

# MATHEMATICAL SCIENCES

**PAPER – II**  
**CODE-01**

OMR Answer Sheet No. :

Roll No. :

(in figures as in Hall Ticket)

Roll Number in words : .....

011445

Question Booklet Sl. No.

Name & Signature of the Invigilator

Time : 2 Hours]

No. of Printed Pages : 32

[Maximum Marks : 200

## Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page.
2. This paper consists of **one hundred (100)** multiple choice type of questions. **All** questions are compulsory.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
  - (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker seal and do not accept an open booklet.
  - (ii) Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
  - (iii) After this verification is over, the Test Booklet Number should be entered on the OMR Answer Sheet and the OMR Answer Sheet Number should be entered on this Test Booklet.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the oval as indicated below on the correct response against each item.

**Example:** (A) (B) (C) (D) where (B) is the correct response.
5. Your responses to the items are to be indicated on the OMR Answer Sheet under Paper – II only. If you mark your response at any place other than in the oval in the OMR Answer Sheet, it will not be evaluated.
6. Rough Work is to be done in the end of this booklet.
7. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. You have to return the original OMR Answer Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are however, allowed to carry original question booklet and duplicate copy of OMR Answer Sheet on conclusion of examination.
9. Use only Blue/Black Ball point pen.
10. Use of any calculator or any electronic devices or log table etc., are prohibited.
11. There shall be no negative marking.

## પરીક્ષાર્થીઓ માટે સૂચનાઓ

1. આ પાનાની ટોચ પર દર્શાવેલી જગ્યામાં તમારો રોલ નંબર લખો.
2. આ પ્રશ્નપત્રમાં બહુવૈકલ્પિક ઉત્તરો ધરાવતા સો (100) પ્રશ્નો આપેલા છે. બધા જ પ્રશ્નો ફરજિયાત છે.
3. પરીક્ષાની શરૂઆતમાં આપને પ્રશ્નપુસ્તિકા આપવામાં આવશે. પ્રથમ પાંચ (૫) મિનિટ દરમિયાન તમારે પ્રશ્નપુસ્તિકા ખોલી અને ફરજિયાતપણે નીચે મુજબ પરીક્ષણ કરવું :
  - (i) પ્રશ્નપુસ્તિકાનો વપરાશ કરવા માટે આ કવર પુષ્ટની ધાર પર આપેલ સીલ સ્ટીકર ફાડી નાખો. કોઈપણ સંજોગોમાં સીલ સ્ટીકર વગરની કે ખુલ્લી પ્રશ્નપુસ્તિકા સ્વીકારશો નહીં.
  - (ii) કવર પુષ્ટ પર છપાયેલ નિર્દેશાનુસાર પ્રશ્નપુસ્તિકાના પ્રશ્નો, પૃષ્ઠો અને સંખ્યાને બરાબર ચકાસી લો. ખામીયુક્ત પ્રશ્નપુસ્તિકા કે જેમાં પ્રશ્નો/ પૃષ્ઠો ઓછાં હોય, બે વાર છપાયા હોય, અનુક્રમમાં અથવા અન્ય કોઈ ફરક હોય અર્થાત કોઈપણ સંજોગોમાં ખામીયુક્ત પ્રશ્નપુસ્તિકા સ્વીકારશો નહીં અને જો ખામીયુક્ત પ્રશ્નપુસ્તિકા મળી હોય તો નિરીક્ષક પાસેથી તુરંત જ બીજી સારી પ્રશ્નપુસ્તિકા મેળવી લેવી. આ માટે ઉમેદવારને પાંચ (૫) મિનિટનો સમયગાળો આપવામાં આવશે. પછીથી, પ્રશ્નપુસ્તિકા બદલવામાં આવશે નહીં કે કોઈ વધારાનો સમયગાળો આપવામાં આવશે નહીં.
  - (iii) આ ચકાસણી સમાપ્ત થાય પછી, પ્રશ્નપુસ્તિકાનો નંબર OMR જવાબ પત્રક પર લખવો અને OMR જવાબ પત્રકનો નંબર પ્રશ્નપુસ્તિકા પર લખવો.
4. પ્રત્યેક પ્રશ્ન માટે ચાર જવાબ વિકલ્પ (A), (B), (C) અને (D) આપવામાં આવેલ છે. તમારે સાચા જવાબના ઓવલ (oval) ને નીચે આપેલ ઉદાહરણ મુજબ પેનથી ભરીને સંપૂર્ણ કાળું કરવાનું રહેશે.

ઉદાહરણ : (A) (B) (C) (D) કે જ્યાં (B) સાચો જવાબ છે.
5. આ પ્રશ્નપુસ્તિકાના પ્રશ્નોના જવાબ અલગથી આપવામાં આવેલ OMR જવાબ પત્રકમાં પેપર-II લપેલ વિભાગમાં જ અંકિત કરવા. જો આપ OMR જવાબ પત્રકમાં આપેલ ઓવલ (oval) સિવાય અન્ય સ્થાને જવાબ અંકિત કરશો તો તે જવાબનું મૂલ્યાંકન કરવામાં આવશે નહીં.
6. કાચું કામ (Rough work) પ્રશ્નપુસ્તિકાના અંતિમ પૃષ્ઠ પર કરવું.
7. જો આપ OMR જવાબ પત્રક નિયત જગ્યા સિવાય અન્ય કોઈપણ સ્થાને, આપનું નામ, રોલ નંબર, ક્ષેત્ર નંબર અથવા એવું કોઈ ચિહ્નકે જેનાથી તમારી ઓળખ થઈ શકે, અંકિત કરશો અથવા અક્ષર ભાષાનો પ્રયોગ કરો, અથવા અન્ય કોઈ અનુચિત સાધનોનો ઉપયોગ કરો, જેમકે અંકિત કરી દીધેલ જવાબ ભૂંસી નાખવો કે સફેદ શાહીનો ઉપયોગ કરી બદલશો તો આપને પરીક્ષા માટે અયોગ્ય જાહેર કરવામાં આવશે.
8. પરીક્ષા સમય પૂરો થઈ ગયા બાદ ઓરીજીનલ OMR જવાબ પત્રક જે તે નિરીક્ષકને ફરજિયાત સોપી દેવું અને કોઈ પણ સંજોગોમાં તે પરીક્ષા ખંડની બહાર લઈ જવું નહીં. પરીક્ષા પૂર્ણ થયા બાદ ઉમેદવાર ઓરીજીનલ પ્રશ્નપુસ્તિકા અને OMR જવાબ પત્રકની ડુપ્લિકેટ કોપી પોતાની સાથે લઈ જઈ શકે છે.
9. માત્ર કાળી / ભૂરી બોલ પોઈન્ટ પેન વાપરવી.
10. કેલ્ક્યુલેટર, લોગ ટેબલ અને અન્ય ઇલેક્ટ્રોનિક યંત્રોનો ઉપયોગ કરવાની મનાઈ છે.
11. ખોટા જવાબ માટે નકારાત્મક ગુણાંકન પ્રથા નથી.



