EXERCISE 8.1

1. In △ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine:

(i) sin A, cos A (ii)

sin C, cos C

Solution:

In a given triangle ABC, right angled at B =∠ B =

90°Given: AB = 24 cm and BC = 7 cm

According to the Pythagoras Theorem,

In a right- angled triangle, the squares of the hypotenuse side is equal to the sum of the squares of the othertwo sides.

By applying Pythagoras theorem, we

get$AC^2=AB^2+BC^2$

$AC^2 = (24)^2+7^2$

$AC^2 =$

(576+49)

$AC^2 =$

$625cm^2$ AC

= √625 = 25

Therefore, AC = 25 cm

(i) To find Sin (A), Cos (A)

We know that sine (or) Sin function is the equal to the ratio of length of the opposite side to the hypotenuseside. So it becomes

Sin (A) = Opposite side /Hypotenuse = BC/AC = 7/25

Cosine or Cos function is equal to the ratio of the length of the adjacent side to the hypotenuse side and itbecomes,

Cos (A) = Adjacent side/Hypotenuse = AB/AC = 24/25

(ii) To find Sin (C), Cos

(C)Sin (C) = AB/AC =

24/25 Cos (C) =

BC/AC = 7/25

2. In Fig. 8.13, find tan P – cot R

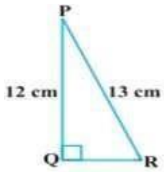

Fig. 8.13

Solution:

In the given triangle PQR, the given triangle is right angled at Q and the given measures are:PR

= 13cm,

PQ = 12cm

Since the given triangle is right angled triangle, to find the side QR, apply the Pythagorean

theoremAccording to Pythagorean theorem,

In a right- angled triangle, the squares of the hypotenuse side is equal to the sum of the squares of the othertwo sides.

$PR^2 = QR^2 + PQ^2$

Substitute the values of PR and

PQ$13^2 = QR^2+12^2$

$169 = QR^2+144$

Therefore, $QR^2 = 169-144$

$QR^2 = 25$

$QR = \sqrt{25} = 5$

Therefore, the side QR = 5 cmTo

find tan P – cot R:

According to the trigonometric ratio, the tangent function is equal to the ratio of the length of the oppositeside to the adjacent sides, the value of tan (P) becomes tan (P) **=** Opposite side /Adjacent side = QR/PQ = 5/12

Since cot function is the reciprocal of the tan function, the ratio of cot function

becomes,Cot (R) = Adjacent side/Opposite side = QR/PQ =

5/12 Therefore, tan (P) – cot (R) = 5/12 – 5/12

= 0Therefore, tan(P) – cot(R)

= 0 **3. If sin A = 3/4, calculate cos A and tan A.**

Solution:

Let us assume a right angled triangle ABC, right angled at

BGiven: Sin A = 3/4

We know that, Sin function is the equal to the ratio of length of the opposite side to the hypotenuse side.Therefore, Sin A = Opposite side /Hypotenuse= 3/4

Let BC be 3k and AC will be 4k

where k is a positive real

number.

According to the Pythagoras theorem, the squares of the hypotenuse side is equal to the sum of the squaresof the other two sides of a right angle triangle and we get,

$AC^2=AB^2+BC^2$

Substitute the value of AC and

BC$(4k)^2=AB^2+(3k)^2$

$16k^2-9k^2$

$=AB^2$

$AB^2=7k^2$

Therefore, AB = √7k

Now, we have to find the value of cos A and tan

AWe know that,

Cos (A) = Adjacent side/Hypotenuse

Substitute the value of AB and AC and cancel the constant k in both numerator and denominator,

we getAB/AC = √7k/4k = √7/4 Therefore, cos (A) = √7/4 tan(A) = Opposite side/Adjacent side

Substitute the Value of BC and AB and cancel the constant k in both numerator and denominator, we get,BC/AB = 3k/√7k = 3/√7

Therefore, tan A = 3/√7

4. Given 15 cot A = 8, find sin A and sec A.

Solution:

Let us assume a right angled triangle ABC, right angled at

BGiven: 15 cot A = 8

So, Cot A = 8/15

We know that, cot function is the equal to the ratio of length of the adjacent side to the opposite side.Therefore,

cot A = Adjacent side/Opposite side = AB/BC = 8/15

Let AB be 8k and BC will be 15k
Where, k is a positive real number.

According to the Pythagoras theorem, the squares of the hypotenuse side is equal to the sum of the squaresof the other two sides of a right angle triangle and we get,

$AC^2=AB^2+BC^2$

Substitute the value of AB and

BC$AC^2 = (8k)^2 + (15k)^2$

$AC^2 = 64k^2 +$

$225k^2$ $AC^2 =$

$289k^2$

Therefore, AC = 17k

Now, we have to find the value of sin A and sec

AWe know that,

Sin (A) = Opposite side /Hypotenuse

Substitute the value of BC and AC and cancel the constant k in both numerator and denominator, we getSin

A = BC/AC = 15k/17k = 15/17

Therefore, sin A = 15/17

Since secant or sec function is the reciprocal of the cos function which is equal to the ratio of the length ofthe hypotenuse side to the adjacent side.

