CBSE Class 10

Mathematics

Previous Year Question Paper 2020

Series: JBB/5 **Code no. 30/5/3**

- Please check that this question paper contains **23** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **40** questions.
- **Please write down the Serial Number of the question in the answer-book before attempting it.**
- 15 minutes of time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer script during this period.

MATHEMATICS (Standard) - Theory

Time Allowed: **3** hours Maximum Marks: **80**

General Instructions:

Read the following instructions very carefully and strictly follow them :

1. Please check that this question paper contains **23** printed pages.
2. The question paper comprises of **four** sections – A, B, C and D. This question paper carries **40** questions. **All** questions are compulsory.
3. **Section A:** Question numbers **1** to **20** comprises of **20** questions of **one** mark each.
4. **Section B:** Question numbers 21 to **26** comprises of **6** questions of **two** mark each.

5. **Section C:** Question numbers **27** to **34** comprises of **8** questions of **three** mark each.

6. **Section D:** Question numbers **35** to **40** comprises of **6** questions of **four** mark each.

7. There is no overall choice in the paper. However, internal choice is provided in 2 questions of one mark, 2 questions of one mark, 2 questions of two marks, 3 questions of three marks and three questions of four marks. You have to attempt only one of the choices in such questions.

8. In addition to this, separate instructions are given with each section and question, wherever necessary.

9. Use of calculators is **not** permitted.

SECTION-A

1. The value(s) of k for which the quadratic equation $2x^2+kx+2=0$ has equal roots, is **1 Mark**

(A) 4

(B) ± 4

(C) -4

(D) 0

Ans: Given quadratic equation is $2x^2+kx+2=0$.

For equal roots, D =0

$\Rightarrow b^2-4ac=0$

Here, a=2, b=k, and c=2

$\Rightarrow k^2-4\times2\times2=0$

$\Rightarrow k^2-16=0$

$k^2=16$

$\therefore k=\pm 4$

Therefore, the values of k is ± 4.

Hence, option (B) is correct.

2. Which of the following is not an A.P.? 1 Mark

(A) -1.2,0.8, 2.8,…

(B) 3. $3+\sqrt{2}$. $3+2\sqrt{2}$. $3+3\sqrt{2}$

(C) $\frac{4}{3},\frac{7}{3},\frac{9}{3},\frac{12}{3},......$

(D) $\frac{-1}{5},\frac{-2}{5},\frac{-3}{5},......$

Ans: (A) We have,

$\Rightarrow a_2-a_1=a_3-a_2=2$

Hence, it is an A.P.

(B) We have,

$\Rightarrow a_2-a_1=a_3-a_2=\sqrt{2}$

Hence, it is an A.P.

(C) We have, $a_2-a_1=1$ and $a_3-a_2=\frac{2}{3}$.

So, $a_2-a_1 \neq a_3-a_2$.

Hence, it is not an A.P.

(D) We have,

$a_2-a_1=a_3-a_2=\frac{-1}{5}$

Hence, it is an A.P.

Therefore, option (C) is correct.

3. The radius of a sphere (in cm) whose volume is $12\pi cm^3$, is 1 Mark

(A) 3

(B) $3\sqrt{3}$

(C) $3^{\frac{2}{3}}$

(D) $\mathbf{3^{\frac{1}{3}}}$

Ans: Volume of a sphere $=\frac{4}{3}\pi r^3$

Given, Volume of the sphere $=12\pi r\ cm^3$

On equating we get,

$\Rightarrow \frac{4}{3}\pi r^3=12\pi r\ cm^3$

$\Rightarrow r^3=\frac{12\pi\times 3}{4\pi}cm^3$

$\Rightarrow r^3=9\ cm^3$

$\Rightarrow r^3=(3)^2\ cm^3$

$\Rightarrow r=(3)^{\frac{2}{3}}\ cm$

Therefore, radius of the sphere is $(3)^{\frac{2}{3}}\ cm$.

Hence, option (C) is correct.

4. The distance between the points (m, -n) and (-m, n) is **1 Mark**

(A) $\mathbf{\sqrt{m^2+n^2}}$

(B) m+n

(C) $\mathbf{2\sqrt{m^2+n^2}}$

(D) $\mathbf{\sqrt{2m^2+2n^2}}$

Ans: Let the points be A(m, -n) and B(-m, n).

From distance formula we get,

$\Rightarrow AB=\sqrt{(-m-m)^2+(n-(-n))^2}$

$\Rightarrow AB=\sqrt{(-2m)^2+(2n)^2}$

$\Rightarrow AB=\sqrt{4m^2+4n^2}$

$\Rightarrow AB=2\sqrt{m^2+n^2}$

Therefore, the distance between the points (m, -n) and (-m, n) is $2\sqrt{m^2+n^2}$.

Hence, option (C) is correct.

5. In Figuire-1, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle QPR=90^\circ$, then length of PQ is **1 Mark**

(A) 3 cm

(B) 4 cm

(C) 2 cm

(D) $2\sqrt{2}$ cm

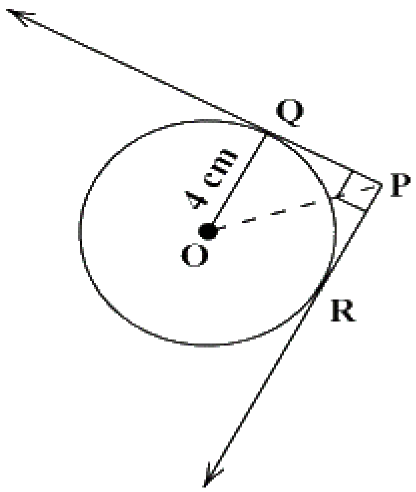


Figure-1

Ans: Given, $\angle QPR=90^\circ$

Since, the line from the centre of the circle bisects the angle between the tangents.

Therefore, $\angle OPQ=45^\circ$.

In ΔPOQ, we have

$$\Rightarrow \tan 45^\circ=\frac{OQ}{PQ}$$

$\Rightarrow 1=\frac{OQ}{PQ}$

$\Rightarrow PQ=OQ$

$\Rightarrow PQ=4\,cm$

Therefore, PQ is 4cm.

Hence, option (B) is correct.

6. On dividing a polynomial p(x) by x^2-4, quotient and remainder are found to be x and 3 respectively. The polynomial p(x) is 1 Mark

(A) $3x^2+x-12$

(B) x^3-4x+3

(C) x^2+3x-4

(D) x^3-4x-3

Ans: As dividend = (divisor × quotient)+remainder

$\Rightarrow p(x)\left[\left(x^2-4\right)\times x\right]+3$

$\Rightarrow p(x)=x^2-4x+3$

Therefore, polynomial p(x) is x^2-4x+3.

7. In Figure-2, DE || BC. If $\frac{AD}{DB}=\frac{3}{2}$ and AE = 2.7 cm, then EC is equal to

1 Mark

(A) 2.0 cm

(B) 1.8 cm

(C) 4.0 cm

(D) 2.7 cm

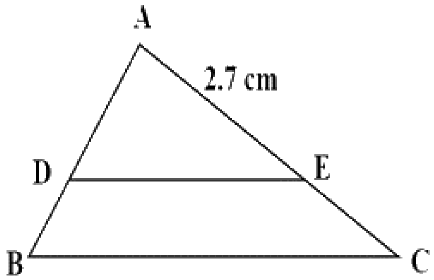


Figure-2

Ans: Given, DE || BC.

Therefore, $\frac{AD}{DB}=\frac{AE}{EC}$

$\Rightarrow \frac{3}{2}=\frac{2.7\text{ cm}}{EC}$

$\Rightarrow EC=\frac{2\times 2.7\text{ cm}}{3}$

$\Rightarrow$ EC = 1.8 cm

Therefore, EC is equal to 1.8 cm.

Hence, option (B) is correct.

8. The point on the x-axis which is equidistant from (-4, 0) and (10, 0) is

1 Mark

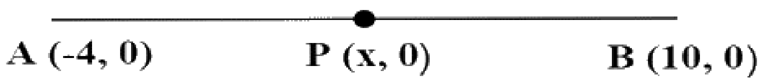
(A) (7, 0)

(B) (5, 0)

(C) (0, 0)

(D) (3, 0)

Ans: Let the point on the x-axis is P(x, 0) which is equidistant from A(-4, 0) and B(10, 0).



We have, AP = PB

Using distance formula, we get

$$\Rightarrow \sqrt{(x-(-4))^2+(0-0)^2}=\sqrt{(10-x)^2+(0-0)^2}$$

Squaring both the sides,

$$\Rightarrow (x+4)^2=(10-x)^2$$

$$\Rightarrow x^2+8x+16=100-20x+x^2$$

$$\Rightarrow 8x+20x=100-16$$

$$\Rightarrow 28x=84$$

$$\Rightarrow x=3$$

Therefore, (3, 0) is equidistant from (-4, 0) and (10, 0).

Hence, option (D) is correct.

Or

The centre of a circle whose end points of a diameter are (-6, 3) and (6, 4) is **1 Mark**

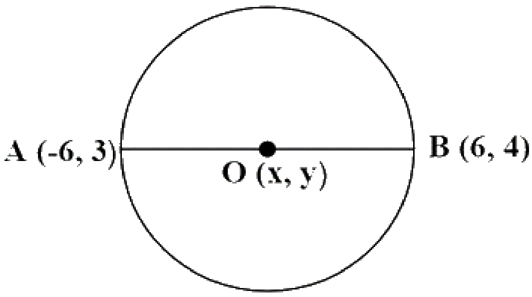
(A) (8, -1)

(B) (4, 7)

(C) $\left(0,\frac{7}{2}\right)$

(D) $\left(4,\frac{7}{2}\right)$

Ans: Let centre be O(x, y) and end points of the diameter be A(-6, 3) and B(6, 4).



Since, centre is the midpoint of diameter. So,

$$\Rightarrow x=\frac{-6+6}{2} \text{ and } y=\frac{3+4}{2}$$

$\Rightarrow x=0 \text{ and } y=\frac{7}{2}$

Therefore, centre of the circle is $\left(0,\frac{7}{2}\right)$.

Hence, option (C) is correct.

9. The pair of linear equations **1 Mark**

$\mathbf{\frac{3x}{2}+\frac{5y}{3}=7 \text{ and } 9x+10y=14}$

(A) consistent

(B) inconsistent

(C) consistent with one solution

(D) consistent with many solutions

Ans: Given $\frac{3x}{2}+\frac{5y}{3}=7$ and $9x+10y=14$

Here, $a_1=\frac{3}{2}, b_1=\frac{5}{3}, c_1=7, a_1=9, b_1=10$ and $c_1=14$.

$$\Rightarrow \frac{a_1}{a_2}=\frac{\frac{3}{2}}{9}, \frac{b_1}{b_2}=\frac{\frac{5}{3}}{10} \text{ and } \frac{c_1}{c_2}=\frac{7}{14}$$

$$\Rightarrow \frac{a_1}{a_2}=\frac{1}{9}, \frac{b_1}{b_2}=\frac{1}{6} \text{ and } \frac{c_1}{c_2}=\frac{1}{2}$$

$$\therefore \frac{a_1}{a_2}=\frac{b_1}{b_1} 1 \frac{c_1}{c_1}$$

Therefore, these lines equation intersect each other at one point and only have one possible solution.

Hence, the pair of linear equation is inconsistent.

Hence, option (B) is correct.

10. In Figure-3, PQ is tangent to the circle with centre at O, at the point B.

If $\angle AOB = 100^o$, then $\angle ABP$ is equal to **1 Mark**

(A) 50o

(B) 40o

(C) 60o

(D) 80o

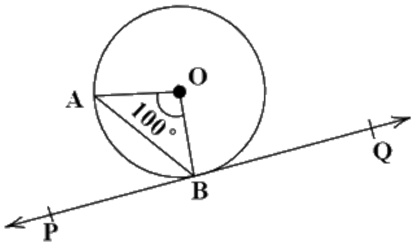


Figure-3

Ans: In ΔAOB, AO=OB

$\therefore \angle OAB = \angle OBA = 40^o$

Since PQ is tangent at the point B, $\angle OBP = 90^o$

$\Rightarrow \angle OBP = \angle OBA + \angle ABP$

$\Rightarrow 90^o = 40^o + \angle ABP$

$\Rightarrow \angle ABP = 90^o - 40^o$

$\Rightarrow ABP = 50^o$

Therefore, $\angle ABP$ is equal to 50^o.

Hence, option (A) is correct.

Fill in the blanks in question numbers 11 to 15.

11. Simplest form of $\frac{1+\tan^2 A}{1+\cot^2 A}$ is __________. **1 Mark**

Ans: $\frac{1+\tan^2 A}{1+\cot^2 A}$ can be simplified as

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$\Rightarrow \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$

$\Rightarrow \tan^2 A$

12. If the probability of an event E happening is 0.023, then $P(\overline{E})$ = _________. 1 Mark

Ans: Given, $P(E) = 0.023$.