DO NOT WRITE HERE



## LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1958	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2096	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2789	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5636	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

No.  
x = 3.14159  
e = 2.71828

log  
0.49715  
0.43429

$\ln x = \log_e x = (1/M) = \log_{10} x$   
 $\log x = \log_{10} x = M \log_e x$

No.  
(1/M) = 2.30259  
M = 0.43429

log  
0.36222  
1/63778



## LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	8
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	8
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	6	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	6	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	6	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	6	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	6	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	6
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	6
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	6
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	6
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	6
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	6
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9966	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	4	4

$p$	1	2	3	4	5	6	7	8	9	10
$\log e^p$	0.4343	0.8686	1.3029	1.7372	2.1715	2.6058	3.0401	3.4744	3.9087	4.3429
$\log e^{p^2}$	1.6657	1.1314	2.6971	2.2628	3.8285	3.3942	4.9599	4.5256	6.0913	5.6571

### ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1664	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2048	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2444	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2558	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	2	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	2	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	2	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	2	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	2	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	2	3	4	5	5	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	2	3	4	5	5	6



## ANTILOGARITHMS

	D	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3259	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3564	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3935	3944	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4405	4415	4425	4435	4445	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4496	4506	4516	4526	4536	4546	4556	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5396	5408	5420	5433	5446	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5496	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	6	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9463	9484	9506	9528	2	4	7	9	11	13	16	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20



## MATHEMATICAL SCIENCES

### Paper – II

1. Consider the set  $E = \left\{ \cos \frac{n\pi}{3} + \sin \frac{n\pi}{3} \mid n \in \mathbb{N} \right\}$ . Then supremum of  $E =$  \_\_\_\_\_ and infimum of  $E =$  \_\_\_\_\_.

(A) 1 and  $-1$

(B)  $\frac{\sqrt{3}}{2} - \frac{1}{2}$  and  $\frac{1}{2} - \frac{\sqrt{3}}{2}$

(C)  $\frac{\sqrt{3}}{2} + \frac{1}{2}$  and  $-\frac{\sqrt{3}}{2} - \frac{1}{2}$

(D)  $\frac{\sqrt{3}}{2} + 1$  and  $-\frac{\sqrt{3}}{2} - 1$

2. Let  $E = \{x_1, x_2, \dots, x_n, \dots\}$  be a countable subset of an open interval  $(a, b)$ . Let  $\{C_n\}$  be a sequence of positive numbers such that  $\sum C_n$  is convergent. Define  $f : (a, b) \rightarrow \mathbb{R}$  as

$f(x) = \sum_{C_n < x} C_n$ . Then which one of the following is false ?

(A)  $f$  is monotonically increasing on  $(a, b)$

(B)  $f$  is discontinuous at every point of  $E$

(C)  $f$  is continuous from the right at every point of  $(a, b)$

(D)  $f$  is continuous at every point of  $(a, b) \setminus E$

3. Which of the following statements are true ?

i. If  $f$  is a bounded real function on  $[a, b]$  and  $f^2$  is Riemann integrable, then  $f$  is also Riemann integrable.

ii. Let  $f$  be define on  $[a, b]$ . If  $f$  has a local minimum at a point  $x \in (a, b)$  then  $f$  is differentiable at  $x$  and  $f'(x) = 0$ .

(A) None of i and ii is true

(B) Only i is true

(C) Only ii is true

(D) Both i and ii are true



4. Let, for  $n = 0, 1, 2, \dots$  define function  $f_n(x)$  on  $\mathbb{R}$  as  $f_n(x) = \frac{x^2}{(1+x^2)^n}$ , and let

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1+x^2 & \text{if } x \neq 0. \end{cases}$$

Then which of the following is true ?

- (A) The series converges pointwise to a function  $f(x)$
  - (B) The series converges uniformly to  $f(x)$
  - (C) The series does not converge for any non-zero  $x$
  - (D) The series converges pointwise to the function  $g(x) = 1 + x^2$  for all  $x$
5. Let  $X = (\mathbb{Q}, d)$  be a metric space where  $\mathbb{Q}$  is the set of all rational numbers and  $d$  is the usual Euclidean metric and let  $E = \{x \in \mathbb{Q} \mid 2 < x^2 < 3\}$ . Then which one of the following statements is false ?
- (A)  $E$  is closed in  $X$
  - (B)  $E$  is bounded in  $X$
  - (C)  $E$  is open in  $X$
  - (D)  $E$  is compact in  $X$
6. Which one of the following improper integrals are divergent ?

i.  $\int_1^{\infty} \frac{dx}{(1+x)\sqrt{x}}$  and

ii.  $\int_1^x \frac{dt}{(t+1)\log(1+t)}$

- (A) Both i and ii are divergent
  - (B) Only i is divergent
  - (C) Only ii is divergent
  - (D) None of the i and ii is divergent
7. Let  $Z = [1, 7]$  and  $d$  be a metric on  $Z$  induced from Euclidean metric on  $\mathbb{R}$ . Let  $E = \left\{1 + \frac{1}{n} \mid n = 1, 2, \dots\right\}$ . Then which one of the following is true ?
- (A)  $E$  is closed in  $(Z, d)$
  - (B)  $E$  is compact in  $(Z, d)$
  - (C)  $E$  is open in  $(Z, d)$
  - (D)  $E$  neither closed nor compact in  $(Z, d)$





8. A function  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  is defined as  $f(x, y) = \begin{cases} x+y & , \text{ if } x=0 \text{ or } y=0 \\ 1 & , \text{ otherwise} \end{cases}$   
Then which one of the following is true ?

(A)  $D_x f(0, 0) = -1$

(B)  $D_y f(0, 0) = -1$

(C) The directional derivative at  $(0, 0)$  in the direction  $\bar{u} = (0, -1)$ ,  $f'(\bar{0}; \bar{u}) = -1$

(D) The directional derivative  $f'(\bar{0}; \bar{u})$  does not exist

9. Consider the function  $f(x, y) = \sqrt{|xy|}$ ,  $\bar{a} = (0, 0)$ .

True statement about this function is

(A) The function is differentiable at  $(0, 0)$

(B) The function is not differentiable at  $(0, 0)$

(C)  $f_x$  is continuous at  $(0, 0)$

(D)  $f_x(0, 0) \neq f_y(0, 0)$

10. If  $u(x, y) = \cos^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$ , then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u =$

(A) 0

(B) 2

(C) -2

(D) 3

11. Consider  $P^4(\mathbb{R})$  of all polynomials of degree  $\leq 3$ . Which of the following sets are linearly independent ?

i.  $\{1 - x - 2x^3, 1 + x^3, 1 + x + 4x^3, x^2\}$

ii.  $\{1 - 2x - x^3, 3x - x^2, 1 + x + x^2 + x^3, 4 + 6x + x^2 + 2x^3\}$

(A) Both i and ii

(B) Only i

(C) Only ii

(D) None of i and ii



12. Let  $U_1$  be a subspace of  $\mathbb{R}^5$  generated by the column vectors  $\{[0 \ 0 \ a \ 0 \ 0]$  and  $[0 \ b \ 0 \ c \ 0]\}$  and  $U_2$  be a subspace of  $\mathbb{R}^5$  generated by column vectors of the form  $\{[d \ 0 \ 0 \ 0 \ 0], [0 \ 0 \ 0 \ e \ 0], [0 \ 0 \ 0 \ 0 \ f]\}$

Then which one of the following is true ?

