$

$$\Rightarrow v = \sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}.$$

49. If a number of identical droplets of water, each of radius r, coalesce to form a single drop of radius R, the resulting rise in the temperature of water is given by (here ρ is the density of water, s its specific heat and σ its surface tension)

(a) $\frac{\sigma}{\rho s}\left(\frac{1}{r}-\frac{1}{R}\right)$ (b) $\frac{3\sigma}{\rho s}\left(\frac{1}{r}-\frac{1}{R}\right)$

(c) $\frac{\sigma}{\rho s}\left(\frac{1}{r}+\frac{1}{R}\right)$ (d) $\frac{3\sigma}{\rho s}\left(\frac{1}{r}+\frac{1}{R}\right)$

Ans. (b)

Sol. Volume conservation

$$x\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3 \Rightarrow R^3 = xr^3 \quad ..(i)$$

Decrease in surface energy = σ(ΔA)

= increase in heat energy.

$$\Rightarrow \sigma\left(x4\pi r^2 - 4\pi R^2\right) = \rho\left(\frac{4}{3}\pi R^3\right)s\,(\Delta T)$$

$$\Rightarrow \Delta T = \frac{3\sigma}{\rho s}\left(\frac{xr^2}{R^3}-\frac{R^2}{R^3}\right)$$

$$\Rightarrow \Delta T = \frac{3\sigma}{\rho s}\left(\frac{1}{r}-\frac{1}{R}\right)$$

$$\Rightarrow \Delta T = \frac{3\sigma}{\rho s}\left(\frac{1}{r}-\frac{1}{R}\right) \quad \left[as\ x = \frac{R^3}{r^3}\right].$$

Multiple Choice Questions

50. The liquid in the capillary tube will rise, if the angle of contact is

(a) 0° (b) 90°

(c) obtuse (d) acute

Ans. (a,d)

Sol. Ascent formula:

$$h = \frac{2T\cos\theta}{r\rho g}$$

Only if θ is acute, cosθ will be positive, meaning the height will be positive and liquid will rise.

If θ is obtuse, cosθ will be negative, meaning the height will be negative and liquid will fall inside the capillary

This means the level of liquid in the capillary will be lesser than the vessel. This happens in case of mercury only if θ = 90° .

51. A capillary tube is immersed vertically in water and the height of the water column is x. When this arrangement is taken into a mine of depth d, the height of the water column is y. If R is the radius of the earth, the ratio x/y is

(a) (1 – d/R) (b) (1 + d/R)

(c) (R – d/R + d) (d) (R + d/R – d)

Ans. (a)

Sol. $h = \frac{2T\cos\theta}{\rho gR}$

Where T = surface tension

θ = angle of contact between capillary and water.

ρ = density of water

g = acc. due to gravity

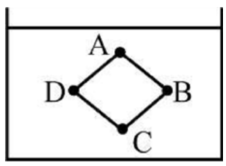
R = radius of capillary

$$h \propto \frac{1}{g}\ (as\ others\ are\ \text{constant})$$

$$\Rightarrow \frac{x}{y} = \frac{g_{at\ depend}}{g_{at\ surface of\ earth}}$$

$$\Rightarrow \frac{x}{y} = \frac{g\left(1-\frac{d}{R}\right)}{g} = \left(1-\frac{d}{R}\right)$$

52. The figure shows a container filled with a liquid of density ρ. Four points A, B, C and D lie on the vertices of a vertical square. Points A and C lie on a vertical line and points B and D lies on a horizontal line. Choose the correct statement(s) about the pressure at the four points.

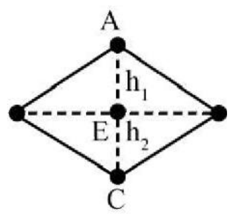


(a) $P_D = P_B$ (b) $P_A < P_B = P_D < P_C$

(c) $P_D = P_B = \frac{P_C - P_A}{2}$ (d) $P_D = P_B = \frac{P_C + P_A}{2}$

Ans. (a,b,d)

Sol. Since B and D are at same horizontal level.

