Physics

Question: If f is the degree of freedom, then $\dfrac{C_P}{C_V}$ for Ideal gas is

Options:

(a) $\gamma = 1 + \dfrac{2}{f}$

(b) $\gamma = 1 + \dfrac{4}{f}$

(c) $\gamma = 1 + \dfrac{6}{f}$

(d) None of these

Answer: (a)

Solution:

We have

$C_V = \dfrac{f}{2}R$ and $C_P - C_V = R$

$C_P - \dfrac{f}{2}R = R$

$C_P = R + \dfrac{f}{2}R$

$C_P = R\left(1 + \dfrac{f}{2}\right)$

Hence,

$\gamma = \dfrac{C_P}{C_V} = \dfrac{R\left(1 + \dfrac{f}{2}\right)}{R\dfrac{f}{2}} = 1 + \dfrac{2}{f}$

$\gamma = 1 + \dfrac{2}{f}$

Question: The graph of $\ln\left(\dfrac{A}{A_0}\right)$ with time is

Options:

(a) $-\lambda t$

(b) λt

(c) λt^2

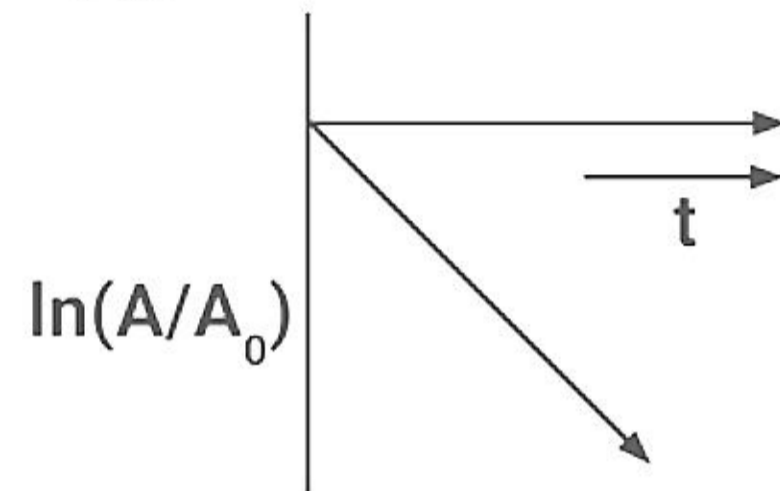(d) $\lambda^2 t^2$

Answer: (a)

Solution:

We know

$A = A_0 e^{-\lambda t}$

$\dfrac{A}{A_0} = e^{-\lambda t}$

Taking log both the sides

$\ell\left(\dfrac{A}{A_0}\right) = \ell n\left(e^{-\lambda t}\right)$

$\ell n\left(\dfrac{A}{A_0}\right) = -\lambda t$

Question: A parallel plate capacitor has a distance of d between the plates. Now a metal sheet of thickness $\dfrac{d}{2}$ is inserted in between. The ratios of new to dd capacitance is?

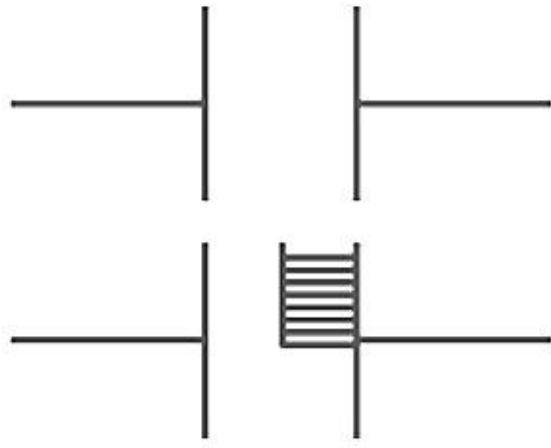**Options:**
(a) 3 : 2
(b) 4 : 7
(c) 2 : 1
(d) 1 : 2

Answer: (c)

Solution:

$C_0 = \dfrac{\varepsilon_0 A}{d}$

$C_1 = \dfrac{\varepsilon_0 A}{d/2}$

$C_1 = 2C$

$\dfrac{C_1}{C} = 2$

Question: R = Radius of nucleus, R_0 = Nuclear constant and A = Mass No. of Nuclic, then graph of θ $\ell\left(\dfrac{R}{R_0}\right)$ with $\ell n A$ is

Options:

(a) $\dfrac{1}{2}\ell A$

(b) $\dfrac{1}{3}\ell A$

(c) $\dfrac{1}{8}\ell A$

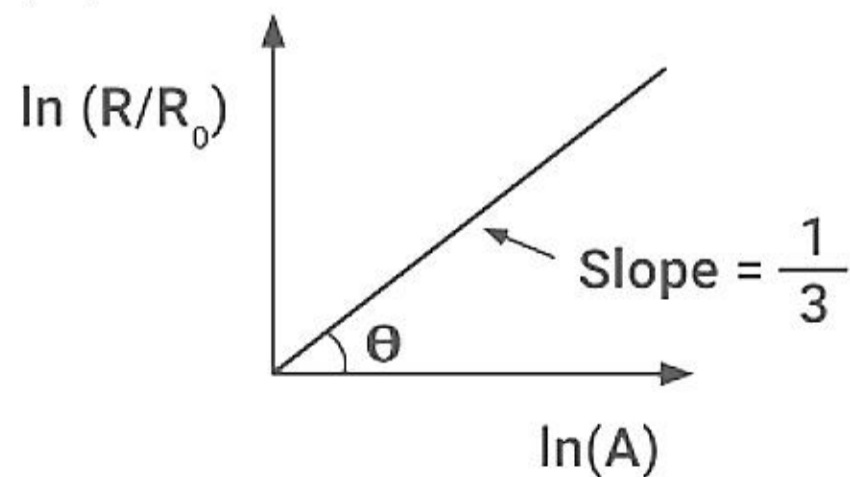(d) None of these

Answer: (b)

Solution:

We know,

$R = R_0 A^{1/3}$

$\dfrac{R}{R_0} = A^{1/3}$

Taking log both the sides

$\ell\left(\dfrac{R}{R_0}\right) = \ell n\left(A^{1/3}\right)$

$\ell\left(\dfrac{R}{R_0}\right) = \dfrac{1}{3}\ell A$

Question: Two satellites revolve around a planet, with radius 3200 km and 800 km what is their ratio of orbital speeds?

Options:

(a) 2 : 3

(b) 2 : 1

(c) 3 : 2

(d) 1 : 2

Answer: (d)

Solution:

$$v = \sqrt{\frac{GM}{r}}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{800}{3200}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Question: Find MOI of following

(A) Solid sphere about diameter

(B) DISC about axis $\perp$ to plane and passing through center.

