

PHYSICS
Joint Entrance Screening Test (JEST - 2017)

Registration No. :

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Question Booklet Series

Z

PLEASE READ THE INSTRUCTIONS CAREFULLY

1. Do not open the seal of the question paper before 10:00 AM.
2. You are given a question paper including a few blank sheets, and a machine readable Optical Mark Reader (OMR) sheet.
3. Enter your registration number on top of this question paper with black/blue pen.
4. Part A contains 15 questions, and carry 3 (three) marks each for correct answer, and -1 (negative one) mark for incorrect answer. Part B contains 10 questions and each carries 3 (three marks). These questions must be answered by integers of 4 digits each. Answer these questions on the OMR by filling in bubbles in the OMR sheet. Note that if the answer is, e.g. 25, you must fill in 0025 and if it is, e.g. 5, you must fill in 0005. If it is 0, you must fill in 0000. If the zeros are not filled in (where required), the answer will be not be credited. There are NO NEGATIVE MARKS for these questions. Part C contains 25 questions, and each carries 1 (one) mark for the correct answer, and -1/3 (negative one third) mark for incorrect answer. Multiple choice questions have only one correct answer.
5. On the OMR sheet, enter the appropriate Question Booklet Series (X, Y or Z) that is mentioned on the top right of the question paper.
6. On the OMR sheet, enter your name, registration number, and signature at the appropriate places. Strictly follow the instructions written on the OMR sheet.
7. On the OMR sheet, completely darken the bubble corresponding to your answer. Strictly follow the instructions written on the OMR sheet.
8. Only non-programmable scientific calculator is allowed, and exchange of calculators among the candidates is not permitted. Use of other items like electronic diary, writing pads, pencil box, beeper, cameras, mobile phones, palmtops, laptops, pagers etc., are not permitted inside the examination hall.
9. For rough work, use only the blank pages attached at the end of the question paper.
10. At the end of the examination, carefully separate the OMR sheet at the marked position, and return the **original copy** of the OMR sheet to the invigilator. Candidates are allowed to take away the **candidates' copy** of the OMR, and the question paper.

List of constants:

Speed of light, c	$3 \times 10^8 \text{ m sec}^{-1}$
Planck's constant, h	$6.626 \times 10^{-34} \text{ Joule sec}$
Boltzmann's constant, k_B	$1.381 \times 10^{-23} \text{ Joule K}^{-1}$
Gas constant, R	$8.314 \text{ Joule K}^{-1} \text{ mol}^{-1}$
Stefan-Boltzmann constant, σ	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Electron charge, e	$1.6 \times 10^{-19} \text{ C}$
Electron mass, m_e	$9.11 \times 10^{-31} \text{ Kg}$
Proton mass, m_p	$1.67 \times 10^{-27} \text{ Kg}$
Acceleration due to gravity, g	9.81 m sec^{-2}
Permittivity of free space, ϵ_0	$8.85 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$
Avogadro's number, N_A	$6.02 \times 10^{23} \text{ mol}^{-1}$
Angstrom, \AA	$10^{-10}\text{m} = 0.1\text{nm}$
1 eV	$1.6 \times 10^{-19} \text{ Joules}$
1 calorie	4.18 Joules

Part-A: 3-Mark Questions

1. Given a matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, which of the following represents $\cos(\pi M/6)$?

(A) $\frac{1}{2} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

(B) $\frac{\sqrt{3}}{4} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

(C) $\frac{\sqrt{3}}{4} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

(D) $\frac{1}{2} \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

2. The wavefunction of a hydrogen atom is given by the following superposition of energy eigenfunctions $\psi_{nlm}(\vec{r})$ (n, l, m are the usual quantum numbers):

$$\psi(\vec{r}) = \frac{\sqrt{2}}{\sqrt{7}}\psi_{100}(\vec{r}) - \frac{3}{\sqrt{14}}\psi_{210}(\vec{r}) + \frac{1}{\sqrt{14}}\psi_{322}(\vec{r}).$$

The ratio of expectation value of the energy to the ground state energy and the expectation value of L^2 are, respectively:

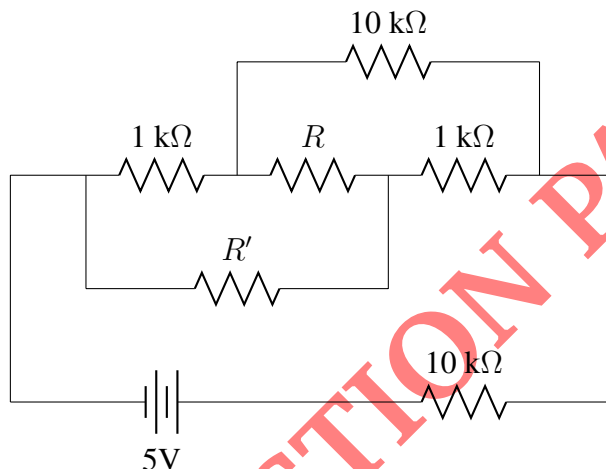
(A) $\frac{229}{504}$ and $\frac{12\hbar^2}{7}$

(B) $\frac{101}{504}$ and $\frac{12\hbar^2}{7}$

(C) $\frac{101}{504}$ and \hbar^2

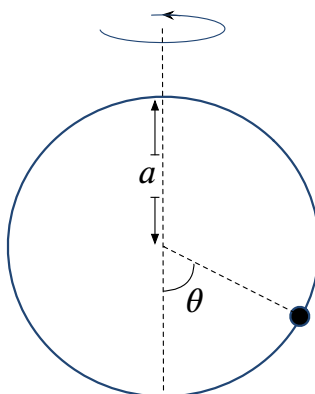
(D) $\frac{229}{504}$ and \hbar^2

3. It is found that when the resistance R indicated in the figure below is changed from $1\text{ k}\Omega$ to $10\text{ k}\Omega$, the current flowing through the resistance R' does not change. What is the value of the resistor R' ?



- (A) $5\text{ k}\Omega$
 (B) $100\ \Omega$
 (C) $10\text{ k}\Omega$
 (D) $1\text{ k}\Omega$

4. A hoop of radius a rotates with constant angular velocity ω about the vertical axis as shown in the figure. A bead of mass m can slide on the hoop without friction. If $g < \omega^2 a$, at what angle θ apart from 0 and π is the bead stationary (i.e., $\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} = 0$)?



- (A) $\tan\theta = \pi g / \omega^2 a$
 (B) $\sin\theta = g / \omega^2 a$
 (C) $\cos\theta = g / \omega^2 a$
 (D) $\tan\theta = g / \pi \omega^2 a$

5. A spin-1/2 particle in a uniform external magnetic field has energy eigenstates $|1\rangle$ and $|2\rangle$. The system is prepared in ket-state $(|1\rangle + |2\rangle)/\sqrt{2}$ at time $t = 0$. It evolves to the state described by the ket $(|1\rangle - |2\rangle)/\sqrt{2}$ in time T . The minimum energy difference between two levels is:
- (A) $h/6T$
 (B) $h/4T$
 (C) $h/2T$
 (D) h/T
6. You receive on average 5 emails per day during a 365-days year. The number of days on average on which you do not receive any emails in that year are:
- (A) More than 5
 (B) More than 2
 (C) 1
 (D) None of the above
7. The H_2 molecule has a reduced mass $M = 8.35 \times 10^{-28}$ kg and an equilibrium internuclear distance $R = 0.742 \times 10^{-10}$ m. The rotational energy in terms of the rotational quantum number J is:
- (A) $E_{\text{rot}}(J) = 7J(J-1)$ meV
 (B) $E_{\text{rot}}(J) = \frac{5}{2}J(J+1)$ meV
 (C) $E_{\text{rot}}(J) = 7J(J+1)$ meV
 (D) $E_{\text{rot}}(J) = \frac{5}{2}J(J-1)$ meV
8. The maximum relativistic kinetic energy of β particles from a radioactive nucleus is equal to the rest mass energy of the particle. A magnetic field is applied perpendicular to the beam of β particles, which bends it to a circle of radius R . The field is given by:
- (A) $3m_0c/eR$
 (B) $\sqrt{2}m_0c/eR$
 (C) $\sqrt{3}m_0c/eR$
 (D) $\sqrt{3}m_0c/2eR$

