SECTION-A

Q1.

$$x^2 \frac{d^2y}{dx^2} = \left\{ 1 + \left(\frac{dy}{dx}\right)^2 \right\}^4$$

ORDER = 2

DEGREE = 1

Q2.

$$f(x) = x+7 \quad ; \quad g(x) = x-7$$

$$\begin{aligned} fog(x) &= f(g(x)) \\ &= f(x-7) \\ &= (x-7)+7 \\ &= x \qquad \forall\ x \in R \end{aligned}$$

$$\frac{d}{dx} fog(x) = \frac{d}{dx}(x) = 1$$

$$\boxed{\frac{d}{dx} fog(x) = 1}$$

4

Q3

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

comparing corresponding elements of each matrix,

$2+y=5$ $\quad$ $2x+2=8$

$\boxed{y=3}$ $\quad$ $\boxed{x=3}$

$x-y=3-3$

$\boxed{x-y=0}$

Q4

$\vec{r} = \vec{a} + \lambda\vec{b}$ $\quad$ where $\vec{a}$ = position vector of pt on line, $\vec{b}$ = parallel vector to line

$$\boxed{\vec{r} = (3\hat{i}+4\hat{j}+5\hat{k}) + \lambda(2\hat{i}+2\hat{j}-3\hat{k})}$$

Also $\vec{r} = (3+2\lambda)\hat{i} + (4+2\lambda)\hat{j} + (5-3\lambda)\hat{k}$

Q5. 2

Q5. $a * b = ab + 1$

i) For all $(a, b) \in (R \times R)$

$\Rightarrow ab + 1 \in R$ $\left[\because \text{If } a, b \in R \Rightarrow ab \in R \Rightarrow (ab+1) \in R\right]$ $\left[\because \text{Multiplication is binary operation}\right]$

$\therefore$ a R b relates to a unique element in R and hence is a binary operation from $R \times R$ to R

ii) For binary operation to be associative $(a * b) * c = a * (b * c)$

$$
\begin{aligned}
LHS &= (a * b) * c \\
&= (ab+1) * c \\
&= (ab+1)c + 1 \\
&= abc + c + 1
\end{aligned}
$$

$$
\begin{aligned}
RHS &= a * (b * c) \\
&= a * (bc+1) \\
&= a(bc+1) + 1 \\
&= abc + a + 1
\end{aligned}
$$

$LHS \neq RHS$

$\therefore$ It is NOT ASSOCIATIVE

Q6.

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$A^2 = AA$$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$A^2 - 5A = A^2 + (-5)A$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + (-5) \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}$$

Q7.

$$A^2 - 5A = A^2 + (-5A)$$

$$= \begin{bmatrix} +5 & -4 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix}$$

Q7. $I = \int \underset{I}{1} . \underset{II}{\sin^{-1}(2x)} \, dx$

$I = [\sin^{-1}(2x)](x) - \int \frac{(x)(2)}{\sqrt{1-4x^2}} \, dx$ [INTEGRATION by parts]

$I = x\sin^{-1}(2x) - \frac{1}{2} \int \frac{2x}{\sqrt{(\frac{1}{2})^2 - x^2}} \, dx$

$I = x\sin^{-1}(2x) + \frac{1}{4} \int \frac{(-8x)}{\sqrt{1-4x^2}} \, dx = x\sin^{-1}(2x) + \frac{1}{4} I_1$ where $I_1 = \int \frac{-8x}{\sqrt{1-4x^2}} \, dx$

Now, $I_1 = \int \frac{dt}{\sqrt{t}}$ let $1-4x^2 = t$

$-8x \, dx = dt$

$= 2\sqrt{t} + C$

$= 2\sqrt{1-4x^2} + C$

$$I = x\sin^{-1}(2x) + \frac{1}{4}\left(2\sqrt{1-4x^2}\right) + C$$

$$\boxed{I = x\sin^{-1}(2x) + \frac{\sqrt{1-4x^2}}{2} + C}$$

Q6

$$y = e^{2x}(a+bx) \quad \text{——(1)}$$

Diff. both sides w.r.t x.

$$\frac{dy}{dx} = e^{2x}(b) + (a+bx)(e^{2x})(2)$$

$$\Rightarrow \frac{dy}{dx} = e^{2x}(b+2a+2bx) \quad \text{——(2)}$$

Diff. both sides w.r.t. x.

$$\frac{d^2y}{dx^2} = e^{2x}(2b) + (b+2a+2bx)(e^{2x})(2)$$

$$= e^{2x}(4b+4a+4bx)$$

$$\frac{d^2y}{dx^2} = 2e^{2x}(2b+2a+2bx) \quad \text{——(3)}$$

Subtract $2 \times$ eqn (2) from eqn (3)

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^{2x}(4b+4a+4bx) - e^{2x}(2b+4a+4bx)$$

$$= e^{2x}(2b)$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 2(e^{2x})(b) \quad \text{--- (4)}$$

In eq^n (3)

$$\frac{d^2y}{dx^2} = e^{2x}(4b + 4a + 4bx)$$

$$= 2\left(\frac{d^2y}{dx^2} - 2\frac{dy}{dx}\right) + 4e^{2x}(a+bx) \quad \text{[From (4)]}$$

$$\frac{d^2y}{dx^2} = 2\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y \quad \text{[From (1)]}$$

∴ The required differential eq^n is

$$\boxed{\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0} \Rightarrow y'' - 4y' + 4y = 0.$$

Where $y'' = \frac{d^2y}{dx^2}$, $y' = \frac{dy}{dx}$, $\cancel{y = e^2}$

P.T.O

Q9 $\sum_{i=0}^{2} P(X_i) = 1$ $\left[\because \sum_{i=0}^{n} P(X_i) = 1\right]$

[Sum of all probabilities = 1]
~~disjoint~~ exhaustive & exclusive

$\Rightarrow k + 2k + 3k = 1$

$\Rightarrow 6k = 1$

$\boxed{k = \frac{1}{6}}$

Q11

Q10 ~~P(A~~ $P(A) = \frac{3}{6} = \frac{1}{2}$ $\left[\because P(E) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of possible outcomes}}\right]$

$P(B) = \frac{3}{6} = \frac{1}{2}$

[$\Rightarrow$ Sample space $S = \{1r, 2r, 3r, 4g, 5g, 6g\}$
$A = \{2r, 4g, 6g\}$ $A \cap B = \{2r\}$
$B = \{1r, 2r, 3r\}$]

$P(A \cap B) = P(\text{no. which is even and red}) = \frac{1}{6}$ [$\because$ No. which is even and red $= \{2r\}$]

$P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Q12

$P(A \cap B) \neq P(A) \cdot P(B)$

$\therefore$ Events ~~are not~~ NOT INDEPENDENT.