(C) HOLLOW cylinder about it axis

(D) RING about axis $\perp$ to plane and passing through center

Options:

(a) $A = \frac{1}{2}mR^2$, $B = mR^2$, $C = \frac{mR^2}{2}$, $D = \frac{2}{5}mR^2$

(b) $A = mR^2$, $B = mR^2$, $C = \frac{mR^2}{2}$, $D = \frac{2}{5}mR^2$

(c) $A = \frac{2}{5}mR^2$, $B = \frac{mR^2}{2}$, $C = mR^2$, $D = mR^2$

(d) None of these

Answer: (c)

Solution:

(A) Moment of inertial solid sphere about its diameter $= \frac{2}{5}mR^2$

(B) Moment of inertia of disc about an axis perpendicular to the plane and passing through the center $= \frac{mR^2}{2}$

(C) Moment of inertia of hollow cylinder about its axis $= mR^2$

(D) Moment of inertia of ring about an axis perpendicular to the plane R passing through center $= mR^2$

Question: If electron, Deutron and Proton and alpha particle are accelerated from rest same $p \cdot d$, then find the descending order of wavelengths

Options:

(a) $\lambda_D > \lambda_e > \lambda_\alpha > \lambda_p$

(b) $\lambda_e > \lambda_p > \lambda_D > \lambda_\alpha$

(c) $\lambda_p > \lambda_e > \lambda_\alpha > \lambda_D$

(d) None of these

Answer: (b)

Solution:

$$E = \frac{hc}{\lambda}$$

$$qV = \frac{hv}{\lambda}$$

$$\lambda = \frac{hc}{qV}$$

$v \rightarrow$ constant, $hv \rightarrow$ constant

$$\tau \propto \frac{1}{9}$$

So the sequence in decreasing order of wavelengths

Question: If the metal has threshold frequency of 5×10^{14} Hz then find out the work function of the metal

Options:

(a) 2.06 ev

(b) 3 ev

(c) 4.04 ev

(d) 3.25 ev

Answer: (a)

Solution:

Work function $(\phi) = h v_0$

$$= \frac{6.67 \times 10^{-34} \times 5 \times 10^4}{1.6 \times 10^{-19}} ev$$

$$= 2.06 ev$$

Question: If $X_a = at + bt^2$ and $X_b = at^2 + bt$ then at what time their velocities will be equal?

Options:

(a) $t = \dfrac{2(a+b)}{a+b}$

(b) $t = \dfrac{2(a+b)}{2a+3b}$

(c) $t = \dfrac{a+b}{2(a+b)}$

(d) none of these

Answer: (c)

Solution:

$$X_a = at + bt^2 \Rightarrow V_a = a + 2bt$$

$$X_b = at^2 - bt \Rightarrow V_b = 2at - b$$

Δ / θ

$$V_a = V_b$$

$$a + 2bt = 2at - b$$

$$a + b = (2a - 2b)t$$

$$t = \frac{a+b}{2(a+b)}$$

Question: A cube of surface area 24 cm^2, heated by 10°C which has coefficient of linear expansion of 5×10^{-4}. Then the change in volume is?

Options:

(a) 0.02 cc

(b) 0.12 cc

(c) 0.01 cc

(d) 0.11 cc

Answer: (b)

Solution:

$$\Delta V = V \times 3\alpha\Delta T$$

$$= 0 \times 10^{-6} \times 3 \times 5 \times 10^{-4} \times 10$$

$$= 120 \times 10^{-10+1}$$

$$= 120 \times 10^{-9}$$

$$= 0.12 \times 11^{-6}$$

$$= 0.12 cc$$

Question: The magnetic field at centre of solenoid is B_0. When no. of turns is halved & current is doubled, what is the new field?

Options:

(a) $2B_0$

(b) B_0

(c) $B_0/2$

(d) $3B_0$

Answer: (b)

Solution:

Initially

$$B_0 = \mu_0 n_1 i_1$$

No. of turns is now halved,
&current is doubled.

$$\therefore B' = \mu_0 n_1 (2) . \frac{i_1}{2}$$

$$= \mu_0 n_2 i_1 = B_0$$

Question: If velocity of E.M wave is 2 x 10^8 ms^{-1} in a medium where $\mu_r = 1$, then find $\in_r$?

Options:

(a) $\dfrac{9}{4}$

(b) $\dfrac{8}{5}$

(c) $\dfrac{8}{9}$

(d) $\dfrac{9}{2}$

Answer: (a)

Solution:

$$\dfrac{v}{c} = \dfrac{1}{\sqrt{\mu_r \in_r}}$$

$$\Rightarrow \in_r = \dfrac{c^2}{v^2 \mu_r} = \dfrac{\left(3 \times 10^8\right)^2}{\left(2 \times 10^8\right)^2 \times 1} = \dfrac{9}{4}$$

Question: Two cells of same *emf* have internal resistance r_1 & r_2 are in series with load R. If potential difference across the 2nd cell is zero, R= ?
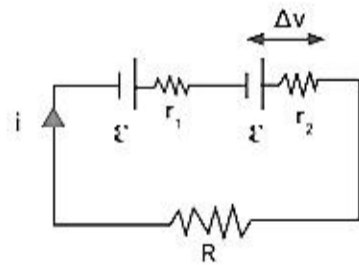
Options:

(a) $R = |r_1 - r_2|$

(b) $R = |r_2 + r_1|$

(c) $R = |r_1 + r_2|$

(d) $R = |r_2 - r_1|$

Answer: (d)

Solution:

$$i = \dfrac{2\varepsilon}{R + r_1 + r_2}$$

$$\Delta v = \varepsilon - i r_2$$

$$0 = \varepsilon - \dfrac{2\varepsilon}{R + r_1 + r_2} r_2$$

$$1 = \dfrac{2 r_2}{R + r_1 + r_2}$$

$$R + r_1 + r_2 = 2 r_2$$

$$R = |r_2 - r_1|$$

Question: Assertion: Two balls are projected such mat may have same range R & have maximum heights H_1 & H_2 respectively man $R = H\sqrt{H_1 H_2}$

Reason: $H_1 = \dfrac{u^2 \sin^2 \theta}{2g}$

$H_2 = \dfrac{u^2 \cos^2 \theta}{2g}$

Options:

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true, but the reason is not the correct explanation of the assertion.

(c) If assertion is true, but reason is false.

(d) If both the assertion and reason are false.

Answer: (a)
Solution:

$R = \dfrac{u^2 \sin^2 \theta}{g}$

Range is same For θ & $(90° - \theta)$

Height $H_1 = \dfrac{u^2 \sin^2 \theta}{2g}$

For $(90 - \theta), H_2 = \dfrac{u^2 \cos^2 \theta}{2g}$

$\sqrt{H_1 H_2} = \dfrac{u^2 \sin\theta \cos\theta}{2g}$

$4\sqrt{H_1 H_2} = \dfrac{2u^2 \sin\theta \cos\theta}{g} = \dfrac{u^2 \sin 2\varepsilon}{g}$

$= R$

Question: For a particle moving in XY plane in UCM speed v and radius r, the acceleration vector at any (r, θ) is?

Options:

(a) $-\cos\theta \hat{i} + \sin\theta \hat{j}$

(b) $-\cos\theta \hat{i} - \sin\theta \hat{j}$

(c) $+\cos\theta \hat{i} - \sin\theta \hat{j}$

(d) $\cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

Answer: (b)

Solution:

Position vector at any instant 't'

$$\vec{r} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$$

$$\therefore \vec{v} = -R\omega\sin\theta\,\hat{i} + R\omega\cos\theta\,\hat{j}$$

$$\& \vec{a} = -R\omega^2\cos\theta\,\hat{i} - R\omega^2\sin\theta\,\hat{j}$$

Since $v = \omega R$ in UCM

$$\vec{a} = \frac{v^2}{R}\left(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right)$$

Question: 27 drops each of 22 volts each combined together to form abig drop, its potential is

Answer: (198)

Solution:

$$27 \times \frac{4\pi r^3}{3} = \frac{4}{3}\pi R^3$$

$$R = 3r$$

$$22 = \frac{KQ}{r}$$

$$V = k \times \frac{27Q}{R}$$

$$= k \times \frac{27Q}{3r}$$

$$= 9\frac{KQ}{r} = 9 \times 22$$

$$V = 198 \text{ Volts}$$

Question: If $\dfrac{I_1}{I_2} = \dfrac{1}{4}$ then $\dfrac{I_{max} + I_{min}}{I_{max} - I_{min}} = \dfrac{5}{x}$ fine value x?