As, $P(E) + P(\overline{E}) = 1$

$\Rightarrow 0.023 + P(\overline{E}) = 1$

$\Rightarrow P(\overline{E}) = 1 - 0.023$

$\Rightarrow P(\overline{E}) = 0.977$

Therefore. $P(\overline{E}) = 0.977$

13. All concentric circles _________ to each other. 1 Mark

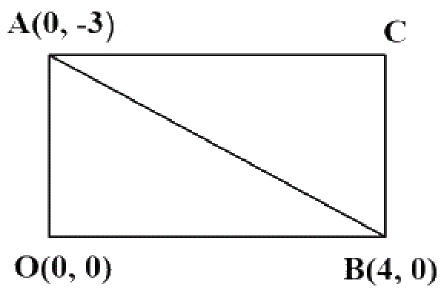
Ans: All concentric circles are similar to each other.

14. The probability of an event that is sure to happen, is _________. 1 Mark

Ans: The probability of an event that is sure to happen is 1.

15. AOBC is a rectangle whose three vertices are A(0, -3), O(0. 0) and B(4, 0). The length of its diagonal is _________. 1 Mark

Ans:



Length of diagonal AB $=\sqrt{(4-0)^2+(0-(-3))^2}$

$=\sqrt{(25)^2+(3)^2}$

$=\sqrt{25}$

$= 5$

Therefore, length of diagonal is 5 units.

Answer the following question numbers 16 to 20.

16. Write the value of $\sin^2 30^\circ+\cos^2 30^\circ$. **1 Mark**

Ans: As, $\sin^2\theta+\cos^2\theta=1$

$\therefore \sin^2 30^\circ+\cos^2 30^\circ=1$

17. Form a quadratic polynomial, the sum and product of whose zeroes are (-3) and 2 respectively. **1 Mark**

Ans: Given, sum of zeroes =-3 and product of zeroes =2

The quadratic equation is given by

x2-(sum of zeroes)x+(product of zeroes) = 0

x2-(-3)x+(2)=0

x2+3x+2=0

Or

Can (x2-1) be a remainder while dividing x^4-3x+5x-9 by (x^3+3)?

Justify your answer with reasons. **1 Mark**

Ans: On dividing x^4-3x+5x-9 by (x^3+3) we get (5x+9) as a remainder.

$$
\begin{array}{r|l}
 & x^2-6 \\
x^2+3 & x^4-3x^2+5x-9 \\
 & \underline{x^4+3x^2} \\
 & -6x^2+5x-9 \\
 & \underline{-6x^2-18} \\
 & 5x+9
\end{array}
$$

Therefore, $\left(x^2-1\right)$ can't be a remainder while dividing x^4-3x^2+5x-9 by $\left(x^2+3\right)$

.

18. Find the sum of the first 100 natural numbers. **1 Mark**

Ans: Let, the sum be $s_{100}=1+2+3+...+100$

Here, a=1, d=1, n=100 and l =100

As, $s_n=\frac{n}{2}\left(a+l\right)$

$\Rightarrow s_{100}=\frac{100}{2}\left(1+100\right)$

$\Rightarrow s_{100}=5050$

Therefore, the sum of the first 100 natural numbers is 5050.

19. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other. **1 Mark**

Ans: Let the other number be x.

Product of number $=LCM\times HCF$

$\Rightarrow 26\times x=182\times 13$

$\Rightarrow x=91$

Therefore, other number is 91.

20. In Figure-4, the angle of elevation of the top of a tower from a point C on the ground, which is 30 m away from the foot of the tower, is 30o. Find the height of the tower. **1 Mark**

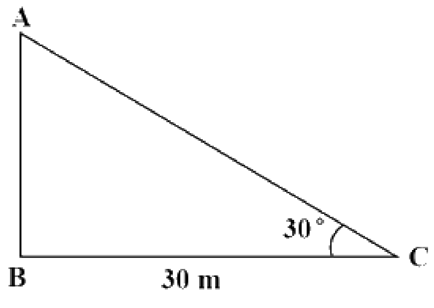


Figure-4

Ans: Let the tower be AB.

Since the tower is vertical, therefore $\angle ABC = 90^{o}$.

In ΔABC, $tab30^{o} = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{30}$

$\Rightarrow AB = 10\sqrt{3}$

Therefore, height of the tower is $10\sqrt{3}$.

SECTION-B

Question numbers 21 to 26 carry 2 marks each.

21. A cone and a cylinder have the same radii but the height of the cone is 3 times that of the cylinder. Find the ratio of their volumes. 2 Marks

Ans: Let radius of cone = radius of cylinder =r and height of cylinder be h, then height of cone will become 3h.

Volume of a cone $= \frac{1}{3}\pi r^2 H$

$\Rightarrow$ Volume of given cone $= \frac{1}{3}\pi r^2 (3h)$

$= \pi r^2 h$

Volume of a cylinder = $\pi r^2 H$

$\Rightarrow$ Volume of given cylinder = $\pi r^2 h$

Therefore, $\frac{\text{Volume of the cone}}{\text{Volume of the cylinder}} = \frac{\pi r^2 h}{\pi r^2 h}$

$\Rightarrow \frac{\text{Volume of the cone}}{\text{Volume of the cylinder}} = \frac{1}{1}$

Therefore, ratio of volume of cone to volume of cylinder is 1:1.

22. In Figure-5, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB+CD=BC+AD$. 2 Marks

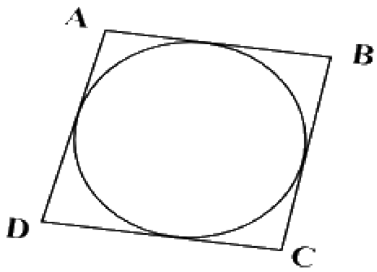


Figure-5

Ans: As we know, length of tangents drawn from an external point are equal. Therefore, we can write

$\Rightarrow AP=AS$ ---(1)

$\Rightarrow BP=BQ$ ---(2)

$\Rightarrow CR=CQ$ ---(3)

$\Rightarrow BR=DS$ ---(4)

On adding equation (1), (2), (3) and (4), we get

$\Rightarrow AP+BP+CR+DR=AS+BQ+CQ+DS$

$\Rightarrow (AP+BP)+(CR+DR)=(AS+DS)+(BQ+CQ)$

$\Rightarrow AB+CD=AD+BC$

Hence, proved that $AB+CD=AD+BC$

Or

In Figure-6, find the perimeter of ΔABC, if $AP=12$ cm. 2 Marks

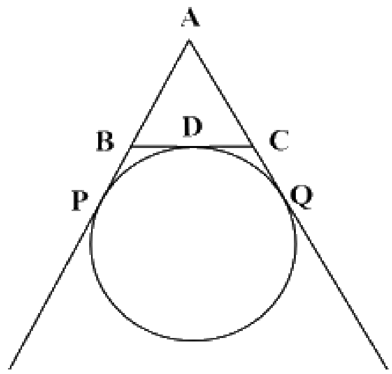


Figure-6

Ans: As we know, tangents drawn from an external point are equal.

Therefore, BD=BP , CD=CQ and AP=Q.

Perimeter of Δ ABC = AB+BC+CA
=AB+BD+CD+AC
=AP+ AQ
=2Ap
=2×12
=24

Therefore, perimeter of Δ ABC is 24 cm.

23.Find the mode of the following distribution:

Marks:	0-12	10-20	20-30	30-40	40-50	50-60
Number of students	4	6	7	12	5	6

2 Marks

Ans: Here, modal class is 30-40.

$$\text{Mode}=l+\frac{f_1-f_0}{2f_1-f_0-f_2}\times h$$

$$\Rightarrow \text{Mode}=30+\frac{12-7}{2\times(12)-7-5}\times 10$$

$$\Rightarrow \text{Mode}=34.16$$

Therefore, mode is 34. 16.

24. In Figure-7, if PQ || BC and PR || CD, prove that $\frac{QB}{AQ}=\frac{DR}{AR}$. 2 Marks

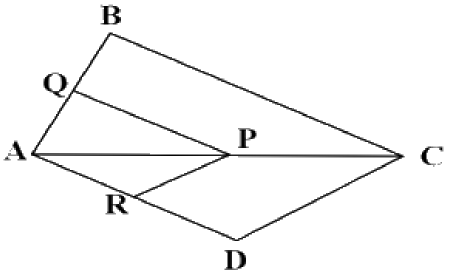


Figure-7

Ans: Since, PQ || BC in ΔABC

By basic proportionality theorem, we get

$$\Rightarrow \frac{AQ}{AB}=\frac{AP}{AC}---(1)$$

In ΔACD, PR || CD

By basic proportionality theorem, we get

$$\Rightarrow \frac{AP}{AC}=\frac{AR}{AD}---(2)$$

From (1) and (2), we get

$$\Rightarrow \frac{AB}{AQ}=\frac{AD}{AR}$$

$$\Rightarrow \frac{AQ+QB}{AQ}=\frac{AR+RD}{AR}$$

$$\Rightarrow 1+\frac{QB}{AQ}=1+\frac{RD}{AR}$$

$$\Rightarrow \frac{QB}{AQ}=\frac{RD}{AR}$$

Hence, proved that $\frac{QB}{AQ}=\frac{RD}{AR}$.

25. Show that $5+2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number. 2 Marks

Ans: Let $5+2\sqrt{7}$ is a rational number.

So, we can write

$$\Rightarrow 5+2\sqrt{7}=\frac{P}{q}$$

$$\Rightarrow \sqrt{7}=\frac{P}{2q}=\frac{5}{2}$$

p, q, 5 and 2 are integers. So, $\frac{P}{2q}-\frac{5}{2}$ is a rational number.

Therefore, $\sqrt{7}$ is also a rational number.

But $\sqrt{7}$ is given to be an irrational number.

This is a contradiction which raised due to our assumption that $5+2\sqrt{7}$ is a rational number.

Therefore, $5+2\sqrt{7}$ is an irrational number.

Or

Check whether 12^n can end with the digit 0 for any natural number n.
2 Marks

Ans: We can write, $12^n=(2^n \times 3)^n$.

If a number ends with 0 then it is divisible by 5. But, prime factorisation of 12^n does not contains 5.

Therefore, 12^n can't end with the digit 0 for any natural number n.

26. If A, B and C are interior angles of a ΔABC, then show that $\cos\left(\frac{B+C}{2}\right)=\sin\left(\frac{A}{2}\right)$. **2 Marks**

Ans: In $\Delta ABC, \angle A+\angle B+\angle C=180^o$ or A+B+C=180o.

B+C=180o-A

L.H.S. $=\cos\left(\frac{B+C}{2}\right)$

$=\cos\left(\frac{180^o-A}{2}\right)$

$=\cos\left(90^o-\frac{A}{2}\right)$

$=\sin\left(\frac{A}{2}\right)$

Therefore, $\cos\left(\frac{B+C}{2}\right)=\sin\left(\frac{A}{2}\right)$.

SECTION-C

Question number 27 to 34 carry 3 marks each.

27. Prove that: $\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=2$ **3 Marks**

Ans: On simplification we get,

$$\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=\left(\left(\sin^2\theta\right)^2-\left(\cos^2\theta\right)^2+1\right)\operatorname{cosec}^2\theta$$

$$\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=\left[\left(\sin^2\theta+\cos^2\theta\right)\left(\sin^2\theta+\cos^2\theta\right)+1\right]\operatorname{cosec}^2\theta$$

As $\sin^2\theta+\cos^2\theta=1$, we get

$$\Rightarrow\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=\left[\left(\sin^2\theta-\cos^2\theta\right)+1\right]\operatorname{cosec}^2\theta$$

$$\Rightarrow\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=\left[1-\cos^2\theta+\sin^2\theta\right]\operatorname{cosec}^2\theta$$

$$\Rightarrow\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2=\left[\sin^2\theta+\sin^2\theta\right]\operatorname{cosec}^2\theta$$

$$\Rightarrow\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=2\sin^2\theta\operatorname{cosec}^2\theta$$

$$\Rightarrow\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=2\sin^2\theta\times\frac{1}{\sin^2\theta}$$

$$\Rightarrow\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=2$$

Hence, proved that $\left(\sin^4\theta-\cos^4\theta+1\right)\operatorname{cosec}^2\theta=2$.

28. Find the sum: (-5)+(-8)+(-11)+…+(-230) **3 Marks**

Ans: a1**=-5**, a2=-8, a3=-11

$\Rightarrow a_2-a_1=a_3-a_2=-3$

It is an A.P., in which first term is -5, common difference is -8 and last term is -230.

$\therefore a_1=-5d,\ d=-3$ and $l=-230$

As $l=a_1+(n-1)d$ where n is number of terms in the A.P.

From this we get,

$\Rightarrow$-230=-5+(n-1)(-3)

$\Rightarrow$-230+5-3=3n+3

$\Rightarrow$-230+3-3=-3n

$\Rightarrow$3n=228

$\Rightarrow$n=76

As we know, sum of the series $(s_n)=\frac{n}{2}(a+l)$

$\Rightarrow(s_n)=\frac{76}{2}(-5-230)$

$\Rightarrow(s_n)=\frac{76}{2}\times(-235)$

$\Rightarrow(s_n)=-8930$

Therefore, the sum of he given series is -8930.

29. Construct a ΔABC with sides BC=6 cm, AB=5 cm and $\angle ABC=60^{o}$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC. **3 Marks**

Ans: Construction of ΔABC:

(1) Draw a line segment AB of length 5 cm.