Sec (A) = Hypotenuse/Adjacent side

Substitute the Value of BC and AB and cancel the constant k in both numerator and denominator, we get,AC/AB = 17k/8k = 17/8

Therefore sec (A) = 17/8

5. Given sec θ = 13/12 Calculate all other trigonometric ratios

Solution:

We know that sec function is the reciprocal of the cos function which is equal to the ratio of the length of thehypotenuse side to the adjacent side

Let us assume a right angled triangle ABC, right angled at

Bsec θ =13/12 = Hypotenuse/Adjacent side = AC/AB

Let AC be 13k and AB will be 12k

Where, k is a positive real number.

According to the Pythagoras theorem, the squares of the hypotenuse side is equal to the sum of the squaresof the other two sides of a right angle triangle and we get,

$AC^2 = AB^2 + BC^2$

Substitute the value of AB and

AC$(13k)^2 = (12k)^2 + BC^2$

$169k^2 = 144k^2 + BC^2$

$169k^2 = 144k^2 +$

BC^2 BC^2 = $169k^2$

- $144k^2$ BC^2=

$25k^2$

Therefore, BC = 5k

Now, substitute the corresponding values in all other trigonometric ratiosSo,

Sin θ = Opposite Side/Hypotenuse = BC/AC =

5/13 Cos θ = Adjacent Side/Hypotenuse = AB/AC

= 12/13 tan θ = Opposite Side/Adjacent Side =

BC/AB = 5/12 Cosec θ = Hypotenuse/Opposite

Side = AC/BC = 13/5cot θ = Adjacent

Side/Opposite Side = AB/BC = 12/5

6. If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠ A = ∠ B.

Solution:

Let us assume the triangle ABC in which CD⊥AB

Give that the angles A and B are acute angles, such

thatCos (A) = cos (B)

As per the angles taken, the cos ratio is

written asAD/AC = BD/BC Now,

interchange the terms, we

getAD/BD = AC/BC

Let take a constant

valueAD/BD = AC/BC = k

Now consider the

equation asAD = k BD .(1)

AC = k BC ...(2)

By applying Pythagoras theorem in △CAD and △CBD we get,CD^2

= BC^2 - BD^2... (3)

CD^2 =AC^2 −AD^2 ....(4)

From the equations (3) and (4) we

get,AC^2−AD^2= BC^2−BD^2

Now substitute the equations (1) and (2) in (3) and

(4)K^2(BC^2−BD^2)=(BC^2−BD^2) k^2=1

Putting this value in equation, we

obtainAC = BC

∠A=∠B (Angles opposite to equal side are equal-isosceles triangle)

7. If cot θ = 7/8, evaluate :

(i) (1 + sin θ)(1 – sin θ)/(1+cos θ)(1-cos θ)

(ii) $\cot^2 \theta$

Solution:

Let us assume a △ABC in which ∠B = 90° and ∠C

= θGiven:

cot θ = BC/AB = 7/8

Let BC = 7k and AB = 8k, where k is a positive real numberAccording

to Pythagoras theorem in △ABC we

get.

$AC^2 = AB^2+BC^2$

$AC^2 = (8k)^2+(7k)^2$

$AC^2 = 64k^2+49k^2$

$AC^2 = 113k^2$

AC = √113 k

According to the sine and cos function ratios, it is written as sin θ = AB/AC = Opposite Side/Hypotenuse = 8k/√113 k = 8/√113 and cos θ = Adjacent Side/Hypotenuse = BC/AC = 7k/√113 k = 7/√113 Now apply the values of sin function and cos function:

$$\text{(i)}\ \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{49}{64}$$

$$\text{(ii)}\ \cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

8. If 3 cot A = 4, check whether $(1-\tan^2 A)/(1+\tan^2 A) = \cos^2 A - \sin^2 A$ or not.

Solution:

Let △ABC in which ∠B=90°

We know that, cot function is the reciprocal of tan function and it is written ascot(A) = AB/BC = 4/3

Let AB = 4k an BC =3k, where k is a positive real number.According

to the Pythagorean theorem,

$AC^2=AB^2+B$

C^2

$AC^2=(4k)^2+($

$3k)^2$

$AC^2=16k^2+9$

$k^2 AC^2=25k^2$

AC=5k

Now, apply the values corresponding to the ratios

tan A = BC/AB = 1/√3
tan(A) = BC/AB = 3/4 Let
BC = 1k and AB = √3 k, sin
(A) = BC/AC = 3/5
Where k is the positive real number of the
cos (A) = AB/AC = 4/5 problemBy
Pythagoras theorem in ΔABC we
Now compare the left hand side(LHS) with right hand side(RHS) get:
$AC^2=AB^2+BC^2$

$AC^2=(\sqrt{3}$

$k)^2+(k)^2$

$AC^2=3k^2+k^2$

$$\text{L.H.S.} = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{7}{25}$$

$$\text{R.H.S.} = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Since, both the LHS and RHS = 7/25

R.H.S. =L.H.S.

