PART : MATHEMATICS

1. If $7 = 5 + \frac{1}{7}(5 + a) + \frac{1}{7^2}(5 + 2a) + ... \infty$ term then the value of a is:

Ans. **(6)**

Sol. $7 = 5 + \frac{1}{7}(5 + a) + \frac{1}{7^2}(5 + 2a) + ... \infty$ .... (i)

multiply with $\frac{1}{7}$ ever term.

$\frac{7}{7} = \frac{5}{7} + \frac{1}{7^2}(5 + a) + ... \infty$ .... (ii)

subtract from equation (i) ....equation (ii)

$7 - \frac{7}{7} = 5 + \frac{1}{7}(a) + \frac{1}{7^2}(a) + ... \infty$

$6 = 5 + \frac{1}{7}a\,[1 + \frac{1}{7} + ... \infty]$

$1 = \frac{1}{7}a\left[\frac{1}{1-\frac{1}{7}}\right]$

$1 = \frac{1}{7}a \times \frac{7}{6}$

$a = 6$

2. If A and B are binomial coefficients of 30th and 12th term of binomial expansion $(1 + x)^{2n-1}$. If 2A = 5B, then the value of n is:

(1) 19 (2) 20 (3) 21 (4) 40

Ans. **(3)**

Sol. $T_{30} = {}^{2n-1}C_{29}.\, x^{29}$, $A = {}^{2n-1}C_{29}$

$T_{12} = {}^{2n-1}C_{11}.\, x^{11}$, $B = {}^{2n-1}C_{11}$

$2A = 5B$

$\Rightarrow$ $2 \times {}^{2n-1}C_{29} = 5.\, {}^{2n-1}C_{11}$

$\Rightarrow$ $\frac{2}{(2n-30)!\,.\,29!} = \frac{5}{(2n-12)!\,.\,11!}$

$n = 21$.

3. In a arithmetic progression S_n represent the sum of n terms and $S_{12} = 57$, $S_{40} = 1030$ then the value of $S_{30} - S_{10}$ is:

Ans. **(515)**

Sol. $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{12} = \frac{12}{2}[2a + 11d] = 57$

$2a + 11d = \frac{19}{2}$ ...(1)

$S_{40} = \frac{40}{2}[2a + 39d] = 1030$

$2a + 39d = \frac{103}{2}$ ...(2)

Equation (2) – (1)

$39d - 11d = \frac{103}{2} - \frac{19}{2}$

$d = \frac{3}{2}$, $a = \frac{-7}{2}$ now

$S_{30} - S_{10} = 15[2a + 29d] - 5[2a + 9d] = 20a + 390d$

$= 20 \times \left(\frac{-7}{2}\right) + 390 \times \left(\frac{3}{2}\right) = 515$

4. Equation of the chord having mid point (3, 1) to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is :

(1) $25x + 5y - 125 = 0$ (2) $48x + 25y - 169 = 0$
(3) $65x + 2y - 12 = 0$ (4) $45x + 4y - 135 = 0$

Ans. **(2)**

Sol. $S_1 = T$

$= \frac{3^2}{25} + \frac{1^2}{16} - 1 = \frac{3x}{25} + \frac{y}{16} - 1$

$= \frac{9}{25} + \frac{1}{16} = \frac{3x}{25} + \frac{y}{16}$

$= 48x + 25y - 169 = 0$

5. Let $A = [a_{ij}]_{2\times2}$ such that $a_{ij} \in \{0, 1\}$. Probability that randomly chosen such matrix A is non-invertible is

(1) $\frac{3}{8}$ (2) $\frac{5}{8}$ (3) $\frac{1}{2}$ (4) $\frac{7}{8}$

Ans. **(2)**

Sol. Total number of matrices $A = 2^4 = 16$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

For non-invertible matrix $|A| = 0$ $\Rightarrow$ $ad - bc = 0$

$ad = bc$

Case 1 $\Rightarrow$ $ad = bc = 0$

$({}^2C_1 + {}^2C_2)({}^2C_1 + {}^2C_2) = 9$

Case 2 $\Rightarrow$ $ad = bc = 1$

$1 \times 1 = 1$

Required probability $= \frac{9+1}{16} = \frac{5}{8}$

6. If system of equations

$x + 2y - 3z = 2$

$2x + \lambda y + 5z = 5$

$4x + 3y + \mu z = 33$

has infinite many solutions then $\lambda + \mu$ is

(1) $\frac{244}{5}$ (2) $\frac{1334}{5}$ (3) $\frac{1296}{5}$ (4) $\frac{4997}{5}$

Ans. **(2)**

Sol. $\Delta = 0 \quad \Delta = \begin{vmatrix} 1 & 2 & -3 \\ 2 & \lambda & 5 \\ 4 & 3 & \mu \end{vmatrix} = 0$

$12\lambda + \lambda\mu - 4\mu + 7 =$ ....(i)