(2) Draw a line segment BC by making an angle of 60o from point B.

(3) Join A and C to get the required ΔABC.

Now, construction of ΔA'BC' whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC are as follows:

(1) Draw a ΔABC with sides BC= 6cm , AB = 5 cm and $\angle ABC=60^{o}$.

(2) On opposite side of vertex A, draw a ray BX making an acute angle with

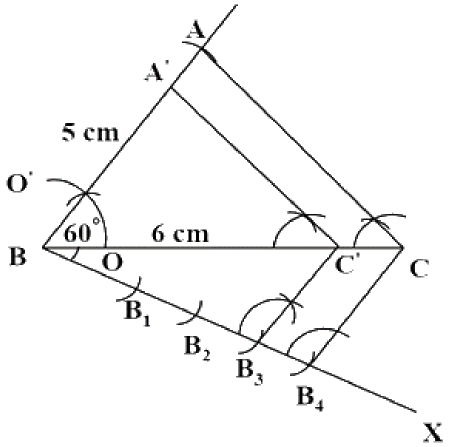
BC.

(3) On the line segment BX, locate four points B1,B2,B3 and B4.

(4) Join B4 and C. Draw a line through B3 parallel to B4C intersecting BC at C

(5) Draw a line parallel to AC through C intersecting AB at A.

Hence, we obtained the required $\Delta A'BC'$.



Or

Draw a circle of radius 3.5 cm . Take a point P outside the circle at a distance of 7 cm from the centre of the circle and construct a pair of tangents to the circle from that point. 3 Marks

Ans: Construction:

(1) Draw a line OP=6 cm.

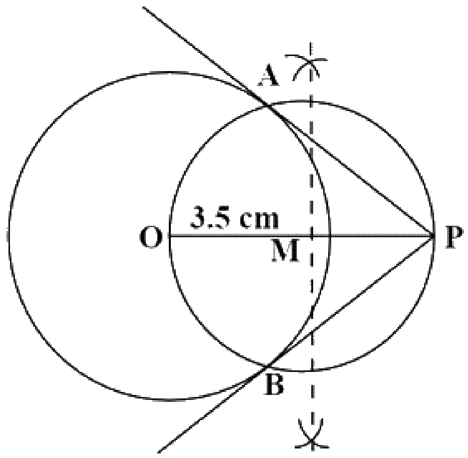
(2) Draw a circle of radius 3.5 cm with O as centre.

(3) Draw a perpendicular bisector of OP that cuts OP at M.

(4) With M as a centre and MP (or OP) as radius, draw a circle which intersects the first circle at A and B.

(5) Join PA and PB.

PA and PB are the required tangents.



30. In Figure-8, ABCD is a parallelogram. A semicircle with a centre O and the diameter AB has been drawn and it passes through D. If AB=12 cm and $OD \perp AB$, then find the area of the shaded region. (Use π=3.14)

3 Marks

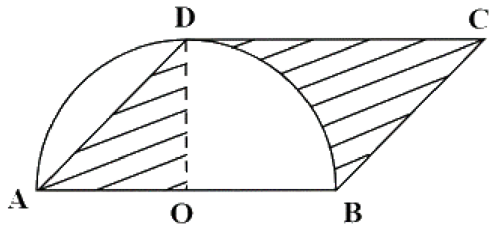


Figure-8

Ans: Given, Diameter = AB = 12 cm

AO=OB=OD=6 cm

$\Rightarrow$ area of OBD = $\frac{1}{4}\times\pi\times(6)^2$

$=28.26\,cm^2$

Area of parallelogram ABCD = base× height

= 12×6

=72 cm2

Area of shaded region = area of ABCD – area of OBD

=(72-28.26) cm2

= 43.74 cm2

Therefore, area of shaded region is 43.74 cm2.

31. Read the following passage and answer the questions given at the end:

Diwali Fair A game in a booth at a Diwali Fair involves using a spinner first. Then, if the spinner stops on an even number, the player is allowed to pick a marble from a bag. The spinner and the marbles in the bag are represented in Figure-9.Prizes are given, when a black marble is picked. Shweta plays the game once.

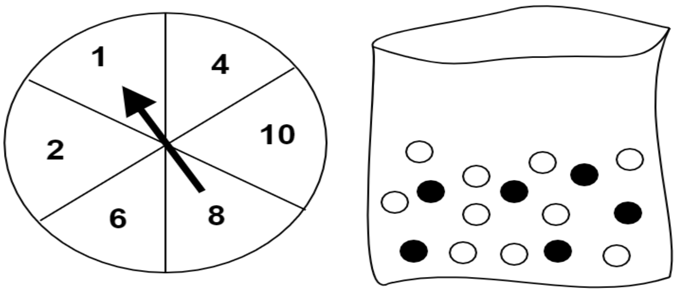


Figure 9

(i) What is the probability that she will be allowed to pick a marble from the bag?

(ii) Suppose she is allowed to pick a marble from the bag, what is the probability of getting a prize, when it is given that the bag contains 20 balls out of which 6 are black? 3 Marks

Ans: The player is allowed to pick a marble from a bag if the spinner stops on an even number.

So, the favourable outcomes are 2, 4, 6, 8 and 10.

Number of favourable outcomes =5

Total number of outcomes =6

(i) The probability that she will be allowed to pick a marble from the bag

$$=\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$=\frac{5}{6}$$

(ii) The bag contains 20 balls out of which 6 are black and prizes are given, when a black marble is picked.

Number of favourable outcomes =6

Total number of outcomes =20

The probability of getting a prize $=\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

$=\frac{6}{20}$

$=\frac{3}{10}$

32. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

3 Marks

Ans: Let the numerator be x and denominator be y. So, fraction is $\frac{x}{y}$.

Given, when 1 is subtracted from the numerator the fraction becomes $\frac{1}{3}$ i.e.,

$\frac{x-1}{y}=\frac{1}{3}$

$\Rightarrow 3(x-1)y$

$\Rightarrow 3x-3=y$

$\Rightarrow 3x-y=3$ ---(1)

Also given, the fraction becomes $\frac{1}{4}$ when 8 is added to its denominator i.e.,

$\frac{x}{y+8}=\frac{1}{4}$

$\Rightarrow 4x=y+8$

$\Rightarrow y=4x-8$---(1)

Putting (2) in (1), we get

3x+4x+8=3

-x=-5

$x=5$

Putting the value of x in (1), we get

$y=4\times5-8$

$y=4\times5-8$

$y=12$

Therefore, the fraction is $\frac{x}{y}$ i.e., $\frac{5}{12}$.

Or

The present age of a father is three years more than three times the age of his son. Three years hence the father's age will be 10 years more than twice the age of the son. Determine their present ages. 3 Marks

Ans: Let the son's age be x.

Given, age of the father is three years more than three times the age of his son.

Therefore, father's age is $3x+3$.

After three years,

Age of the son $=x+3$

Age of the father $=3x+3+3$ ---(1)

But, according to question, after three years the father's age will be 10 years more than twice the age of the son.

Age of the father $10+2(x+3)$ ---(2)

From (1) and (2), we get

$\Rightarrow 3x+6=10+2(x+3)$

$\Rightarrow 3x+6=10+2x+6$

$\Rightarrow x=10$

So, the present age of son is 10 years.

Present age of father $=3\times10+3$

$=33$ years

Therefore, the present age of son is 10 years and father is 33 years.

33. Find the ratio in which the y-axis divides the line segment joining the points (6, -4) and (-2, 7). Also find the point of intersection. 3 Marks

Ans: Let the y-axis divides the line segment joining the points (6, -4) and (-2, -7) in the ratio k:1 and the point be (0, y).

From section formula we know that if a point (x, y) divides the line joining the points (x1, y1) and (x2, y2) in the ratio m : n, then $\left(x,y\right)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$

Using this, we get

$\Rightarrow 0=\frac{k\times\left(-2\right)+1\times 6}{k+1}$

$\Rightarrow -2k+6=0$

$\Rightarrow 2k=6$

$\Rightarrow k=3$

Therefore, the y-axis divides the line segment joining the points (6, -4) and (-2, -7) in 3:1.

Now,

Coordinate of point of intersection $=\left(0,\frac{3\times\left(-7\right)+1\times\left(-4\right)}{3+1}\right)$

$=\left(0,\frac{-25}{4}\right)$

Therefore, point of intersection is $\left(0,\frac{-25}{4}\right)$.

Or

Show that the points (7, 10), (-2, 5) and (3, -4) are vertices of an isosceles right triangle. 3 Marks

Ans: Let A(7, 10), B(-2, 5) and C(3, -4).

Using distance formula,

$\Rightarrow AB=\sqrt{\left(-2-7\right)^2+\left(5-10\right)^2}$

$=\sqrt{\left(-9\right)^2+\left(-5\right)^2}$

$=\sqrt{106}$

$\Rightarrow BC=\sqrt{(3+2)^2+(-4-5)^2}$

$=\sqrt{(5)^2+(-9)^2}$

$=\sqrt{106}$

$\Rightarrow AC=\sqrt{(3-7)^2+(-4-10)^2}$

$=\sqrt{(-4)^2+(-14)^2}$

$=\sqrt{212}$

We can see that AB = BC.

Also,

$\Rightarrow AB^2+BC^2=106+106$

= 212

= AC2

Therefore, by Pythagoras theorem ΔABC is a right-angled triangle.

Hence, the points (7, 10) (-2, 5) and (3, -4) are vertices of an isosceles right triangle.

34. Use Euclid Division Lemma to show that the square of any positive integer is either of the form 3q or 3q+1 for some integer q. 3 Marks

Ans: We know from Euclid Division Lemma that if a and b are two positive integers then a=bm+r where $0\leq 0r<b$.

Now, let the positive integer be a and b=3.

r is an integer greater than or equal to zero and less than 3. Therefore, r can be either 0,1 or 2.

For r=0, the equation becomes

$\Rightarrow a=3m+0$

$\Rightarrow a=3m$

Squaring both the sides,

$\Rightarrow a^2=(3m)^2$

$\Rightarrow a^2=3(3m)^2$

Let $3m^2=q$.

$\Rightarrow a^2=3q$

For r=1, the equation becomes

$\Rightarrow a=3m+1$

Squaring both the sides,

$\Rightarrow a^2=(3m+1)^2$

$\Rightarrow a^2=9m^2+6m+1$

$\Rightarrow a^2=3(3m^2+2m)+1$

Let $q=3m^2+2m$.

$\Rightarrow a^2=3q+1$

For r=2, the equation becomes

$\Rightarrow a=3m+2$

Squaring both the sides,

$\Rightarrow a^2=(3m+2)^2$

$\Rightarrow a^2=9m^2+12m+4$

$\Rightarrow a^2=9m^2+12m+3+1$

$\Rightarrow a^2=3(3m^2+4m+1)+1$

Let $q=3m^2+4m+1$.

$\Rightarrow a^2=3q+1$

Hence proved that square of any positive integer is either of the form 3q or 3q+1 for some integer q.

SECTION-D

Question numbers 35 to 40 carry 4 marks each.

35. Sum of the areas of two squares is 544m2. If the difference of their perimeter is 32m, find the sides of the two squares. 4 Marks

Ans: Let the sides of two squares be x and y where x>y.

Perimeter of first square =4x

Perimeter of second square =4y

Given, the difference of their perimeter is 32m.

$\Rightarrow$4x-4y=32

$\Rightarrow$x-y=8

$\Rightarrow$x=5+y

Area of first square =x2

Area of second square =y2

Given, sum of the areas of two squares is 544 m2.

$\Rightarrow x^2+y^2=544$

Putting x=8+y, we get

$\Rightarrow (y+8)^2+y^2=544$

$\Rightarrow y^2+16y+64+y^2=544$

$\Rightarrow 2y^2+16y-480=0$

$\Rightarrow y^2+8y-240=0$

$\Rightarrow y^2+20y-12y-240-0$

$\Rightarrow y(y+20)-12(y+20)=0$

$\Rightarrow (y+20)(y-12)=0$

$\Rightarrow (y=-20)$ or $(y=12)$

Length cannot be negative. So, $y \neq -20$.

$\therefore$ y=12m

As, x=8+y

x=8+12

x=20

$\therefore x=20$

Hence, the sides of two squares are 20m and 12m.

Or

A motorboat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. 4 Marks

Ans: Let the speed of the stream be x km/h.

Given, speed of boat in still water is 18 km/h$^{18\ \text{km/h}}$.

Speed of boat in downstream = speed of boat in still water + speed of the stream

$=(18+x)$ km/h

Speed of boat in upstream = speed of boat in still water - speed of the stream

$=(18-x)$ km/h

As given that motorboat takes 1 hour more to go upstream than to return downstream to the same spot.