9. The central force which results in the orbit $r = a(1 + \cos \theta)$ for a particle is proportional to:

- (A) r
- (B) r^2
- (C) r^{-2}
- (D) None of the above

10. A gas of N molecules of mass m is confined in a cube of volume $V = L^3$ at temperature T . The box is in a uniform gravitational field $-g\hat{z}$. Assume that the potential energy of a molecule is $U = mgz$, where $z \in [0, L]$ is the vertical coordinate inside the box. The pressure $P(z)$ at height z is:

- (A) $P(z) = \frac{N}{V} \frac{mgL}{2} \frac{\exp\left(-\frac{mg(z-L/2)}{k_B T}\right)}{\sinh\left(\frac{mgL}{2k_B T}\right)}$
- (B) $P(z) = \frac{N}{V} \frac{mgL}{2} \frac{\exp\left(-\frac{mg(z-L/2)}{k_B T}\right)}{\cosh\left(\frac{mgL}{2k_B T}\right)}$
- (C) $P(z) = \frac{k_B T N}{V}$
- (D) $P(z) = \frac{N}{V} mgz$

11. A transistor in common base configuration has ratio of collector current to emitter current β and ratio of collector to base current α . Which of the following is true?

- (A) $\beta = \alpha/(\alpha + 1)$
- (B) $\beta = (\alpha + 1)/\alpha$
- (C) $\beta = \alpha/(\alpha - 1)$
- (D) $\beta = (\alpha - 1)/\alpha$

12. The energy of a particle is given by $E = |p| + |q|$, where p and q are the generalized momentum and coordinate, respectively. All the states with $E \leq E_0$ are equally probable and states with $E > E_0$ are inaccessible. The probability density of finding the particle at coordinate q , with $q > 0$ is:

- (A) $(E_0 + q)/E_0^2$
- (B) q/E_0^2
- (C) $(E_0 - q)/E_0^2$
- (D) $1/E_0$

13. Consider a quantum particle of mass m in one dimension in an infinite potential well, *i.e.*, $V(x) = 0$ for $-a/2 < x < a/2$, and $V(x) = \infty$ for $|x| \geq a/2$. A small perturbation, $V'(x) = 2\epsilon|x|/a$, is added. The change in the ground state energy to $O(\epsilon)$ is:
- (A) $\frac{\epsilon}{2\pi^2}(\pi^2 - 4)$
 (B) $\frac{\epsilon}{2\pi^2}(\pi^2 + 4)$
 (C) $\frac{\epsilon\pi^2}{2}(\pi^2 + 4)$
 (D) $\frac{\epsilon\pi^2}{2}(\pi^2 - 4)$
14. The strength of magnetic field at the center of a regular hexagon with sides of length a carrying a steady current I is:
- (A) $\frac{\mu_0 I}{\sqrt{3}\pi a}$
 (B) $\frac{\sqrt{6}\mu_0 I}{\pi a}$
 (C) $\frac{3\mu_0 I}{\pi a}$
 (D) $\frac{\sqrt{3}\mu_0 I}{\pi a}$
15. An ideal gas with adiabatic exponent γ undergoes a process in which its pressure P is related to its volume V by the relation $P = P_0 - \alpha V$, where P_0 and α are positive constants. The volume starts from being very close to zero and increases monotonically to P_0/α . At what value of the volume during the process does the gas have maximum entropy?
- (A) $\frac{P_0}{\alpha(1+\gamma)}$
 (B) $\frac{\gamma P_0}{\alpha(1-\gamma)}$
 (C) $\frac{\gamma P_0}{\alpha(1+\gamma)}$
 (D) $\frac{P_0}{\alpha(1-\gamma)}$

Part-B: 3-Mark Questions

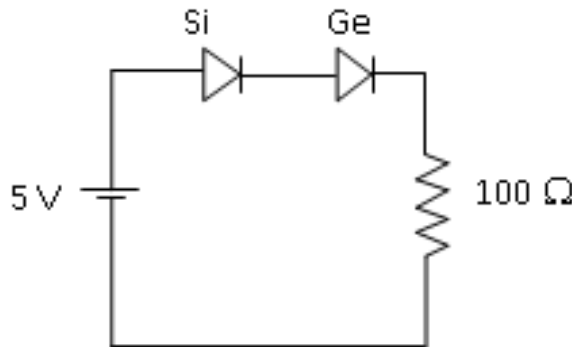
16. A proton is confined in an infinite square well of width 4 femtometer. Calculate the energy of the photon emitted when the proton undergoes a transition from the excited state ($n = 11$) to the ground state ($n = 1$). Find the answer only up to the first four significant digits.