Answer: (4)

Solution:

$$\frac{I_1}{I_2} = \frac{1}{4} \Rightarrow I_2 = 4I_1$$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

$$= \frac{\left(\sqrt{I_1} + \sqrt{4I_1}\right)^2}{\left(\sqrt{I_1} - \sqrt{4I_1}\right)^2} = \frac{9}{1}$$

$$\Rightarrow I_{max} = 9 I_{min}$$

$$\therefore \frac{I_{max} + I_{min}}{I_{max} - I_{min}} = \frac{9+1}{9-1} = \frac{10}{8}$$

$$= 5/4$$

$$\therefore x = 4$$

Question: when temperature is increased the susceptibility of para and ferro materials will?

Answer: ($x \propto \dfrac{1}{T - T_C}$)

Solution:

Para: $x \propto \dfrac{1}{T}$

Ferro turns to para after curie's temp. and then $x \propto \dfrac{1}{T - T_C}$

Question: Current electricity characteristics of npn transistor is given as below. If input resistance is 60 Ohm, Output resistance is 200 Ohm find voltage gain
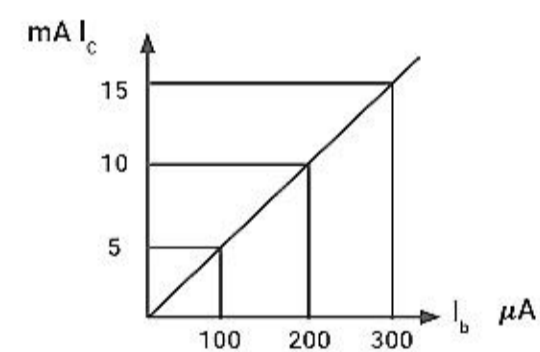
Answer: (500/3)

Solution:

From graph,

$I_C = 15 \text{mA}$

$I_B = 300 \mu A$

$R_i = 60 \Omega$

$R_0 = 200 \Omega$

$$\therefore A_v = \frac{I_C R_0}{I_B R_i} = \frac{15 \times 10^{-3} \times 200}{300 \times 10^{-6} \times 60}$$
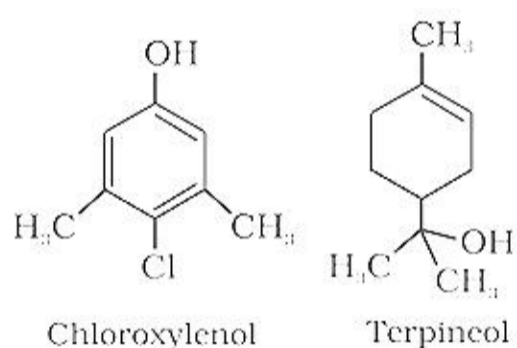
$$= \frac{500}{3}$$

Chemistry

Question: Dettol has 2 structure one has 6 and other has 2 pi electrons. What is name of structure with 2 pi electrons?

Options:

(a) Chloroxylenol

(b) Terpineol

(c) Bithionol

(d) Salvarsan

Answer: (b)

Solution: Dettol is a mixture of chloroxylenol and terpineol. Chloroxylenol has 6 pi electrons and terpineol has 2 pi electrons

Chloroxylenol   Terpineol

Question: Assertion: Water with BOD value 17 is considered polluted

Reason: The amount of oxygen required by the bacteria to break down the inorganic and organic matter in certain volume of water

Options:

(a) Both assertion and reason are true, reason is correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not a correct explanation the assertion.

(c) Assertion is true, but reason is false.

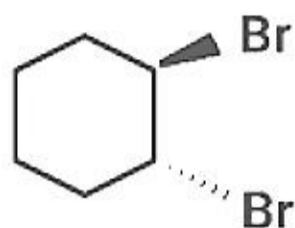(d) Assertion is false, but reason is true.

Answer: (c)

Solution: The amount of oxygen required by bacteria to break down the organic matter present in a certain volume of a sample of water, is called Biochemical Oxygen Demand (BOD). Highly polluted water could have a BOD value of 17 ppm or more.
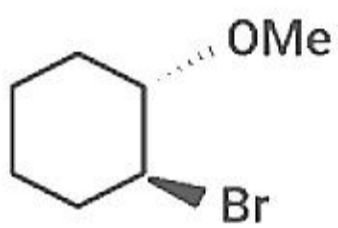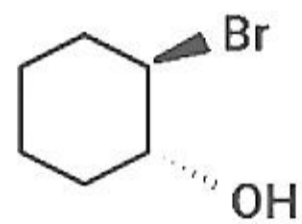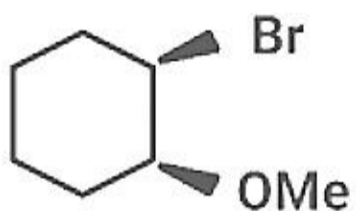
Question:

Cyclohexene $\xrightarrow[\text{CH}_3\text{OH}]{\text{Br}_2}$

Options:

(a)

Br (wedge), Br (dash)

(b)

OMe (dash), Br (wedge)

(c)

Br (wedge), OH (dash)
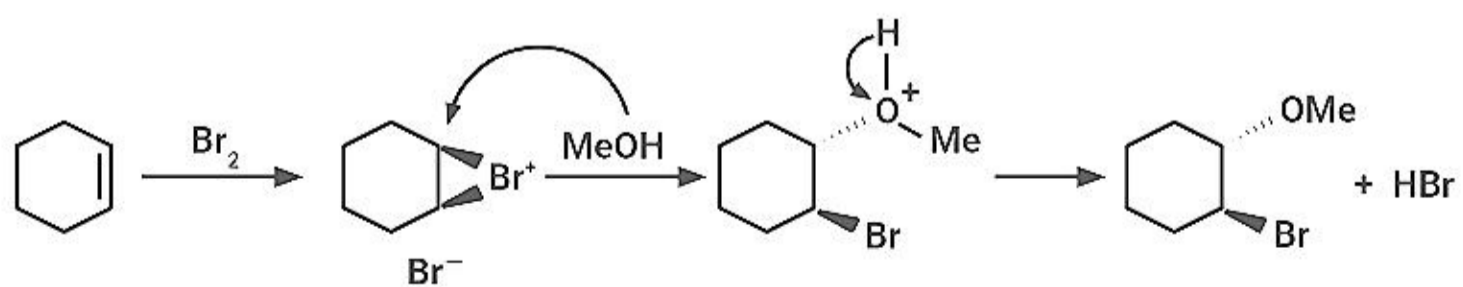
(d)

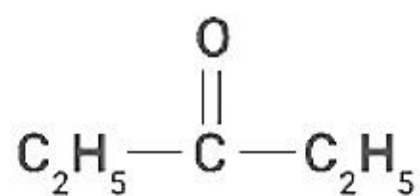Br (wedge), OMe (wedge)
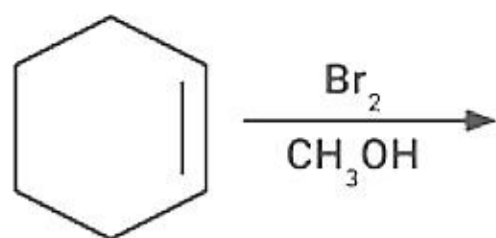
Answer: (b)

Solution:

Question: Which of following does not give enamine on reaction with 2° amine?