Q11 $\vec{a} = 2\hat{i}+3\hat{j}+\hat{k}$, $\vec{b} = \hat{i}-2\hat{j}+\hat{k}$, $\vec{c} = -3\hat{i}+\hat{j}+2\hat{k}$

$[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c})$

$= \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ where $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k} = 2\hat{i}+3\hat{j}+\hat{k}$

$\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k} = \hat{i}-2\hat{j}+\hat{k}$

$\vec{c} = c_1\hat{i}+c_2\hat{j}+c_3\hat{k} = -3\hat{i}+\hat{j}+2\hat{k}$

$= \begin{bmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{bmatrix}$

$= 2(-4-1) - 3(2+3) + 1(1-6)$

$= -10-15-5$

$\therefore \boxed{[\vec{a}\ \vec{b}\ \vec{c}] = -30}$

Q12 $I = \int \dfrac{\tan^2 x \cdot \sec^2 x}{1-\tan^6 x}\, dx$

let $\tan^3 x = t$

$3\tan^2 x \sec^2 x\, dx = dt$

Q13.

$$\cancel{I = \int \frac{t^2\,dt}{1-t^6} = \int \frac{t^2}{1-(t^3)^2}\,dt}$$

$$\cancel{\text{let } t^3 = m}$$

$$\cancel{3t^2\,dt}$$

$$I = \frac{1}{3}\int \frac{dt}{(1)^2 - t^2}$$

$$= \frac{1}{3}\cdot\frac{1}{2.(1)}\log\left|\frac{1+t}{1-t}\right| + C \qquad \left[\because \int \frac{dx}{a^2-x^2} = \log\left|\frac{a+x}{a-x}\right| + c\right]$$

$$\boxed{I = \frac{1}{6}\log\left|\frac{1+\tan^3 x}{1-\tan^3 x}\right| + c}$$

SECTION-C

P.T.O

Q13. $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$

$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right), AB < 1$

$\Rightarrow \tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4}$ and $(2x)(3x) < 1$

$6x^2 < 1$

$-\frac{1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$

taking tan on both side

$\Rightarrow \frac{5x}{1-6x^2} = 1$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow 6x^2 + 6x - x - 1 = 0$

$\Rightarrow (6x-1)(x+1) = 0$

$\Rightarrow x = \frac{1}{6}$ or $x = -1$

$\therefore x = -1$ is not a solution $[\because -1 \notin (-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}})]$

$\Rightarrow \boxed{x = \frac{1}{6}}$ is ~~giv~~ sol^n.

VERIFICATION

For $x = 1/6$

LHS : $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}\left(\frac{5}{5}\right) = \pi/4 =$ RHS.

LHS = RHS

$\Rightarrow x = 1/6$ is a sol^n

For $(x) = -1$

$\tan^{-1}(-2) - \tan^{-1}(-3) = \frac{\pi}{2} - \cot^{-1}(-2) - \frac{\pi}{2} + \cot^{-1}(-3)$ ⇒

$= \pi - \cot^{-1}(3) - \pi + \cot^{-1}(2)$

$= \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2}-\frac{1}{3}}{1+\frac{1}{6}}\right) = \tan^{-1}\left(\frac{1}{7}\right) \neq \text{RHS}$

LHS ≠ RHS

∴ $x = -1$ is NOT solⁿ

Q15

Q14. $\log(x^2+y^2) = 2\tan^{-1}\frac{y}{x}$.

Diff. both sides w.r.t x.

$\frac{1}{x^2+y^2}\left(2x + 2y\frac{dy}{dx}\right) = \frac{2}{1+\frac{y^2}{x^2}} \cdot \frac{d}{dx}\left(\frac{y}{x}\right)$ $\left[\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} ; \frac{d}{dx}(\log x) = \frac{1}{x}\right]$

⇒ $\frac{x + yy'}{x^2+y^2} = \frac{x^2}{x^2+y^2} \frac{(xy' - y)}{x^2}$ $\left[\text{where } y' = \frac{dy}{dx}\right]$

⇒ $x + yy' = xy' - y$

⇒ $x + y = y'(x - y)$

$\Rightarrow \quad y' = \dfrac{x+y}{x-y}$

$$\boxed{\frac{dy}{dx} = \frac{x+y}{x-y}}$$

Hence proved.

Q15 $\quad I = \displaystyle\int \frac{3x+5}{x^2+3x-18}\, dx$

$$3x+5 = a\left[\frac{d}{dx}(x^2+3x-18)\right] + b$$

$$= a(2x+3) + b$$

comparing coefficients on both sides,

$3 = 2a \qquad\qquad 3a+b = 5$

$\boxed{a = \frac{3}{2}} \qquad\qquad b = 5 - 3\left(\frac{3}{2}\right) = \frac{1}{2} \quad \Rightarrow \boxed{b = \frac{1}{2}}$