Hence, **$(1-\tan^2 A)/(1+\tan^2 A) = \cos^2 A - \sin^2 A$** is proved

9. In triangle ABC, right-angled at B, if tan A = 1/√3 find the value of:

(i) sin A cos C + cos A sin C

(ii) cos A cos C – sin A sin C

A=4

k_2

AC

= 2k

Now find the values of cos A,

Sin ASin A = BC/AC = 1/2

Cos A = AB/AC = √3/2

Then find the values of cos C and sin

CSin C = AB/AC = √3/2

Cos C = BC/AC = 1/2

Now, substitute the values in the given problem

(i) sin A cos C + cos A sin C = (1/2) ×(1/2)+ √3/2 ×√3/2 = 1/4 + 3/4

= 1(ii) cos A cos C – sin A sin C = (√3/2)(1/2) – (1/2) (√3/2) = 0

10. In △ PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos Pand tan P

Solution:

In a given triangle PQR, right angled at Q, the following measures

arePQ = 5 cm

PR + QR = 25 cm

Now let us assume, QR

= xPR = 25-QR

PR = 25- x

According to the Pythagorean

Theorem,$PR^2 = PQ^2 + QR^2$Substitute

the value of PR

as x$(25- x)^2 = 5^2 + x^2$

$25^2 + x^2 - 50x = 25 + x^2$

$625 + x^2 - 50x - 25 - x^2 = 0$

-50x = -

600 x= -

600/-50

x = 12 =

QR

Now, find the value of

PRPR = 25- QR

Substitute the value of

PR = 13

Now, substitute the value to the given problem

(1) sin p = Opposite Side/Hypotenuse = QR/PR = 12/13

(2) Cos p = Adjacent Side/Hypotenuse = PQ/PR = 5/13

(3) tan p =Opposite Side/Adjacent side = QR/PQ = 12/5

11. State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) sec A = 12/5 for some value of angle A.

(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A.

(v) sin θ = 4/3 for some angle θ.

Solution:

(i) The value of tan A is always less than

1.Answer: **False**

Proof: In ΔMNC in which ∠N =

90°,MN = 3, NC = 4 and MC =

5

Value of tan M = 4/3 which is greater than 1.

The triangle can be formed with sides equal to 3, 4 and hypotenuse = 5 as it will follow the Pythagorastheorem.

$MC^2 = MN^2 +$

NC^2

$5^2 = 3^2 + 4^2$

25=9+16

$25 = 25$

(ii) sec A = 12/5 for some value of angle AAnswer:

True

Justification: Let a ΔMNC in which ∠N = 90º,

MC=12k and MB=5k, where k is a positive real

number.By Pythagoras theorem we get,

$MC^2=MN^2+N$

C^2

$(12k)^2=(5k)^2+$

NC^2

$NC^2+25k^2=14$

$4k^2$

(iii) cos A is the abbreviation used for the cosecant of angle A.Answer: **False**

Justification: Abbreviation used for cosecant of angle M is cosec M. cos M is the abbreviation used forcosine of angle M. **(iv)** cot A is the product of cot and A.Answer: **False**

Justification: cot M is not the product of cot and M. It is the cotangent of ∠M.

(v) sin θ = 4/3 for some angle θ.Answer**: False**

Justification: sin θ = Opposite/Hypotenuse

We know that in a right angled triangle, Hypotenuse is the longest side.

∴ sin θ will always less than 1 and it can never be 4/3 for any value of θ.

EXERCISE 8.2

1. Evaluate the following:

(i) sin 60° cos 30° + sin 30° cos 60°

(ii) 2 $\tan^2 45° + \cos^2 30° – \sin^2 60$

(iii) $\dfrac{\cos 45°}{\sec 30° + \text{cosec } 30°}$

(iv) $\dfrac{\sin 30° + \tan 45° - \text{cosec } 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Solution:

(i) sin 60° cos 30° + sin 30° cos 60°

First, find the values of the given trigonometric

ratiossin 30° = 1/2 cos 30° =

√3/2sin 60°

= 3/2 cos

60°= 1/2

Now, substitute the values in the given problem sin 60° cos 30° + sin 30° cos

60° = √3/2 ×√3/2 + (1/2) ×(1/2) = 3/4+1/4 = 4/4 =1

(ii) 2 $\tan^2 45° + \cos^2 30° - \sin^2 60$

We know that, the values of the trigonometric ratios

cos 30° = √3/2

tan 45° = 1

Substitute the values in the given problem

2 $\tan^2 45° + \cos^2 30° - \sin^2 60 = 2(1)^2 + (\sqrt{3}/2)^2 - (\sqrt{3}/2)^2$

2 $\tan^2 45° + \cos^2 30° - \sin^2 60$ = 2 + 0

2 $\tan^2 45° + \cos^2 30° - \sin^2 60$ = 2

(iii) cos 45°/(sec 30°+cosec 30°)We know that, cos 45° = 1/√2

sec 30° = 2/√3cosec 30° = 2

Substitute the values, we get

$$\frac{\cos 45^\circ}{\sec 30^\circ+\operatorname{cosec} 30^\circ}=\frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2}=\frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$=\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2+2\sqrt{3}}$$

$$=\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2(1+\sqrt{3})}=\frac{\sqrt{3}}{2\sqrt{2}(1+\sqrt{3})}=\frac{\sqrt{3}}{2\sqrt{2}(\sqrt{3}+1)}$$