- i.  $\mathbb{R}^5 = U_1 \oplus U_2$
- ii.  $\dim U_1 = \dim U_2$

- (A) Both i and ii are true
- (B) Only i is true
- (C) Only ii is true
- (D) None of i and ii is true

13. Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}$  be a linear transformation defined by  $T([x_1, x_2, x_3, x_4]) = x_1 + x_2 + x_3 + x_4$ .

Then one of the basis for  $\ker(T)$  is

- (A)  $\{[1, 1, 1, 1]\}$
- (B)  $\{[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0]\}$
- (C)  $\{[1, 0, 0, 0], [0, 1, 0, 1], [0, 0, 1, 1]\}$
- (D)  $\{[1, 0, 0, -1], [0, 1, 0, -1], [0, 0, 1, -1]\}$

14. Consider a matrix  $M = \begin{bmatrix} -2 & 3 & 1 \\ 2 & 1 & 3 \\ -1 & 4 & 6 \end{bmatrix}$ . Then adjoint of M is

(A)  $\begin{bmatrix} -6 & -14 & 8 \\ -15 & -11 & 8 \\ 9 & 5 & -8 \end{bmatrix}$

(B)  $\begin{bmatrix} -6 & -15 & 9 \\ -14 & -11 & 5 \\ 8 & 8 & -8 \end{bmatrix}$

(C)  $\begin{bmatrix} -6 & -15 & 5 \\ -14 & -11 & -8 \\ 8 & 8 & 9 \end{bmatrix}$

(D)  $\begin{bmatrix} 6 & 15 & -5 \\ 14 & 11 & 8 \\ -8 & -8 & -9 \end{bmatrix}$



15. Which of the following statements are true ?

- i. If A and B are two matrices with A non singular then AB and BA have the same set of eigen values.
- ii. If A is an  $n \times n$  matrix with characteristics polynomial  $f(\lambda)$  then coefficient of  $\lambda^{n-1}$  in  $f(\lambda)$  is  $(- \text{trace of A})$ .

- (A) Both i and ii are true
- (B) Only i is true
- (C) Only ii is true
- (D) Neither i nor ii is true

16. The quadratic form  $Q([x, y]) = 8x^2 - 6xy + 3y^2$  is

- (A) Positive definite
- (B) Negative definite
- (C) Neither positive nor negative definite
- (D) Positive semidefinite

17. Let  $M = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ . Then the Jordan form of A is

(A)  $\begin{bmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(B)  $\begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(C)  $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(D)  $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$



18. Let  $X$  be an inner product space with norm  $\| \cdot \|$ . Then for elements  $x, y, z$  of  $X$  which of the following identities do not hold ?

(A)  $\langle x, x \rangle = \|x\|^2$

(B)  $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$

(C)  $\|z-x\|^2 + \|z-y\|^2 = \frac{1}{2}\|x-y\|^2 + 2\left\|z - \frac{1}{2}(x+y)\right\|^2$

(D)  $\|x+y\|^2 = \|x\|^2 + \|y\|^2$

19. Let  $X$  be the inner product space of all real-valued continuous functions on  $[0, 2\pi]$  with inner product defined as  $\langle x, y \rangle = \int_0^{2\pi} x(t)y(t)dt$ . Then the set  $E = \left\{ \frac{\cos nt}{\sqrt{\pi}} \mid n = 0, 1, 2, \dots \right\}$  is

(A) Orthonormal subset of  $X$

(B) Orthogonal but not an orthonormal subset of  $X$

(C) Orthonormal but not orthogonal subset of  $X$

(D) Not an orthogonal subset of  $X$

20. Let  $X = \mathbb{R}^2$  and define  $N(x, y) = |x| + |y|, \forall x, y \in \mathbb{R}$ . Then  $(X, N)$  is

(A) A normed linear space

(B) An inner product space inducing the same norm

(C) Not a normed linear space

(D)  $N$  is a norm that satisfies parallelogram law

21. The bilinear transformation  $f(z) = \frac{z+1}{z-1}$  maps the upper half plane to the region defined by

(A)  $|z| < 1$

(B)  $|z| > 1$

(C)  $\text{Im}(z) > 0$

(D)  $\text{Im}(z) < 0$

22. The series  $\sum_{n=0}^{\infty} e^{nz}$  converges

(A) for all  $z \in \mathbb{C}$

(B) for all  $z \in \mathbb{C}$  such that  $|z| < 1$

(C) for all  $z \in \mathbb{C}$  such that  $\text{Re}(z) < 0$

(D) for all  $z \in \mathbb{C}$  such that  $\text{Im}(z) < 0$



23. If  $\sum_{n=0}^{\infty} a_n z^n$  converges at  $1$ ,  $-1$  and  $1+i$  and does not converge at  $1-i$ , then the radius of convergence of  $\sum a_n (z-1)^n$  is

- (A)  $> \sqrt{2}$  (B)  $= 1$   
(C)  $= \sqrt{2}$  (D)  $< 1$

24. Let  $f: \mathbb{C} \rightarrow \mathbb{C}$  be analytic such that  $|f(z)| \leq |s-z|^3$  for all  $z, s \in \mathbb{C}$  such that  $|s-z|=1$ . Then

- (A)  $f$  is a constant function  
(B)  $f(z) = az + b$  for all  $z \in \mathbb{C}$  where  $a, b \in \mathbb{C}$  such that  $a$  can be different from  $0$   
(C)  $f(z) = az^2 + bz + c$  for all  $z \in \mathbb{C}$  where  $a, b, c \in \mathbb{C}$  such that  $a$  can be different from  $0$   
(D)  $f$  is a trigonometric function

25. Let  $f$  be analytic on  $|z| < 1$  such that  $f\left(\frac{1}{n}\right) = \frac{n+2n^2}{n^2+1}$  for infinitely many  $n \in \mathbb{N}$ . Then

- (A)  $f$  is a constant function  
(B)  $f$  is a non-constant polynomial in the domain  
(C)  $f(z) = \frac{z+2z^2}{z^2+1}$  for all  $z$   
(D)  $f(z) = \frac{z+2}{1+z^2}$  for all  $z$

26. Let  $f$  be analytic on  $0 < |z| < 1$  such that  $\lim_{z \rightarrow 0} \frac{1}{f(z)} = 0$ . Then

- (A)  $0$  is a removable singularity of  $f$   
(B)  $0$  is a pole of  $f$   
(C)  $0$  is an essential singularity of  $f$   
(D) There is  $n \in \mathbb{N}$  such that  $f(z) = \frac{1}{z^n}$  for all  $z$



27. If  $f: \mathbb{Z}_6 \rightarrow S_3$  and  $g: S_3 \rightarrow \mathbb{Z}_6$  be group homomorphisms. Then
- (A)  $f$  and  $g$  are trivial
  - (B)  $f$  is one-one, but  $g$  is not one-one
  - (C)  $g$  is one-one but  $f$  is not one-one
  - (D) neither  $f$  nor  $g$  is one-one
28. The number of elements of order 6 in  $S_5$  is
- (A) 0 (none)
  - (B) 20
  - (C) 24
  - (D) 30
29. Let  $F$  be a field with 64 elements. If  $p(x) = x^8 - 1$  and  $q(x) = x^7 + 1$ , then
- (A)  $p(x)$  and  $q(x)$  have roots in  $F$
  - (B)  $p(x)$  has a root in  $F$  and  $q(x)$  has no root in  $F$
  - (C)  $q(x)$  has a root in  $F$  and  $p(x)$  has no root in  $F$
  - (D) neither  $p(x)$  nor  $q(x)$  has a root in  $F$
30. The class equation of  $S_3 \times \mathbb{Z}_2$  is
- (A)  $1 + 1 + 2 + 2 + 3 + 3 = 12$
  - (B)  $1 + 1 + 4 + 6 = 12$
  - (C)  $1 + 1 + 2 + 2 + 6 = 12$
  - (D)  $1 + 1 + 4 + 3 + 3 = 12$
31. The number of mutually non-isomorphic abelian groups of order 720 is
- (A) 8
  - (B) 10
  - (C) 12
  - (D) 6
32. The ring  $\mathbb{Z}_{12}[x]$  is
- (A) a field
  - (B) a Euclidean domain and not a field
  - (C) an integral domain and not a Euclidean domain
  - (D) not an integral domain