$\Delta z = \begin{vmatrix} 1 & 2 & 2 \\ 2 & \lambda & 5 \\ 4 & 3 & 33 \end{vmatrix} = 0$

$\lambda = \frac{19}{5}$

from (i) $\mu = 263$

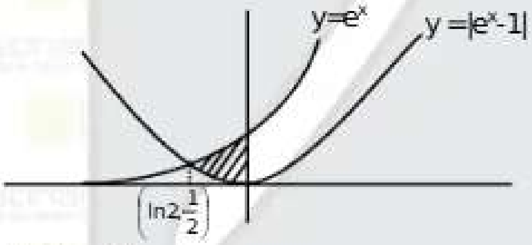
$\lambda + \mu = \frac{19}{5} + 263 = \frac{1334}{5}$

7. The area bounded by $y = e^x$, $y = |e^x - 1|$ and y-axis is ____

(1) $\ln 2$ (2) $1 + \ln 2$ (3) $1 - \ln 2$ (4) $2\ln 2$

Ans. **(3)**

Sol.



$e^x = 1 - e^x$

$e^x = \frac{1}{2}$

$x = -\ln 2$

required area $= \int_{-\ln 2}^{0} (e^x - (1 - e^x))dx = \left[2e^x - x\right]_{-\ln 2}^{0}$

$= 2 - (2e^{-\ln 2} + \ln 2) = 2 - 1 - \ln 2 = 1 - \ln 2$

8. $f(x) = [x] + |x - 2|$; $-2 < x < 3$

and m = number of points of discontinuity, and

n = number of points of non differentiability.

Then the value of m + n is:

(1) 8 (2) 7 (3) 9 (4) – 10

Ans. **(1)**

Sol. $f(x) = \begin{cases} -x \ ; & x \in (-2,-1) \\ 1-x \ ; & x \in [-1, 0) \\ 2-x \ ; & x \in [0, 1) \\ 3-x \ ; & x \in [1, 2) \\ x \ ; & x \in [2, 3) \end{cases}$

As we can see that function is discontinuous at −1, 0, 1 and 2.
Therefore, function is non-differentiable at −1, 0, 1 and 2.
Therefore, m = 4 and n = 4.
Then, m + n = 8.

9. There is a group A of 5 boys and 3 girls and another group B of 5 boys and 6 girls. How many ways can we invite 4 boys and 4 girls for party with 5 from group A and 3 from group B.

(1) 2850 (2) 2550 (3) 3150 (4) 3450

Ans. **(3)**

Sol. Group A ⇒ 5B, 3G
Group B ⇒ 5B, 6G
4 Boys and 4girls invite
5 from group A and 3 from group B

Case (i)	In group A	4B 1G	In B	0B 3G
	In group A	3B 2G	In B	1B 2G
	In group A	2B 3G	In B	2B 1G
	In group A	1B 4G	In B	3B 0G (not possible)

We have three cases than.

$^5C_4 \times {^3C_1} \times {^5C_0} \times {^6C_3} = 5 \times 3 \times 1 \times 20 = 300$

$^5C_3 \times {^3C_2} \times {^5C_1} \times {^6C_2} = 10 \times 3 \times 5 \times 15 = 2250$

$^5C_2 \times {^3C_3} \times {^5C_2} \times {^6C_1} = 10 \times 1 \times 10 \times 6 = 600$

$300 + 2250 + 600 = 3150$

10. $2\cos x \dfrac{dy}{dx} = \sin 2x - 2y\sin x$, y(x) = y and y(0) = 0, then find the value of $y'\left(\dfrac{\pi}{4}\right) + y\left(\dfrac{\pi}{4}\right)$ is:

(1) $\dfrac{1}{2}$ (2) $\dfrac{1}{\sqrt{2}}$ (3) $-\dfrac{1}{2}$ (4) $-\dfrac{1}{\sqrt{2}}$

Ans. **(2)**

Sol. $(2\cos x)\, y'(x) = \sin 2x - 2y(x)\sin x$

Put $x = \dfrac{\pi}{4}$

$$2 \times \frac{1}{\sqrt{2}}\, y'\left(\frac{\pi}{4}\right) = 1 - 2\, y\left(\frac{\pi}{4}\right) \times \frac{1}{\sqrt{2}}$$

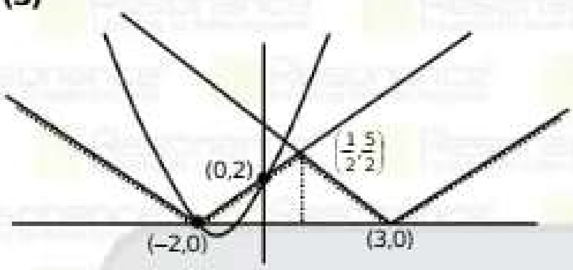
$$y'\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

11. Number of real solutions of the equation $x^2 + 3x + 2 = \min\{|x-3|, |x+2|\}$ is ____

(1) 0 (2) 1 (3) 2 (4) 3

Ans. **(3)**

Sol.