Now, rationalize the terms we get,

$$=\frac{\sqrt{3}}{2\sqrt{2}(\sqrt{3}+1)}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}=\frac{3-\sqrt{3}}{2\sqrt{2}\,(3-1)}=\frac{3-\sqrt{3}}{2\sqrt{2}\,(2)}$$

Now, multiply both the numerator and denominator by √2 , we get

$$=\frac{3-\sqrt{3}}{2\sqrt{2}\,(2)}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{3\sqrt{2}-\sqrt{3}\sqrt{2}}{8}=\frac{3\sqrt{2}-\sqrt{6}}{8}$$

Therefore, cos 45°/(sec 30°+cosec 30°) = (3√2 – √6)/8

(iv) $$\frac{\sin 30^\circ+\tan 45^\circ-\operatorname{cosec} 60^\circ}{\sec 30^\circ+\cos 60^\circ+\cot 45^\circ}$$

We know that,

sin 30° = 1/2 tan 45° = 1 cosec 60° = 2/√3 sec 30° = 2/√3 cos 60° = 1/2 cot 45° = 1

Substitute the values in the given problem, we get

$$\frac{\sin 30°+\tan 45°-\text{cosec } 60°}{\sec 30°+\cos 60°+\cot 45°}=\frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1}=\frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}}$$

Now, cancel the term 2√3, in numerator and denominator, we get

$$=\frac{\sqrt{3}+2\sqrt{3}-4}{4+\sqrt{3}+2\sqrt{3}}=\frac{3\sqrt{3}-4}{3\sqrt{3}+4}$$

Now, rationalize the terms

$$=\frac{3\sqrt{3}-4}{3\sqrt{3}+4}\times\frac{3\sqrt{3}-4}{3\sqrt{3}-4}$$

$$=\frac{27-12\sqrt{3}-12\sqrt{3}+16}{27-12\sqrt{3}+12\sqrt{3}+16}=\frac{27-24\sqrt{3}+16}{11}=\frac{43-24\sqrt{3}}{11}$$

Therefore,

$$\frac{\sin 30°+\tan 45°-\text{cosec } 60°}{\sec 30°+\cos 60°+\cot 45°}=\frac{43-24\sqrt{3}}{11}$$

(v) $\frac{5\cos^2 60°+4\sec^2 30°-\tan^2 45°}{\sin^2 30°+\cos^2 30°}$

We know

that,cos

60° = 1/2

sec 30° =
2/√3tan

45° = 1 sin

30° = 1/2

cos 30° =

√3/2

= (5/4+16/3-1)/(1/4+3/4)

= (15+64-12)/12/(4/4)

= 67/12

2. Choose the correct option and justify your choice :(i) 2tan 30°/1+tan^{2}30° =

(A) sin 60° (B) cos 60° (C) tan 60° (D) sin 30°(ii) $1-\tan^2 45°/1+\tan^2 45°$ =

(A) tan 90° (B) 1 (C) sin 45° (iii) sin 2A = 2 (D) 0

sin A is true when A =

(A) 0° (B) 30° (C) 45° (D) 60°

(iv) $2\tan 30°/1-\tan^2 30°$ =

(A) cos 60° (B) sin 60° (C) tan 60° (D) sin 30°

Solution:

(i) (A) is correct.

Substitute the of tan 30° in the given equationtan

30° = 1/√3

$2\tan 30°/1+\tan^2 30° = 2(1/\sqrt{3})/1+(1/\sqrt{3})^2$

= (2/√3)/(1+1/3) = (2/√3)/(4/3)

= 6/4√3 = √3/2 = sin 60°

The obtained solution is equivalent to the trigonometric ratio sin 60°

(ii) (D) is correct.

Substitute the of tan 45° in the given equationtan

45° = 1

$1-\tan^2 45°/1+\tan^2 45° = (1-1^2)/(1+1^2)$

= 0/2 = 0

The solution of the above equation is 0.

(iii) (A) is correct.

To find the value of A, substitute the degree given in the options one by onesin

2A = 2 sin A is true when A = 0°

As sin 2A = sin 0° = 0

2 sin A = 2 sin 0° = 2 × 0

= 0or,

Apply the sin 2A formula, to find the degree

valuesin 2A = 2sin A cos A ⇒2sin A cos A

= 2 sin A

⇒ 2cos A = 2 ⇒ cos A = 1

Now, we have to check, to get the solution as 1, which degree value has to be applied.When

0 degree is applied to cos value, i.e., cos 0 =1

Therefore, ⇒ A = 0°

(iv) (C) is correct.

Substitute the of tan 30° in the given equationtan

30° = 1/√3

2tan30°/1-tan²30° = 2(1/√3)/1-(1/√3)²

= (2/√3)/(1-1/3) = (2/√3)/(2/3) = √3 = tan 60°

The value of the given equation is equivalent to tan 60°.