P.T.O

16

$$I = \int \frac{\frac{3}{2}(2x+3) + \frac{1}{2}}{x^2+3x-18}\,dx$$

$$= \frac{3}{2}\int \frac{(2x+3)\,dx}{x^2+3x-18} + \frac{1}{2}\int \frac{dx}{x^2+3x-18}$$

$$= \frac{3}{2} I_1 + \frac{1}{2} I_2 .$$

$$I_1 = \int \frac{2x+3}{x^2+3x-18}\,dx \qquad I_2 = \int$$

let $x^2+3x-18 = t$

$(2x+3)\,dx = dt$

$$I_1 = \int \frac{dt}{t} = \log|t| + c_1$$

$$I_1 = \log|x^2+3x-18| + c_1$$

Ans, $$I_2 = \int \frac{dx}{x^2+3x-18} = \int \frac{dx}{x^2+3x+\left(\frac{3}{2}\right)^2 - 18 - \left(\frac{3}{2}\right)^2}$$

$$= \int \frac{dx}{\left(x+\frac{3}{2}\right)^2 - 18 - \frac{9}{4}} = \int \frac{dx}{\left(x+\frac{3}{2}\right)^2 - \frac{81}{4}}$$

Q16

$$I_1 = -\int_a^0 f(a-t)\,dt \qquad \Rightarrow$$

$$= \int_0^a f(a-t)\,dt \qquad \left[\because \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx\right]$$

$$I_1 = \int_0^a f(a-x)\,dx \qquad \left[\because \int_a^b f(t)\,dt = \int_a^b f(x)\,dx\right] \qquad \Rightarrow$$

LHS = RHS

Hence proved

Q17

$$I = \int_0^\pi \frac{x \sin x}{1+\cos^2 x}\,dx \qquad \text{---}(1)$$

$$I = \int_0^\pi \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}\,dx \qquad \left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right] \qquad \therefore$$

$$I = \int_0^\pi \frac{(\pi-x)\sin x}{1+\cos^2 x}\,dx \qquad \text{---}(2)$$

Adding eqn (1), (2)

$$2I = \int_0^\pi \frac{\pi \sin x}{1+\cos^2 x}\,dx$$

$$I_2 = \int \frac{dx}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}$$

$$= \frac{1}{2 \cdot \frac{9}{2}} \log \left| \frac{\left(x+\frac{3}{2}\right) - \frac{9}{2}}{\left(x+\frac{3}{2}\right) + \frac{9}{2}} \right| + c_2$$

$$= \frac{1}{9} \log \left| \frac{x-3}{x+6} \right| + c_2$$

$$I = \frac{3}{2} \log |x^2+3x-18| + c_1 + \frac{1}{18} \log \left| \frac{x-3}{x+6} \right| + c_2$$

$$I = \frac{3}{2} \log |x^2+3x-18| + \frac{1}{18} \log \left| \frac{x-3}{x+6} \right| + c$$ where $c = c_1 + c_2 =$ constt.

Q16 T.P: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

LHS : $I_1 = \int_0^a f(x)\,dx$

let $x = a - t$ When $x = 0$ $t = a$

$dx = -dt$ $x = a$ $t = 0$

$$\Rightarrow \quad 2I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\, dx$$

let $\cos x = t$ when $x = 0$ $t = 1$

$-\sin x\, dx = dt$ $x = \pi$ $t = -1$

$$\Rightarrow \quad I = \frac{\pi}{2} \int_1^{-1} \frac{-dt}{1+t^2}$$

$$= -\frac{\pi}{2}\left[\tan^{-1} t\right]_1^{-1} \cancel{= -\frac{\pi}{2}\left[-\frac{\pi}{4} - \frac{\pi}{4}\right] = \frac{\pi}{2} \cdot \frac{\pi}{2}}$$

$$\boxed{I = \frac{\pi^2}{4}}$$

$$\therefore \quad \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} = \frac{\pi^2}{4}$$

Q17

$\vec{A} = \hat{i} + \hat{j} + \hat{k}$

$\vec{B} = 2\hat{i} + 5\hat{j} + 0\hat{k}$

$\vec{C} = 3\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{D} = \hat{i} - 6\hat{j} - \hat{k}$

Q.18.

$\vec{AB}$ = Position vector of $\vec{B}$ − Position vector of $\vec{A}$

$\vec{m} = \hat{i} + 4\hat{j} - \hat{k}$

$\vec{CD}$ = Position vector of $\vec{D}$ − Position vector of $\vec{C}$

$\vec{n} = -2\hat{i} - 8\hat{j} + 2\hat{k}$

Angle b/w $\vec{AB}$ and $\vec{CD}$ = θ

$$\cos\theta = \frac{\vec{m}\cdot\vec{n}}{|\vec{m}|\,|\vec{n}|}$$

$$= \frac{m_1n_1 + m_2n_2 + m_3n_3}{\sqrt{m_1^2+m_2^2+m_3^2}\,\sqrt{n_1^2+n_2^2+n_3^2}}$$

$$= \frac{-2 - 32 - 2}{\sqrt{1+16+1}\,\sqrt{4+64+4}}$$

$$= \frac{-36}{\sqrt{18}\,\sqrt{72}} = \frac{-36}{(3\sqrt{2})(6\sqrt{2})} = \frac{-36}{18\times 2}$$

$$= \frac{-36}{36} = -1$$

$\theta = \cos^{-1}(-1) \Rightarrow \boxed{\theta = \pi}$

∴ Since, $\vec{AB}$ and $\vec{CD}$ are antiparallel,

they ARE COLLINEAR.

Q18. $\Delta = \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$

$\Delta = \begin{vmatrix} a+b & a+b & -(a+b) \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$ $R_1 \to R_1 + R_2$

$= (a+b) \begin{vmatrix} 1 & 1 & -1 \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$ Taking $(a+b)$ common from R_1

$= (a+b) \begin{vmatrix} 1 & 1 & -1 \\ -(b+c) & b+c & b+c \\ -b & -a & a+b+c \end{vmatrix}$ $R_2 \to R_2 + R_3$

$= (a+b)(b+c) \begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -b & -a & a+b+c \end{vmatrix}$ Taking $(b+c)$ common from R_2

$= (a+b)(b+c) \begin{vmatrix} 0 & 0 & -1 \\ 0 & 2 & 1 \\ a+c & b+c & a+b+c \end{vmatrix}$ $C_1 \to C_1 + C_3$, $C_2 \to C_2 + C_3$

$$\Delta = (a+b)(b+c)(c+a) \begin{vmatrix} 0 & 0 & -1 \\ 0 & 2 & 1 \\ 1 & b+c & a+b+c \end{vmatrix}$$ Taking $(a+c)$ common from C_1

Expanding along C_1

$\Delta = (a+b)(b+c)(c+a)(2)$

$\Delta = 2(a+b)(b+c)(c+a)$

Hence proved.

