Top 10 JEE Main Binomial Theorem Questions with Solutions

The top 10 JEE Main Binomial Theorem questions with answers are listed below for students to practise to prepare the Binomial Theorem topics.

Question 1: If $(1 + a^x)^n = 1 + 8x + 24x^2 +$ —, then the value of a and n are?

Answer: As given $(1 + a^x)^n = 1 + 8x + 24x^2 +$ —---?

On comparing same coefficients, we have

$= {}^na\ (n - 1)\ a = 48$

$8\ (8 - a) = 48$

$8 - a = 6$

$= a = 2$

$= n = 4$

Question 2: If the coefficient of x^7 in $(ax^2 + [1/bx])^{11}$ is equal to the coefficient of $x-7$ in $(ax - 1/bx^2)11$, then ab = ___.

Answer: In the expansion of $(ax^2 + [1/bx])^{11}$, the general term is

$T^{r+1} = {}^{11}Cr\ (ax^2)^{11-r}\ (1/bx)^r$

$= {}^{11}C_r * a^{11-r} * [\ 1/br\ ] * x^{22-3r}$

For x^7, we must have $22 – 3r = 7$

$r = 5$, and the coefficient of $x^7 = {}^{11}C_5 * a^{11-5} * [1/b^5] = {}^{11}C_5 * a^6 * b^5$

Similarly, in the expansion of $(ax^{-1}bx^2)^{11}$, the general term is

$T^{r+1} = {}^{11}Cr * (-1)^r * [a^{11} - r/b^r\ ] * x^{11-3r}$

For x-7 we must have, $11 – 3r = -7$, $r = 6$, and the coefficient of $x-7$ is ${}^{11}C_6 * [a^5/b^6] = {}^{11}C_5 * a^5 * b^6$.

As given, ${}^{11}C_5\ [a^6/b^5] = {}^{11}C_5 * [a^5/b^6]$

$= ab = 1$

Question 3: In the polynomial (x − 1) (x − 2) (x − 3) . . . . . (x − 100), what is the coefficient of x^{99}?

Answer: (x − 1) (x − 2) (x − 3) . . . . . (x − 100)

Number of terms = 100

Coefficient of x^{99} = (x − 1) (x − 2) (x − 3) . . . . . (x − 100)

= (−1 −2 −3 − — −100)

= − (1 + 2 + — +100)

= − [100 * 101 / 2

= – 5050

Question 4: In the expansion of (x + a)n, the sum of odd terms is P and sum of even terms is Q, then the value of $(P^2 - Q^2)$ will be _________.

Answer: $(x + a)^n = x^n + {}^nC_1x^{n-1} a + ... = (x^n + nC^2x^{n-2}a^2 + ... + ({}^nC_1x^{n-1}a + {}^nC_3x^{n-3}a_3 + —)$ = P+Q

$(x - a)^n = P - Q$

As the terms are altered,

$P^2 - Q^2 = (P + Q) (P - Q) = (x + a)^n (x - a)^n$

$P^2 - Q^2 = (x^2 - a^2)^n$

Question 5: If the sum of the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ is a and if the sum of the coefficients in the expansion of $(1 +x^2)^n$ is b, then what is the relation between a and b?

Answer: We have a = sum of the coefficient in the expansion of $(1 - 3x + 10x2)^n$ = (1 − 3 + 10)n = $(8)^n$

$=(1 - 3x + 10x^2)^n = (2)3^n$, [Putting x = 1] .

Now, b = sum of the coefficients in the expansion of $(1 + x^2)^n = (1 + 1)^n$ = 2n.

Clearly, $a = b^3$

Question 6: $(1 + x)^n - n^{x - 1}$ is divisible by?

1. A) 2x
2. B) x^2
3. C) $2x^3$

4. D) All of these

Answer: $(1 + x)^n = 1 + n^x + ([n (n − 1)] / [2!]) * x^2 + ([n (n − 1) (n − 2)] / [3!]) * x^3 + —$

$=(1 + x)^n − n^{x − 1} = x^2 [([n (n − 1)] / [2!]) + ([n (n − 1) (n − 3)] / [3!]) * x + —]$

=From above it is clear that $(1 + x)^n − nx − 1$ is divisible by x^2.