So, we can write

$\Rightarrow$ Time taken for upstream = time taken for downstream +1

As $\text{Time}=\frac{\text{Distance}}{\text{Speed}}$, we can write

$$\Rightarrow \frac{\text{Distance covered in upstream}}{\text{Speed of upstream}}=\frac{\text{Distance covered in downstream}}{\text{Speed of downstream}}+1$$

$$\Rightarrow \frac{24}{18-x}=\frac{24}{18+x}+1$$

$$\Rightarrow 24(18+x)=24(18-x)+18(18-x)(18+x)$$

On simplifying we get,

$$\Rightarrow (24\times18)+24x=(24\times18)-24x+((18\times18)-s^2)$$

$$\Rightarrow x^2+48x-324=0$$

$$\Rightarrow x^2+54x-6x-324=0$$

$$\Rightarrow x(x+54)-6(x+54)=0$$

$$\Rightarrow (x+54)(x-6)=0$$

$\Rightarrow (x=-54)$ or $(x=6)$

Speed of stream cannot be negative. So, $x \neq -54$.

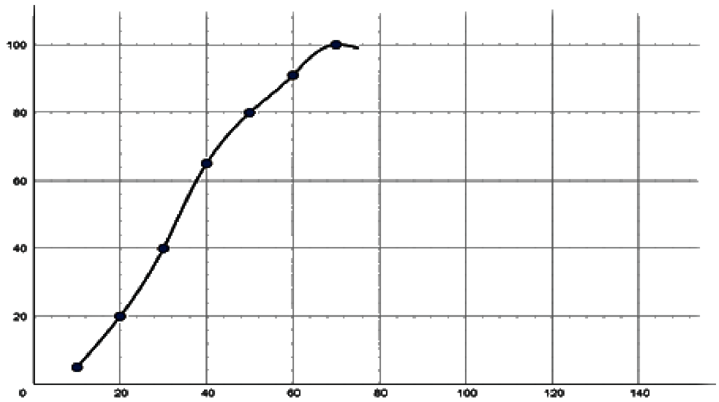
Therefore, speed of stream is 6 km/h.

36. For the following data, draw a 'less than' ogive and hence find the median of the distribution.

Age (in years)	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**
Number of persons:	**5**	**15**	**20**	**25**	**15**	**11**	**9**

4 Marks

Ans: Plotting age (in years) on the x-axis and cumulative frequency on y-axis.



Age	Numbers of persons
Less than 10	5
Less than 20	20
Less than 30	40
Less than 40	65
Less than 50	80
Less than 60	91
Less than 70	100

Here, we have N=100

$\Rightarrow \frac{N}{2}=50$

From the curve we get x-ordinate as 33.5 when ordinate is 50.

Therefore, the median of the given distribution is 33.5.

Or

The distribution given below shows the number of wickets taken by bowlers in one-day cricket matches. Find the mean and the median of the number of wickets taken.

Number of wickets:	**20-60**	**60-100**	**100-140**	**140-180**	**180-220**	**220-260**
Number of bowlers:	**7**	**5**	**16**	**12**	**2**	**3**

4 Marks

Ans: Assuming a=120 and h=40.

Number of wickets:	**Number of bowlers (f)**	**xi**	$u_i = \frac{(x_i - a)}{h}$	$f_i u_i$	**cf**
20-60	7	40	-2	-14	7
60-100	5	80	-1	-5	12
100-140	16	120	0	0	28
140-180	12	160	1	12	40
180-220	2	200	2	4	42
220-260	3	240	3	9	45

We get, $\sum f_i = 40$ and $\sum f_i u_i = 6$.

$$\text{Mean} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 120 + \frac{6 \times 40}{45}$$

$$= 125.33$$

We have, N=45

$$\Rightarrow \frac{N}{2} = 22.5$$

Therefore, Median class = 100-140, Cumulative frequency = 28, i = 100, cf=12, f=16 and h=40.

$$\text{Median}=l+\left(\frac{\frac{N}{2}\text{-cf}}{f}\right)\times h$$

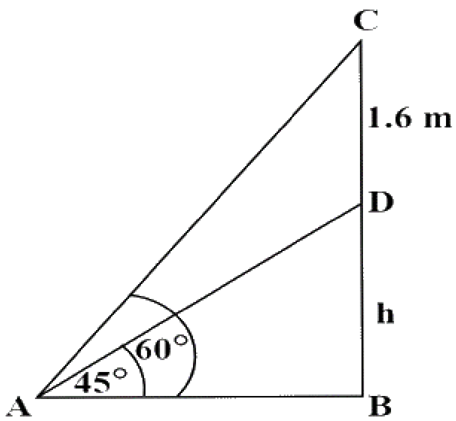
$$\Rightarrow \text{Median}=100+\left(\frac{22.5-12}{16}\right)\times 40$$

= 126.25

Therefore, the mean is 125.33 and the median is 126.25 of the number of wickets taken.

37. A statue 1.6 m tall, stand on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60o and from the same point the angle of elevation of the top of the pedestal is 45o. Find the height of the pedestal. (Use $\sqrt{3}$=1.73) 4 Marks

Ans: Let the height of pedestal be h metres.



In Δ ABD, we have

$$\Rightarrow \tan 45^\circ=\frac{BD}{AB}$$

$$\Rightarrow 1=\frac{h}{AB}$$

$$\Rightarrow AB=h\text{---}(1)$$

In ΔABC, we have

$$\Rightarrow \tan 60^{o} = \frac{BC}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{BD+DC}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{h+1.6}{AB}$$

$$\Rightarrow AB = \frac{h+1.6}{\sqrt{3}} \text{---}(2)$$

From (1) and (2), we get

$$\Rightarrow h = \frac{h+1.6}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}h = h+1.6$$

$$\Rightarrow \left(\sqrt{3}-1\right)h = 1.6$$

$$\Rightarrow h = \frac{1.6}{\left(\sqrt{3}-1\right)}$$

$$\Rightarrow h = \frac{1.6}{(1.73-1)}$$

$$\Rightarrow h = 2.19$$

Therefore, the height of the pedestal is 2.19 m.

38. Obtain other zeroes of the polynomial

$p(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$ if two zeroes are $\sqrt{5}$ and $-\sqrt{5}$. 4 Marks

Ans: Two zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

So, we can write, $x=\sqrt{5}$ and $x=-\sqrt{5}$.

We get, $x-\sqrt{5}$ and $x+\sqrt{5}=0$.

Multiplying both the factors we get,

$\Rightarrow \left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right)=0$

$\Rightarrow x^2-5=0$

x^2-5 is a factor of $p\left(x\right)=2x^4-x^3-11x^2+5x+5$.

Dividing $2x^4-x^3-11x^2+5x+5$ by x^2-5, we get the quotient as $2x^2-x-1$.

On factorising $2x^2-x-1$, we get

$\Rightarrow 2x^2-2x+x-1=0$

$\Rightarrow 2x\left(x-1\right)+1\left(x-1\right)=0$

$\Rightarrow \left(x-1\right)\left(2x+1\right)=0$

$\Rightarrow x=1,\ x=-\dfrac{1}{2}$

Therefore, other two zeroes of the polynomial are 1 and $-\dfrac{1}{2}$.

Or

What minimum must be added to $2x^3-3x^2+6x+7$ so that the resulting polynomial will be divisible by x^2-4x+8? 4 Marks

Ans:

$$
\begin{array}{r|l}
 & \quad\quad\quad\ \ 2x+5 \\
\hline
x^2-4x+8 & 2x^3-3x^3+6x+7 \\
 & \underline{2x^3-8x+16x} \\
 & \ \ 5x^2-10x+7 \\
 & \underline{5x^2-20x+40} \\
 & \quad\ 10x-33
\end{array}
$$

For $2x^3-3x^3+6x+7$ to be divisible by x^2-4x+8, remainder should be zero when we divide $2x^3-3x^3+6x+7$ by x^2-4x+8.

Dividing $2x^3-3x^3+6x+7$ by x^2-4x+8, we get the remainder as 10x-33.

Therefore, we have to add –(10x-33) i.e., 33-10x so that the resulting polynomial will be divisible by x2-4x+8.

39. In a cylindrical vessel of radius 10 cm, containing some water, 9000

small spherical balls are dropped which are completely immersed in water which raises the water level. If each spherical ball is of radius 0.5 cm, then find the rise in the level of water in the vessel. 4 Marks

Ans: Given, Radius of spherical balls $=0.5$ cm

Radius of cylindrical vessel $=10$ cm

As we know, Volume of a sphere $=\frac{4}{3}\times\pi\times(\text{radius})^3$

$\Rightarrow$ Volume of 9000 spherical balls $=9000\times\frac{4}{3}\times\pi\times(\text{radius of spherical ball})^3$

$\Rightarrow$ Volume of 9000 spherical balls $=9000\times\frac{4}{3}\times\pi\times(0.5)^3$

$\Rightarrow$ Volume of cylinder $=\pi\times(\text{radius of cylinder})^2\times(\text{rise in the level})$

Now, Volume of cylinder $=$ Volume of 9000 spherical balls

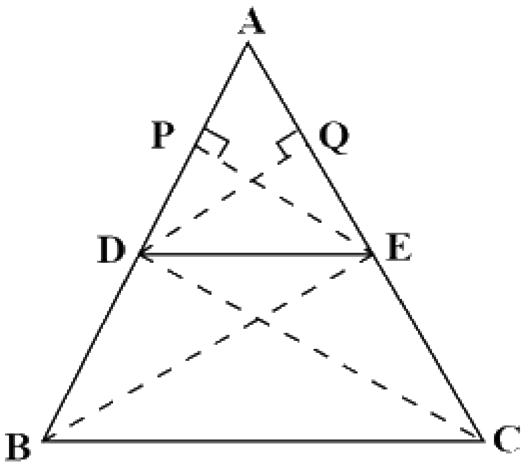
$\Rightarrow \pi\times(10)^2\times(\text{rise in the level})=9000\times\frac{4}{3}\times\pi\times(0.5)^3$

$\Rightarrow$ Rise in the level = 15 cm

Therefore, the rise in the level of water in the vessel is 15 cm.

40. If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, prove that the other two sides are divided in the same ratio. 4 Marks

Ans: Consider a ΔABC in which DE is drawn parallel to BC which intersects the side AB and AC at D and E respectively.



We have to prove $\frac{AD}{DB}=\frac{AE}{EC}$.

Construction: Join BE and CD and draw $DQ \perp AC$ and $EP \perp AC$.

Area of $\Delta ADE=\frac{1}{2}\times base\times height$

$\Rightarrow$ Area of $\Delta ADE=\frac{1}{2}\times AD\times EP$

Also,

$\Rightarrow$ Area of $\Delta ADE=\frac{1}{2}\times AE\times DQ$

Similarly,

$\Rightarrow$ Area of $\Delta BDE=\frac{1}{2}\times BD\times EP$

$\Rightarrow$ Area of $\Delta DEC=\frac{1}{2}\times EC\times DQ$

On taking ratio we get,

$$\Rightarrow \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE}=\frac{\frac{1}{2}\times AD\times EP}{\frac{1}{2}\times BD\times EP}$$

$$\Rightarrow \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE}=\frac{AD}{BD}----(1)$$

Similarly,

$$\Rightarrow \frac{\text{Area of } \Delta \text{ADE}}{\text{Area of } \Delta \text{DEC}} = \frac{\frac{1}{2} \times \text{AE} \times \text{DQ}}{\frac{1}{2} \times \text{EC} \times \text{DQ}}$$

$$\Rightarrow \frac{\text{Area of } \Delta \text{ADE}}{\text{Area of } \Delta \text{DEC}} = \frac{\text{AE}}{\text{EC}} ---(2)$$

Also, in Δ BDE and Δ DEC has the same base DE and between the same parallel lines BC and DE.

So, Area of Δ BDE = Area of Δ DEC ----(3)

Therefore, from equation (1), (2), and (3) we get

$$\Rightarrow \frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$$

Hence proved that if a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, then the other two sides are divided in the same ratio.

Class X
Mathematics
(CBSE 2019)

Time: 3Hr
Marks : 80

GENERAL INSTRUCTIONS :
(i) All questions are compulsory.
(ii) This question paper consists of 30 questions divided into four ***sections - A, B, C*** *and* ***D****.*
(iii) ***Section A*** *contains 6 questions of 1 mark each.*
Section B *contains 6 questions of 2 marks each.*
Section C *contains 10 questions of 3 marks each.*
Section D *contains 8 questions of 4 marks each.*
(iv) There is no overall choice. However, an internal choice has been provided in two questions of 1 mark, two questions of 2 marks, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculator is not permitted.

Section A

Question numbers 1 to 6 carry 1 mark each.

1. Find the coordinates of a point A, where AB is diameter of a circle whose centre is $(2,-3)$ and B is the point $(1,4)$.

Solution: Let the centre be O and coordinates of point A be (x,y)

$\frac{x+1}{2}=2$ [By Mid-point formula]

Implies that

$x=3$

$\frac{y+4}{2}=-3$

$\therefore$ Coordinates of $A=(3,-10)$

2. For what values of k, the roots of the equation $x^2+4x+k=0$ are real?

OR

Find the value of k for which the roots of the equation $3x^2-10x+k=0$ are reciprocal of each other.

Solution:

$x^2+4x+k=0$

$\therefore$ Roots of given equation are real,

$D\geq 0$

Implies that

$(4)^2 - 4 \times k \geq 0$

Implies that

$-4k \geq -16$

Implies that

$k \leq 4$

$\therefore$ k has all real values ≤ 4

OR

$3x^2 - 10x + k = 0$

$\because$ Roots of given equation are reciprocal of each other.