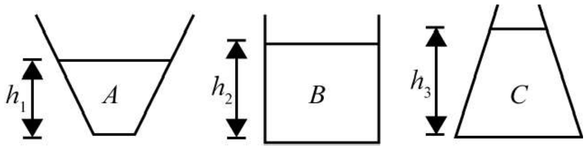


$P_B = P_D = P_E$

$P_E = P_A + \rho gh_1$

$P_C = P_A + \rho g(h_1 + h_2)$

53. Equal volumes of liquid are poured in the three vessels A, B and C ($h_1 < h_2 < h_3$). All the vessels have same base area. Select the correct alternatives.



(a) The force on the base will be maximum in vessel A.

(b) The force on the base will be maximum in vessel C.

(c) Net force exerted by the liquid in all the three vessels is equal.

(d) Net force exerted by the liquid in vessel A is maximum.

Ans. (b,c)

Sol. Force on base = $(P_{base})(A_{ua})$

So as $h_1 < h_2 < h_3$

So, $P_1 < P_2 < P_3$

Hence $F_A = (pgh_1)A$

$F_B = (pgh_2)A$

$F_C = (pgh_3)A$

54. A spherical pot is more than half filled with water as shown in the figure. Choose the correct statement(s) about the forces exerted by water on the pot.

(a) It is always normal to the surface of pot.

(b) Everywhere it acts in the downward direction.

(c) The net horizontal force on the pot is zero.

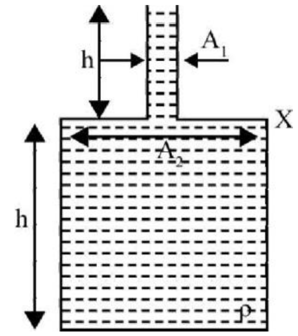
(d) The net vertical force on the pot is in the downward direction.

Ans. (a,c,d)

Sol. Pressure always acts normal to the surface hence force would be normal. Also since this liquid is at rest hence net force in horizontal direction will be zero.

In vertical direction liquid will exert its weight.

55. The vessel shown in the figure has two sections of areas of cross section A_1 and A_2. A liquid of density p fills both the sections, up to a height h in each. Neglect atmospheric pressure.



(a) The pressure at the base of the vessel is 2hpg.

(b) The force exerted by the liquid on the base of the vessel is $2hpgA_2$.

(c) The weight of the liquid is $< 2hpgA_2$

(d) The walls of the vessel at the level X exert a downward force $hpg(A_2 - A_1)$ on the liquid.

Ans. (a,b,c,d)

Sol. Pressure at the base = p(g)(2h) = 2pgh

Force at the base = $2pgh\ (A_2)$

Weight of liquid = $pgh(A_1) + (pgh)A_2$

$= pgh(A_1 + A_2) < 2hpgA_2$

Net force of liquid = 0

$\Rightarrow W. + F_x = F_B$

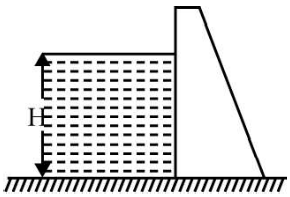
F_x = Force from surface X.

F_B = Force on liquid from base.]

$\Rightarrow pg(A_1 + A_2)h_2 + F_X = 2pghA_2$

$\Rightarrow F_X = pghA_2 - pghA_1$

56. A wall of length l supports water to a height h as shown figure. Choose the correct statement(s). Take p as the density of water.



(a) The force exerted by water on the wall per unit length is $1/2\ pgH^2 l$

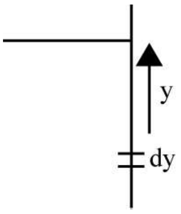
(b) The force exerted by water on the wall is $1/2\ pgH^2 l$

(c) The point of application of the resultant force acts 2H/3 below the free surface.

(d) The point of application of the resultant force acts H/3 below the free surface.