3. If tan (A + B) = √3 and tan (A – B) = 1/√3 ,0° < A + B ≤ 90°; A > B, find A and B.

Solution: tan

(A + B) = √3

Since √3 =

tan 60°

Now substitute the degree value

⇒ tan (A + B) = tan

60°(A + B) = 60° …

(i)

The above equation is assumed as

equation (i)tan (A – B) = 1/√3 Since

1/√3 = tan 30°

Now substitute the degree value

⇒ tan (A – B) = tan 30°

(A – B) = 30° … equation (ii) Now add

the equation (i) and (ii), we getA + B

+ A – B = 60° + 30° Cancel the terms

B2A = 90°

A= 45°

Now, substitute the value of A in equation (i) to find the value of

B45° + B = 60°

B = 60° – 45°

B = 15°

Therefore A = 45° and B = 15°

4. State whether the following are true or false. Justify your answer.

(i) sin (A + B) = sin A + sin B.

(ii) The value of sin θ increases as θ increases.

(iii) The value of cos θ increases as θ increases.

(iv) sin θ = cos θ for all values of θ.

(v) cot A is not defined for A = 0°.

Solution: (i)

False.

Let us take A = 30° and B = 60°, then

Substitute the values in the sin (A + B) formula, we

getsin (A + B) = sin (30° + 60°) = sin 90° = 1 and,

sin A + sin B = sin 30° + sin 60°

= 1/2 + √3/2 = 1+√3/2

Since the values obtained are not equal, the solution is false.

(ii) True. Justificat ion:

According to the values obtained as per the unit circle, the values of sin

are:sin 0° = 0 sin 30° = 1/2 sin 45° = 1/√2 sin 60° = √3/2sin

90° = 1

Thus the value of sin θ increases as θ increases. Hence, the statement is true

(iii) False.

According to the values obtained as per the unit circle, the values of cos

are:cos 0° = 1 cos 30° = √3/2cos 45° = 1/√2 cos 60° = 1/2 cos 90° = 0

Thus, the value of cos θ decreases as θ increases. So, the statement given above is false.

(iv) False sin θ = cos θ, when a right triangle has 2 angles of (π/4). Therefore, the above statement is false.

(v) True.

Since cot function is the reciprocal of the tan function, it is also written

as:cot A = cos A/sin A Now substitute A = 0° cot 0° = cos 0°/sin 0° =

1/0 = undefined.Hence, it is true

EXERCISE 8.3

1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Solution:

To convert the given trigonometric ratios in terms of cot functions, use trigonometric formulasWe know that, $cosec^2A - cot^2A = 1$ $cosec^2A = 1 + cot^2A$

Since cosec function is the inverse of sin function, it is written as

$1/sin^2A = 1 + cot^2A$ Now, rearrange the terms, it becomes$sin^2A = 1/(1+cot^2A)$

Now, take square roots on both sides, we getsin A = $\pm 1/(\sqrt{(1+cot^2A)}$

The above equation defines the sin function in terms of cot function Now, to express sec function in terms of cot function, use this formula$sin^2A = 1/ (1+cot^2A)$ Now, represent the sin function as cos function$1 - cos^2A = 1/ (1+cot^2A)$ Rearrange the terms, $cos^2A = 1 - 1/(1+cot^2A)$

$\Rightarrow cos^2A = (1-1+cot^2A)/(1+cot^2A)$

Since sec function is the inverse of cos function,

$\Rightarrow 1/sec^2A = cot^2A/(1+cot^2A)$

Take the reciprocal and square roots on both sides, we get

$\Rightarrow sec A = \pm\sqrt{(1+cot^2A)}/cotA$

Now, to express tan function in terms of cot functiontan A = sin A/cos A and cot A = cos A/sin A

Since cot function is the inverse of tan function, it is rewritten astan A = 1/cot A

2. Write all the other trigonometric ratios of ∠A in terms of sec A.

Solution:

Cos A function in terms of sec A:

sec A = 1/cos A ⇒ cos A = 1/sec

A sec A function in terms of sec

A:

$\cos^2 A + \sin^2 A = 1$ Rearrange the

terms$\sin^2 A = 1 - \cos^2 A$ $\sin^2 A = 1 -$

$(1/\sec^2 A)$ $\sin^2 A = (\sec^2 A - 1)/\sec^2 A$

$\sin A = \pm \sqrt{(\sec^2 A - 1)}/\sec A$ cosec

A function in terms of sec A:

sin A = 1/cosec A ⇒cosec A = 1/sin A

cosec A = $\pm \sec A/\sqrt{(\sec^2 A - 1)}$ Now,

tan A function in terms of sec A:

$\sec^2 A - \tan^2 A = 1$

Rearrange the terms

⇒ $\tan^2 A = \sec^2 A$

$- 1$ $\tan A = \sqrt{(\sec^2 A - 1)}$ cot A

function in terms of sec A:

tan A = 1/cot A ⇒ cot A

= 1/tan A cot A =

$\pm 1/\sqrt{(\sec^2 A - 1)}$

3. Evaluate:

(i) $(\sin^2 63° + \sin^2 27°)/(\cos^2 17° + \cos^2 73°)$
(ii) sin 25° cos 65° + cos 25° sin 65°

