The Top 10 JEE Main Complex Number Questions with Solutions

The Top 10 JEE Main complex number questions with solutions is listed below and comprises all the major sub topics like Roots of Complex Numbers, Complex Conjugate Roots Theorem, Complex Exponential Form with detailed solution step by step.

Q1: Let a complex number z, |z| ≠ 1, satisfy log1/√2 [(|z| + 11) / (|z| – 1)2] ≤ 2. Then, the largest value of |z| is equal to?
(a) 8
(b) 5
(c) 6
(d) 7
Sol: Option (d)
Given that |z| ≠ 1.
log1/√2 [(|z| + 11) / (|z| – 1)2] ≤ 2
[(|z| + 11) / (|z| – 1)2] ≥ 1 / 2
2|z| + 22 ≥ (|z| – 1)2
2|z| + 22 ≥ |z|2 – 2|z| + 1
|z|2 – 4|z| – 21 ≤ 0
(|z| – 7) (|z| + 3) ≤ 0
|z| ≤ 7
The largest value of |z| = 7

Q 2: The least value of |z| where z is a complex number which satisfies the inequality , i = √-1, is equal to
(a) 2
(b) 3
(c) 8
(d) √5
Sol: Option (b)
2[{(|z| + 3) (|z| – 1)} / {|z| + 1}] ≥ log√2(16)
2[{(|z| + 3) (|z| – 1)} / {|z| + 1}] ≥ 23 [{(|z| + 3) (|z| – 1)} / {|z| + 1}] ≥ 3
|z|2 + 2 |z| – 3 ≥ 3 |z| + 3
|z|2 – |z| – 6 ≥ 0
(|z| – 3) (|z| + 2) ≥ 0
|z|min = 3
The least value of |z| = 3

Q 3: If |z1 + z2|2 = |z1|2 + |z2|2 , then z1/z2 is
(a) purely imaginary
(b) purely real
(c) zero of purely imaginary
(d) neither real nor imaginary
Sol: option (a)

Given that |z1 + z2|2 = |z1|2 + |z2|2
So z1/z2 is purely imaginary.

Que 4: If z and w are two complex numbers such that |z| ≤ 1, |w|≤ 1 and, Then, z is equal to
(a) 1 or i
(b) i or -i
(c) i or -1
(d) 1 or -1
Sol: Option (d)
Given |z| ≤ 1 and |w| ≤ 1
Let z = x1 + iy1
w = x2 + iy2
So x12 + y12 ≤ 1
x22 + y22 ≤ 1
Therefore, |z + iw| = |x1 + y1+ i(x2+iy2)| = 2
(x1 – y1)2 + (y2+x2)2 = 4 ….(1)
Also |z + iw| = |x1 + y1 + i(x2+ iy2)| = 2
(x1 – y2)2+(y1 – x2)2 = 4 ….(2)
Subtract (2) – (1)
(y1 – x2)2-(y1 + x2)2 = 0
y12 + x22– 2y1x2– y12– x22– 2y1x2 = 0
y1x2 = 0
y1 = 0
x12 ≤1
-1≤ x ≤1
So z = 1+ i 0 or -1 + i0

Que 5: If 3/(2+ cos θ+i sin θ) = a+ib, then [(a-2)2+b2] is
(a) 0
(b) -1
(c) 1
(d) 2
Sol: Option (c)
Given that 3/(2+ cos θ+i sin θ) = a+ib
(3/(2+ cos θ+ i sin θ))((2 + cos θ)-i sin θ)/(2 + cos θ)- i sin θ) ((6 + 3 cos θ)- i 3 sin θ)/((2+cos θ)2+sin2 θ)
((6 + 3 cos θ)- i(3 sin θ))/(5 + 4 cos θ)
Comparing with a+ib, we get;
a = (6+3 sin θ)/(5+4 cos θ)
b = -3 sin θ/(5+4 cos θ)
(a – 2)2+ b2 = [(6 + 3 sin θ)/(5 + 4 cos θ)] – 22+ (-3 sin θ/(5+4 cos θ))2
= ((-4 – 5 cos θ)2+ 9 sin2θ)/(5 + 4 cos θ)2
= ((4 +5 cos θ)2+ 9 sin2θ)/(5 + 4 cos θ)2
= (16 + 40 cos θ + 25 cos2θ+ 9 sin2θ)/(5 + 4 cos θ)2