Ans. (b,c)

Sol.



Pressure at the strip = pgy

Force at that strip, dF = $pgy \times ldy$

Torque on that strip due to liquid pressure = (pgy^2/dy)

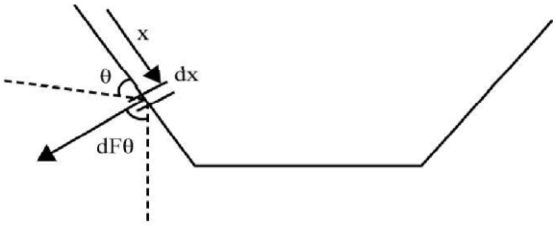
$$F = \int dF = \int_0^H pgyldy = \frac{p\,gH^2 l}{2}.$$

$$t = \int dt = \int_0^H pgy^2\, ldy = \frac{p\,gH^2 l}{3}$$

If point of application of force we surface is d then
F × d = t

$$\Rightarrow d = \frac{t}{F} = \frac{\frac{pgH^3 l}{3}}{\frac{p\,gH^2 l}{2}} = \frac{2H}{3}$$

Force per unit length of wall = $\frac{pgH^2}{2}$



Consider strip at a depth (x sin θ) as shown small force on this element,

dF = pg(x sin θ)(bdx)

Horizontal force = $\int dF \sin\theta$

$$= \int_0^H pgb\left(\sin^2\theta\right)xdx$$

$$= pgb\left(\sin^2\theta\right)\left[\frac{x^2}{2}\right]_0^{H/\sin\theta}$$

$$= \frac{p\,gH^2}{2}.$$

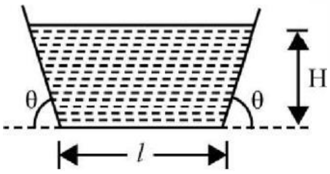
$$\text{Vertical force} = \int dF\cos\theta = \int_0^{\frac{H}{\sin\theta}} pbg\sin\theta\cos\theta\, xdx$$

$$= pbg\sin\theta\cos\theta\left(\frac{x^2}{2}\right)_0^{\frac{H}{\sin\theta}}$$

$$= \frac{pbg\sin\theta\cos\theta}{2}\left[\frac{H^2}{\sin^2\theta}\right]$$

$$\frac{p\,gbH^2\cot\theta}{2}.$$

57. The tank shown in figure has the base area (l × b). It is filled with a liquid of density p to a height H. Choose the correct alternative (s).



(a) The force at the base of the container is,
F = 1/2pgH (l + 2H cot θ) b.

(b) The horizontal component of the force acting on the inclined wall is $F_h = 1/2\ pgH^2b$

(c) The vertical component of the force acting on the inclined wall is $F_v = \frac{1}{2}pgbH^2 \operatorname{cosec}\theta$.

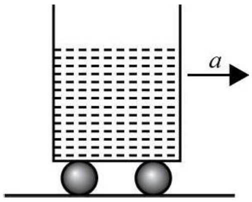
(d) The vertical component of the force acting on the inclined wall is $F_v = \frac{1}{2}pgbH^2\cot\theta$.

Ans. (b,d)

Sol. Pressure at base = pgH

Force at base = $pghlb$

58. An open vessel containing liquid is moving with constant acceleration a on a levelled horizontal surface. For this situation mark out the correct statement(s).



(a) The maximum pressure is at the leftmost bottom corner.

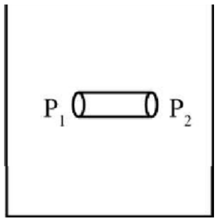
(b) Along a horizontal line within the liquid as we move from left to right the pressure decreases.

(c) The pressure at all points on a line making an angle of $\tan^{-1}\left(\frac{a}{g}\right)$ with horizontal would be same.

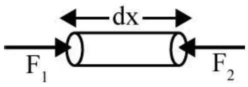
(d) Along a horizontal line within the liquid as we move from left to right, the pressure remains same.