Solution:

(i) $(\sin^2 63° + \sin^2 27°)/(\cos^2 17° + \cos^2 73°)$

To simplify this, convert some of the sin functions into cos functions and cos function into sin function and itbecomes,

$= [\sin^2(90°-27°) + \sin^2 27°] / [\cos^2(90°-73°) + \cos^2 73°)]$

$= (\cos^2 27° + \sin^2 27°)/(\sin^2 27° + \cos^2 73°)$

= 1/1 =1 (since $\sin^2 A + \cos^2 A = 1$)

Therefore, $(\sin^2 63° + \sin^2 27°)/(\cos^2 17° + \cos^2 73°)$

(ii) sin 25° cos 65° + cos 25° sin 65°

To simplify this, convert some of the sin functions into cos functions and cos function into sin function and itbecomes,

= sin(90°-25°) cos 65° + cos (90°-65°) sin 65°

= cos 65° cos 65° + sin 65° sin 65°

= $\cos^2 65° + \sin^2 65°$ = 1 (since $\sin^2 A + \cos^2 A$

= 1) Therefore, sin 25° cos 65° + cos 25°

sin 65° = 1

4. Choose the correct option. Justify your choice.
(i) 9 $\sec^2 A$ – 9 $\tan^2 A$ =
(A) 1 (B) 9 (C) 8 (D) 0
(ii) (1 + tan θ + sec θ) (1 + cot θ – cosec θ)
(A) 0 (B) 1 (C) 2 (D) – 1
(iii) (sec A + tan A) (1 – sin A) =
(A) sec A (B) sin A (C) cosec A (D) cos A

(iv) 1+$\tan^2 A$/1+$\cot^2 A$ =

(A) $\sec^2 A$ (B) -1 (C) $\cot^2 A$ (D) $\tan^2 A$

Solution: (i)

(B) is

correct.

Justification:

Take 9 outside, and it becomes9

$\sec^2 A$ - 9 $\tan^2 A$

= 9 ($\sec^2 A$ - $\tan^2 A$)

= 9×1 = 9 (∵ sec2 A - tan2 A = 1)

Therefore, 9 $\sec^2 A$ - 9 $\tan^2 A$ = 9

(ii) (C) is

correct

Justification:

(1 + tan θ + sec θ) (1 + cot θ - cosec

θ) We know that, tan θ = sin θ/cos

θsec θ = 1/ cos θ

cot θ = cos θ/sin

θcosec θ = 1/sin

Now, substitute the above values in the given problem, we get

= (1 + sin θ/cos θ + 1/ cos θ) (1 + cos θ/sin θ – 1/sin θ)

Simplify the above equation,

= (cos θ +sin θ+1)/cos θ × (sin θ+cos θ-1)/sin θ

= $(\cos\theta+\sin\theta)^2-1^2/(\cos\theta \sin\theta)$

= $(\cos^2\theta + \sin^2\theta + 2\cos\theta \sin\theta -1)/(\cos\theta \sin\theta)$

= (1+ 2cos θ sin θ -1)/(cos θ sin θ) (Since $\cos^2\theta + \sin^2\theta = 1$)

= (2cos θ sin θ)/(cos θ sin θ) = 2

Therefore, (1 + tan θ + sec θ) (1 + cot θ – cosec θ) =2

(iii) (D) is

correct.

Justification:

We know

that, Sec

A= 1/cos A

Tan A = sin A / cos A

Now, substitute the above values in the given problem, we get

(secA + tanA) (1 – sinA)

= (1/cos A + sin A/cos A) (1 – sinA)

= (1+sin A/cos A) (1 – sinA)

= $(1 - \sin^2 A)/\cos A$

= $\cos^2 A/\cos A = \cos A$

Therefore, (secA + tanA) (1 – sinA) = cos A

(iv) (D) is correct.

We know

that, $\tan^2 A$

= $1/\cot^2 A$

Now, substitute this in the given problem, we get

$1+\tan^2 A/1+\cot^2 A$

= $(1+1/\cot^2 A)/1+\cot^2 A$

= $(\cot^2 A+1/\cot^2 A)\times(1/1+\cot^2 A)$

= $1/\cot^2 A = \tan^2 A$

So, $1+\tan^2 A/1+\cot^2 A = \tan^2 A$

5. Prove the following identities, where the angles involved are acute angles for which theexpressions are defined.

(i) $(\text{cosec } \theta - \cot \theta)^2 = (1-\cos \theta)/(1+\cos \theta)$

(ii) $\cos A/(1+\sin A) + (1+\sin A)/\cos A = 2 \sec A$

(iii) $\tan \theta/(1-\cot \theta) + \cot \theta/(1-\tan \theta) = 1 + \sec \theta \text{ cosec } \theta$

(iv) $(1 + \sec A)/\sec A = \sin^2 A/(1-\cos A)$

[Hint : Simplify LHS and RHS separately]

(v) $(\cos A - \sin A+1)/(\cos A + \sin A - 1) = \text{cosec } A + \cot A$, using the identity $\text{cosec}^2 A = 1+\cot^2 A$.