33. The last (unit) digit of  $2023^{2023}$  is
- (A) 1 (B) 3  
(C) 7 (D) 9
34. The Galois group of  $(x^2 - 2)(x^2 - 3)$  over  $\mathbb{Q}$  is
- (A)  $\mathbb{Z}_2 \times \mathbb{Z}_2$  (B)  $\mathbb{Z}_4$   
(C)  $\mathbb{Z}_2$  (D)  $\mathbb{Z}_8$
35. Let  $X = \left\{ \frac{1}{n} \mid n \in \mathbb{N} \right\}$ ,  $Y = \mathbb{N}$  and  $Z = \mathbb{Z}$  be subspaces of  $\mathbb{R}$  with usual metric topology. Then
- (A)  $X$  and  $Y$  are homeomorphic to  $Z$   
(B)  $X$  is homeomorphic to  $Z$  and  $Y$  is not homeomorphic to  $Z$   
(C)  $Y$  is homeomorphic to  $Z$  and  $X$  is not homeomorphic to  $Z$   
(D) Neither  $X$  nor  $Y$  is homeomorphic to  $Z$
36. Let  $\tau$  be the topology on  $\mathbb{R}$  generated by the basis  $\{(a, b] \mid a, b \in \mathbb{Q}\}$ . If for every  $n \in \mathbb{N}$ ,  $x_n = \frac{1}{n}$  and  $y_n = \sqrt{2} + \frac{1}{n}$ , then
- (A)  $x_n \xrightarrow{n} 0$  and  $y_n \longrightarrow \sqrt{2}$  in  $\tau$   
(B)  $x_n \longrightarrow 0$  and  $(y_n)$  does not converge in  $\mathbb{R}$  in the topology  $\tau$   
(C)  $y_n \longrightarrow \sqrt{2}$  and  $(x_n)_n$  does not converge in  $\mathbb{R}$  in the topology  $\tau$   
(D) neither  $(x_n)$  nor  $(y_n)$  converges in  $\mathbb{R}$  in the topology  $\tau$
37. Let  $X$  and  $Y$  be topological spaces and  $f : X \rightarrow Y$  be a continuous map. Then
- (A)  $Y$  is compact if  $X$  is compact  
(B)  $Y$  is connected if  $X$  is connected  
(C)  $f$  is an open map if  $X$  is compact and  $Y$  is Hausdorff  
(D)  $f$  is closed map if  $X$  is compact and  $Y$  is Hausdorff



38. If  $X, Y \subset \mathbb{R}$  and  $Z = \{x+y \mid x \in X, y \in Y\}$ . Then (in  $\mathbb{R}$ )
- (A)  $Z$  is open if  $X$  is open
  - (B)  $Z$  is closed if  $X$  is closed
  - (C)  $Z$  is compact if  $X$  is compact
  - (D)  $Z$  is connected if  $X$  is connected
39. Let  $X$  and  $Y$  be topological spaces. Which one of the following is false ?
- (A) If  $X$  and  $Y$  are first countable,  $X \times Y$  is first countable
  - (B) If  $X$  and  $Y$  are second countable,  $X \times Y$  is second countable
  - (C) If  $X$  and  $Y$  are regular,  $X \times Y$  is regular
  - (D) If  $X$  and  $Y$  are normal,  $X \times Y$  is normal
40. Let  $X$  be the subspace  $[0, 1]$  of  $\mathbb{R}_l$  ( $\mathbb{R}$  with the topology generated by  $\{[a, b] \mid a, b \in \mathbb{R}\}$ ). Then
- (A)  $X$  is compact
  - (B)  $X$  is connected
  - (C)  $X$  is normal
  - (D)  $X$  is regular
41. The differential equation  $x \frac{dy}{dx} - (\ln x)y^2 + y = 0, x > 0$ , is
- (A) Bernoulli's equation of degree 2
  - (B) An exact differential equation
  - (C) A homogeneous differential equation
  - (D) Reducible to an exact differential equation
42. The growth of a population in a closed environment is given to be governed by the differential equation,  $\frac{dN}{dt} = aN - bN^2, N > 0, a > 0, b > 0$  where  $N(t)$  represents the population at any time  $t$ .
- If  $N = N_0$ , when  $t = 0$ , then  $\lim_{t \rightarrow \infty} N(t)$  is
- (A)  $\frac{a}{b^2}$
  - (B)  $\frac{a}{b}$
  - (C)  $\frac{b}{a}$
  - (D)  $\frac{b^2}{a}$





43. For the initial value problem  $\frac{dy}{dx} = y^2 + \cos x$ ,  $x > 0$ ,  $y(0) = 0$  the largest interval of existence for the solution, predicted by Picard's theorem, is

(A)  $\left[0, \frac{1}{4}\right]$

(B)  $\left[0, \frac{1}{3}\right]$

(C)  $\left[0, \frac{1}{2}\right]$

(D)  $[0, 1]$

44. If  $L_1(D) = 3xD$ ,  $L_2(D) = 2D$ ,  $D \equiv \frac{d}{dx}$  are any two differential operators, then the differential operator  $(L_1L_2 - L_2L_1)(D)$ , is

(A)  $6D$

(B)  $4D$

(C)  $-4D$

(D)  $-6D$

45. Consider the differential equation  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$  and the two functions

$$\phi_1(x) = \frac{1}{2}(e^x + e^{3x}) \text{ and } \phi_2(x) = \frac{1}{2}(e^{2x} + e^{3x})$$

The correct statement concerning the above two functions is

(A)  $y = 2\phi_1(x) - 3\phi_2(x)$ , is a solution of the given O.D.E.

(B)  $y = 2\phi_1(x) + \phi_2(x)$ , is a solution of the given O.D.E.

(C)  $y = \phi_1(x)$  and  $y = \phi_2(x)$  are both solutions of the given O.D.E., but their linear combination  $\{2\phi_1(x) + \phi_2(x)\}$  is not a solution of the given O.D.E.