Q19 $y = \sin t$ $\qquad$ $x = \cos t + \log(\tan(t/2))$

Diff. w.r.t t $\qquad$ Diff. w.r.t t

$$\boxed{\frac{dy}{dt} = \cos t} \text{ --- (1)} \qquad \frac{dx}{dt} = -\sin t + \frac{1}{\tan t/2} \cdot \sec^2 t/2 \cdot \frac{1}{2}$$

Diff w.r.t $\cancel{d}t$

$$= -\sin t + \frac{\cos t/2}{2\sin t/2 \cos^2 t/2}$$

$$\frac{d^2y}{dt^2} = -\sin t$$

$$= -\sin t + \frac{1}{\sin t} \quad [\because 2\sin A\cos A = \sin 2A]$$

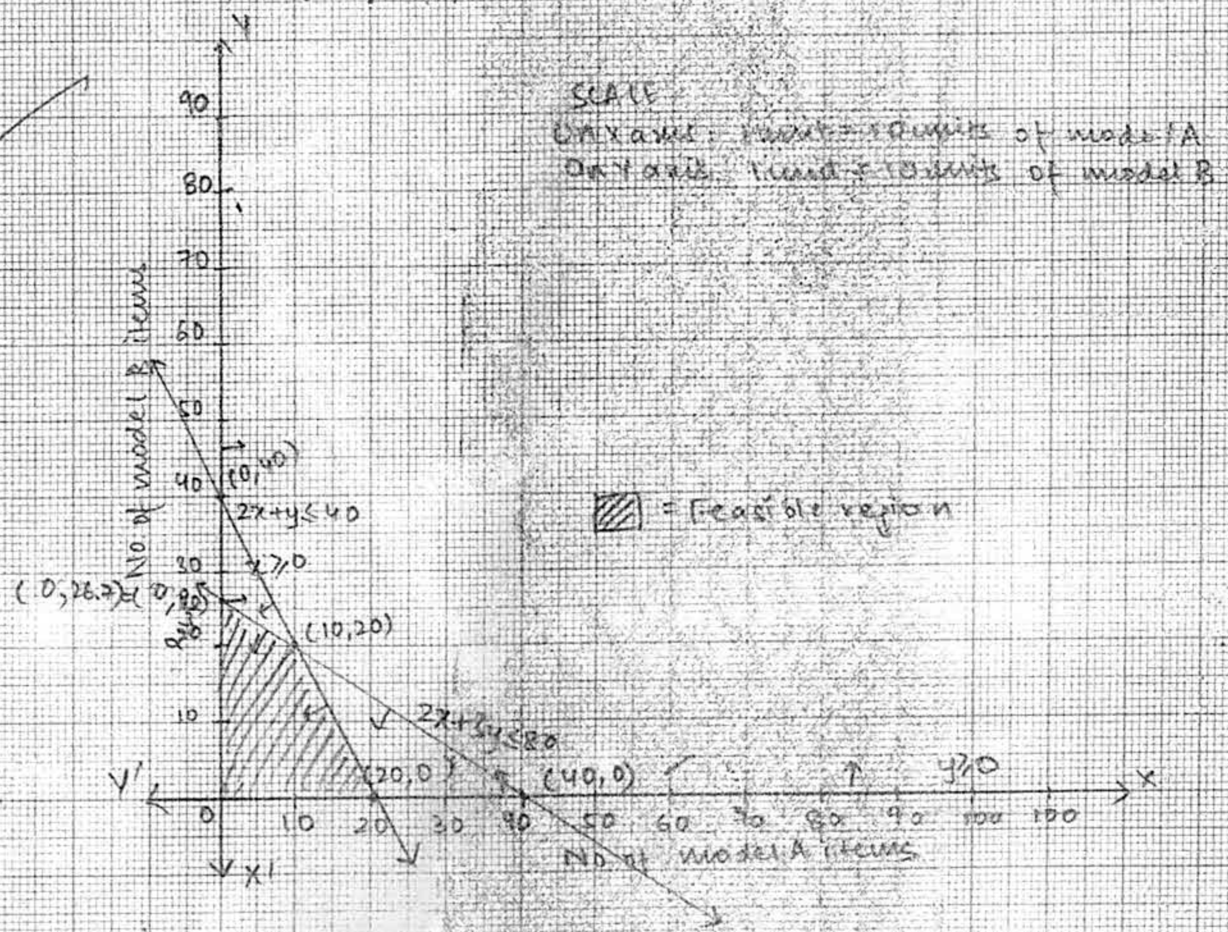
$$\left.\frac{d^2y}{dt^2}\right]_{t=\pi/4} = -\sin\frac{\pi}{4}$$