17. The expression

$$2 \times \left(\frac{1}{\pi^4} + \frac{1}{(3\pi)^4} + \frac{1}{(5\pi)^4} \dots \right)^{-1}$$

is equal to :

18. A solid insulating ball of radius $a = 1 \text{ cm}$ is surrounded by a conducting spherical shell with an inner radius of $b = 2 \text{ cm}$ and outer radius $c = 2.2 \text{ cm}$. The inner ball has a charge $Q_1 = 10 \mu\text{C}$, which is uniformly distributed throughout its volume. The conducting spherical shell contains a charge of $Q_2 = -10 \mu\text{C}$. Determine the electrostatic potential (in Volts) at a point $r = 1.5 \text{ cm}$. Consider $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$.
19. What is the smallest possible time necessary to freeze 2 kg of water at 273 K if a 50 watt motor is available and the outside air (hot reservoir) is at 300 K.
20. A bead slides along a smooth wire bent in the shape of parabola $z = cr^2$. The bead rotates in a circle of radius 10 cm, when the wire is rotating about its verticle symmetry axis with angular velocity 10 radians/s. Take $g = 10 \text{ m/s}$. Find the value of c .
21. A 5 V supply is across a series combination of a Si diode, a Ge diode and a 100Ω resistor. Find the power dissipated in the resistor.



22. A point like charge $Q = 10 \mu\text{C}$ is split into two charges $Q_1 \mu\text{C}$ and $(Q - Q_1) \mu\text{C}$. What should be the magnitude of Q_1 so as to obtain maximal repulsive force between the two charges?
23. Consider the matrix $M =$

$$\begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & 2 \\ 2 & 2 & -2 \end{pmatrix}$$

The magnitude of the product of the minimum and maximum eigenvalues of the matrix H given by

$$\sum_{m=1}^N \mathbf{1}^{m-1} \otimes M \otimes \mathbf{1}^{N-m}$$

(where $\mathbf{1}^k = 1 \otimes 1 \otimes 1 \otimes \dots$ k times and \otimes denotes a direct product and $N = 8$) is :

24. A particle (in one dimension) is described by the wave function $\psi(x) = 0$ for $x < 0$ and $\psi(x) = Ce^{-x}(1 - e^{-x})$ for $x > 0$, where x is in nanometers and C is a constant. Calculate the average position for the particle. Express your answer in units of $10^{-3}nm$.
25. A spacecraft in an orbit about earth has the speed of 10,160 m/s at a perigee of 6680 km from earth's center. What speed does the spacecraft have at apogee of 42,200 km?

Part-C: 1-Mark Questions

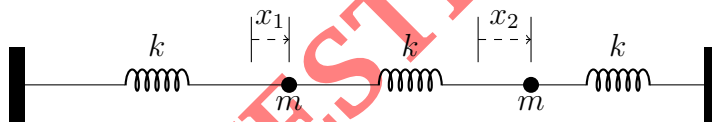
26. The *adjoint* of a differential operator $\frac{d}{dx}$ acting on a wavefunction $\psi(x)$ for a quantum mechanical system is:
- (A) $\frac{d}{dx}$
(B) $-i\hbar\frac{d}{dx}$
(C) $-\frac{d}{dx}$
(D) $i\hbar\frac{d}{dx}$
27. In Millikan's oil-drop experiment an oil drop of radius r , mass m and charge $q = 6\pi\eta r(v_1 + v_2)/E$ is moving upwards with a terminal velocity v_2 due to an applied electric field of magnitude E , where η is the coefficient of viscosity. The acceleration due to gravity is given by:
- (A) $g = 6\pi\eta r v_1/m$
(B) $g = 3\pi\eta r v_1/m$
(C) $g = 6\pi\eta r v_2/m$
(D) $g = 3\pi\eta r v_2/m$
28. The electric field $\vec{E} = E_0 \sin(\omega t - kz)\hat{x} + 2E_0 \sin(\omega t - kz + \pi/2)\hat{y}$ represents:
- (A) a linearly polarized wave
(B) a right-hand circularly polarized wave
(C) a left-hand circularly polarized wave
(D) an elliptically polarized wave