Options:

(a)

$$C_2H_5-\overset{\overset{\displaystyle O}{\|}}{C}-C_2H_5$$

(b)

$$+\overset{\overset{\displaystyle O}{\|}}{C}+$$
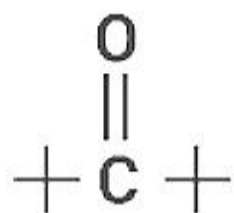
(c)

(d)

Answer: (b)

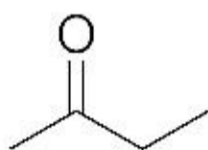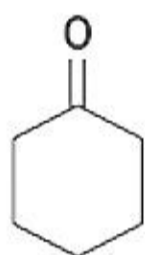**Solution:** Due to absence of alpha hydrogens in this molecule, it will not form enamine on reaction with 2° amine.

Question: Assertion: The amphoteric behavior of H_2O is explained by Lewis's acid base theory

Reason: H_2O acts as acid with NH_3 and base with H_2S

Options:

(a) Both assertion and reason are true, reason is correct explanation of assertion.

(b) Both assertion and reason are true, but reason is not a correct explanation the assertion.

(c) Assertion is true, but reason is false.

(d) Assertion is false, but reason is true.

Answer: (d)

Solution: The amphoteric behavior of H_2O is explained by Bronsted acid base theory and H_2O acts as acid with NH_3 & as a base with H_2S

Question: Which of the following has the least spin only magnetic moment?

Options:

(a) Fe^{3+}

(b) Fe^{2+}

(c) Cu^{2+}

(d) Ni^{2+}

Answer: (c)

Solution: The ion having least number of unpaired electrons will have least spin only magnetic moment. Fe^{3+} has 5 unpaired electrons, Fe^{2+} has 4 unpaired electrons, Cu^{2+} has 1 and Ni^{2+} has 2 unpaired electrons.

Question: What is the correct order of electron gain enthalpy of Cl, F, Te, Po

Options:

(a) Cl > F > Te > Po

(b) F > Cl > Te > Po

(c) F < Cl < Te < Po

(d) Cl < F < Te < Po

Answer: (a)

Solution:

Electron gain enthalpy

F $\Rightarrow$ –333

Cl $\Rightarrow$ –349

Te $\Rightarrow$ –190

Po $\Rightarrow$ –174

Cl > F > Te > Po

Question: Order of reduction potential of Cl_2, I_2, Ag^+, Na^+, Li^+

Options:

(a) $Cl_2 > Li^+ > I_2 > Na^+ > Ag^+$

(b) $I_2 > Li^+ > Cl_2 > Na^+ > Ag^+$

(c) $Na^+ > Li^+ > I_2 > Ag^+ > Cl_2$

(d) $Cl_2 > Ag^+ > I_2 > Na^+ > Li^+$

Answer: (d)

Solution:

Question: Heat of combustion of benzene and acetylene is given as ΔH_1, and ΔH_2, What is the expression for ΔH when acetylene is isomerized to benzene
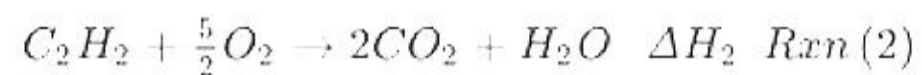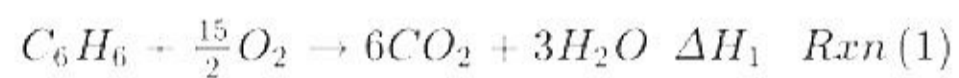
Options:

(a) $\Delta H_2 - \Delta H_1$

(b) $3\Delta H_2 - \Delta H_1$

(c) $\Delta H_2 - 3\Delta H_1$

(d) $3\Delta H_2 + \Delta H_1$

Answer: (b)

Solution:

$$C_6H_6 + \frac{15}{2}O_2 \rightarrow 6CO_2 + 3H_2O \quad \Delta H_1 \quad Rxn\,(1)$$
$$C_2H_2 + \frac{5}{2}O_2 \rightarrow 2CO_2 + H_2O \quad \Delta H_2 \quad Rxn\,(2)$$
$$\overline{3C_2H_2 \rightarrow C_6H_6 \quad \Delta H_3 \quad\quad Rxn\,(3)}$$
$$(3) = 3 \times (2) - (1)$$
$$\therefore \Delta H_3 = 3.\,(\Delta H_2) - \Delta H_1$$

Question: Given K_{sp} of Bi_2S_3. What is formula for S?

Options:

(a) $\left(\dfrac{K_{sp}}{4}\right)^{\frac{1}{2}}$

(b) $\left(\dfrac{K_{sp}}{27}\right)^{\frac{1}{3}}$

(c) $\left(K_{sp}\right)^{\frac{1}{5}}$

(d) $\left(\dfrac{K_{sp}}{108}\right)^{\frac{1}{5}}$

Answer: (d)

Solution:

Question: Why sodium extract of Halide is boiled with HNO_3 before testing for halogen?

Options:

(a) To remove other ions which may interfere in the test

(b) To make solution acidic

(c) To remove turbidity

(d) To convert Fe^{2+} to Fe^{3+}

Answer: (a)

Solution: If nitrogen or sulphur is also present in the compound, the sodium fusion extract is first boiled with concentrated nitric acid to decompose cyanide or sulphide of sodium formed during Lassaigne's test. These ions would otherwise interfere with silver nitrate test for halogens.

Question: Compare Debroglie Wavelength of proton, neutron, electron and alpha particle

Options:

(a) $\lambda_\alpha < \lambda_p \simeq \lambda_n < \lambda_e$

(b) $\lambda_e < \lambda_h \simeq \lambda_p < \lambda_\alpha$

(c) $\lambda_p < \lambda_n \simeq \lambda_\alpha < \lambda_e$

(d) $\lambda_n < \lambda_p \simeq \lambda_c < \lambda_\alpha$
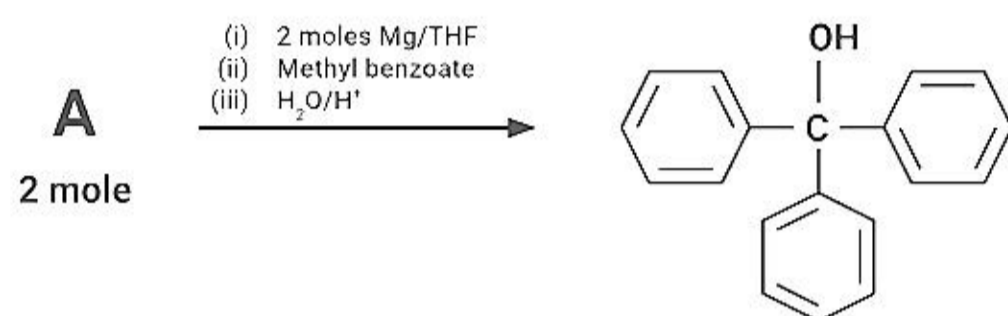
Answer: (a)