Ans. (a,b,c)

Sol. Consider a cylindrical element as shown.



$F_1 - F_2 = ma$



$\Rightarrow P_1A - P_2A = \rho(Adx)a \Rightarrow P_1 - P_2 = \rho a dx$

$So, \frac{dp}{dx} = \rho a$

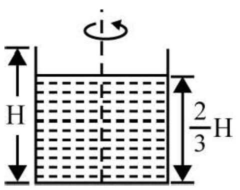
$Also \frac{dp}{dy} = \rho g$

So if dp is same then

$\frac{dy}{dx} = \frac{\rho a}{\rho g} = \frac{a}{g}$

i.e. pressure would be constant along line having slope $\tan^{-1}\left(\frac{a}{g}\right)$ with horizontal.

59. A circular cylinder of radius r and height H is filled with water to a height 2/3 H. It starts rotating about its axis with constantly increasing angular speed. Choose the correct alternatives.



(a) At all speeds, shape of the free surface is parabolloid.
(b) The free surface touches first the brim of cylinder and then the base of the cylinder.
(c) The free surface cannot touch the base without spilling water
(d) The free surface touches the brim as well as base at the same instant.

Ans. (a,b,c)

Sol. When the container is rotated then

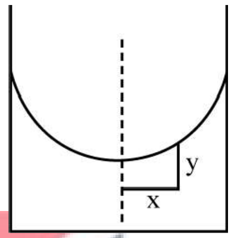
$\frac{dp}{dx} = \rho w^2$

$\frac{dp}{dy} = \rho g$

$\Rightarrow \frac{dy}{dx} = \frac{w^2x}{g}$

$\Rightarrow y = \frac{w^2x^2}{2g} + C$

So shape be parabolloid as shown



Also parabolloid first touch brim and then base are centre of curve is along +y. Thus, base won't be touched unless water is spilled out.

60. Water flows steadily through a horizontal pipe of a variable cross-section. If the pressure of water is P at a point where the velocity of flow is v, what is the pressure at another point where the velocity of flow is 2v; ρ being the density of water ?

(a) $P - \frac{3}{2}\rho v^2$ (b) $P + \frac{3}{2}\rho v^2$

(c) $P - 2\rho v^2$ (d) $P + 2\rho v^2$

Ans. (a)

Sol. According to Bernoulli's theorem

$P + \rho gh + \frac{\rho v^2}{2} = coust$

So since height is same b/w the points hence.

$P + \frac{1}{2}\rho v^2 = p^1 + \frac{1}{2}\rho(2v)2 \Rightarrow p^1 = p - \frac{3\rho v^2}{2}.$

$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$

$P + \frac{1}{2}\rho V^2 = P_2 + \frac{1}{2}\rho(2V)^2$

$P + \frac{1}{2}\rho V^2 - \frac{1}{2}\rho \times 4V^2 = P_2$

$P_2 = P - \frac{3}{2}\rho V^2$

61. If the velocity head of a stream of water is equal to 10 cm then its speed of flow is approximately

(a) 1.0 m/s (b) 1.4 m/s

(c) 140 m/s (d) 10 m/s

Ans. (b)

Sol. Velocity head = 10 cm = 0.1 m

Using the formula.

$$\text{Velocity head} = \frac{v^2}{2g}$$

v^2 = Velocity head × 2g

$v^2 = 0.1 \times 2 \times 9.8$

$$v^2 = \frac{196}{100}$$

$$v = \frac{14}{10}$$

v = 1.4 m/s

62. A tank is filled to a height H. The range of water coming out of a hole which is a depth H/4 from the surface of water level is

(a) $\frac{2H}{\sqrt{3}}$ (b) $\frac{\sqrt{3}H}{2}$

(c) $\sqrt{3}H$ (d) $\frac{3H}{4}$

Ans. (b)