Total number of solutions = 2

12. Consider the differential equation $x^2 \frac{dy}{dx} = 2xy + 3$ such that y (1) = 4 then the value of 2(y (2)) is

Ans. **(39)**

Sol. Divide by x^2

$$\frac{dy}{dx} - \frac{2}{x}y = \frac{3}{x^2}$$

$$\text{I.F.} = e^{\int \frac{-2}{x}dx} = e^{-2\ell nx} = e^{\ell nx^{-2}} = \frac{1}{x^2}$$

$$y.\ \text{I. F.} = \int \frac{3}{x^2}.\ \text{I.F. } dx$$

$$\frac{y}{x^2} = 3\int \frac{dx}{x^4}$$

$$\frac{y}{x^2} = \frac{-1}{x^3} + C$$

$$y(1) = 4 \quad \Rightarrow \quad 4 = -1 + C \Rightarrow \quad C = 5$$

$$\frac{y}{x^2} = \frac{-1}{x^3} + 5$$

$$y = -\frac{1}{x} + 5x^2$$

$$y(2) = -\frac{1}{2} + 20 = \frac{39}{2}$$

$$2y(2) = 39$$

13. If

$$\lim_{x\to 0}\begin{vmatrix} a+\frac{\sin x}{x} & 1 & b \\ a & 1+\frac{\sin x}{x} & b \\ a & 1 & b+\frac{\sin x}{x} \end{vmatrix} = \lambda a + \mu b + C$$

where λ and μ are the coefficient of a, b and c is constant then find the value of $(\lambda + \mu + c)^2$

Ans. **(16)**

Sol.

$$\Rightarrow \begin{vmatrix} a+1 & 1 & b \\ a & 1+1 & b \\ a & 1 & b+1 \end{vmatrix} = \lambda a + \mu b + C$$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ a & 1 & b+1 \end{vmatrix} = \lambda a + \mu b + C$$

$C_2 \to C_1 + C_2$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ a & a+1 & b+1 \end{vmatrix} = \lambda a + \mu b + C$$

$a + b + 2 = \lambda a + \mu b + C$

So $\lambda = 1, \mu = 1, C = 2$

Now, $(\lambda + \mu + C)^2 = 16$

14. If $f : (-\infty, \infty) \to (-\infty, 1)$, and $f(x) = \dfrac{2^x - 2^{-x}}{2^x + 2^{-x}}$, then f(x) is

(1) one-one and onto
(2) one-one and into
(3) many-one and onto
(4) many-one and into

Ans. **(2)**

Sol. $f(x) = \dfrac{2^{2x} - 1}{2^{2x} + 1} = \dfrac{2^{2x} + 1 - 2}{2^{2x} + 1} = 1 - \dfrac{2}{2^{2x} + 1}$

2^{2x} is one-one so f(x) is one-one

For $x \in (-\infty, \infty)$

$2^x \in (0, \infty)$

$2^{2x} + 1 \in (1, \infty)$

$\dfrac{1}{2^{2x} + 1} \in (0, 1)$

$\dfrac{-2}{2^{2x} + 1} \in (-2, 0)$

$1 - \dfrac{2}{2^{2x} + 1} \in (-1, 1)$

Range of f(x) is $(-1, 1)$ but codomain of f(x) is $(-\infty, 1)$ so f(x) is into.

15. Given $\dfrac{x^2}{a^2} \dfrac{-y^2}{b^2} = 1$ is a hyperbola with latus rectum $12\sqrt{5}$ and eccentricity $\sqrt{\dfrac{5}{2}}$ and another hyperbola $\dfrac{x^2}{A^2} \dfrac{-y^2}{B^2} = -1$ with latus rectum $15\sqrt{2}$. If the product of transverse axis of both the hyperbola is $100\sqrt{10}$, eccentricity of the later hyperbola is:

(1) 0
(2) 2
(3) $\sqrt{\dfrac{13}{5}}$
(4) $\sqrt{\dfrac{11}{5}}$

Ans. **(4)**

Sol. $\frac{x^2}{a^2}\frac{-y^2}{b^2}=1$

$\frac{2b^2}{a}=12\sqrt{5}$ .... (i)

$\sqrt{1+\frac{b^2}{a^2}}=\sqrt{\frac{5}{2}}$ .... (ii)

from (i) and (ii)

$a=4\sqrt{5}$ and $b^2=120$

$\frac{x^2}{A^2}\frac{-y^2}{B^2}=-1$

$\frac{2A^2}{B}=15\sqrt{2}$ .... (iii)

(since product of transverse axis = $100\sqrt{10}$

$(2a).(2B)=100\sqrt{10}$ .... (iv)

from (iii) & (iv)

$A^2=\frac{375}{4}$ and $B^2=\frac{625}{8}$

$e_2=\sqrt{1+\frac{A^2}{B^2}}=\sqrt{\frac{11}{5}}$ (substituting the value of A^2 and B^2 from the above equation).