Trick: $(1 + x)^n − nx − 1$, put n = 2 and x = 3;

Then 42 − 2 * 3 − 1 = 9 is not divisible by 6, 54 but divisible by 9, which is given by option (b) i.e., $x^2 = 9$.

Question 7: If the three consecutive coefficients in the expansion of (1 + x)n are 28, 56 and 70, then the value of n is __?

Answer: Let the three consecutive coefficients be ${}_nC^{r−1} = 28$, ${}_nC^r = 56$ and ${}_nC^{r+1} = 70$, so that

${}_nC^r / {}_nC^{r−1} = [n − r + 1] / [r] = 56 / 28 = 2$ and ${}_nC^{r+1} / {}_nC^r = [n − r] / [r + 1] = 70 / 56 = 5 / 4$

This gives n + 1 = 3r and 4n − 5 = 9r

4n − 5 / n + 1 = 3

= n = 8

Question 8: Let $R = (5√5 + 11)^{2n + 1}$ and f = R − [R], where [.] denotes the greatest integer function. The value of R * f is

Answer: Since (5√5 − 11) (5√5 + 11) = 4

5√5 − 11 = 4 / (5√5 + 11), Because $0 < 5√5 − 11 < 1 = 0 < (5√5 − 11)^{2n + 1} < 1$, for positive integer n.

Again, $(5√5 + 11)^{2n + 1} − (5√5 − 11)^{2n + 1} = 2^{2n+1}C1 (5√5)^{2n} * 11 + 2^{n + 1}C^3(5√5)^{2n − 2} × 113 + — +$

$2n + 1C^{2n+1}\ 11^{2n+1}\}$

$= 2 \{^{2n+1}C_1(125)^n * 11 + 2^{n+1}C^3(125)n−1\ 113 + — + 2^{n+1}C^{2n+1}\ 11^{2n+1}\}$

= 2k, (for some positive integer k)

Let f′= (5√5 − 11)2n + 1 , then [R] + f − f′=2k

f − f′ = 2k − [R]

= f − f′ is an integer.

But, $0 \le f < 1; 0 < f' < 1$

$= -1 < f - f' < 1$

$f - f' = 0$ (integer)

$f = f'$

Therefore, $Rf = Rf' = (5\sqrt{5} + 11)2n + 1 * (5\sqrt{5} - 11)^{2n + 1}$

$= ([5\sqrt{5}]2 + 112)2^{n + 1}$

$= 42^{n+1}$

Question 9: The digit in the unit place of the number (183!) + 3183 is __?

Ans: We know that n! terminates in 0 for n^3 at 5 and 34^n terminator in 1, (because 34 = 81)

Therefore, 3180 = (34)45 terminates in 1.

Also 33 = 27 terminates in 7

3183 = 3180 * 33 terminates in 7.

183! + 3183 terminates in 7 i.e. the digit in the unit place = 7.

Question 10: If the coefficients of pth, $(p + 1)^{th}$ and $(p + 2)^{th}$ terms in the expansion of $(1 + x)^n$ are in A.P., then find the equation in terms of n.

Answer: Coefficient of p^{th}, $(p + 1)^{th}$ and $(p + 2)^{th}$ terms in expansion of $(1 + x)^n$ are ${}^nC_{p-1}$, ${}^nC_p$,${}^nC_{p+1}$. Then ${}^{2n}C_p = {}^nC_{p-1} + {}^nC_{p+1}$

$= n^2 - n(4p + 1) + 4p^2 - 2 = 0$

Trick: Let p = 1, hence ${}_nC^0$, ${}_nC^1$ and ${}_nC^2$ are in A.P.

$= 2 * {}_nC^1 = {}_nC^0 + {}_nC^2$

$= 2^n = 1 + [n (n - 1)] / [2]$

$= 4^n = 2 + n^2 - n$

$= n^2 - 5^n + 2 = 0$