Sol. Horizontal Range = $2\sqrt{h.h'}$

h = depth of the hole below the free surface of the liquid

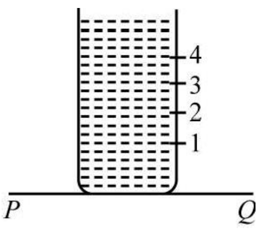
h^1 = height of the hole above the bottom of the tank

Here, h = H/4

$$h' = H - \frac{H}{4} = \frac{3H}{4}$$

$$\text{Ramge} = 2\sqrt{\frac{H}{4} \times \frac{3H}{4}} = \frac{\sqrt{3}H}{2}$$

63. A cylindrical vessel of 90 cm height is kept filled up to the brim. It has four holes 1, 2, 3 and 4 which are, respectively, at height of 20 cm, 30 cm, 40 cm and 50 cm from the horizontal floor PQ. The water falling at the maximum horizontal distance from the vessel comes from

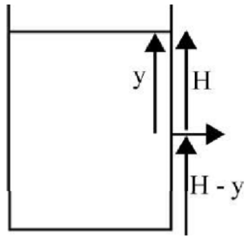


(a) hole number 4 (b) hole number 3

(c) hole number 2 (d) hole number 1

Ans. (a,b)

Sol. $Range = 2\sqrt{y(H - y)}$



$Hole\ 1\ R = 2\sqrt{20 \times 70} = 20\sqrt{14}\ cm$

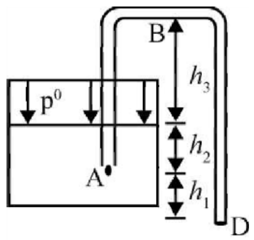
$Hole\ 2\ R = 2\sqrt{30 \times 60} = 20\sqrt{18}\ cm$

$Hole\ 3\ R = 2\sqrt{40 \times 50} = 20\sqrt{20}\ cm$

$Hole\ 4\ R = 2\sqrt{50 \times 40} = 20\sqrt{20}\ cm$

So $R_4 = R_3$ = maximum

64. The figure shows a siphon tube removing liquid from a container. Choose the correct statements.



(a) The siphon tube removes liquid only when $h_1 > 0$

(b) The velocity of flow is $v = \sqrt{2g(h_1 + h_2)}$

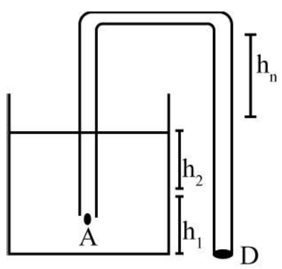
(c) The pressure at the point B is $p_B = p_0 - \rho g h_3$

(d) The pressure at the point D is p_0

Ans. (b,d)

Sol. Point D is exposed to atmosphere so $P_D = P_0$

Applying bernoulli's theorem between A and D we can write [considering reference point for PE to be D.]



$P + pgh + \frac{1}{2}pv^2 = \text{Constant}$

$(P_0 + pgh_2) + \frac{1}{2}pV_2^2 + pgh_1 = P_0 + \frac{1}{2}pV^2$

$\Rightarrow \frac{1}{2}pV^2 = pg(h_1 + h_2)$

$\Rightarrow v = \sqrt{2g(h_1 + h_2)}$

For $v > 0 \Rightarrow h_1 + h_2 > 0 \Rightarrow h_1 > -h_2$

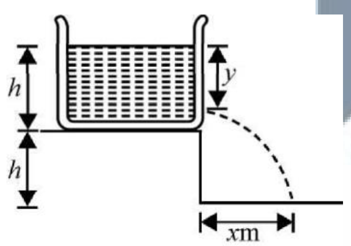
Applying bernoulli's theorem b/w A and D we can write.