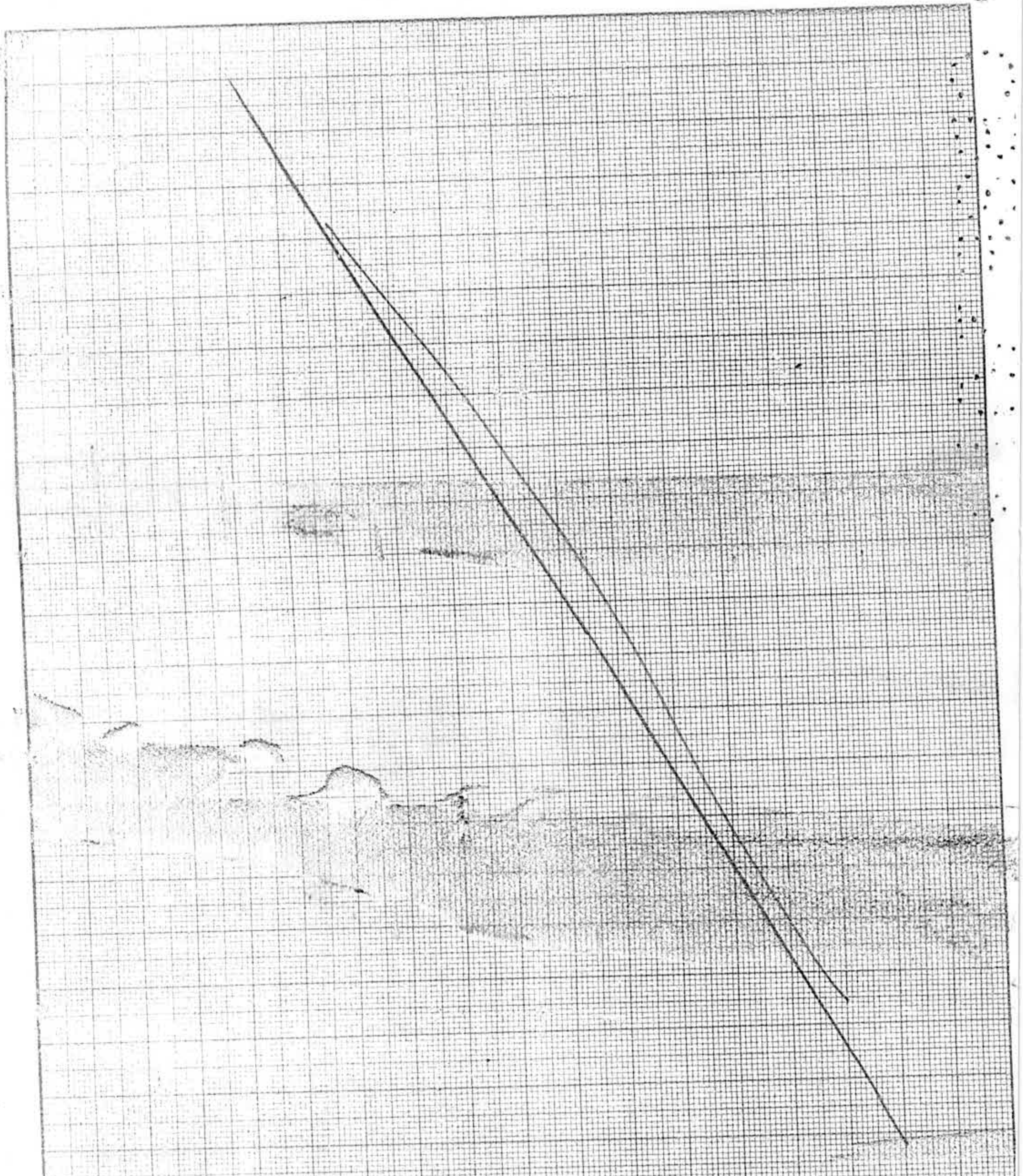
$$\boxed{\frac{dx}{dt} = -\sin t + \frac{1}{\sin t}} \text{ --- (2)} \Rightarrow \frac{dx}{dt} = \frac{\cos^2 t}{\sin t}$$

$$\boxed{\left.\frac{d^2y}{dt^2}\right]_{t=\pi/4} = -\frac{1}{\sqrt{2}}}$$

P.T.O

Q 29


SCALE
On X axis: 1 unit = 10 units of model A
On Y axis: 1 unit = 10 units of model B
No. of model B items
No. of model A items
(0,40)
2x+y≤40
x≥0
(0, 26.7)
(10,20)
2x+3y≤80
(20,0)
(40,0)
y≥0
= Feasible region



Q19 Contd...

~~Dif~~ Divide eqn (1) by (2)

$$\frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t + \frac{1}{\sin t}}$$

$$\frac{dy}{dx} = \frac{\cos t . \sin t}{1-\sin^2 t} = \frac{\sin t}{\cos t} = \tan t$$

Diff. w.r.t x.

$$\frac{d^2y}{dx^2} = \frac{d(\tan t)}{dt} \frac{dt}{dx}$$

$$= \sec^2 t . \frac{1}{dx/dt} = \frac{\sec^2 t}{-\sin t + \frac{1}{\sin t}}$$

$$= \frac{\sec^2 t . \sin t}{1-\sin^2 t} = \frac{\sin t}{\cos^2 t . \cos^2 t} = \frac{\sin t}{\cos^4 t}$$

$$= \sin t . \sec^4 t$$

$$\left.\frac{d^2y}{dx^2}\right]_{t=\pi/4} = \frac{1}{\sqrt{2}} (\sqrt{2})^4 = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\therefore \boxed{\left.\frac{d^2y}{dx^2}\right]_{t=\pi/4} = 2\sqrt{2}}$$

Q20. $R = \{(a,b) : a \leq b\}$

REFLEXIVE :

Every element $a \in \mathbb{R}$ is equal to itself

$\Rightarrow a = a$

$\Rightarrow a \leq a$ is true

$\therefore (a,a) \in R$ for all $a \in \mathbb{R}$ where $\mathbb{R}$ = set of all Real nos

The relation is REFLEXIVE R = Relation.

TRANSITIVE :

For all $(a,b) \in R$ and $(b,c) \in R$

$a \leq b$ and $b \leq c$ where $a, b, c \in \mathbb{R}$

$\Rightarrow a \leq c$

$\Rightarrow (a,c) \in R$

$\therefore$ The ~~set is tr~~ relation is TRANSITIVE

$\because$ for $(a,b),(b,c) \in R$, $\cancel{(a,c) \in R}$ $(a,c) \in R$

Q21.

SYMMETRIC : For relation to be symmetric,
for all $(a, b) \in R$, (b, a) should also exist in R

$\cancel{a \leq b \nRightarrow}$ $a \leq b$
$b \nleq a$ → This relation is true only $a = b = 1$.