Let the roots be α and $\frac{1}{\alpha}$

Product of roots = $\frac{\text{c}}{\text{a}}$

Implies that

$\alpha, \frac{1}{\alpha} = \frac{k}{3}$

$\therefore$ $k = 3$.

3. Find A if $\tan\ 2A = \cot(A - 24°)$

OR

Find the value of $(\sin^2 33° + \sin^2 57°)$

Solution:

Given:

$\tan 2A = \cot(A - 24°)$

Implies that $\tan 2A = \tan[90° - (A - 24°)]$

Implies that $\tan 2A = \tan[90° - A + 24°]$

Implies that $\tan 2A = \tan[114° - A]$

Implies that $2A = 114° - A$

Implies that $3A = 114°$

Implies that $A = \frac{114°}{3}$

Implies that $A = 38°$

OR

Given:

$\sin^2 33° + \sin^2 57°$

$= \sin^2 33° + [\cos(90° - 57°)]^2$

$= \sin^2 33° + \cos^2 33°$

$= 1$

4. How many two digits numbers are divisible by 3?

Solution:

Two digits numbers divisible by 3 are

$12, 15, 18, \ldots\ldots, 99.$

$a = 12, d = 15 - 12 = 3$

Implies that

$T_n = 99$

Implies that

$a + (n-1)d = 99$

Implies that

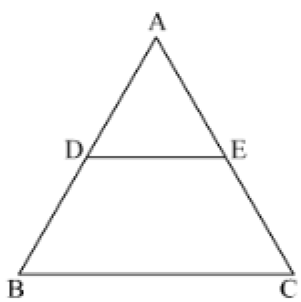
$12 + (n-1)3 = 99$

Implies that

$n = 30$

Number of two digit numbers divisible by 3 are 30.

5.In Fig. 1, $DE \parallel BC, AD = 1\ cm$ and $BD = 2\ cm$. What is the ratio of the ar (ΔABC) to the ar (ΔADE)?



Solution:

$DE \parallel BC$

$\Delta ADE \sim \Delta ABC$ [By AA similarity]

$\therefore \dfrac{ar(\Delta ABC)}{ar(\Delta ADE)} = \dfrac{AB^2}{AD^2}$ [By area similarity theorem]

$\Rightarrow \dfrac{ar(\Delta ABC)}{ar(\Delta ADE)} = \dfrac{3^2}{1^2}$

$\Rightarrow \dfrac{ar(\Delta ABC)}{ar(\Delta ADE)} = \dfrac{9}{1}$

6. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Sol. Rational number lying between $\sqrt{2}$ and $\sqrt{3}$ is $1.5 = \dfrac{15}{10} = \dfrac{3}{2}$

$\left[\because \sqrt{2} \sim 1.414 \text{ and } \sqrt{3} \sim 1.732\right]$

Section B

7. Find the HCF of 1260 and 7344 using Euclid's algorithm.

OR

Show that every positive odd integer is of the form $(4\text{q}+1)$ or $(4\text{q}+3)$**, where q is some integer.**

Since 7344 > 1260

$7344 = 1260 \times 5 + 1044$

Since remainder $\neq 0$

$1260 = 1044 \times 1 + 216$

$1044 = 216 \times 4 + 180$

$216 = 180 \times 1 + 36$

$180 = 36 \times 5 + 0$

The remainder has now become zero.

$\therefore$ HCF of 1260 and 7344 is 36.

OR

Let a be positive odd integer

Using division algorithm on a and b = 4

$a = 4q + r$ Since $0 \leq r < 4$, the possible remainders are 0, 1, 2 and 3

$\therefore$ a can be 4q or 4q + 1 or 4q + 2 or 4q + 3, where q is the quotient

Since a is odd, a cannot be 4q and 4q + 2

$\therefore$ Any odd integer is of the form 4q + 1 or 4q + 3, where q is some integer.

8. Which term of the AP 3, 15, 27, 39, ... will be 120 more than its 21st term?

OR

If S_n**, the sum of first n terms of an AP is given by** $S_n = 3n^2 - 4n$**, find the nth term.**

Solution:

Given AP is

3, 15, 27, 39 ....

where a = 3, d = 15 – 3 = 12

Let the nth term be 120 more than its 21st term.

$t_n = t_{21} + 120$

$\Rightarrow 3 + (n-1)12 = 3 + 20 \times 12 + 120$

$\Rightarrow (n-1) \times 12 = 363 - 3$

$\Rightarrow (n-1) = \frac{360}{12}$

$\therefore n = 31$

Hence, the required term is

$t_{31} = 3 + 30 \times 12$

$= 363$

OR

$S_n = 3n^2 - 4n$

Let S_{n-1} be sum of $(n-1)$ terms

$t_n = S_n - S_{n-1}$

$= (3n^2 - 4n) - \left[3(n-1)^2 - 4(n-1)\right]$

$= (3n^2 - 4n) - \left[3n^2 - 6n + 3 - 4n + 4\right]$

$= 3n^2 - 4n - 3n^2 + 10n - 7$

$\therefore t_n = 6n - 7$

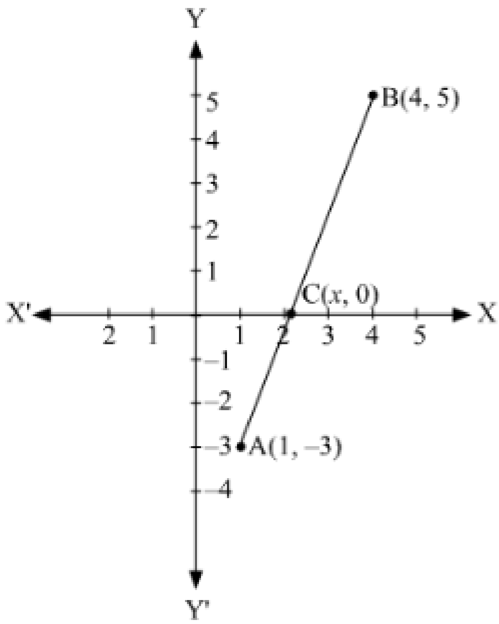
Therefore, required nth term $= 6n - 7$

9. Find the ratio in which the segment joining the points (1, –3) and (4, 5) is divided by x-axis? Also find the coordinates of this point on x-axis.

Solution:

Let $\mathrm{C}(x, 0)$ divides the line segment joining the points $\mathrm{A}(1,-3)$ and $\mathrm{B}(4,5)$ in $k:1$ ratio.

By section formula,



$(x, y) = \left(\dfrac{mx_2 + nx_1}{m+n}, \dfrac{my_2 + ny_1}{m+n}\right)$

Implies that

$(x, 0) = \left(\dfrac{4k + 1\times 1}{k+1}, \dfrac{5k + 1\times(-3)}{k+1}\right)$

Implies that

$(x, 0) = \left(\dfrac{4k+1}{k+1}, \dfrac{5k-3}{k+1}\right)$

Implies that

$\dfrac{5k-3}{k+1} = 0$

Implies that

$5k - 3 = 0$

Implies that

$5k = 3$

$k = \frac{3}{5}$

and $x = \frac{4k+1}{k+1} = \frac{4\times\frac{3}{5}+1}{\frac{3}{5}+1}$

$\Rightarrow x = \frac{\frac{12+5}{5}}{\frac{3+5}{5}}$

$\Rightarrow x = \frac{17}{8}$

Therefore, coordinates of point P are $\left(\frac{17}{8}, 0\right)$.

10. A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game.

Solution:

Total possible outcomes are (HHH), (HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT) i.e., 8. [½]

The favourable outcomes to the event E 'Same result in all the tosses' are TTT, HHH. Therefore, the number of favourable outcomes = 2

$\because \mathrm{P}(\mathrm{E}) = \frac{2}{8} = \frac{1}{4}$

Hence, probability of losing the game $= 1 - \mathrm{P}(\mathrm{E})$

$= 1 - \frac{1}{4} = \frac{3}{4}$

11. A die is thrown once. Find the probability of getting a number which (i) is a prime number (ii) lies between 2 and 6.

Solution:

Total outcomes = 1, 2, 3, 4, 5, 6 Prime numbers = 2, 3, 5 Numbers lie between 2 and 6 = 3, 4, 5

(i) P (Prime Numbers) $= \frac{3}{6} = \frac{1}{2}$

(ii) P (Numbers lie between 2 and 6) $= \frac{3}{6} = \frac{1}{2}$

12. Find c if the system of equations $cx + 3y + (3-c) = 0, 12x + cy - c = 0$ has infinitely many solutions?

Solution:

$cx + 3y + (3-c) = 0$ ....(i)

$12x + cy - c = 0$ ....(ii)

For infinitely many solutions $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}$

$\Rightarrow \frac{c}{12} = \frac{3}{c}$ or

$\Rightarrow c^2 = 36$

$\Rightarrow c = \pm 6$

$\frac{3}{c} = \frac{3-c}{-c}$

$\Rightarrow c(c-6) = 0$

$\Rightarrow c = 0, 6$

Hence the value of $c = 6$.

Section C

Question numbers 13 to 22 carry 3 marks each.

13. Prove that $\sqrt{2}$ is an irrational number.

Solution:

Let $\sqrt{2}$ be rational.

$\therefore \sqrt{2} = \frac{p}{q}$ where p and q are co-prime integers and , q $\neq$0

Implies that $\sqrt{2}q = p$

$2q^2 = p^2 ....(\text{i})$

$\Rightarrow 2 \text{ divides } p^2$

$\Rightarrow 2 \text{ divides } p ....(\text{A})$

Let $p = 2c$ for some integer c

$p^2 = 4c^2$

$\Rightarrow 2q^2 = 4c^2$

$\Rightarrow q^2 = 2c^2$

$\Rightarrow 2 \text{ divides } q^2$

$\Rightarrow 2 \text{ divides } q ....(\text{B})$

From (A) and (B), we get

$\therefore$ 2 is common factor of both p and q. But this contradicts the fact that p and q have no common factor other than 1

$\therefore$ Our supposition is wrong Hence, $\sqrt{2}$ is an irrational number.

14. Find the value of k such that the polynomial $x^2 - (k+6)x + 2(2k-1)$ has sum of its zeros equal to half of their product.

Solution:

For given polynomial $x^2 - (k+6)x + 2(2k-1)$

Here

$\text{a} = 1, b = -(k = 6), c = 2(2k-1)$

Given that:

$\because$ Sum of zeroes = $\frac{1}{2}$ (product of zeroes)

$\Rightarrow \frac{-\left[-(k+6)\right]}{1} = \frac{1}{2} \times \frac{2(2k-1)}{1}$

$\Rightarrow k+6 = 2k-1$

$\Rightarrow 6+1 = 2k-k$

$\Rightarrow k = 7$

Therefore, the value of $k = 7$

15. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

OR

A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator. Find the fraction.

Solution:

Let the sum of ages of two sons be x years

Age of man $= 3x$ years

After 5 years age of the man $= (3x+5)$ years

Sum of ages of two sons $= (x+10)$ years

Given, $(3x+5) = 2(x+10)$

$\Rightarrow (3x+5) = 2x+20$

$\Rightarrow x = 15$

Hence $3x = 3(15) = 45$

Thus the age of the man(father) is 45 years.

OR

Let's assume the fraction be $\frac{x}{y}$

First condition:

$\frac{x-2}{y} = \frac{1}{3}$

$\Rightarrow 3x-6 = y$

$\Rightarrow 3x-y = 6 \ldots\ldots\ldots\ldots(1)$

Second condition:

$\frac{x}{y-1} = \frac{1}{2}$

$\Rightarrow 2x = y-1$

$\Rightarrow 2x-y = -1 \ldots\ldots\ldots\ldots(2)$

Using eliminated method:

Multiplying (2) by -1 and then adding (1) and (2)

$\Rightarrow 3x - y = 6$

$\Rightarrow -2x + y = 1$

$\Rightarrow x = 7$

Now,from (1),

$\Rightarrow 3x - y = 6$

$\Rightarrow 3(7) - y = 6$

$\Rightarrow 21 - y = 6$

$\Rightarrow y = 15$

Hence the required fraction is $\frac{7}{15}$.

16. Find the point on y-axis which is equidistant from the points (5, – 2) and (– 3, 2).

OR

The line segment joining the points A(2, 1) and B(5, – 8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by 2x – y + k = 0, find the value of k.

Solution:

Since the point is on y-axis so,X-coordinate is zero

Let the point be (0, y)

It's distance from A(5, –2) and B(– 3, 2) are equal

$\therefore \sqrt{(0-5)^2 + (y+2)^2} = \sqrt{(0+3)^2 + (y-2)^2}$

$\Rightarrow 25 + y^2 + 4y + 4 = 9 + y^2 - 4y + 4 \quad [\text{squaring both sides}]$

$\Rightarrow 4y + 29 = -4y + 13$

$\Rightarrow 4y + 4y = 13 - 29$

$\Rightarrow 8y = -16 \therefore y = \frac{-16}{8} = -2$

Thus, the point is $(0, -2)$.

OR

As line segment AB is trisected by the points P and Q. Therefore,

Case I: When AP : PB = 1 : 2.