= (16 + 40 cos θ + 16 cos2θ+ 9 (sin2θ + cos2θ))/(5 + 4 cos θ)2
= (16 + 40 cos θ + 16 cos2θ+ 9)/(5 + 4 cos θ)2
= (16 cos2θ+40 cos θ+ 25)/(5 + 4 cos θ)2
= (4 cos θ+5)2/(5+4 cos θ)2
= 1

Q 6: If it is a real number, then the argument of sin θ + i cos θ is
(a) π- tan-1 (4/3)
(b) – tan-1 (3/4)
(c) π- tan-1 (4/3)
(d) tan-1 (4/3)
Sol: Option (a)
Let z is real.
4 sin θ + 3 cos θ = 0
tan θ = -3/4 [θ lies in 2nd quadrant]
arg (sin θ+ i cosθ) = π + tan-1 (cos θ /sin θ) π –
tan-1(4/3)

Q 7: If √(x+iy) = ±(a+ib), then √(-x-iy) is equal to
(a) ±(b+ia)
(b) ±(a-ib)
(c) (ai+b)
(d) None of these
Sol: Option (d)
Given that √(x+iy) = ±(a+ib)
Squaring both sides, we get;
(x+iy) = (a+ib)2
= a2 + 2aib – b2
= a2– b2 + 2aib
Comparing real and imaginary part, we get;
x = a2-b2
y = 2ab
-x – iy = -a2 + b2– i(2ab)
-x – iy = (-b + ia)2
Taking square root, we get;
√(-x-iy) = ±(-b+ia)

Q 8: The point in the set { z ∈ C: arg ((z-2)/(z-6i)) = π/2} (where C denotes the set of all complex number) lie on the curve which is
(a) hyperbola
(b) pair of lines
(c) parabola
(d) circle
Sol: Option (d)

Given that arg $((z-2)/(z-6i) = \pi/2$
Arg $(z-2) - \arg(z-6i) = \pi/2$
$z = x+iy$
$\arg ((x-2)+iy) - \arg(x+(y-6)i = \pi/2$
We know, $\tan^{-1}A - \tan^{-1}B = \tan^{-1}[(A-B)/(1+AB)]$
$x(x-2) + y(y-6) = 0$
$x^2 - 2x + 1 + y^2 - 6y + 9 - 10 = 0$
$(x-1)^2 + (y-3)^2 = \sqrt{(10)}^2$
The point lies on a circle.

Q 9: The complex number $(-\sqrt{3}+3i)(1-i)/(3+\sqrt{3}i)(i)(\sqrt{3}+\sqrt{3}i)$, when represented in the argand diagram, lies in
(a) in the second quadrant
(b) in the first quadrant
(c) on the Y-axis (imaginary axis)
(d) on the X-axis (real axis)
Sol: Option (c)
Given $z = (-\sqrt{3}+3i)(1-i)/(3+\sqrt{3}i)(i)(\sqrt{3}+\sqrt{3}i)$
Taking $\sqrt{3}$ outside
$= \sqrt{3}(-1+\sqrt{3}i)(1-i)/(\sqrt{3})^2(\sqrt{3}+i)(i)(1+i)$
$= (1/\sqrt{3})(-1+\sqrt{3}i)(1-i)/(-1+i\sqrt{3})(1+i)$
$= (1-i)/\sqrt{3}(1+i)$
Multiply the numerator and denominator with $(1-i)$
$= (1-i)(1-i)/\sqrt{3}(1+i)(1-i)$
$= (1-i)^2/\sqrt{3}(1+1)$
$= (1-2i-1)/2\sqrt{3}$
$= -2i/2\sqrt{3}$
$= -i/\sqrt{3}$, purely imaginary
So, z lies on the Y-axis.

Q 10: The complex numbers $\sin x + i \cos 2x$ and $\cos x - i \sin 2x$ are conjugate to each other for
(a) $x = n\pi$
(b) $x = (n+½)\pi$
(c) $x = 0$
(d) No values of x
Sol: Option (d)
Given that $\sin x + i \cos 2x$ and $\cos x - i \sin 2x$ are conjugate to each other.
$\sin x - i \cos 2x = \cos x - i \sin 2x$
Comparing real and imaginary parts
$\sin x = \cos x$ ..(i)
$\cos 2x = \sin 2x$…(ii)
From equation (i) $\tan x = 1$
From equation (ii) $\tan 2x = 1$
$2 \tan x/(1-\tan^2 x) = 1$, not satisfied by $\tan x = 1$.

Therefore, no value of x is possible.