(D)  $\{2\phi_1(x) + \phi_2(x)\}$  is not a solution of the given O.D.E., because  $\phi_1(x)$ ,  $\phi_2(x)$  are both a linear combination of exponential functions



46. The partial differential equation of the family of surfaces,  $Z = (x + y) + f(xy)$ , is  
(A)  $xp - yq = x - y$  (B)  $xp - yq = 0$  (C)  $xp + yq = 0$  (D)  $xp + yq = x + y$

47. A solution of the first order PDE  $p^2 + q^2 = x + y$  is

- (A)  $z = \frac{2}{3}(a + 2x)^{3/2} + \frac{2}{3}(y - 2a)^{3/2} + b$  (B)  $z = \frac{2}{3}(a + x)^{3/2} + \frac{2}{3}(y - a)^{3/2} + b$   
(C)  $z = \frac{1}{3}(a + x)^{3/2} + \frac{1}{3}(y - a)^{3/2} + b$  (D)  $z = \frac{1}{3}(a + x)^{3/2} - \frac{1}{3}(y - a)^{3/2} + b$

where  $b$  is a constant

48. Which one of the following represents one dimensional heat equation ?

- (A)  $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$  (B)  $\frac{\partial^2 u}{\partial x^2} + k \frac{\partial^2 u}{\partial y^2} = 0$   
(C)  $\frac{\partial^2 u}{\partial t^2} = k \frac{\partial^2 u}{\partial x^2}$  (D)  $\left(\frac{\partial u}{\partial t}\right)^2 = k \frac{\partial^2 u}{\partial x^2}$

49. The solution of the PDE  $(z_x^2 + z_y^2)y = zz_y$ , is

- (A)  $z^2 = (\alpha + cx)^2 + c^2 y^2$  (B)  $z = (\alpha + cx)^2 + c^2 y^2$   
(C)  $z^2 + (\alpha + cx)^2 + c^2 y^2 = 0$  (D)  $z = (\alpha + cx) + c^2 y^2$

50. Suppose that the basic Gauss elimination algorithm is used to solve the system of equations :

$$7x_1 - 7x_2 + x_3 = 12$$

$$-21x_1 + 21x_2 + 10x_3 = 3$$

$$14x_1 + 0x_2 + 2x_3 = 2$$

Then the algorithm will

- (A) Successfully complete the solution  
(B) Terminate in the beginning of the first step  
(C) Terminate in the beginning of the second elimination step  
(D) Terminate in the beginning of the back-substitution process



51. If a sequence  $\{x_n\}$  of approximations to a root of the equation  $x^3 + 3x^2 - 1$  in the interval  $[-3, -2]$  is generated using Newton-Raphson method and  $x_0 = -3$  then  $x_2 =$
- (A)  $-3.0006$  (B)  $-2.899$   
(C)  $-2.879$  (D)  $-3.1$

52. The percent error in the estimation of  $\int_1^2 \sqrt{x} dx$  using Simpson's  $\frac{1}{3}$  rule over 4 equidistant points is
- (A) 1.244% (B) 15.18%  
(C) 12.44% (D) 1.518%

53. For finding the approximate solution of an initial value problem :  $\frac{dy}{dt} = f(t, y)$ ,  $y(t_0) = y_0$  by Runge-Kutta method of order 2, the required iterative scheme is

- (A)  $y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)$ , where  $k_1 = f(t_i, y_i)$  and  $k_2 = f\left(t_i + \frac{h}{2}, y_i + h\frac{k_1}{2}\right)$   
(B)  $y_{i+1} = y_i + hf(t_i, y_i)$   
(C)  $y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)$ , where  $k_1 = f(t_i, y_i)$ ,  $k_2 = f\left(t_i + \frac{h}{2}, y_i + hk_1\right)$   
(D)  $y_{i+1} = y_i + \frac{h}{2}(k_1 + k_2)$ , where  $k_1 = f(t_i, y_i)$ ,  $k_2 = f(t_i + h, y_i + hk_1)$

54. In order to show that the shortest distance between the two points  $(x_1, y_1)$  and  $(x_2, y_2)$  in  $\mathbb{R}^2$ , which one of the variational problem should be solved ?

- (A) Minimize  $J[y(x)] = \int_{x_1}^{x_2} [1 + y'^2(x)] dx$   
(B) Minimize  $J[y(x)] = \int_{x_1}^{x_2} [1 + y'^2(x)]^{\frac{1}{2}} dx$   
(C) Minimize  $J[y(x)] = \int_{x_1}^{x_2} [1 + y'^2(x)]^{\frac{1}{2}} dx$   
(D) Minimize  $J[y(x)] = \int_{x_1}^{x_2} [1 + y^2(x)] dx$



55. The eigenvalues and eigenfunctions of  $y(x) = \lambda \int_{-1}^1 (x+t)y(t)dt$  are

(A)  $\lambda = +\frac{\sqrt{3}}{2}, \varphi_1(x) = \frac{\sqrt{3}}{2}(1-\sqrt{3}x); \lambda = -\frac{\sqrt{3}}{2}, \varphi_2(x) = -\frac{\sqrt{3}}{2}(1+\sqrt{3}x)$

(B)  $\lambda = \frac{\sqrt{3}}{2}, \varphi_1(x) = \frac{\sqrt{3}}{2}(1+\sqrt{3}x); \lambda = -\frac{\sqrt{3}}{2}, \varphi_2(x) = -\frac{\sqrt{3}}{2}(1-\sqrt{3}x)$

(C)  $\lambda = \frac{\sqrt{3}}{2}, \varphi_1(x) = -\frac{\sqrt{3}}{2}(1-\sqrt{3}x); \lambda = -\frac{\sqrt{3}}{2}, \varphi_2(x) = \frac{\sqrt{3}}{2}(1+\sqrt{3}x)$

(D)  $\lambda = \frac{\sqrt{3}}{2}, \varphi_1(x) = -\frac{\sqrt{3}}{2}(1+\sqrt{3}x); \lambda = -\frac{\sqrt{3}}{2}, \varphi_2(x) = \frac{\sqrt{3}}{2}(1-\sqrt{3}x)$

56. For the initial value problem  $y''(x) - y(x) = 0$  with initial conditions  $y(0) = 0, y'(0) = 1$ , the corresponding integral equation is

(A)  $y(x) = x + \int_0^1 (x-t)y(x) dt$

(B)  $y(x) = x + \int_0^x (t-x)y(t)dt$

(C)  $y(x) = 1 + \int_0^x (1-x)y(x)dx$

(D)  $y(x) = x + \int_0^x (x-t)y(t)dt$

57. A uniform rod AB moves so that A and B have velocities  $\bar{u}, \bar{v}$  respectively, at any moment  $t$ . If  $M$  is the mass of the rod, then the kinetic energy of the rod, at that moment is

(A)  $\frac{1}{6}M[\bar{u}^2 + \bar{v}^2 + \bar{u} \cdot \bar{v}]$

(B)  $\frac{1}{4}M[\bar{u}^2 + \bar{v}^2 + \bar{u} \cdot \bar{v}]$

(C)  $\frac{1}{2}M[\bar{u}^2 + \bar{v}^2 + \bar{u} \cdot \bar{v}]$

(D)  $\frac{1}{2}M[\bar{u}^2 + \bar{v}^2 + 2\bar{u} \cdot \bar{v}]$

58. The kinetic and potential energies of a particle of unit mass, moving in a plane, are given by  $T = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2), V = -\frac{\mu}{r}$ , where  $(r, \theta)$  are polar coordinates.