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $(\sin \theta - 2\sin^3\theta)/(2\cos^3\theta - \cos \theta) = \tan \theta$
(viii) $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7+\tan^2 A+\cot^2 A$
(ix) $(\text{cosec } A - \sin A)(\sec A - \cos A) = 1/(\tan A+\cot A)$
[Hint : Simplify LHS and RHS separately] (x) $(1+\tan^2 A/1+\cot^2 A) = (1-\tan A/1-\cot A)^2 = \tan^2 A$

Solution:

(i) $(\text{cosec } \theta - \cot \theta)^2 = (1-\cos \theta)/(1+\cos \theta)$

To prove this, first take the Left-Hand side (L.H.S) of the given equation, to prove the Right Hand Side(R.H.S)

L.H.S. = $(\text{cosec } \theta - \cot \theta)^2$

The above equation is in the form of $(a-b)^2$, and

expand it Since $(a-b)^2 = a^2 + b^2 - 2ab$ Here

$a = \text{cosec } \theta$ and $b = \cot \theta$

$= (\text{cosec}^2\theta + \cot^2\theta - 2\text{cosec } \theta \cot \theta)$

Now, apply the corresponding inverse functions and equivalent ratios to simplify

$= (1/\sin^2\theta + \cos^2\theta/\sin^2\theta - 2\cos \theta/\sin^2\theta)$

$= (1 + \cos^2\theta - 2\cos \theta)/(1 - \cos^2\theta)$

$= (1-\cos \theta)^2/(1 - \cos\theta)(1+\cos \theta)$

$= (1-\cos \theta)/(1+\cos \theta)$ = R.H.S.

Therefore, $(\text{cosec } \theta - \cot \theta)^2 = (1-\cos \theta)/(1+\cos$

$\theta)$ Hence proved.

(ii) (cos A/(1+sin A)) + ((1+sin A)/cos A) = 2 sec ANow, take the L.H.S of the given equation.

L.H.S. = (cos A/(1+sin A)) + ((1+sin A)/cos A)

= [$\cos^2$A + (1+sin A)^2]/(1+sin A)cos A

= ($\cos^2$A + $\sin^2$A + 1 + 2sin A)/(1+sin A) cos A Since $\cos^2$A + $\sin^2$A = 1, we can

write it as

= (1 + 1 + 2sin A)/(1+sin A) cos A

= (2+ 2sin A)/(1+sin A)cos A

= 2(1+sin A)/(1+sin A)cos A

= 2/cos A = 2 sec A = R.H.S. L.H.S.

= R.H.S.

(cos A/(1+sin A)) + ((1+sin A)/cos A) = 2 sec AHence proved.

(iii) tan θ/(1-cot θ) + cot θ/(1-tan θ) = 1 + sec θ cosec θ

L.H.S. = tan θ/(1-cot θ) + cot θ/(1-tan θ) We know that tan θ =sin θ/cos θcot θ = cos θ/sin θ

Now, substitute it in the given equation, to convert it in a simplified form

= [(sin θ/cos θ)/1-(cos θ/sin θ)] + [(cos θ/sin θ)/1-(sin θ/cos θ)]

= [(sin θ/cos θ)/(sin θ-cos θ)/sin θ] + [(cos θ/sin θ)/(cos θ-sin θ)/cos θ]

= $\sin^2$θ/[cos θ(sin θ-cos θ)] + $\cos^2$θ/[sin θ(cos θ-sin θ)] = $\sin^2$θ/[cos θ(sin θ-cos θ)] – $\cos^2$θ/[sin θ(sin θ-cos θ)]

= 1/(sin θ-cos θ) [($\sin^2$θ/cos θ) – ($\cos^2$θ/sin θ)]

= 1/(sin θ-cos θ) × [($\sin^3$θ – $\cos^3$θ)/sin θ cos θ]

= [(sin θ-cos θ)($\sin^2$θ+$\cos^2$θ+sin θ cos θ)]/[(sin θ-cos θ)sin θ cos θ]

= (1 + sin θ cos θ)/sin θ cos θ

= 1/sin θ cos θ + 1 = 1 + sec θ cosec θ =

R.H.S.Therefore,

L.H.S.

= R.H.S.

Hence proved

(iv) (1 + sec A)/sec A = $\sin^2$A/(1-

cos A) First find the simplified form

of L.H.S

L.H.S. = (1 + sec A)/sec A

Since secant function is the inverse function of cos function and it is written as

= (1 + 1/cos A)/1/cos A

= (cos A + 1)/cos A/1/cos A

Therefore, (1 + sec A)/sec A = cos A + 1 R.H.S.

= $\sin^2$A/(1-cos A)

We know that $\sin^2$A = (1 – $\cos^2$A), we get

= (1 – $\cos^2$A)/(1-cos A)

= (1-cos A)(1+cos A)/(1-cos A)

Therefore, $\sin^2$A/(1-cos A)= cos A + 1

L.H.S. = R.H.S.

Hence proved

(v) (cos A–sin A+1)/(cos A+sin A–1) = cosec A + cot A, using the identity cosec^2A = 1+$\cot^2$A. With the help of identity function, cosec^2A = 1+$\cot^2$A, let us prove the above equation.