29. An ideal gas has a specific heat ratio $C_P/C_V = 2$. Starting at a temperature T_1 the gas undergoes an isothermal compression to increase its density by a factor of two. After this an adiabatic compression increases its pressure by a factor of two. The temperature of the gas at the end of the second process would be:
- (A) $T_1/2$
 (B) $\sqrt{2} T_1$
 (C) $2 T_1$
 (D) $T_1/\sqrt{2}$
30. Suppose yz plane forms the boundary between two linear dielectric media I and II with dielectric constant $\epsilon_I = 3$ and $\epsilon_{II} = 4$, respectively. If the electric field in region I at the interface is given by $\vec{E}_I = 4\hat{x} + 3\hat{y} + 5\hat{z}$, then the electric field \vec{E}_{II} at the interface in region II is:
- (A) $4\hat{x} + 3\hat{y} + 5\hat{z}$
 (B) $4\hat{x} + 0.75\hat{y} - 1.25\hat{z}$
 (C) $-3\hat{x} + 3\hat{y} + 5\hat{z}$
 (D) $3\hat{x} + 3\hat{y} + 5\hat{z}$
31. Given the condition $\nabla^2\Phi = 0$, the solution of the equation $\nabla^2\Psi = k\vec{\nabla}\Phi \cdot \vec{\nabla}\Phi$ is given by:
- (A) $\Psi = k\Phi^2/2$
 (B) $\Psi = k\Phi^2$
 (C) $\Psi = k\Phi\ln\Phi$
 (D) $\Psi = k\Phi\ln\Phi/2$
32. Circular fringes are obtained with a Michelson interferometer using 600nm laser light. What minimum displacement of one mirror will make the central fringe from bright to dark?
- (A) 600 nm
 (B) 300 nm
 (C) 150 nm
 (D) 120 Å

33. If \vec{k} is the wavevector of incident light ($|\vec{k}| = 2\pi/\lambda$, λ is the wavelength of light) and \vec{G} is a reciprocal lattice vector, then the Bragg's law can be written as:

- (A) $\vec{k} + \vec{G} = 0$
- (B) $2\vec{k} \cdot \vec{G} + G^2 = 0$
- (C) $2\vec{k} \cdot \vec{G} + k^2 = 0$
- (D) $\vec{k} \cdot \vec{G} = 0$

34. For the coupled system shown in the figure, the normal coordinates are $x_1 + x_2$ and $x_1 - x_2$, corresponding to the normal frequencies ω_0 and $\sqrt{3}\omega_0$, respectively.



At $t = 0$, the displacements are $x_1 = A$, $x_2 = 0$, and the velocities are $v_1 = v_2 = 0$. The displacement of the second particle at time t is given by:

- (A) $x_2(t) = \frac{A}{2} \left(\cos(\omega_0 t) + \cos(\sqrt{3}\omega_0 t) \right)$
- (B) $x_2(t) = \frac{A}{2} \left(\cos(\omega_0 t) - \cos(\sqrt{3}\omega_0 t) \right)$
- (C) $x_2(t) = \frac{A}{2} \left(\sin(\omega_0 t) - \sin(\sqrt{3}\omega_0 t) \right)$
- (D) $x_2(t) = \frac{A}{2} \left(\sin(\omega_0 t) - \frac{1}{\sqrt{3}} \sin(\sqrt{3}\omega_0 t) \right)$

35. How much force does light from a 1.8 W laser exert when it is totally absorbed by an object?

- (A) 6.0×10^{-9} N
- (B) 0.6×10^{-9} N
- (C) 6.0×10^{-8} N
- (D) 4.8×10^{-9} N

36. An electron confined within a thin layer of semiconductor may be treated as a free particle inside an infinitely deep one-dimensional potential well. If the difference in energies between the first and the second energy levels is δE , then the thickness of the layer is:

(A) $\sqrt{\frac{3\hbar^2\pi^2}{2m\delta E}}$

(B) $\sqrt{\frac{2\hbar^2\pi^2}{3m\delta E}}$

(C) $\sqrt{\frac{\hbar^2\pi^2}{2m\delta E}}$

(D) $\sqrt{\frac{\hbar^2\pi^2}{m\delta E}}$

37. The half-life of a radioactive nuclear source is 9 days. The fraction of nuclei which are left undecayed after 3 days is:

(A) $7/8$

(B) $1/3$

(C) $5/6$

(D) $1/2^{1/3}$

38. Self inductance per unit length of a long solenoid of radius R with n turns per unit length is:

(A) $\mu_0\pi R^2n^2$

(B) $2\mu_0\pi R^2n$

(C) $2\mu_0\pi R^2n^2$

(D) $\mu_0\pi R^2n$

39. A gas contains particles of type A with fraction 0.8, and particles of type B with fraction 0.2. The probability that among 3 randomly chosen particles at least one is of type A is:

(A) 0.8

(B) 0.25

(C) 0.33

(D) 0.992

40. The number of different Bravais lattices possible in two dimensions is:

- (A) 2
- (B) 3
- (C) 5
- (D) 6

41. The output intensity I of radiation from a single mode of resonant cavity obeys

$$\frac{d}{dt}I = -\frac{\omega_0}{Q}I,$$

where Q is the quality factor of the cavity and ω_0 is the resonant frequency. The form of the frequency spectrum of the output is:

- (A) Delta function
- (B) Gaussian
- (C) Lorentzian
- (D) Exponential

42. For a quantum mechanical harmonic oscillator with energies, $E_n = (n + 1/2)\hbar\omega$, where $n = 0, 1, 2 \dots$, the partition function is:

- (A) $\frac{e^{\hbar\omega/k_B T}}{e^{\hbar\omega/k_B T} - 1}$
- (B) $e^{\hbar\omega/2k_B T} - 1$
- (C) $e^{\hbar\omega/2k_B T} + 1$
- (D) $\frac{e^{\hbar\omega/2k_B T}}{e^{\hbar\omega/k_B T} - 1}$

43. If the direction with respect to a right-handed cartesian coordinate system of the ket vector $|z, +\rangle$ is $(0, 0, 1)$, then the direction of the ket vector obtained by application of rotations: $\exp(-i\sigma_z\pi/2)\exp(i\sigma_y\pi/4)$, on the ket $|z, +\rangle$ is (σ_y, σ_z) are the Pauli matrices):

- (A) $(0, 1, 0)$
- (B) $(1, 0, 0)$
- (C) $(1, 1, 0)/\sqrt{2}$
- (D) $(1, 1, 1)/\sqrt{3}$

44. In the ground state of hydrogen atom, the most probable distance of the electron from the nucleus, in units of Bohr radius a_0 is:

- (A) $1/2$
- (B) 1
- (C) 2
- (D) $3/2$

45. For operators P and Q , the commutator $[P, Q^{-1}]$ is:

- (A) $Q^{-1} [P, Q] Q^{-1}$
- (B) $-Q^{-1} [P, Q] Q^{-1}$
- (C) $Q^{-1} [P, Q] Q$
- (D) $-Q [P, Q] Q^{-1}$

46. The mean value of random variable x with probability density $p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp[-(x^2 + \mu x)/(2\sigma^2)]$, is:

- (A) 0
- (B) $\mu/2$
- (C) $-\mu/2$
- (D) σ

47. A spin $1/2$ particle is in a state $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$, where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of S_z operator. The expectation value of the spin angular momentum measured along x direction is:

- (A) \hbar
- (B) $-\hbar$
- (C) 0
- (D) $\hbar/2$

48. A semicircular piece of paper is folded to make a cone with the centre of the semicircle as the apex. The half-angle of the resulting cone would be:
- (A) 90°
(B) 60°
(C) 45°
(D) 30°
49. If the Rydberg constant of an atom of finite nuclear mass is αR_∞ , where R_∞ is the Rydberg constant corresponding to an infinite nuclear mass, the ratio of the electronic to nuclear mass of the atom is:
- (A) $(1 - \alpha)/\alpha$
(B) $(\alpha - 1)/\alpha$
(C) $(1 - \alpha)$
(D) $1/\alpha$
50. A cylindrical shell of mass m has an outer radius b and an inner radius a . The moment of inertia of the shell about the axis of the cylinder is:
- (A) $\frac{1}{2}m(b^2 - a^2)$
(B) $\frac{1}{2}m(b^2 + a^2)$
(C) $m(b^2 + a^2)$
(D) $m(b^2 - a^2)$

JEST SAMPLE QUESTION PAPER

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