If  $p_r, p_\theta$  be the corresponding momentum coordinates, the Hamiltonian  $H(r, \theta, p_r, p_\theta)$ , governing the motion of the particle is

(A)  $\left(p_r^2 + \frac{1}{r^2}p_\theta^2\right) - \frac{\mu}{r}$

(B)  $\frac{1}{2}\left(p_r^2 + \frac{p_\theta^2}{r^2}\right) - \frac{\mu}{r}$

(C)  $\frac{1}{2}\left(p_r^2 + p_\theta^2\right) + \frac{\mu}{r}$

(D)  $2\left(p_r^2 + \frac{p_\theta^2}{r^2}\right) - \frac{\mu}{r}$



59. If a uniform solid sphere of radius  $R$  is in motion on a horizontal fixed plane, without slipping, then the condition of rolling, at the point of contact of the plane, in terms of  $\vec{r}, \vec{\Omega}, R$ : where  $\vec{r}$  denotes the position vector of the centre of the sphere from a fixed point on the plane,  $\vec{\Omega}$  denotes the rotational angular velocity of the sphere about its centre and  $R$  denotes the radius of the sphere, is

- (A)  $\dot{\vec{r}} + R\vec{\Omega} \times \vec{i} = \vec{0}$  (B)  $\dot{\vec{r}} + R\vec{\Omega} \times \vec{j} = \vec{0}$   
 (C)  $\dot{\vec{r}} - R\vec{\Omega} \times \vec{k} = \vec{0}$  (D)  $\dot{\vec{r}} + R\vec{\Omega} \times \vec{k} = \vec{0}$

Where  $\vec{i}, \vec{j}, \vec{k}$  are unit vectors along the axes at 'O', which is the origin of the coordinates.

60. A rigid body is moving about a point 'O' of itself, which is kept fixed. If  $\vec{\Gamma}(o)$  be the moment of the external forces acting on the body, about the fixed point 'O' and  $\vec{\Omega}$  be the instantaneous angular velocity of the body, then the rate of change of the kinetic energy 'T' of the rigid body is given by

- (A)  $\frac{dT}{dt} = (\vec{\Omega} \cdot \vec{\Gamma}(o))$  (B)  $\frac{dT}{dt} = \frac{1}{2}(\vec{\Omega} \cdot \vec{\Gamma}(o))$   
 (C)  $\frac{dT}{dt} = \frac{1}{4}(\vec{\Omega} \cdot \vec{\Gamma}(o))$  (D)  $\frac{dT}{dt} = 2(\vec{\Omega} \cdot \vec{\Gamma}(o))$

61. Let  $X$  and  $X - Y$  are uncorrelated variables then  $V(X - Y)$  equals

- (A)  $V(X) - V(Y)$  (B)  $V(Y) - V(X)$  (C)  $V(X) + V(Y)$  (D)  $2V(X) + V(Y)$

62. In answering a question on a multiple choice test, each question having  $m$  ( $>1$ ) multiple choice answers, the probability that an examinee knows an answer is 'p', otherwise he guesses. Assuming that an examinee answer a question correctly, the probability that he knows the answer is

- (A)  $\frac{p}{1+(m-1)p}$  (B)  $\frac{mp-1}{mp}$  (C)  $\frac{mp}{1+p(m-1)}$  (D)  $\frac{mp}{1+(m+1)p}$

63. Let the cumulative distribution function of a random variable  $X$  is

$$F(x) = \begin{cases} 0, & \text{if } x < 1 \\ \frac{(x+1)}{3}, & \text{if } 1 \leq x < 2 \\ 1, & \text{if } x \geq 2 \end{cases}$$

Then,  $E(X)$  is

- (A)  $\frac{4}{3}$  (B)  $\frac{1}{2}$  (C)  $\frac{1}{3}$  (D)  $\frac{7}{6}$



64. Let  $\rho$  denote the correlation coefficient between the random variables  $X$  and  $Y$ . Let  $V(X) = V(Y)$ . Then the correlation coefficient between  $X$  and  $X + Y$  is

- (A)  $\sqrt{\frac{1+\rho}{2}}$       (B)  $\sqrt{1+\rho}$       (C)  $\sqrt{1+2\rho}$       (D)  $\sqrt{\frac{1-\rho}{2}}$

65. Let the random variable  $X$  have density  $f(x) = \begin{cases} 1-|1-x|, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$  then,  $P\left(\frac{1}{2} \leq X \leq \frac{3}{2}\right)$  is

- (A)  $\frac{3}{4}$       (B)  $\frac{1}{2}$       (C)  $\frac{1}{4}$       (D)  $\frac{2}{3}$

66. Let  $X$  and  $Y$  be two independent Poisson random variables with  $E(X) = E(Y)$ . Let  $\alpha = P(X \leq 2 | X + Y = 5)$ . Then  $\alpha$  becomes

- (A) less than  $\frac{1}{2}$   
(B) greater than  $\frac{1}{2}$   
(C)  $\frac{1}{2}$   
(D) undeterministic because  $E(X)$  is unknown

67. The joint distribution of two random variables  $X$  and  $Y$  is given below

$X \backslash Y$	1	2
0	$\frac{1}{4}$	$\frac{1}{4}$
1	$\frac{1}{4}$	$\frac{1}{4}$

Let  $B_1 = \{X < \frac{1}{2}, Y > 1\}$  and  $B_2 = \{x \geq \frac{1}{2}, Y > 1\}$

Then which of the following results holds ?

- (A) The events in both  $B_1$  and  $B_2$  are Independent  
(B) Only in  $B_1$ , the events are independent  
(C) Only in  $B_2$ , the events are independent  
(D) In both,  $B_1$  and  $B_2$ , the events are not independent



68. The average of a group of 20 observations is 16. If 2 observations with average 10 are discarded and 6 observations with average 10 are included in the group, then what is the percentage change in the average of the group ?

- (A) 0.0% (B) 6.25% decrease  
(C) 6.25% increase (D) 1.0% decrease

69. A subset is selected at random from  $\{1, 2, \dots, 10\}$ . The probability that it contains the elements 3 and 7 is

- (A)  $\frac{1}{2}$  (B)  $\frac{1}{4}$   
(C)  $\frac{1}{5}$  (D)  $\frac{1}{6}$

70. Let  $X$  and  $Y$  be independent random variables each having uniform  $U(0, 1)$  distribution. Define  $U = \min(X, Y)$  and  $V = \max(X, Y)$ . Consider the following statements.

$$T_1 : P(V \leq x) = x^2$$

$$T_2 : E(V) > E(U)$$

Which of the above two statements is/are correct ?

- (A)  $T_1$  only (B)  $T_2$  only  
(C)  $T_1$  and  $T_2$  both (D) Neither  $T_1$  nor  $T_2$

71. Let  $\Omega$  is a set of points  $W$ ,  $C_i = 1, 2, \dots, 7$  are subsets of  $\Omega$ , and  $(\Omega, \mathcal{F}, P) = (B_i, \frac{i}{10}, i = 1, 2, 3, 4)$  is a probability model with  $B_1 = C_1 \cup C_7$ ,  $B_2 = C_2 \cup C_8$ ,  $B_3 = C_3 \cup C_5$  and  $B_4 = C_4$ . Then the distribution of a random variable  $X(w) = (v - 4)^2$  for  $w \in C_v$ ,  $v = 1, 2, \dots, 7$  is