For eg: $\frac{1}{2} \leq 1 \Rightarrow \left(\frac{1}{2}, 1\right) \in R$

but $1 \nleq \frac{1}{2}$ $\therefore \left(1, \frac{1}{2}\right) \notin R$

$\therefore$ Relation is NOT SYMMETRIC

Q21

$y = \sqrt{3x-2}$

$\left.\frac{dy}{dx}\right]_{(x_1,y_1)} = \frac{3}{2\sqrt{3x_1-2}} = \frac{3}{2y_1} =$ slope of tangent $= m$

Acc. to quesn $m = 2$ [$\because$ Slope of $(4x-2y+5=0)$ is $m_1 = 2$. and ||el line have equal slope]

$\Rightarrow \frac{3}{2y_1} = 2 \Rightarrow \boxed{y_1 = \frac{3}{4}}$

$\Rightarrow \frac{3}{2\sqrt{3x_1-2}} = 2 \Rightarrow 3 = 4\sqrt{3x_1-2}$

$\Rightarrow 9 = 16(3x_1-2)$ squaring both side

$\Rightarrow 3x_1 - 2 = \frac{9}{16} \Rightarrow x_1 = \frac{\cancel{45}}{48} = \frac{41}{48} \Rightarrow \boxed{x_1 = \frac{41}{48}}$

$x_1 = \frac{41}{48}$

$\Rightarrow y_1 = \sqrt{3x_1 - 2} = \sqrt{\frac{3 \times 41}{48} - 2} = \sqrt{\frac{41-32}{16}} = \pm\frac{3}{4} = \left|\frac{3}{4}\right| = \sqrt{\frac{9}{16}}$

At $\boxed{\left(\frac{41}{48}, \frac{3}{4}\right)}$ and $\left(\frac{41}{48}, \frac{-3}{4}\right)$, ~~slope of~~ tangent is parallel to given line.

Not possible

(But $\because y \nless 0 \Rightarrow y \geq 0$)

EQUATION OF TANGENT

For $x_1 = 41/48$, $y_1 = 3/4$

$y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$

$y - \frac{3}{4} = 2x - \frac{41}{24}$

$2x - y - \frac{41}{24} + \frac{18}{24} = 0$

$\Rightarrow 2x - y - \frac{23}{24} = 0$

$\Rightarrow \boxed{48x - 24y - 23 = 0}$

For $x_1 = 41/48$, $y_2 = \cancel{3/4}$

$\cancel{y + 3/4 = 2(x - 41/48)}$

$\cancel{2x - y - \frac{41}{24} - \frac{18}{24} = 0}$

$\cancel{\boxed{48x - 24y - 59 = 0}}$

NOT POSSIBLE

X

Slope of normal $= \dfrac{-1}{\left[\frac{dy}{dx}\right]_{(x_1,y_1)}} = \dfrac{-1}{2}$

EQUATION OF NORMAL :

At $\left(\frac{41}{48}, \frac{3}{4}\right)$

$y - \frac{3}{4} = \frac{-1}{2}\left(x - \frac{41}{48}\right)$

$\Rightarrow y - \frac{3}{4} = \frac{-x}{2} + \frac{41}{96}$

$\Rightarrow y + \frac{x}{2} - \frac{3}{4} - \frac{41}{96} = 0$

$\Rightarrow y + \frac{x}{2} + \left(\frac{-72-41}{96}\right) = 0$

$\Rightarrow \boxed{96y + 48x - 113 = 0}$

~~At~~ At $\left(\frac{41}{48}, \frac{-3}{4}\right)$

$y + \frac{3}{4} = \frac{-1}{2}\left(x - \frac{41}{48}\right)$

$\Rightarrow y + \frac{3}{4} + \frac{x}{2} - \frac{41}{96} = 0$

$\Rightarrow y + \frac{x}{2} + \frac{72-41}{96} = 0$

$\Rightarrow \boxed{96y + 48x + 31 = 0}$

NOT POSSIBLE

$\frac{1}{24}$

72

P.T.O

Q22 $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$

$$\frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{4x^2}{1+x^2}$$

It is linear DE of form $\frac{dy}{dx} + Py = Q$

$P = \frac{2x}{1+x^2}$ $\quad Q = \frac{4x^2}{1+x^2}$

$$I.F = e^{\int P\,dx} = e^{\ln|1+x^2|} = 1+x^2$$

Sol^n of DE :

$$y(1+x^2) = \int \frac{4x^2(1+x^2)}{1+x^2}\,dx + C$$

$$= \int 4x^2\,dx + C$$

$$\boxed{y(1+x^2) = \frac{4x^3}{3} + C}$$

Q23.

$x=0, y=0$

$\therefore c=0$

$\Rightarrow \boxed{3y(1+x^2)=4x^3}$

Q23

Line I

$\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$

$\Rightarrow \frac{x-1}{-3} = \frac{y-2}{\lambda/7} = \frac{z-3}{2} = \beta$

Line II

$\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$

$\frac{x-1}{\frac{-3\lambda}{7}} = \frac{y-5}{1} = \frac{z-6}{-5} = \mu$

Direction ratios are $\underset{(a_1, a_2, a_3)}{\left(-3, \frac{\lambda}{7}, 2\right)}$ and $\underset{(b_1, b_2, b_3)}{\left(\frac{-3\lambda}{7}, 1, -5\right)}$ respectively

For lines to be $\perp$, $\cancel{\vec{a_1}.\vec{a_2}=0}$ $\vec{a_1}.\vec{b}=0$ where ($\vec{a}, \vec{b}$ are llel vectors of lines)

$a_1b_1 + a_2b_2 + a_3b_3 = 0$

$\frac{9\lambda}{7} + \frac{\lambda}{7} - 10 = 0$

$\frac{10\lambda}{7} = 10$

$\boxed{\lambda = 7}$

$\therefore$ For $\lambda=7$, lines are $\perp$.

Q24.

Any point of line I is $(-3\beta+1,\ \beta+2,\ 2\beta+3)$

line II is $(-3\mu+1,\ \mu+5,\ -5\mu+6)$

For lines to intersect, they should be equal.