$P_0 + pgh_2 = P_B + \frac{1}{2}pV^2 + pgh_3$

$\Rightarrow P_0 + pgh_2 = P_B + \frac{1}{2}p(2g(h_1 + h_2)) + pgh_3$

$\Rightarrow P_B = P_0 - (pgh_1 + pgh_3)$

65. A tank is filled upto a height h with a liquid and is placed on a platform of height h from the ground. To get maximum range x_m a small hole is punched at a distance of y from the free surface of the liquid. Then



(a) $x_m = 2h$ (b) $x_m = 1.5h$

(c) $y = h$ (d) $y = 0.75h$

Ans. (a,c)

Sol. Velocity of liquid jet coming out $= \sqrt{2gy}$

$x = \sqrt{2gy} \times T = \sqrt{2gy} \times \sqrt{\frac{2(2h - y)}{g}}$

$\Rightarrow x = 2\sqrt{y(2h - y)}$

For x to be maximum y (2h - y) should be maximum.

i.e. $\frac{d}{dy}(y(2h - y)) = 0$

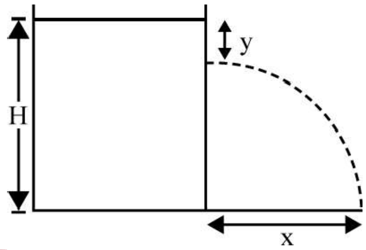
$\Rightarrow \frac{d}{dy}(2hy - y^2) = 0$

$\Rightarrow 2h - 2y = 0$

$\Rightarrow y = h$

$x_m = 2\sqrt{h(2h - h)} = 2h$

66. A cylindrical vessel is filled with a liquid up to a height H. A small hole is made in the vessel at a distance y below, the liquid surface as shown in figure. The liquid emerging from the hole strike the ground at distance x



(a) x is equal if hole is at depth y or H – y

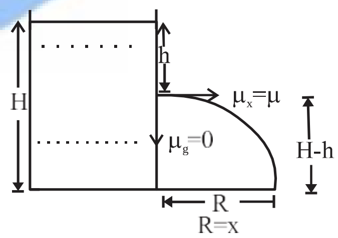
(b) x is maximum for $y = \frac{H}{2}$

(c) Both (a) and (b) are correct

(d) Both (c) and (d) are wrong

Ans. (c)

Sol.



Vertical:-

$y_f - y_l = u_g t + \frac{1}{2}a_y t^2$

$O - (H - h) = 0(t) + \frac{1}{2}(-g)t^2$

$t = \sqrt{\frac{2(H - h)}{g}}$

Horizontal Range:

$$x = ut$$

$$= \sqrt{2gh}\sqrt{\frac{(H-h)}{g}}$$

$$x = 2\sqrt{h(H-h)}$$

Range become maximum

$x \rightarrow$ max.

$h(H-h) \rightarrow$ max.

$$\frac{d}{dh}(H-h)h = 0$$

$$\frac{d}{dh} = \left(hH - h^2\right) = 0$$

$$\frac{d}{dh}(hH) - \frac{d}{dh}\left(h^2\right) = 0$$

$H(1) - 2h = 0$

$H = 2h$

$h = H/2$

$$X = 2\sqrt{h(H-h)}$$

$$Put\ h = \frac{H}{2}$$

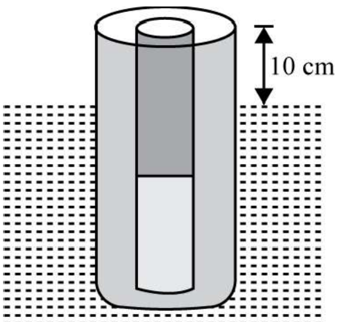
$$X_{max} = 2\sqrt{\frac{H}{2}\left(H - \frac{H}{2}\right)}$$

$$X_{max} = 2\sqrt{\frac{H}{2} \times \frac{H}{2}}$$

$X_{max} = H.$

Numberic Value Type Questions

67. A tube with both ends open floats vertically in water. Oil with a density 800 kg/m^3 is poured into the tube. The tube is filled with oil upto the top end while in equilibrium. The length of the tube outside the water is 10 cm. Determine the depth (in cm) upto which the oil will be filled in tube.