Solution:

$$\lambda = \frac{h}{mv}$$

Proton, neutron, Electron, α – particle

$m_{\alpha-\text{particle}} > m_p \simeq n_n > m_e$

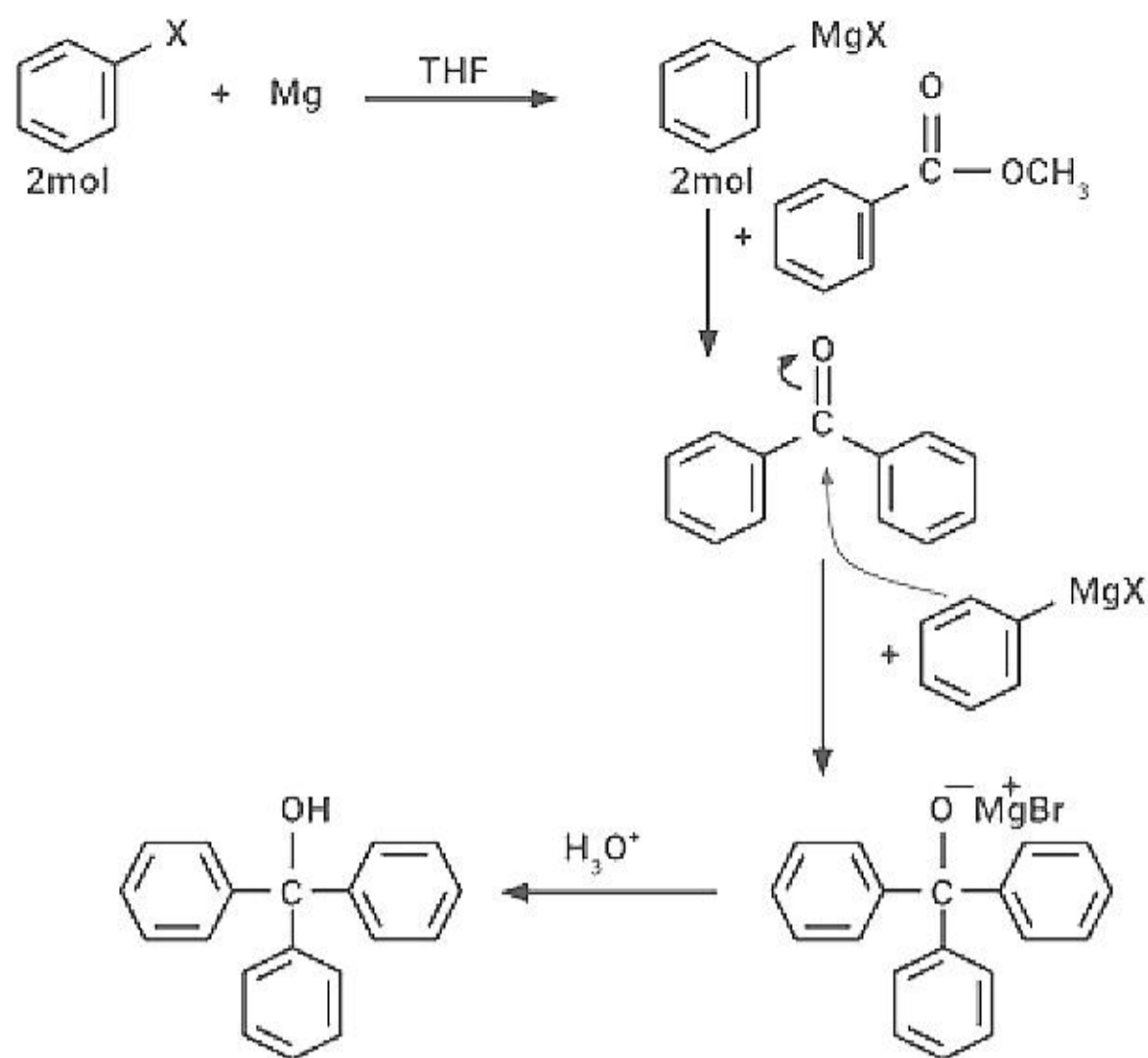$\lambda_{\alpha-\text{particle}} < \lambda_p \simeq \lambda_n < \lambda_e$

Question:

A 2 mole

(i) 2 moles Mg/THF
(ii) Methyl benzoate
(iii) H$_2$O/H$^+$

What is A?

Options:

(a) Bromobenzene

(b) Benzylbromide

(c) Diphenylbromide

(d) Methylbromide

Answer: (a)

Solution:

Question: The minimum energy needed to eject an electron from Pt where threshold frequency was given

Options:

(a) Work function

(b) Kinetic energy

(c) Atomic energy

(d) Mechanical energy

Answer: (a)

Solution:

w_0 (minimum energy to eject electron is work function) = $h\nu_0$ (Threshold frequency)

$h = 6.6 \times 10^{-34}$ J/s

Question: Which of the following give biuret test?

Options:

(a) Glycine

(b) Alanine

(c) both (a) and (b)

(d) None of these

Answer: (d)

Solution: The name of the test comes from the name of the compound, biuret, which gives this test. The reaction is not absolutely specific for peptide bond because many compounds containing two carbonyl groups linked through nitrogen or carbon atoms give a positive result.

Question: $PCl_5 + H_2O \rightarrow A + HCl$

$A + 3H_2O \rightarrow B + HCl$

Find the number of ionizable hydrogen in B

Answer: 3.00

Solution:

$PCl_5 + H_2O \rightarrow POCl_3 + HCl$

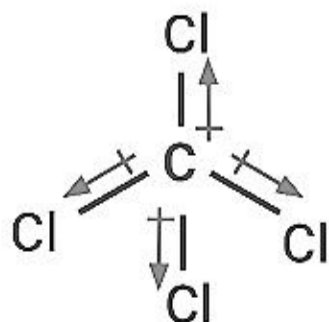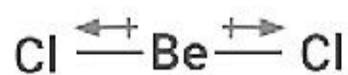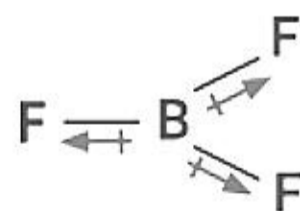$POCl_3 + H_2O \rightarrow H_3PO_4 + HCl$
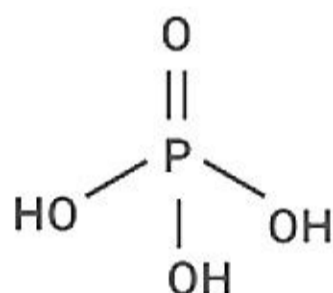
Question: How many of the following species have zero dipole moment?

BF_3, $BeCl_2$, NH_3, H_2O, CCl_4, HCl

Answer: 3.00

Solution: BF_3, $BeCl_2$, CCl_4

Question: Novalac has molar mass 393. Find no of monomers in it.