L.H.S. = (cos A–sin A+1)/(cos A+sin A–1)

Divide the numerator and denominator by sin A, we get

= (cos A–sin A+1)/sin A/(cos A+sin A–1)/sin A

We know that cos A/sin A = cot A and 1/sin A = cosec A

= (cot A – 1 + cosec A)/(cot A+ 1 – cosec A)

= (cot A – cosec^2A + $\cot^2$A + cosec A)/(cot A+ 1 – cosec A) (using cosec^2A – $\cot^2$A = 1

= [(cot A + cosec A) – (cosec^2A – $\cot^2$A)]/(cot A+ 1 – cosec A)

= [(cot A + cosec A) – (cosec A + cot A)(cosec A – cot A)]/(1 – cosec A + cot A)

= (cot A + cosec A)(1 – cosec A + cot A)/(1 – cosec A + cot A) =

cot A + cosec A = R.H.S.

Therefore, (cos A–sin A+1)/(cos A+sin A–1) = cosec A + cot A

Hence Proved

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

L.H.S = $\sqrt{\frac{1+\sin A}{1-\sin A}}$

First divide the numerator and denominator of L.H.S. by cos A,

$$= \sqrt{\frac{\frac{1}{\cos A}+\frac{\sin A}{\cos A}}{\frac{1}{\cos A}-\frac{\sin A}{\cos A}}}$$

We know that 1/cos A = sec A and sin A/ cos A = tan A and it becomes,

= √(sec A+ tan A)/(sec A-tan

A) Now using rationalization,

we get

$$= \sqrt{\frac{\sec A+\tan A}{\sec A-\tan A}} \times \sqrt{\frac{\sec A+\tan A}{\sec A+\tan A}}$$

$$= \sqrt{\frac{(\sec A + \tan A)^2}{\sec^2 A - \tan^2 A}}$$

= (sec A + tan A)/1

= sec A + tan A =

R.H.SHence proved

(vii) (sin θ – 2sin^{3}θ)/(2cos^{3}θ-cos θ) = tan θ

L.H.S. = (sin θ – 2sin^{3}θ)/(2cos^{3}θ – cos θ)

Take sin θ as in numerator and cos θ in denominator as outside, it becomes

= [sin θ(1 – 2sin^{2}θ)]/[cos θ(2cos^{2}θ-

1)] We know that sin^{2}θ = 1-cos^{2}θ

= sin θ[1 – 2(1-cos^{2}θ)]/[cos θ(2cos^{2}θ -1)]

= [sin θ(2cos²θ -1)]/[cos θ(2cos²θ -1)]

= tan θ = R.H.S.

Hence proved

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7+\tan^2 A+\cot^2 A$

L.H.S. = $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$ It is of the form $(a+b)^2$, expand it

$(a+b)^2 = a^2 + b^2 + 2ab$

= $(\sin^2 A + \operatorname{cosec}^2 A + 2 \sin A \operatorname{cosec} A) + (\cos^2 A + \sec^2 A + 2 \cos A \sec A)$

= $(\sin^2 A + \cos^2 A) + 2 \sin A(1/\sin A) + 2 \cos A(1/\cos A) + 1 + \tan^2 A + 1 + \cot^2 A$

= $1 + 2 + 2 + 2 + \tan^2 A + \cot^2 A$

= $7+\tan^2 A+\cot^2 A$ = R.H.S.

Therefore, $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7+\tan^2 A+\cot^2 A$

Hence proved.

(ix) (cosec A – sin A)(sec A – cos A) = 1/(tan A+cotA)

First, find the simplified form of L.H.S

L.H.S. = (cosec A – sin A)(sec A – cos A)

Now, substitute the inverse and equivalent trigonometric ratio forms

= (1/sin A – sin A)(1/cos A – cos A)

= $[(1-\sin^2 A)/\sin A][(1-\cos^2 A)/\cos A]$

= $(\cos^2 A/\sin A)\times(\sin^2 A/\cos A)$

= cos A sin A

Now, simplify the R.H.S

R.H.S. = 1/(tan A+cotA)

= 1/(sin A/cos A +cos A/sin A)

= $1/[(\sin^2 A+\cos^2 A)/\sin A \cos A]$

= cos A sin A

L.H.S. = R.H.S.

(cosec A – sin A)(sec A – cos A) = 1/(tan A+cotA)

Hence proved

(x) $(1+\tan^2 A/1+\cot^2 A) = (1-\tan A/1-\cot A)^2 = \tan^2 A$

L.H.S. = $(1+\tan^2 A/1+\cot^2 A)$

Since cot function is the inverse of tan function,

= $(1+\tan^2 A/1+1/\tan^2 A)$

= $1+\tan^2 A/[(1+\tan^2 A)/\tan^2 A]$

Now cancel the $1+\tan^2 A$ terms, we get

= $\tan^2 A$

$(1+\tan^2 A/1+\cot^2 A)$ =

$\tan^2 A$ Similarly, (1-

tan A/1-cot A)^2 =

$\tan^2 A$ Hence proved