Ans. (0050)

Sol.
$$h_{oil} = \frac{S_{water}}{S_{oil}} h_{water}$$

$$h_0 = \frac{S_w}{S_0} h_w$$

$$= \frac{800}{1000} \times (h_0 - 0.1)$$

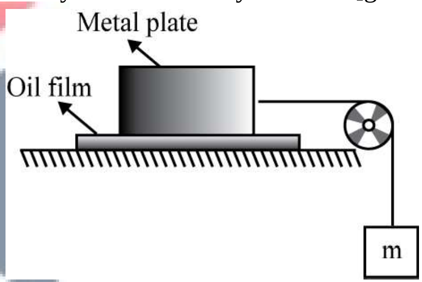
$h_0 = 0.8\,h_0 - 0.08$

$h_0 = 0.4$

depth = 0.4 + 0.1 = 0.5m

= 50 cm

68. A rectangular metal plate has dimensions of 10 cm × 20cm. A thin film of oil separates the plate from a fixed horizontal surface. The separation between the rectangular plate and the horizontal surface is 0.2 mm. An ideal string is attached to the plate and passes over an ideal pulley to a mass m. When m = 125 g, the metal plate moves at constant speed of 5 cm/s across the horizontal surface. Find the coefficient of viscosity of oil in 10^{-1} dyne-s/cm^2. [g = 10 m/s^2]



Ans. (0025)

Sol. As plate moves with constant velocity, the net force acting on plate is zero.

For block of mass m, mg = T

For plate T = F_v (F_v = viscous force)

$$\eta = \frac{mg}{A\frac{dv}{dy}} = \frac{125 \times 10 \times 100}{10 \times 20 \times 250}$$

= 2.5 dyne-s/cm^2

= 25 × 10^{-1} dyne -s/cm^2

69. What is the excess pressure inside a bubble of soap solution s of radius 5.00 mm ? Given that the surface tension of soap solution at the temperature (20°C) is 2.50 × 10^{-2} N/m. If an air bubble of the same dimension were formed at a depth of 4.0 cm inside a container containing soap solution (relative density 1.20), what would be the pressure inside the bubble (in 10^{-2} atm) ? (1 atm = 1.01 × 10^5 Pa)

Ans. 100.4

Sol. Excess pressure inside soap bubble,

$$P = \frac{4S}{R} = \frac{4 \times 2.5 \times 10^{-2}}{5 \times 10^{-3}}$$

$\Rightarrow P = 20\,Pa$

Excess pressure inside air bubble,

$$P' = \frac{2S}{R} = \frac{2 \times 2.5\times10^{-2}}{5\times10^{-3}} = 10\,Pa$$

Total pressure = $P' + h\rho g + p_{atm}$
= 10 + (1.01 × 10^5) + (0.04 × 1.2 × 10^3 × 9.8)

= 1.014 × 10^5 Pa

ASSERTION & REASON

(A) Statement I is true, Statement II is true and Statement II is a correct explanation for Statemetn I.

(B) Statement I is true, Statement II is true and Statement II is NOT the correct explanation for Statement I.

(C) Statement I is true, Statement II is false.

(D) Statement I is false, Statement II is true.

70. **Statement I :** Pascal's law is the working principal of a hydraulic lift.

Statement II : Pressure is equal to thrust acting per unit area.

(a) A (b) B

(c) C (d) D

Ans. (b)

Sol. According pascal's law, if pressure is increased is at one point, same pressure is increased at each and every point.

71. **Statement I :** To float, a body must displace liquid whose weight is greater than the actual weight of the body.

Statement II : The body will experience no net downward force, in the case of floating.

(a) A (b) B

(c) C (d) D

Ans. (c)

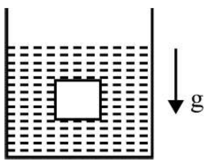
Sol. Buoyant force (F_B) = weight of liquid displaced.

For a body to float $F_B \geq$ weight of body

Body experiences net downword force when it is floating

72. **Statement I :** A block is immersed in a liquid inside a beaker, which is falling freely. Buoyant force acting on block is zero.