Answer: 3.00

Solution:
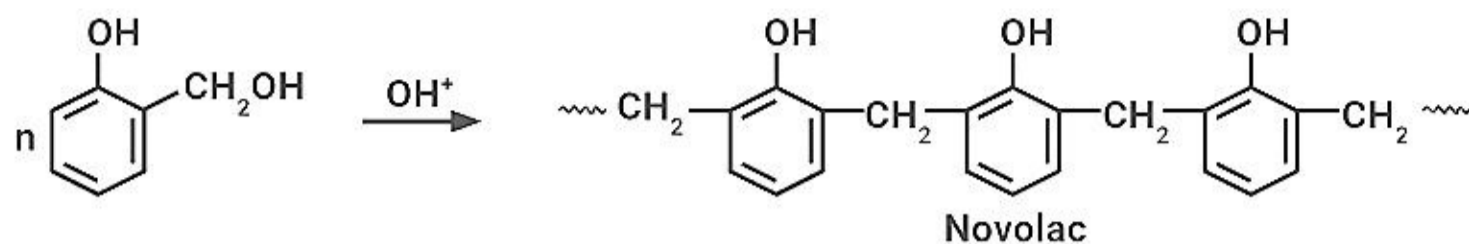
Question: Lead nitrate on heating gives PbO, O_2 and an unknown gas A which dimerises to form B. Find number of bridged O atoms in B

Answer: 0.00

Solution:

$$Pb(NO_3)_2 \xrightarrow{\Delta} PbO + O_2 + \underset{(A)}{NO_2}$$

$$\underset{(A)}{2NO_2} \rightarrow \underset{(B)}{N_2O_4}$$

Number of bridged oxygen atoms in N_2O_4 is 0 (zero)

MATHEMATICS

Question: $A = \{x \in R : |x+1| < 2\}, B = \{x \in R : |x-1| \geq 2\}$ then:
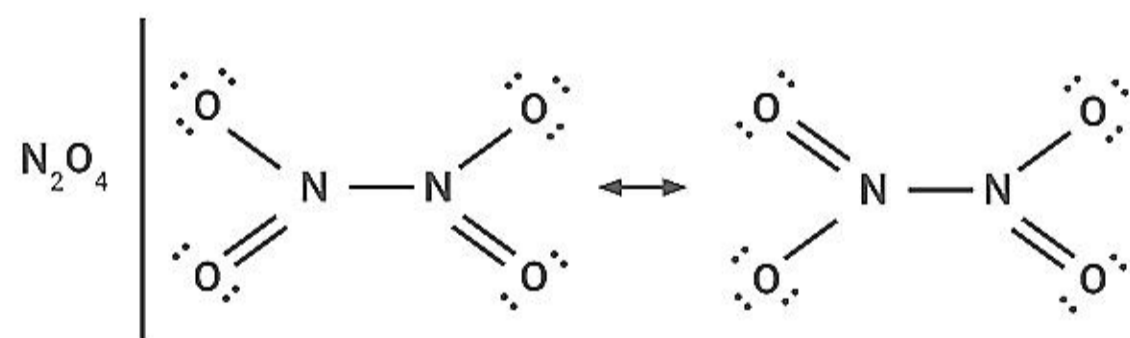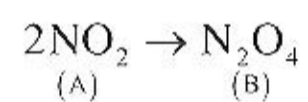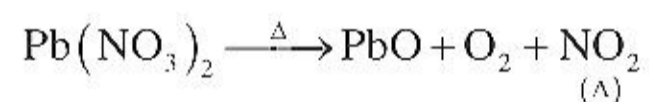
Options:

(a) $A \cup B = R - [1,3]$

(b) $A \cap B = (-1,1)$

(c) $A \cap B = (-3,-1]$

(d) $B - A = R - (-3,1)$

Answer: (c)

Solution:

$A : |x+1| < 2 \Rightarrow x \in (-3,1)$

$B : |x-1| \geq 2 \Rightarrow x \in (-\infty,-1] \cup [3,\infty)$

$\therefore A \cup B = (-\infty,1] \cup [3,\infty)$

$A \cap B = (-3,-1]$

$B - A = (-\infty,-3] \cup [3,\infty)$

Question: $y = y(x)$ is the solution of the differential equation $2x^2 \dfrac{dy}{dx} - 2xy + 8y^2 = 0$, $y(e) = \dfrac{e}{3}$, then $y(1)$ is equal to:

Options:

(a) $\dfrac{2}{3}$

(b) 3

(c) $\dfrac{3}{2}$

(d) -1

Answer: (d)

Solution:

$2x^2 \cdot \dfrac{dy}{dx} - 2xy = -8y^2$

$\Rightarrow \dfrac{dy}{dx} - \dfrac{y}{x} = \dfrac{-4y^2}{x^2}$

$$\frac{-1}{y^2} \cdot \frac{dy}{dx} + \frac{1}{xy} = \frac{4}{x^2}$$

Put $\frac{1}{y} = t$; $\frac{-1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

$$\Rightarrow \frac{dt}{dx} + \frac{t}{x} = \frac{4}{x^2}$$

$$\Rightarrow I.F. = e^{\int \frac{1}{x}dx} = x$$

$$\therefore t(x) = \int \frac{4}{x}dx = 4\ln x + c$$

$$\Rightarrow \frac{x}{y} = 4\ln x + c$$

$$\therefore y(e) = \frac{e}{3} \Rightarrow c = -1$$

$$\therefore \frac{x}{y} = 4\ln x - 1$$

$$\Rightarrow y(1) = -1$$

Question: $y = x+1$ intersect the ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$ at the point P and Q. PQ is the diameter of circle then the value of $(3r)^2$

Answer: 20.00

Solution:
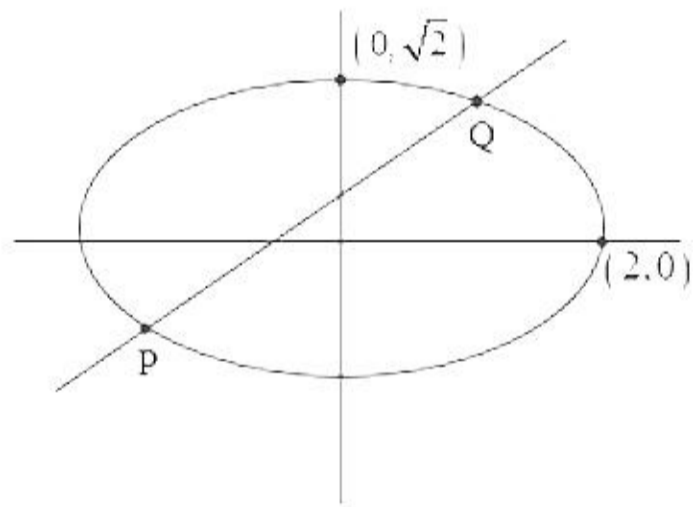
$$\frac{x^2}{4} + \frac{y^2}{2} = 1; \ y = x + 1$$