Then, coordinates of P are

$\left\{\frac{1\times 5 + 2\times 2}{1+2}, \frac{1\times(-8) + 1\times 2}{1+2}\right\}$

Implies that

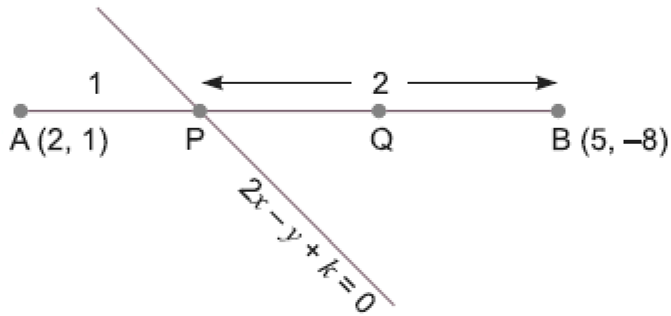
$P(3, -2)$

Since the point $P(3, -2)$ lies on the line

$2x - y + k = 0$

$\Rightarrow 2\times 3 - (-2) + k = 0$

$\Rightarrow k = -8$



17. Prove that : $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

OR

Prove that: $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2.$

Solution:

$\text{L.H.S}: (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$= sin^2\theta + cosec^2\theta + 2 + cos^2\theta + sec^2\theta + 2 \qquad \left[\because \sin\theta = \frac{1}{\operatorname{cosec}\theta} \text{ and } \cos\theta = \frac{1}{\sec\theta}\right]$

$= \sin^2\theta + \cos^2\theta + 1 + \cot^2\theta + 1 + \tan^2\theta + 4 \qquad \left[\because \operatorname{cosec}^2\theta + 1 + \cot^2\theta \text{ and } \sec^2\theta = 1 + \tan^2\theta\right]$

$= \sin^2\theta + \cos^2\theta + 1 + \cot^2\theta + 1 + \tan^2\theta + 4 \qquad \left[\because \operatorname{cosec}^2\theta + 1 + \cot^2\theta \text{ } and \text{ } \sec^2\theta = 1 + \tan^2\theta\right]$

$= 1 + 1 + 1 + 4 + \tan^2\theta + \cot^2\theta \qquad \left[\because \cos^2\theta + \sin^2\theta = 1\right]$

$= 7 + \tan^2\theta + \cot^2\theta$

L.H.S=R.H.S

OR

$\text{L.H.S}: \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$

$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$

$= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A \cdot \cos A}$

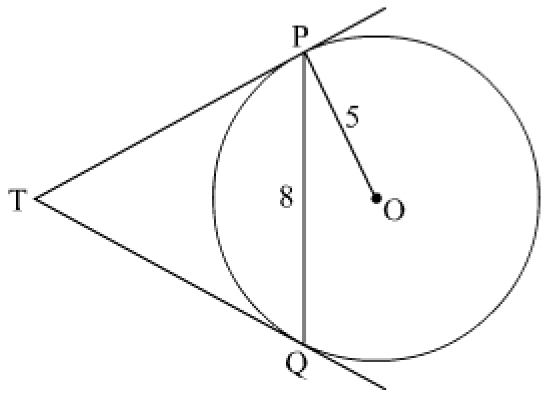
$= \frac{\sin^2 A + \cos^2 A + 2\sin A \cdot \cos A - 1}{\sin A \cdot \cos A}$

$= \frac{1 + 2\sin A \cdot \cos A - 1}{\sin A \cdot \cos A}$

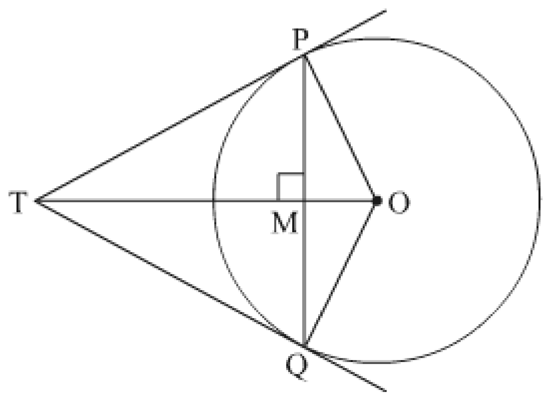
$= 2$

Hence, L.H.S=R.H.S.

18.In Fig. , PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP.



Solution:



Given radius, $OP = OQ = 5\text{ cm}$

Length of chord, $PQ = 4\text{ cm}$

$OT \perp PQ$,

$\therefore PM = MQ = 4\text{ cm}$ [Perpendicular draw from the centre of the circle to a chord bisect the chord]

In right ΔOPM,

$OP^2 = PM^2 + OM^2$

$\Rightarrow 5^2 = 4^2 + OM^2$

$\Rightarrow OM^2 = 25 - 16 = 9$

Hence $OM = 3\text{cm}$

In right ΔPTM,

$PT^2 = TM^2 + PM^2 \rightarrow (1)$

$\angle OPT = 90^\circ$ [Radius is perpendicular to tangent at point of contact]

In right ΔOPT,

$OT^2 = PT^2 + OP^2 \rightarrow (2)$

From equations (1) and (2), we get

$OT^2 = \left(TM^2 + PM^2\right) + OP^2$

$\Rightarrow \left(TM + OM\right)^2 = \left(TM^2 + PM^2\right) + OP^2$

$\Rightarrow TM^2 + OM^2 + 2\times TM\times OM = TM^2 + PM^2 + OP^2$

$\Rightarrow OM^2 + 2\times TM\times OM = PM^2 + OP^2$

$\Rightarrow 3^2 + 2\times TM\times 3 = 4^2 + 5^2$

$\Rightarrow 9 + 6TM = 16 + 25$

$\Rightarrow 6TM = 32$

$\Rightarrow TM = \frac{32}{6} = \frac{16}{3}$

Equation (1) becomes,

$PT^2 = TM^2 + PM^2$

$= \left(\frac{16}{3}\right)^2 + 4^2$

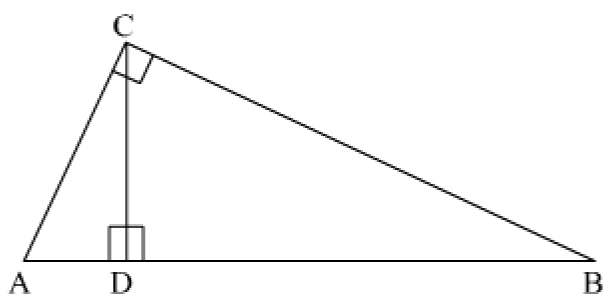
$= \left(\frac{256}{9}\right) + 16 = \frac{(256+144)}{9}$

$= \left(\frac{400}{9}\right) = \left(\frac{20}{3}\right)^2$

Hence $PT = \frac{20}{3}$

Thus, the length of tangent PT is $\frac{20}{3}$ cm.

19.In Fig. 3, ∠ACB = 90° and CD ⊥ AB, prove that CD2 = BD × AD. [3] A B C D Fig. 3



OR

If P and Q are the points on side CA and CB respectively of Δ ABC, right angled at C, prove that (AQ2 + BP2) = (AB2 + PQ2)

Solution:

Given that : $CD \perp AB$

$\angle ACB = 90^\circ$

To Prove : $CD^2 = BD\times AD$

Using Pythagoras Theorem in ΔACD

$AC^2 = AD^2 + CD^2 ......(1)$

Using Pythagoras Theorem in ΔCDB

$CB^2 = BD^2 + CD^2 ......(2)$

Similarly in ΔABC,

$AB^2 = AC^2 + BC^2 .....(3)$

As AB = AD + DB

Since, AB=AD+BD.....(4)

Squaring both sides of equation (4), we get

$(AB)^2 = (AD + BD)^2$

Since, $AB^2 = AD^2 + BD^2 + 2 \times BD \times AD$

From equation (3) we get

$AC^2 + BC^2 = AD^2 + BD^2 + 2 \times BD \times AD$

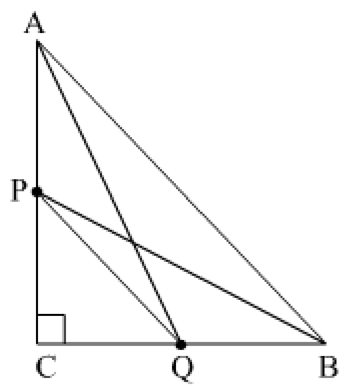
Substituting the value of AC^2 from equation (1) and the value of BC^2 from equation (2), we get

$AD^2 + CD^2 + BD^2 + CD^2 = AD^2 + BD^2 + 2 \times BD \times AD$

Since, $2\ CD^2 = 2 \times BD \times AD$

Hence, $CD^2 = BD \times AD$

OR



Using the Pythagoras theorem in ΔABC,

ΔACQ, ΔBPC, ΔPCQ, we get

$AB^2 = AC^2 + BC^2 .....(1)$

$AQ^2 = AC^2 + CQ^2 ..... (2)$

$BP^2 = PC^2 + BC^2 ..... (3)$

$PQ^2 = PC^2 + CO^2 ..... (4)$

Adding the equations (2) and (3) we get

$AQ^2 + BP^2 = AC^2 + CQ^2 + PC^2 + BC^2$

$= (AC^2 + BC^2) + (CQ^2 + PC^2)$

$= AB^2 + PQ^2$

As L.H.S $= AQ^2 + BP^2$

$= AB^2 + PQ^2$ = R.H.S

20. Find the area of the shaded region in Fig. 4, if ABCD is a rectangle with sides 8 cm and 6 cm and O is the centre of circle. (Take π = 3.14)

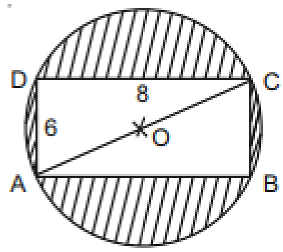


Fig. 4

Solution:

Here, diagonal AC also represents the diameter of the circle.

Using Pythagoras theorem:

$$AC = \sqrt{AB^2 + BC^2}$$

$$AC = \sqrt{8^2 + 6^2}$$

$$AC = \sqrt{64 + 36}$$

$$AC = \sqrt{100}$$

$$AC = 10$$

$\therefore$ Radius of the circle, $OC = \frac{AC}{2} = 5cm$

Area of the shaded region = Area of the circle – Area of rectangle

$$= \pi r^2 - AB \times BC$$

$$= \pi (OC)^2 - AB \times BC$$

$$= 3.14 \times 5^2 - 8 \times 6$$

$$= 78.5 - 48$$

$$= 30.5$$

Therefore,the area of shaded region is $30.5cm^2$.

21. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed?

Solution: Width of the canal = 6 m

Depth of the canal = 1.5 m

Length of the water column formed in $\frac{1}{2}$ hr

= 5 km or 5000 m

$\therefore$ Volume of water flowing in $\frac{1}{2}$ hr

= Volume of cuboid of length 5000 m, width 6 m and depth 1.5 m.

= 5000 × 6 × 1.5 = 45000 m^3

On comparing the volumes,

Volume of water in field = Volume of water coming out from canal in 30 minutes.

Irrigated area × standing water = 45000.

Irrigated Area $\frac{45000}{\frac{8}{100}}$ $\quad [\because 1 \text{ m}=100 \text{ cm}]$

$\frac{45000 \times 100}{8} = 5,62,500 \text{m}^3$

22. Find the mode of the following frequency distribution.

Class	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**
Frequency	**8**	**10**	**10**	**16**	**12**	**6**	**7**

Solution:

Class	Frequency
0-10	8
10-20	10
20-30	$10 \rightarrow f_0$
30-40	$16 \rightarrow f_1$
40-50	$12 \rightarrow f_2$
50-60	6
60-70	7

Here, 30 - 40 is the modal class, and I = 30, h = 10

$\therefore \text{Mode}=I+\left(\frac{f_0-f_1}{2f_1-f_0-f_2}\right)\times h$

$=30+\left(\frac{16-10}{2\times 16-10-12}\right)\times 10$

$=30+\frac{16}{10}\times 10=30+6=36$

Section D

23. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

OR

A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream.

Determine the speed of the stream and that of the boat in still water.

Solution:

Let the time in which tap with longer and smaller diameter can fill the tank separately be x hours and y hours respectively.