Statement II : In case of freely falling liquid there is no pressure difference between any two points.



(a) A (b) B

(c) C (d) D

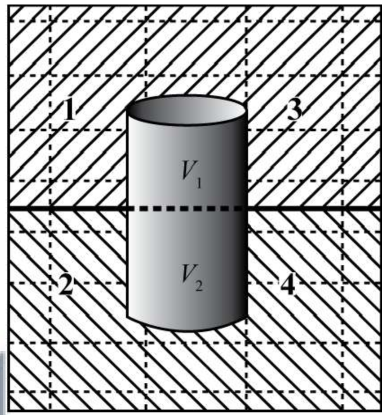
Ans. (a)

Sol. $\Delta P = (\rho\, g_{eff}\, h)$

For free fall $g_{eff} = 0$ hence no pressure difference.

73. **Statement I :** When a body floats such that its parts are immersed into two immiscible liquids then force exerted by liquid I is of magnitude $\rho_1 v_1 g$.

Statement II : Total buoyant force = $\rho_1 v_1 g + \rho_2 v_2 g$.

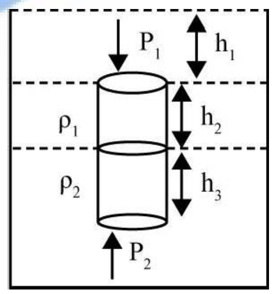


(a) A (b) B

(c) C (d) D

Ans. (d)

Sol. $F_B = (p_2 - p_1)A$



$= [\rho_1 g(h_1 + h_2) + \rho_2 g(h_3)] - [\rho_1 g h_1]A$
$= \rho_1 g h_2 A + \rho_2 g h_3 A$
$= \rho_1 V_1 g + \rho_2 v_2 g$

74. **Statement I :** All the raindrops hit the surface of the earth with the same constant velocity.

Statement II : An object falling through a viscous medium eventually attains a terminal velocity.

(a) A (b) B

(c) C (d) D

Ans. (d)

Sol. All rain drops are of different mass and hence attain different speeds.

Match the Column

75. Match the column I and column II -

	Column-I		**Column-II**
(A)	If the radius of soap bubble A is fource times that of another soap bubble B, then the ratio of excess pressuren (P_B/P_A) will be	(P)	16 : 9
(B)	If two small drops of mercury, each of radius R coalesce to form a single large drop, the ratio of the total surface energy before and after change will be	(Q)	$2^{1/3}:1$
(C)	The enrgy required to blow a bubble of radius 4 cm and 3cm in the same liquid is in the ratio of	(R)	4 : 1
(D)	Two soap bubbles are blown. In the first bubble excess pressure in 4 times that of the second soap bubble. The ratio of radii of first to second soap bubble.	(S)	1 : 4

Ans. (A→ R); (B → Q);(C → P);(D → S)

Sol. (A) $P_{ext} \propto \frac{1}{r}$

So $\frac{P_B}{P_A} = \frac{r_A}{r_B} = \frac{4}{1}$

(B) $2\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$

$\Rightarrow 2r^3 = R^3$

Surface energy = T(surface area)

Where T = surface tension.

$$so\ \frac{U_i}{U_f} = \frac{T\left(2\times 4\pi r^2\right)}{\left(4\pi R^2\right)} = \frac{2r^2}{R^2} = \frac{2}{(2)^{\frac{2}{3}}} = \frac{2^{\frac{1}{3}}}{1}$$

(c) Energy = TΔS $\Rightarrow$ E $\propto$ r^2

$$So\ \frac{E_4}{E_3} = T\frac{(4)^2}{(3)^2} = \frac{16}{9}$$

(D) $P_{excess} \propto \frac{1}{r} \Rightarrow \frac{P_{1st}}{P_{2nd}} = \frac{r_{end}}{r_{1st}} = 4 \Rightarrow \frac{r_{1st}}{r_{2nd}} = \frac{1}{4}$.