$$\Rightarrow x^2 + 2(x+1)^2 = 4$$

$$\Rightarrow 3x^2 + 4x - 2 = 0$$

$$x = \frac{-4 \pm \sqrt{16+24}}{6} = \frac{-4 \pm 2\sqrt{10}}{6}$$

$$x_1 = \frac{-2+\sqrt{10}}{3}; \ x_2 = \frac{-2-\sqrt{10}}{3}$$

$$y_1 = \frac{1+\sqrt{10}}{3}; \ y_2 = \frac{1-\sqrt{10}}{3}$$

$$\therefore PQ = 2r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{\frac{40+10}{9}} = \frac{2}{3}\sqrt{20}$$

$$\therefore r = \frac{\sqrt{20}}{3}$$

$$\Rightarrow (3r)^2 = 20$$

Question: $1 + 2.3 + 3.3^2 + ... + 10.3^9 =$

Answer: $\dfrac{1 + 19.3^{10}}{4}$

Solution:

$$s = 1 + 2.3 + 3.3^2 + 4.3^3 + ... + 10.3^9$$

$$-(3s = \qquad 1.3 + 2.3^2 + 3.3^3 + ... + 9.3^9 + 10.3^{10})$$

$$-2s = 1 + 3 + 3^2 + 3^3 + ...3^9 - 10.3^{10}$$
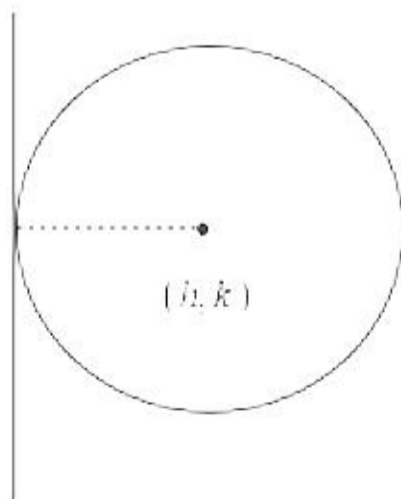
$$= \frac{3^{10} - 1}{2} - 10.3^{10}$$

$$-2s = \frac{-1 - 19.3^{10}}{2}$$

$$\Rightarrow s = \frac{1 + 19.3^{10}}{4}$$

Question: A circle touching y-axis and the line $x + y = 0$ find the locus of the centre.

Answer: $x^2 - y^2 = 2xy$

Solution:

$h = r$
Also, $x + y = 0$ is tangent

$$\therefore \left| \frac{h+k}{\sqrt{2}} \right| = r = h$$

$$x^2 + y^2 + 2xy - 2x^2 = 0$$

$$\Rightarrow x^2 - y^2 - 2xy = 0$$

Question: Find coefficient of x^{101} in $(5+x)^{500} + x.(5+x)^{499} + x^2.(5+x)^{498} + \ldots + x^{500}$

Answer: $^{501}C_{101}(5)^{399}$

Solution:

$$(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + \ldots + x^{500}(x+5)^0$$

$$\Rightarrow (5+x)^{500} \frac{\left[\left(\frac{x}{5+x} \right)^{501} - 1 \right]}{\left(\frac{x}{5+x} - 1 \right)}$$

$$\Rightarrow (x+5)^{500} \left[\frac{(5+x)^{501} - x^{501}}{5(x+5)^{500}} \right]$$

$$\Rightarrow \frac{1}{5} \left[(x+5)^{501} - x^{501} \right]$$

$$\Rightarrow \text{Coeff. of } x^{101} = \frac{1}{5} \cdot {}^{501}C_{101}(5)^{400} = {}^{501}C_{101}(5)^{399}$$

Question:

$$x = 12(1 + \sin t \cos t)$$

$$y = 12(1 + \sin t)^2$$

If at $P(x_0, y_0)$ the tangent makes angle $\frac{\pi}{3}$ with +ve direction of x-axis then $y_0 = ?$

Answer: 3.00

Solution:

$$\frac{dy}{dx} = \sqrt{3} = \frac{12 \times 2(1 + \sin t)\cos t}{12\left(\frac{2\cos 2t}{2} \right)}$$

$$\sqrt{3} \cos 2t = 2\cos t (1 + \sin t)$$

$$\sqrt{3} \cos 2t - \sin 2t = 2\cos t$$

$$\frac{\sqrt{3}}{2} \cos 2t - \frac{1}{2} \sin 2t = \cos t$$

$$\cos\left(2t + \frac{\pi}{6} \right) = \cos t$$

$2t + \dfrac{\pi}{6}$ may be equal to t

$$2t + \frac{\pi}{6} = t$$

$$t = \frac{-\pi}{6}$$

$$y_0 = 12\left(1 - \frac{1}{2}\right)^2 = 3$$

Question: $\displaystyle\lim_{x \to \frac{\pi}{2}} \tan^2 x \left(\left(2\sin^2 x + 4\sin x + 3\right)^{\frac{1}{2}} - \left(\sin^2 x + 6\sin x + 2\right)^{\frac{1}{2}} \right)$

Answer: 0.00

Solution:

Given, $\displaystyle\lim_{x \to \frac{\pi}{2}} \tan^2 x \left(\left(2\sin^2 x + 4\sin x + 3\right)^{\frac{1}{2}} - \left(\sin^2 x + 6\sin x + 2\right)^{\frac{1}{2}} \right)$

$$\lim_{x \to \frac{\pi}{2}} \tan^2 x \left[\sqrt{2\sin^2 x + 4\sin x + 3} - \sqrt{\sin^2 x + 6\sin x + 2} \right]$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}} \tan^2 x \left[\frac{\sin^2 x - 2\sin x + 1}{\sqrt{2\sin^2 x + 4\sin x + 3} + \sqrt{\sin^2 x + 6\sin x + 2}} \right]$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}} \frac{1}{6} \frac{(\sin x - 1)^2}{\cot^2 x}$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}} \frac{2(\sin x - 1).\cos x}{2\cot x \cdot \left(-\text{cosec}^2 x\right)} \cdot \frac{-1}{6}$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}} \frac{-1}{6}(\sin x - 1)\sin^3 x$$

$$\Rightarrow 0$$

Question: Find b if $12 \times \displaystyle\int_3^b \frac{1}{(x^2 - 4)(x^2 - 1)} = \ln\left(\frac{49}{40}\right)$ & $b > 3$.

Answer: 6.00

Solution:

$$12 \times \int_3^b \frac{1}{(x^2 - 4)(x^2 - 1)} = \ln\left(\frac{49}{40}\right)$$

$$4 \times \int\limits_{3}^{b} \left(\frac{1}{x^2-4} - \frac{1}{x^2-1} \right) dx = \ln\left(\frac{49}{40} \right)$$

$$4 \left[\frac{1}{2(2)} \ln\left| \frac{x-2}{x+2} \right| - \frac{1}{2} \ln\left| \frac{x-1}{x+1} \right| \right]_{3}^{b} = \ln\left(\frac{49}{40} \right)$$

$$\ln\left| \frac{b-2}{b+2} \right| - 2\ln\left| \frac{b-1}{b+1} \right| - \ln\frac{1}{5} + 2\ln\frac{1}{2} = \ln\left(\frac{49}{40} \right)$$

As $b > 3$

$$\ln\left[\left(\frac{b-2}{b+2} \right)\left(\frac{b+1}{b-1} \right)^2 \right] = \ln\left(\frac{49}{40} \times \frac{1}{5} \times 4 \right)$$

$$\left(\frac{b-2}{b+2} \right)\left(\frac{b+1}{b-1} \right)^2 = \frac{49}{50} = \frac{1}{2}\left(\frac{7}{5} \right)^2$$

$$= \left(\frac{6-2}{6+2} \right)\left(\frac{6+1}{6-1} \right)^2$$

$$\Rightarrow b = 6$$

Question: Find slope of VP where V is vertex of $y = x - x^2$ & P is point of tangency of $y = 4 + kx$ & $y = x - x^2$.