$\frac{1}{x}+\frac{1}{y}=\frac{8}{15}$ ....(i)

And $x=y-2$ ....(ii)

On substituting $x=y-2$ from (ii) in (i), we get

$\frac{1}{y-2}+\frac{1}{y}=\frac{8}{15}$

$\Rightarrow \frac{y+y-2}{y^2-2y}=\frac{8}{15}$

$\Rightarrow 15(2y-2)=8(y^2-2y)$

$\Rightarrow 30y-30=8y^2-16y$

$\Rightarrow 8y^2-46y+30=0$

$\Rightarrow 4y^2-20y-3y+15=0$

$\Rightarrow (4y-3)(y-5)=0$

$\Rightarrow y=\frac{3}{4}, y=5$

Substituting values of y in (ii), we get

$x=\frac{3}{4}-2$

$x=\frac{-5}{4}$

$\therefore x\neq\frac{-5}{4}$

(time cannot be negative)

Hence, the time taken by tap with longer diameter is 3 hours and the time taken by tap with smaller diameter is 5 hours, in order to fill the tank separately

OR

Let the speed of boat is x km/h in still water

And stream y km/h

According to question,

$\frac{30}{x-y}+\frac{44}{x+y}=10$

And

$\frac{40}{x-y}+\frac{55}{x+y}=13$

Let $\frac{1}{x-y}=u$ and $\frac{1}{x+y}=v$

$30u+44v=10$.....(i)

$40u+55v=13$....(ii)

On solving equation (i) and (ii) we get,

$u = \frac{1}{5} \Rightarrow x - y = 5 \qquad \rightarrow \quad \text{(iii)}$

$\text{v} = \frac{1}{11} \Rightarrow x + y = 11 \qquad \rightarrow \quad \text{(iv)}$

On solving equation (iii) and (iv) we get,

$x = 8\,\text{km/h}$

$y = 3\text{km/h}$

24. If the sum of first four terms of an AP is 40 and that of first 14 terms is 280. Find the sum of its first n terms.

Solution:

Given that: $S_4 = 40$ and $S_{14} = 280$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_4 = \frac{4}{2}\left[2a + (4-1)d\right] = 40$$

$$\Rightarrow 2a + 3d = 20 ....\text{(i)}$$

$$S_{14} = \frac{14}{2}\left[2a + (14-1)d\right] = 280$$

$$\Rightarrow 2a + 13d = 40 ....\text{(ii)}$$

$$\text{(ii)-(i)},$$

$$10d = 20 \Rightarrow d = 2$$

Substituting the value of d in (i),we get

$$2a + 6 = 20 \Rightarrow a = 7$$

Sum of first n terms,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[14 + (n-1)2\right]$$

$$= n(7 + n - 1)$$

$$= n(n+6)$$

$$= n^2 + 6n$$

Therefore, $S_n = n^2 + 6n$

$$\text{L.H.S} = \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$$

$= \frac{\tan A - 1 + \sec A}{\tan A + 1 - \sec A}$ (Dividing numerator & denominator by $\cos A$)

$$= \frac{(\tan A + \sec A) - 1}{(\tan A - \sec A) + 1}$$

$$=\frac{\{(\tan A+\sec A)-1\}(\tan A-\sec A)}{(\tan A-\sec A)+1(\tan A-\sec A)}$$
$$=\frac{(\tan^2 A+\sec^2 A)-(\tan A-\sec A)}{\{\tan A-\sec A+1\}(\tan A-\sec A)}$$
$$=\frac{-1-\tan A+\sec A}{\{\tan A-\sec A+1\}(\tan A-\sec A)}$$
$$=\frac{-1}{\tan A-\sec A}$$
$$=\frac{1}{\sec A-\tan A}$$
LHS=RHS

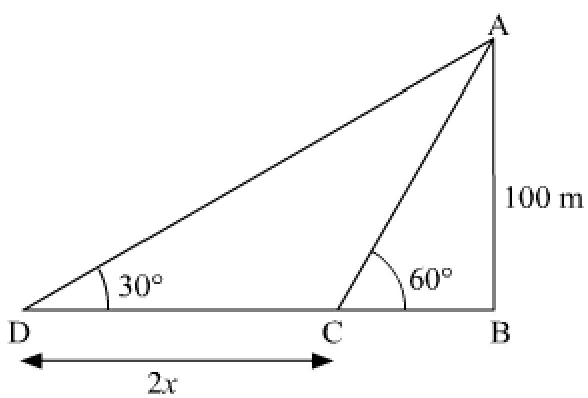
Hence proved.

26. A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. $\left[\text{Use}\sqrt{3}=1.732\right]$

OR

Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

Solution:



AB is a lighthouse of height 100m. Let the speed of boat be x metres per minute. And CD is the distance which man travelled to change the angle of elevation.

Therefore,

$\text{CD}=2x\ [\because \text{Distance=Speed}\times\text{Time}]$

$\tan(60°) = \frac{AB}{BC}$

$\sqrt{3} = \frac{100}{BC}$

$\Rightarrow BC = \frac{100}{\sqrt{3}}$

$\tan(30°) = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{BD}$

$BD = 100\sqrt{3}$

CD=BD-BC

$2x = 100\sqrt{3} - \frac{100}{\sqrt{3}}$

$2x = \frac{300-100}{\sqrt{3}}$

$\Rightarrow x = \frac{200}{2\sqrt{3}}$

$\Rightarrow x = \frac{100}{\sqrt{3}}$

Using,

$\sqrt{3} = 1.73$

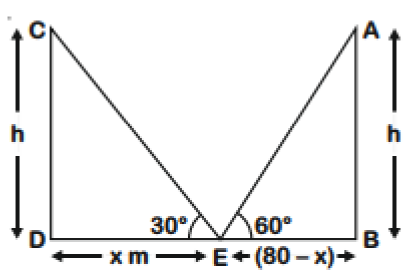
$x = \frac{100}{1.73} \approx 57.80$

Hence,the speed of the boat is 57.80 meters per minute.

OR

Let the poles be AB, CD each of height h meter and E is the point between the poles on the road.

Let ∠AEB = 60°, ∠CED = 30° and DE be x meter.



$\therefore \text{BE} = (80 - x)\text{m}$

In ΔAEB,

$$\tan 60^\circ = \frac{AB}{BE}$$

$$\Rightarrow \sqrt{3} = \frac{h}{(20 - x)}$$

$$\Rightarrow h = \sqrt{3}(80 - x)\text{m..........(i)}$$

In ΔCDE,

$$\tan 30^\circ = \frac{CD}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow h = \frac{x}{\sqrt{3}}\text{m........(ii)}$$

From equation (i) and (ii), we get

$$\frac{x}{\sqrt{3}} = \sqrt{3}(80 - x)$$

$$\Rightarrow x = 240 - 3x$$

$$\Rightarrow 4x = 240$$

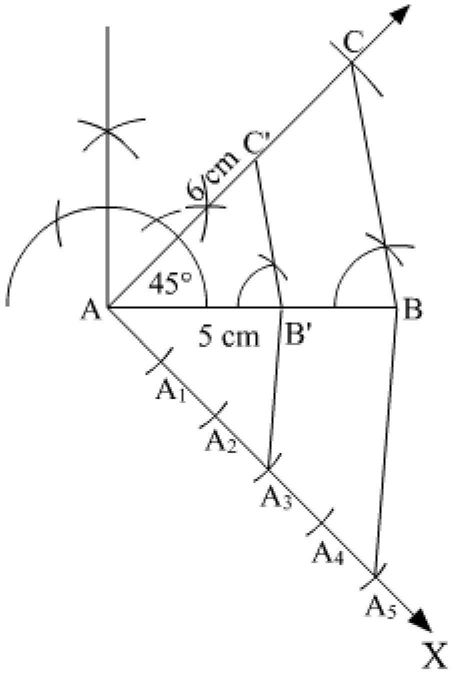
$$\Rightarrow x = 60\ m$$

Put value of x in equation (ii), we get

$h = 20\sqrt{3}\text{m, DE=60m and BE=20m}$

Hence, the heights of each pole is $20\sqrt{3}\text{m}$ and distance of the point from the poles are 60 m and 20 m.

27. Construct a ΔABC in which CA = 6 cm, AB = 5 cm and ∠BAC = 45º. Then construct a triangle whose sides are 3 5 of the corresponding sides of ΔABC.
Solution:



Steps of construction:

1. Draw AB = 5 cm. With A as centre, draw $\angle BAC = 45°$. Join BC. $\angle ABC$ is thus formed.
2. Draw AX such that $\angle BAX$ is an acute angle.
3. Cut 5 equal arcs $AA_1, A_1A_2, A_2A_3, A_3A_4$ and A_4A_5 .
4. Join A_5 to B and draw a line through A_3 parallel to A_5B which meets AB at B'.

Here, AB' = $\frac{3}{5}$AB

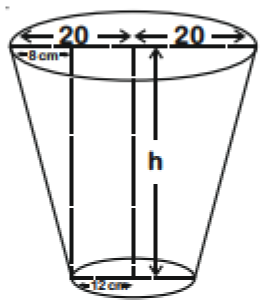
5.Now draw a line through B' parallel to BC which joins AC at C'.

Here, B'C' = $\frac{3}{5}$BC and AC'=$\frac{3}{5}$AC

Thus, AB'C' is the required triangle.

28. A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm3. The radii of the top and bottom of circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of the metal sheet used in making it. (Use π = 3.14)

Solution:



Let the height of the bucket be h cm and slant height be l cm.
Here

$r_1 = 20\ cm$

$r_2 = 12\ cm$

And capacity of bucket = 12308.8 cm3

We know that capacity of bucket $= \dfrac{\pi h}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$

$= 3.14 \times \dfrac{h}{3}\left(400 + 144 + 240\right)$

$= 3.14 \times \dfrac{h}{3} \times 784$

So we have $= 3.14 \times \dfrac{h}{3} \times 784 = 12308.8$

$h = \dfrac{12308.8 \times 3}{3.14 \times 784}$

$= 15\,\text{cm}$

Now, the slant height of the frustum,

$l = \sqrt{h^2 + \left(r_1 - r_2\right)^2}$

$= \sqrt{15^2 + 8^2}$

$= \sqrt{289}$

$= 17\,\text{cm}$

Area of metal sheet used in making it

$= \pi r_2^2 + \pi\left(r_1 + r_2\right)^2$

$= 3.174 \times \left[144 + \left(20 + 12\right) \times 17\right]$

$= 2160.32\,\text{cm}^2$

29. Prove that in a right angle triangle, the square of the hypotenuse is equal the sum of squares of the other two sides.

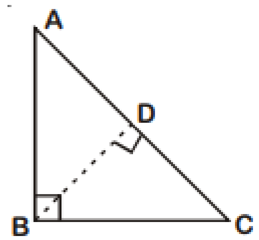
Solution:

Given : A right triangle ABC in which ∠B = 90°

To Prove: $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

i.e. $AC^2 = AB^2 + BC^2$

Construction : From B, draw $BD \perp AC$



In ΔABC and ΔADB

$\angle BAC = \angle DAB$ [Common]

$\angle ABC = \angle ADB$ [Each 90°]

$\therefore \Delta ABC \sim \Delta ADB$ [By AA similarity]

$\Rightarrow \frac{AB}{AC} = \frac{AD}{AB}$

$\Rightarrow AB^2 = AD \times AC \ldots(i)$

Similarly, $\Delta ABC \sim \Delta BDC$

$\Rightarrow \frac{BC}{DC} = \frac{AC}{BC}$

$\Rightarrow BC^2 = AC \times DC \ldots(ii)$

On Adding (i) and (ii), we get

$AB^2 + BC^2 = AD \times AC + AC \times DC$

$\Rightarrow AB^2 + BC^2 = AC(AD + DC)$

$\Rightarrow AB^2 + BC^2 = AC \times AC$

$\Rightarrow AC^2 = AB^2 + BC^2$

30. If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2.

Class	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**	**Total**
Frequency	f_1	**5**	**9**	**12**	f_2	**3**	**2**	**40**

OR

The marks obtained by 100 students of a class in an examination are given below.

Marks	**Number of students**
0-5	**2**
5-10	**5**
10-15	**6**
15-20	**8**
20-25	**10**
25-30	**25**

30-35	**20**
35-40	**18**
40-45	**4**
45-50	**2**

Draw 'a less than' type cumulative frequency curves (ogive). Hence find median.
Solution:

Class	Frequency	Cumulative Frequency
0-10	f_1	f_1
10-20	5	$5+f_1$
20-30	9	$14+f_1$
30-40	12	$26+f_1$
40-50	f_2	$26+f_1+f_2$
50-60	3	$29+f_1+f_2$
60-70	2	$31+f_1+f_2$
Total=40=n		