$-3\beta+1 = -3\mu+1$ $\mu+5=\beta+2$ $2\beta+3 = -5\mu+6$

$3\mu-3\beta=0$ $\mu-\beta+3=0$ — (3) $2\beta+5\mu=3$ — (2)

$\mu=\beta$ — (1)

From (1), (2)

$7\mu=3$

$\boxed{\mu=3/7}$ $\boxed{\beta=\frac{3}{7}}$

In eqn(3)

$\frac{3}{7}-\frac{3}{7}+3 \neq 0$

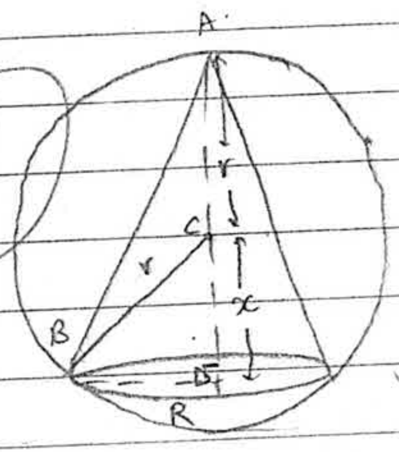
$\therefore$ Values donot satisfy eqn(3)

$\Rightarrow$ LINES do NOT intersect.

P.T.O

SECTION-D

Q24.



Consider a sphere of radius r.

Then, height of cone = $r + x = H$

Radius of cone = $\sqrt{r^2 - x^2} = R$

Volume of cone = $V = \frac{1}{3}\pi R^2 H$

$$V = \frac{1}{3}\pi (r^2 - x^2)(r + x)$$

$$= \frac{\pi}{3}(r^3 - rx^2 + xr^2 - x^3).$$

$$\frac{dV}{dx} = \frac{\pi}{3}(-2rx + r^2 - 3x^2)$$

For critical pt $\frac{dV}{dx} = 0$

$$r^2 - 2rx - 3x^2 = 0$$

$$r^2 - 3rx + rx - 3x^2 = 0$$

$$(r + x)(r - 3x) = 0$$

x cannot be -ve $\Rightarrow \boxed{\frac{r}{3} = x}$ is a critical pt

$$\frac{d^2V}{dx^2} = \frac{\pi}{3}(-2r-6x)$$

$$\left.\frac{d^2V}{dx^2}\right]_{x=r/3} = \frac{\pi}{3}\left[-2r-2r\right] = -\frac{4r\pi}{3} < 0$$

Using Second Derivative test, $\therefore$ $\boxed{x = \frac{r}{3}}$ is a ~~m~~ point of maxima

$H = r + x$

$\boxed{H = \frac{4r}{3}}$ Hence proved

Max volume of cone $= V_{max} = \frac{\pi}{3}(r+x)(r^2-x^2)$

$$= \frac{\pi}{3}\left(\frac{4r}{3}\right)\left(r^2-\frac{r^2}{9}\right)$$

$$= \frac{4\pi r^3 \cdot 8}{81} = \frac{32\pi r^3}{81} \text{ (unit)}^3$$

$$\boxed{V_{max} = \frac{32\pi r^3}{81} \text{ (unit)}^3}$$

Q25. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$$\text{adj}\, A = \begin{bmatrix} 0 & +2 & 1 \\ -1 & -9 & -5 \\ 2 & +23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$|A| = 2(0) + 3(-2) + 5(1)$$
$$= 0 - 6 + 5 = -1$$

$$\cancel{+A} \quad A^{-1} = \frac{1}{|A|} \text{adj}\, A = -1 \begin{bmatrix} 0 & -1 & 2 \\ \cancel{2} & -9 & 23 \\ \cancel{+1} & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Given Eq^n:

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

Q26.

$AX = B$

~~Post~~ Premultiply A^{-1} on both sides

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\frac{x}{y}$ $\boxed{x = 1}$

$\boxed{y = 2}$

$\boxed{z = 3}$

Q26 R. Eu

Q26.

A = ~~the~~ Event of choosing a defective item.

E_1 = Item produced by A

E_2 = Item produced by B

E_3 = Item produced by C.

$P(E_1) = \frac{50}{100} = \frac{1}{2}$, $P(E_2) = \frac{30}{100} = \frac{3}{10}$, $P(E_3) = \frac{20}{100} = \frac{1}{5}$

$P\left(\frac{A}{E_1}\right) = \frac{1}{100}$, $P\left(\frac{A}{E_2}\right) = \frac{5}{100} = \frac{5}{100}$; $P\left(\frac{A}{E_3}\right) = \frac{7}{100}$.

Using Baye's Thm,

$$A\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3).P(A/E_3)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{100}}{\frac{1}{2} \times \frac{1}{100} + \frac{3}{10} \times \frac{5}{100} + \frac{1}{5} \times \frac{7}{100}}$$

$$= \frac{\frac{1}{200}}{\frac{1}{200} + \frac{3}{200} + \frac{7}{500}} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{3}{2} + \frac{7}{5}} = \frac{\frac{1}{2}}{2 + \frac{7}{5}} = \frac{\frac{1}{2}}{\frac{17}{5}}$$

$\boxed{P(E_1/A) = \frac{5}{34}}$ = Probability that defective item was produced by A

Q27. let $\vec{n} = A\hat{i} + B\hat{j} + C\hat{k}$ of the required plane

Eqⁿ of plane is $A(x-2) + B(y-2) + C(z+1) = 0$ —— (3)

The plane also passes through $(3,4,2)$ & $(7,0,6)$

$$A + 2B + 3C = 0 \quad — (1)$$

$$5A - 2B + 7C = 0 \quad — (2)$$

Adding (1), (2)

$$6A + 10C = 0$$

$$\boxed{A = -\frac{5C}{3}}$$

In eqn(2) $-\frac{25C}{3} + 7C = 2B$

$$-\frac{4C}{2 \times 3} = B$$

$$\Rightarrow \boxed{B = -\frac{2C}{3}}$$

$$\vec{n}_1 = -\frac{5C}{3}\hat{i} - \frac{2C}{3}\hat{j} + C\hat{k} = -\frac{C}{3}(5\hat{i} + 2\hat{j} - 3\hat{k})$$

Q29. let x be no. of A models
y be no. of B models.