Answer: $\dfrac{-3}{2}, \dfrac{5}{2}$

Solution:

$$y = x - x^2$$

$$\frac{dy}{dx} = 1 - 2x$$

Vertex $v\left(\dfrac{1}{2}, \dfrac{1}{4} \right)$

Let point of tangency is (x_1, y_1)

$$\Rightarrow y_1 = x_1 - x_1^2 \quad \dots(1)$$

Tangent $y - y_1 = (1 - 2x_1)(x - x_1)$

It passes through $(0, 4)$

$$4 - y_1 = (1 - 2x_1)(-x_1)$$

$$4 - x_1 + x_1^2 = -x_1 + 2x_1^2 \quad \text{(From eqn. (1))}$$

$$x_1^2 = 4$$

$$x_1 = \pm 2$$

So, r can be $(2, -2)$ or $(-2, -6)$

Slope of VP $= \dfrac{-2-\dfrac{1}{4}}{2-\dfrac{1}{2}}$ or $\dfrac{-6-\dfrac{1}{4}}{-2-\dfrac{1}{2}}$

$= \dfrac{-1}{4} \times \dfrac{2}{3}$ or $\dfrac{25}{4} \times \dfrac{2}{5}$

$= \dfrac{-3}{2}$ or $\dfrac{5}{2}$

Question: $ax^2 - 2bx + 15 = 0$ has a repeated root α. $x^2 - 2bx + 22$ has roots α and β then $\alpha^2 + \beta^2 = ?$

Answer: $\dfrac{137}{2}$

Solution:

$ax^2 - 2bx + 15 = 0 \Rightarrow \alpha = \dfrac{b}{a}; \; \alpha^2 = \dfrac{15}{a}$

$\Rightarrow b^2 = 15a$

$x^2 - 2bx + 22 = 0$

$\Rightarrow \alpha + \beta = 2b; \; \alpha\beta = 22$

$\therefore \beta = 2b - \alpha = 2b - \dfrac{15}{b}; \; \alpha = \dfrac{15}{b}$

$\therefore \alpha\beta = \dfrac{15}{b}\left(2b - \dfrac{15}{b}\right) = 30 - \dfrac{225}{b^2} = 22$

$\Rightarrow b = \dfrac{15}{2\sqrt{2}}$

$\therefore \alpha = 2\sqrt{2}, \; \beta = \dfrac{11\sqrt{2}}{2}$

$\Rightarrow \alpha^2 + \beta^2 = 8 + \dfrac{121}{2} = \dfrac{137}{2}$

Question: If there is a biased dice with number 2, 4, 8, 16, 32, 32. Probability of appearing of number n is $\dfrac{1}{n}$. Then the probability of sum 48 in 3 throws is

Answer: $\dfrac{7}{16^3}$

Solution:

$s = \{2, 4, 8, 16, 32, 32\}$

$P(2) = \dfrac{1}{2}, P(4) = \dfrac{1}{4}, P(8) = \dfrac{1}{8}, P\left(\dfrac{1}{16}\right) = \dfrac{1}{16}, P(32) = \dfrac{1}{32}$

For sum = 48 in 3 throw, possible cases are

$\{8, 8, 32\}, \{16, 16, 16\}$

$\therefore$ Required probability $= \dfrac{1}{8} \times \dfrac{1}{8} \times \dfrac{1}{32} \times 3 + \dfrac{1}{16} \times \dfrac{1}{16} \times \dfrac{1}{16} = \dfrac{7}{16^3}$

Question: If mean deviation of first n natural numbers from mean is $\dfrac{5(n+1)}{n}$, where n is odd then $n = ?$

Answer: 21.00

Solution:

Mean deviation about mean of set of first n natural number when n is odd $= \dfrac{n^2 - 1}{4n} = \dfrac{5(n+1)}{n}$

$\Rightarrow n^2 - 1 = 20n + 20$
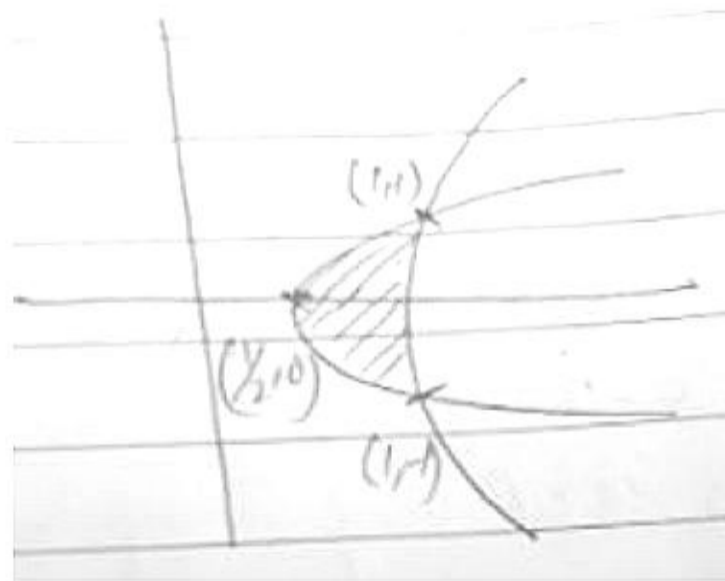
$\Rightarrow n^2 - 20n - 21 = 0$

$\Rightarrow (n - 21)(n + 1) = 0$

$\Rightarrow n = 21$

Question: Find area bounded by $y^2 = 2x - 1$ and $y^2 = 4x - 2$

Answer: $\dfrac{1}{3}$

Solution:

$y^2 = 2\left(x - \dfrac{1}{2}\right)$; $y^2 = 4\left(x - \dfrac{3}{4}\right)$

Area $= \displaystyle\int_{-1}^{1} \left(\dfrac{y^2 + 3}{4}\right) - \left(\dfrac{y^2 + 1}{2}\right) dy$

$= \left\{ \dfrac{1}{4}\left[\dfrac{y^3}{3} + 3y\right] - \dfrac{1}{2}\left[\dfrac{y^3}{3} + y\right] \right\}_{-1}^{1}$

$$= \frac{1}{4}\left[\left(\frac{10}{3}+\frac{10}{3}\right)-2\left(\frac{4}{3}+\frac{4}{3}\right)\right]$$

$$= \frac{1}{4}\left[\frac{20}{3}-\frac{16}{3}\right]=\frac{1}{3}$$

Question: Find value of $2\sin 12 - \sin 72$

Answer: $\sqrt{3}\left(\dfrac{1-\sqrt{5}}{4}\right)$

Solution:

$2\sin 12 - \sin 72 = \sin 12 + \sin 12 - \sin 72$

$= \sin 12 + 2\sin(-30)\cos(42) = \sin 12 - \cos 42$

$= \sin 12 - \sin 48 = 2\sin(-18)\cos(30)$

$= -\sqrt{3}\sin 18 = \sqrt{3}\left(\dfrac{1-\sqrt{5}}{4}\right)$