- (A)  $\begin{bmatrix} 0 & 1 & 4 & 9 \\ 0.4 & 0.3 & 0.2 & 0.1 \end{bmatrix}$  (B)  $\begin{bmatrix} 0 & 1 & 4 & 9 \\ 0.1 & 0.2 & 0.3 & 0.4 \end{bmatrix}$   
(C)  $\begin{bmatrix} 0 & 1 & 4 & 9 \\ 0.3 & 0.4 & 0.2 & 0.1 \end{bmatrix}$  (D)  $\begin{bmatrix} 0 & 1 & 4 & 9 \\ 0.1 & 0.2 & 0.4 & 0.3 \end{bmatrix}$



72. Let  $X$  be a random variable with moment generating function  $M_X(t) = (1 - 4t)^{-5}$ ,  $t < \frac{1}{4}$ . Then the lower bounds for  $P(X < 40)$ , using Chebyshev's inequality and Markov's inequality, respectively, are

(A)  $\frac{4}{5}$  and  $\frac{5}{6}$

(B)  $\frac{5}{6}$  and  $\frac{5}{6}$

(C)  $\frac{5}{6}$  and  $\frac{1}{2}$

(D)  $\frac{4}{5}$  and  $\frac{1}{2}$

73. Let  $(X, Y)$  be a random vector having the joint moment generating function

$$M(t_1, t_2) = \left( \frac{1}{2}e^{-t_1} + \frac{1}{2}e^{t_1} \right)^2 \left( \frac{1}{2} + \frac{1}{2}e^{t_2} \right)^2, (t_1, t_2) \in \mathbb{R}^2. \text{ Then } P(|X + Y| = 2) \text{ equals}$$

(A)  $\frac{1}{3}$

(B)  $\frac{1}{4}$

(C)  $\frac{1}{5}$

(D)  $\frac{1}{6}$

74. Let  $\{X_n\}_{n \geq 1}$  be a sequence of independent and identically distributed random variables with probability density function

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Then, the value of the  $\lim_{n \rightarrow \infty} P\left(\frac{-1}{n} \sum_{i=1}^n \log X_i \leq 1 + \frac{1}{\sqrt{n}}\right)$  is equal to

(A) 0

(B)  $\frac{1}{2}$

(C)  $\Phi(1)$ ,  $\Phi(\cdot)$  is the cdf of  $N(0, 1)$  distribution

(D)  $\Phi(2)$

75. Let  $X_1, X_2, \dots, X_n$  ( $n \geq 2$ ) be a random sample from a distribution with probability density function  $f(x, \theta) = \frac{3x^2}{\theta} e^{-x^3/\theta}$ ,  $x > 0, \theta > 0$ ;  $\theta$  is unknown. If  $T = \sum_{i=1}^n X_i^3$ , then which of the following statements is true ?

(A)  $\frac{n-1}{T}$  is the unique UMVUE of  $\frac{1}{\theta}$

(B)  $\frac{n}{T}$  is the unique UMVUE of  $\frac{1}{\theta}$

(C)  $\frac{T}{n}$  is the MLE of  $\frac{1}{\theta}$

(D)  $\frac{n}{T}$  is the MLE of  $\theta$





76. Let  $X_1, X_2, \dots, X_n$  be a random sample from the distribution having probability density function  $f(x, \theta) = \theta(1-x)^{\theta-1}$ ,  $0 < x < 1, \theta > 0$ .

To test  $H_0 : \theta = 1$  against  $H_1 : \theta > 1$ , the UMP test of size  $\alpha$  ( $0 < \alpha < 1$ ) would reject  $H_0$  if

(A)  $-\sum_{i=1}^n \log(1-X_i)^2 \leq \chi_{2n, 1-\alpha}^2$

(B)  $-\sum_{i=1}^n \log(1-X_i)^2 \leq \chi_{n, 1-\alpha}^2$

(C)  $-\sum_{i=1}^n \log(1-X_i)^2 \leq \chi_{n, \alpha}^2$

(D)  $-\sum_{i=1}^n \log(1-X_i)^2 \leq \chi_{2n, \alpha}^2$

Here  $P(\chi_n^2 \geq \chi_{n, \alpha}^2) = \alpha$

77. Let  $X_1, X_2, \dots, X_n$  be a random sample from  $U(\theta - 0.5, \theta + 0.5)$  distribution,  $\theta \in \mathbb{R}$ .

If  $X_{(1)} = \min\{X_1, X_2, \dots, X_n\}$  and  $X_{(n)} = \max\{X_1, X_2, \dots, X_n\}$  then, which one of the following statements is not a MLE of  $\theta$ ?

(A)  $\frac{1}{2}(3X_{(n)} - X_{(1)} - 2)$

(B)  $\frac{1}{2}(X_{(1)} + X_{(n)})$

(C)  $\frac{1}{4}(3X_{(1)} + X_{(n)} + 1)$

(D)  $\frac{1}{4}(X_{(1)} + 3X_{(n)} - 1)$

78. Consider the following two probability density functions (pdfs).

$$f_0(x) = \begin{cases} 2x, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases} \text{ and } f_1(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Let  $X$  be a random variable having pdf  $f_0$  or  $f_1$ . Considering the test to, test

$H_0 : X \sim f_0(x)$  versus  $H_1 : X \sim f_1(x)$  at level  $\alpha = 0.05$ , which of the following observed value of  $X$ , the MP test rejects  $H_0$ .

(A) 0.28

(B) 0.25

(C) 0.23

(D) 0.18



79. Let  $X_1, X_2, X_3, Y_1, Y_2, Y_3, Y_4$  be iid  $N(\mu, \sigma^2)$  random variables. Let

$$\bar{X} = \frac{1}{3} \sum_{i=1}^3 X_i \text{ and } \bar{Y} = \frac{1}{4} \sum_{j=1}^4 Y_j. \text{ The distribution of } \frac{\sqrt{15} (\bar{X} - \bar{Y})}{\sqrt{\sum_{i=1}^3 (X_i - \bar{X})^2 + \sum_{j=1}^4 (Y_j - \bar{Y})^2}}$$
 is same

as that of KW, where

- (A)  $K = 2, W \sim t_{(7)}$  (B)  $K = \frac{1}{2}, W \sim t_{(7)}$   
 (C)  $K = \frac{1}{2}, W \sim t_{(5)}$  (D)  $K = 2, W \sim t_{(5)}$

80. Let  $X_1, X_2, \dots, X_n$  follows  $N(\mu, 1)$  distribution and let prior distribution of  $\mu$  be  $N(0, 1)$ .

Under squared error loss function, the Bayes estimator of  $\mu$  is

- (A)  $\frac{\sum_{i=1}^n X_i}{n}$  (B)  $\frac{\sum_{i=1}^n X_i}{n+1}$   
 (C)  $\frac{\sum_{i=1}^n X_i^2}{n}$  (D)  $\frac{\sum_{i=1}^n X_i^2}{n+1}$

81. Let  $X_1, X_2, \dots, X_n$  ( $n \geq 3$ ) be a random sample from Poisson distribution with mean

$\theta > 0$ . Define  $T = \sum_{i=1}^n X_i$ , then UMVUE of  $\theta^3 e^{-2\theta}$  is

- (A)  $\frac{T}{n} \left( \frac{T}{n} - 1 \right) \left( \frac{T}{n} - 2 \right) \left( 1 - \frac{2}{n} \right)^{T-3}$   
 (B)  $\frac{T(T-1)(T-2)(n-2)^{T-3}}{n^T}$   
 (C)  $T(T-1)(T-2) \left( \frac{n-2}{n} \right)^{T-3}$   
 (D)  $e^{-\frac{2T}{n}} \left( \frac{T}{n} \right)^3$



82. Consider a Markov chain with transition probability matrix P given as

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

Then  $\lim_{n \rightarrow \infty} p_{23}^{(n)}$  equals

- (A)  $\frac{1}{3}$  (B)  $\frac{1}{2}$   
 (C) 0 (D) 1

83. Let  $Y_1, Y_2, Y_3$  be uncorrelated observations with common variance  $\sigma^2$  and  $E(Y_1) = \theta_0 + \theta_1$ ,  $E(Y_2) = \theta_0 + \theta_2$  and  $E(Y_3) = \theta_0 + \theta_3$ , where  $\theta_i$ 's are unknown parameters. In the frame work of the linear model, which of the following is true ?