Objective $Z = 15x + 10y$ (Maximize)
Subject to constraints: $x, y \geq 0$
$2x + y \leq 40$ (Skilled man working hrs)
$2x + 3y \leq 80$ (Semi-skilled man working hrs)

$2x + y \leq 40$ | $2x + 3y \leq 80$
$2x + y = 40$ | $2x + 3y = 80$

x	0	20
y	40	0

x	10	40
y	20	0

Zero test : TRUE | Zero test : TRUE

GRAPH: On graph paper.

Eq^n of line BC:

$(y-2) = \frac{5}{-2}(x-6)$

$\Rightarrow y-2 = -\frac{5}{2}x + 15$

$\Rightarrow \boxed{y_2 = -\frac{5}{2}x + 17}$

Eq^n of line AC

$(y-5) = \frac{3}{-4}(x-2)$

$y-5 = -\frac{3x}{4} + \frac{3}{2}$

$\boxed{y_3 = -\frac{3x}{4} + \frac{13}{2}}$

Area of shaded region is required area $A = ar(ABDE) + ar(BCFD) - ar(ACFE)$

$$A = \int_2^4 y_1\,dx + \int_4^6 y_2\,dx - \int_2^6 y_3\,dx$$

$$= \int_2^4 (x+3)\,dx + \int_4^6 \left(-\frac{5x}{2} + 17\right) dx - \int_2^6 \left(-\frac{3x}{4} + \frac{13}{2}\right) dx$$

$$A = \left[\frac{x^2}{2} + 3x\right]_2^4 + \left[17x - \frac{5x^2}{4}\right]_4^6 - \left[\frac{13x}{2} - \frac{3x^2}{8}\right]_2^6$$

$$= [8 + 12 - 2 - 6] + [102 - 45 - 68 + 20] - \left[39 - \frac{27}{\ } - 13 + \frac{3}{2}\right]$$

$$= 12 + 9 - 14$$

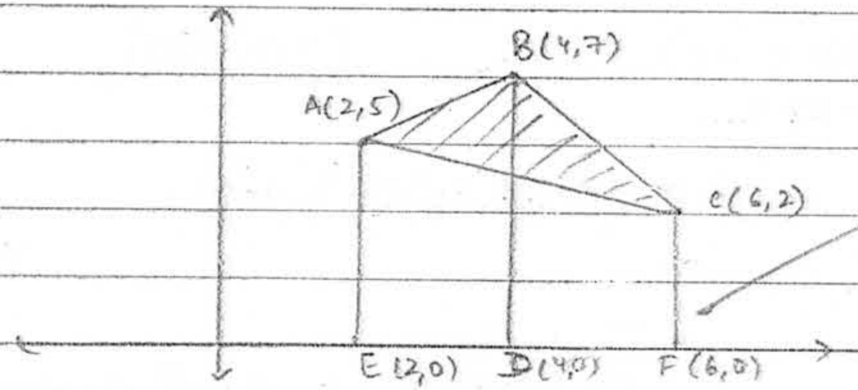
$\boxed{A = 7 \text{ sq. units}}$

∴ Eqⁿ of other plane is

$$\boxed{\vec{r}\cdot(5\hat{i}+2\hat{j}-3\hat{k}) = 23} \rightarrow \text{Vector eq}^n \Rightarrow \boxed{\vec{r}\cdot(5\hat{i}+2\hat{j}-3\hat{k})-23=0}$$

$$5x+2y-3z=23 \rightarrow \text{Cartesian eq}^n$$

Q28.



Eqⁿ of general line:

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

Eqⁿ of line AB : ~~Eqⁿ of li~~

$$(y-5) = \frac{2}{2}(x-2)$$

$$\boxed{y = x+3}$$

Direction ratios of normal to the plane are

$$\left(\frac{-5c}{3}, \frac{-2c}{3}, c\right) \equiv \cancel{(5c, 2c, -3c)}\ (5, 2, -3)$$

$\therefore\ \vec{n} = 5\hat{i} + 2\hat{j} - 3\hat{k}$

Eq^n: of plane: $5(x-2) + 2(y-2) - 3(z+1) = 0$ [From(3)]

$5x + 2y - 3z - 10 - 4 - 3 = 0$

$\boxed{5x + 2y - 3z = 17}$ is cartesian eq^n of plane.

VECTOR EQN:

$\boxed{\vec{r}.\vec{n} = d} \Rightarrow \left[\vec{r} - (2\hat{i} + 2\hat{j} - \hat{k})\right].\vec{n} = 0$ $\left[\because (\vec{r} - \vec{a}).\vec{n} = 0 \text{ is eq}^n \text{ of plane}\right]$

$\vec{r}(5\hat{i} + 2\hat{j} - 3\hat{k}) - (2\hat{i} + 2\hat{j} - \hat{k})(5\hat{i} + 2\hat{j} - 3\hat{k}) = 0$

$\Rightarrow \boxed{\vec{r}(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17}$ is vector eq^n of plane.

For plane parallel to above plane, $\vec{n}_2 = \vec{n} = 5\hat{i} + 2\hat{j} - 3\hat{k}$

$(\vec{r} - \vec{a}_1).\vec{n} = 0$

Here $\vec{a}_1 = 4\hat{i} + 3\hat{j} + \hat{k}$

$\left[\vec{r} - (4\hat{i} + 3\hat{j} + \hat{k})\right].(5\hat{i} + 2\hat{j} - 3\hat{k}) = 0$

$\vec{r}(5\hat{i} + 2\hat{j} - 3\hat{k}) - (20 + 6 - 3) = 0$

Corner pt	$Z = 15x + 10y$	
$(0,0)$	$Z = 0$	
~~$(\frac{80}{3}, 0)$~~ $(0, \frac{80}{3})$	$Z = \cancel{400} \; 0 + \frac{800}{3} \simeq 266.7 \simeq 267$	
$(10, 20)$	$Z = 150 + 200 = 350$	(MAX)
$(20, 0)$	$Z = 300$	

$\therefore$ No. of model A = 10

No. of model B = 20

Maximum profit = ₹ 350