- (A)  $\sum_{i=1}^3 \theta_i$  is estimable and error sum of squares is zero  
 (B) Each of  $\theta_0, \theta_1, \theta_2$  and  $\theta_3$  is individually estimable but error sum of squares is not zero  
 (C)  $\theta_1 - \theta_2, \theta_1 - \theta_3$  and  $\theta_3 - \theta_2$  are each estimable and error sum of squares is zero  
 (D)  $\theta_1 - \theta_2, \theta_1 - \theta_3$  and  $\theta_3 - \theta_2$  are each estimable and error sum of squares is not zero

84. Consider the model  $Y_i = i\beta + \epsilon_i, i = 1, 2, 3$ , where  $\epsilon_1, \epsilon_2$  and  $\epsilon_3$  are independent with mean 0 and variance  $\sigma^2, 2\sigma^2$  and  $3\sigma^2$  respectively. Which of the following is the best linear unbiased estimator of  $\beta$  ?

- (A)  $(Y_1 + 2Y_2 + 3Y_3)/6$  (B)  $\frac{6}{11} \left( Y_1 + \frac{Y_2}{2} + \frac{Y_3}{3} \right)$   
 (C)  $(3Y_1 + 2Y_2 + Y_3)/10$  (D)  $(Y_1 + Y_2 + Y_3)/6$



85. Let  $Y = (Y_1, Y_2, \dots, Y_n)'$  have  $N_n(0, I)$  multivariate normal distribution. Which of the following is the covariance matrix of the conditional distribution of  $Y$  given  $\sum_{i=1}^n Y_i$ ? ( $l$  denote the  $n \times 1$  vector with all elements 1.)

- (A)  $l$                       (B)  $I + \frac{ll'}{n}$                       (C)  $I - \frac{ll'}{n}$                       (D)  $\frac{ll'}{n}$

86. Let  $\underline{X}' = (X_1, X_2, X_3, X_4, X_5) \sim N_5(\underline{\mu}, \Sigma)$  and eigenvalues of the correlation matrix are (0.5, 0.3, 0.3, 3.5, 0.4). Then the percentage variance that the first principal component accounts, out of the total variances is

- (A) 50%                      (B) 60%                      (C) 70%                      (D) 80%

87. Consider the following statements :

$T_1$  : The bias ratio estimator for estimating  $\bar{Y}$  becomes zero if the regression of  $Y$  on  $X$  passes through origin.

$T_2$  : The ratio estimator of population mean is more efficient than sample mean when  $\rho_{X,Y} > 0$ , where  $\rho_{X,Y}$  is the correlation coefficient between  $Y$  and  $X$ .

Which of the above statements is/are correct ?

- (A)  $T_1$  only                      (B)  $T_2$  only  
(C) Both  $T_1$  and  $T_2$                       (D) Neither  $T_1$  nor  $T_2$

88. The following observations were recorded from two populations with distribution functions  $F_1(x)$  and  $F_2(x)$ .

Population – I : 3, 7; Population – II : 2, 5, 8, 9

The Mann-Whitney test statistic  $U$  for testing  $H_0 : F_1(x) = F_2(x)$  has the value

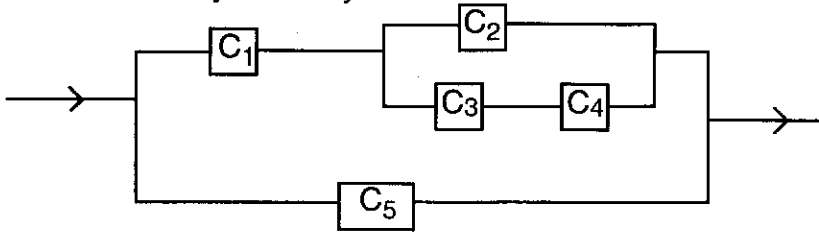
- (A) 02                      (B) 03                      (C) 06                      (D) 08

89. The density function of a random variable  $X$  is given by  $f(x, \theta) = \theta e^{-\theta x}$ ,  $x > 0$ ,  $\theta > 0$ . The asymptotic confidence interval with confidence coefficient 0.95 for parameter  $\theta$  is

- (A)  $\bar{x} \left( 1 \pm \frac{1.96}{\sqrt{n}} \right)$                       (B)  $\bar{x} \pm \frac{1.96}{\sqrt{n}} \frac{1}{\bar{x}}$   
(C)  $\sqrt{n} \bar{x} \pm \frac{1.96}{\sqrt{n}} \bar{x}$                       (D)  $\left( 1 \pm \frac{1.96}{\sqrt{n}} \right) \frac{1}{\bar{x}}$



90. The equi-reliable components with reliability 0.5 are connected in the following system. Then, reliability of the system is



- (A)  $\frac{21}{32}$       (B)  $\frac{7}{8}$       (C)  $\frac{20}{34}$       (D)  $\frac{7}{16}$
91. To estimate population mean, if allowable difference in the estimate is 2 and population standard deviation is 6, then the necessary sample size with 99 per cent precision is approximately [Given  $\Phi(2.576) \approx 0.995$ ]
- (A) 40      (B) 50      (C) 55      (D) 60
92. In the following blocks of  $2^3$  factorial design, the confounded treatment combination is  
 B – I : ((1), c, ab, abc)  
 B – II : (a, b, ac, bc)
- (A) AC      (B) BC      (C) AB      (D) ABC
93. Let  $X(t)$  = number of customers in the system at time  $t$  in  $M | M | C$  queueing model, with  $C = 3$ , arrival rate  $\lambda > 0$  and survival rate  $\mu > 0$ . Which of the following statements is true ?
- (A)  $\{X_{(t)}\}$  is birth and death process with constant birth and death rates  
 (B) If  $X_t$  has a stationary distribution then  $\lambda > 3\mu$   
 (C) If  $\lambda < 3\mu$ , then stationary distribution is a geometric distribution with parameter  $\frac{\lambda}{3\mu}$   
 (D) The number of customers undergoing service at time  $t$  is  $\min\{X(t), 3\}$
94. If FPC is ignored,  $V(P_{st})$  under stratified random sampling is

(A)  $\sum_h \left( \frac{W_h^2 P_h Q_h}{N_h} \right)$       (B)  $\sum_h \left( \frac{W_h^2 P_h Q_h}{n_h} \right)$

(C)  $\sum_h \left( \frac{W_h P_h Q_h}{N_h} \right)$       (D)  $\sum_h \left( \frac{W_h^2 P_h Q_h}{N_h^2} \right)$





**Space for Rough Work**





**Space for Rough Work**