$f_1+5+9+12+f_2+3+2=40$

$f_1+f_2=40-31=9 \ ...(i)$

Median=32.5 Given

Since,Median class is 30-40

$l=30, h=10, \text{cf}=14+f_1, f=12$

Median $= l+\left(\dfrac{\frac{n}{2}-\text{cf}}{f}\right)\times h$

$$32.5=30\left[\dfrac{\frac{40}{2}-(14+f_1)}{12}\right]\times 10$$

$$2.5=\frac{10}{12(20-14-f_1)}$$

$3=6-f_1$

$f_1=3$

On putting in (i),

$f_1+f_2=9$

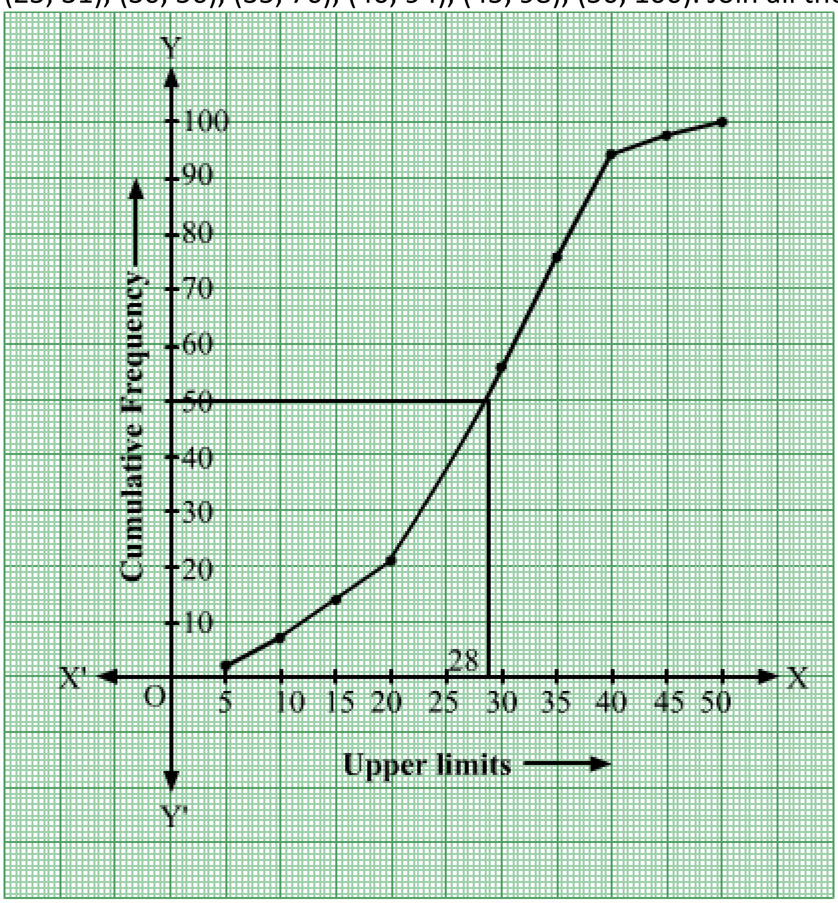
$f_2=9-3 \qquad [\because f_1=3]$

$=6$

OR

Marks	Number of students	Marks less than	Cumulative frequency
0-5	2	Less than 5	2
5-10	5	Less than 10	7
10-15	6	Less than 15	13
15-20	2	Less than 20	21
20-25	10	Less than 25	31
25-30	25	Less than 30	56
30-35	20	Less than 35	76
35-40	18	Less than 40	94
40-45	4	Less than 45	98
45-50	2	Less than 50	100

Let us now plot the points corresponding to the ordered pairs (5, 2), (10, 7), (15, 13), (20, 21), (25, 31), (30, 56), (35, 76), (40, 94), (45, 98), (50, 100). Join all the points by a smooth curve.



Locate $\frac{n}{2} = \frac{100}{2} = 50$ on Y-axis

From this point draw a line parallel to X-axis cutting the curve at a point. From this point, draw a perpendicular to X-axis. The point of intersection of perpendicular with the X-axis determines the median of the data. Therefore median = 28.8

X - CBSE BOARD - 2018

CODE (30/3)

Date: 28.03.2018 **Mathematics - Question Paper Solutions**

SECTION A

1. What is the value of $(\cos^2 67° - \sin^2 23°)$?

Ans. $\cos^2 67^0 - \sin^2 23^0$

as $\cos(90^0 - \theta) = \sin\theta$

Let $\theta = 23^0$

$\cos(90^0 - 23^0) = \sin 23^0$

$\cos 67^0 = \sin 23^0$

$\therefore \cos^2 67^0 = \sin^2 23^0$

$\therefore \cos^2 67^0 - \sin^2 23^0 = 0$

2. In an AP, if the common difference $(d) = -4$, and the seventh term (a_7) is 4, then find the first term.

Ans. $a_7 = 4$

$a + 6d = 4 \quad (as\ a_n = a + (n-1)d)$

but $d = -4$

$a + 6(-4) = 4$

$a + (-24) = 4$

$a = 4 + 24 = 28$

Therefore first term $a = 28$

3. Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then find $\frac{\text{ar } \Delta ABC}{\text{ar } \Delta PQR}$.

Ans. $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$ (Ratio of area of similar triangle is equal to square of their praportional sides)

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

4. What is the HCF of smallest prime number and the smallest composite number ?

Ans. Smallest prime number is 2.

Smallest composite number is 4

Therefore HCF is 2.

5. Find the distance of a point $P(x, y)$ from the origin.

Ans. Using distance formual

$$\ell(OP) = \sqrt{(x-0)^2 + (y-0)^2}$$

$$\ell(OP) = \sqrt{x^2 + y^2}$$

6. If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k.

Ans. $\because$ x = 3 is one of the root of $x^2 - 2kx - 6 = 0$

$$(3)^2 - 2k(3) - 6 = 0$$

$$9 - 6k - 6 = 0$$

$$3 - 6k = 0$$

$$3 = 6k$$

$$k = \frac{3}{6} = \frac{1}{2}$$

SECTION B

7. Two different dice are tossed together. Find the probability :

(i) of getting a doublet

(ii) of getting a sum 10, of the numbers on the two dice.

Ans. Sample space $= S = \{(1,1)(1,2).......,(6,6)\}$

$n(s) = 36$

i) A = getting a doublet

A = {(1, 1), (2, 2) ...., (6, 6)}

$n(A) = 6$

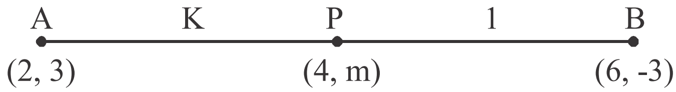
$\therefore \ P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

ii) B = getting sum of numbers as 10.
B = {(6, 4), (4, 6), (5, 5)}
$n(B) = 3$

$\therefore \ P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

8.Find the ratio in which $P(4, m)$ divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence find m.

Ans. Suppose the point P(4, m) divides the line segment joining the points A(2, 3) and B(6, -3) in the ratio K : 1.



Co-ordinates of point $P \equiv \left(\frac{6K+2}{K+1}, \frac{-3K+3}{K+1} \right)$

But the co-ordinates of point P are given as (4, m)

$\frac{6K+2}{K+1} \Rightarrow 4$ ......(1) and

$\frac{-3K+3}{K+1} = m$ ......(2)

$6K + 2 = 4K + 4$
$2K = 2$
$K = 1$

Putting K = 1 in eq. (2)

$\frac{-3(1)+3}{1+1} = m$

$\therefore \ m = 0$

Ratio is 1 : 1 and m = 0
i.e. P is the mid point of AB

9. An integer is chosen at random between 1 and 100. Find the probability that it is :

(i) divisible by 8

(ii) not divisible by 8

Ans. An integer is chosen at random from 1 to 100
Therefore n(S) = 100
(i) Let A be the event that number chosen is divisible by 8

$\therefore \ A = \{8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96\}$

$\therefore \; n(A) = 12$

Now, P (that number is divisible by 8) $= P(A) = \frac{n(A)}{n(S)}$

$$= \frac{12}{100} = \frac{6}{50} = \frac{3}{25}$$

$$\boxed{P(A) = \frac{3}{25}}$$

(ii) Let 'A' be the event that number is not divisible by 8.

$\therefore \; P(A') = 1 - P(A)$

$= 1 - \frac{3}{25}$ $\qquad \boxed{P(A') = \frac{22}{25}}$

10. In figure.1, ABCD is a rectangle. Find the values of x and y.

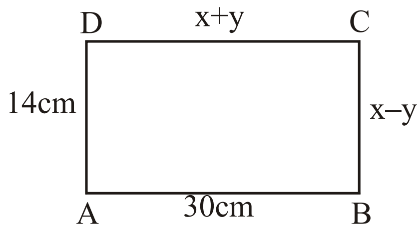


Figure 1

Ans. Since it is a rectangle

$\ell(AB) = \ell(CD)$

$x + y = 30$ ...(i)

$\ell(AD) = \ell(BC)$

$x - y = 14$ ...(ii)

Adding (1) and (2), we get

$2x = 44$

$x = 22$

Putting $x = 22$ in equation (i)

$22 - y = 14 \Rightarrow 22 - 14 = y$

$\therefore \; y = 8$

$\therefore \; x = 22$ and $y = 8$

11. Find the sum of first 8 multiples of 3.

Ans. First 8 multiples of 3 are

3, 6, 9, 12, 15, 18, 21, 24

The above sequence is an A.P.

$a = 3$, $d = 3$ and last term $l = 24$

$$S_n = \frac{n}{2}(a+l) = \frac{8}{2}[3+24] = 4(27)$$

$$S_n = 108$$

12. Given that $\sqrt{2}$ is irrational, prove that $\left(5+3\sqrt{2}\right)$ is an irrational number.

Ans. Let us assume that $\left(5+3\sqrt{2}\right)$ is rational. Then there exist co-prime positive integers a and b such that

$$5+3\sqrt{2} = \frac{a}{b}$$

$$3\sqrt{2} = \frac{a}{b} - 5$$

$$\sqrt{2} = \frac{a-5b}{3b}$$

$\Rightarrow \sqrt{2}$ is rational. [$\because$ a, b are integers , $\because \frac{a-5b}{3b}$ is rational].

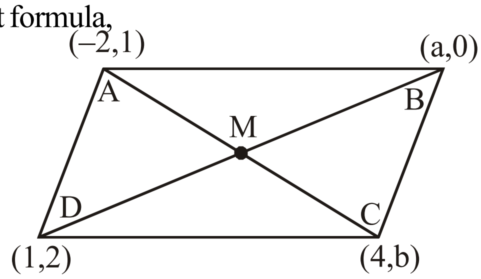
This contradicts the fact that $\sqrt{2}$ is irrational.
So our assumption is incorrect.

Hence, $\left(5+3\sqrt{2}\right)$ is an irrational number.

SECTION C

13. If $A(-2, 1)$, $B(a, 0)$, $C(4, b)$ and $D(1, 2)$ are the vertices of a parallelogram $ABCD$, find the values of a and b. Hence find the lengths of its sides.

Ans. M is midpoint of AC and BD using midpoint formula,



$$\left(\frac{-2+4}{2}, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{2+0}{2}\right)$$

$$\left(\frac{2}{2}, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{2}{2}\right)$$

$\therefore\ \frac{2}{2}=\frac{a+1}{2}\Rightarrow a+1=2\Rightarrow a=1$

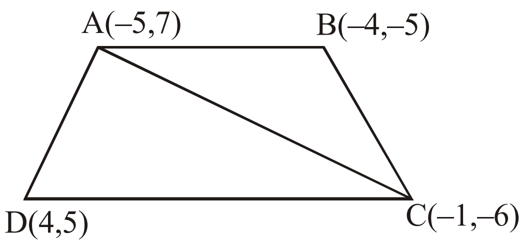
and $\frac{1+b}{2}=\frac{2}{2}\Rightarrow 1+b=2\Rightarrow b=1$

OR

If $A(-5, 7)$, $B(-4, -5)$, $C(-1, -6)$ and $D(4, 5)$ are the vertices of quadrilateral, find the area of the quadrilateral $ABCD$.

Ans. $A(\Delta ABC)=\frac{1}{2}\left(x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right)$

If $A=\left(x_1, y_1\right), B=\left(x_2, y_2\right), C=\left(x_3, y_3\right)$ are vertices of ΔABC.



$A(\square ABCD)=A(\Delta ABC)+A(\Delta ADC)$ ....(i)

$A(\square ABC)=\frac{1}{2}\left[-5(-5+6)-4(-6-7)-1(7+5)\right]$

$=\frac{1}{2}[-5+52-12]$

$=\frac{1}{2}[35]$

$=\frac{35}{2}$ Sq.units

$A(\Delta ADC)=\frac{1}{2}\left[-5(5+6)+4(-6-7)-1(7-5)\right]$

$=\frac{1}{2}[-55-52-2]$

$=\frac{-109}{2}$

$\because$ Area cannot be negative.

$\therefore\ A(\Delta ADC)=\frac{109}{2}$ sq.units

$$\therefore\ A(\square ABCD) = \frac{35}{2} + \frac{109}{2} = \frac{144}{2} = 72 \text{ sq. units}$$

14. Find all zeroes of the polynomial $(2x^4 - 9x^3 + 5x^2 + 3x - 1)$ if two of its zeroes are $(2+\sqrt{3})$ and $(2-\sqrt{3})$.

Ans. It is given that $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are two zeros of $f(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

$$\{x-(2+\sqrt{3})\}\{x-(2-\sqrt{3})\} = (x-2-\sqrt{3})(x-2+\sqrt{3})$$

$$= (x-2)^2 - (\sqrt{3})^2$$

$$= x^2 - 4x + 1$$

$\therefore\ (x^2 - 4x + 1)$ is a factor of $f(x)$

$$\begin{array}{r|l}
 & 2x^2 - x - 1 \\
x^2 - 4x + 1 & 2x^4 - 9x^3 + 5x^2 + 3x - 1 \\
 & 2x^4 - 8x^3 + 2x^2 \\
 & (-)\quad (+)\quad (-) \\ \hline
 & -x^3 + 3x^2 + 3x - 1 \\
 & -x^3 + 4x^2 - x \\
 & (+)\ (-)\ (+) \\ \hline
 & -x^2 + 4x - 1 \\
 & -x^2 + 4x - 1 \\
 & (+)(-)(+) \\ \hline
 & 0 \\ \hline
\end{array}$$

Let us now divide $f(x)$ by $x^2 - 4x + 1$

We have,

$$\therefore\ f(x) = (x^2 - 4x + 1)(2x^2 - x - 1)$$

Hence, other two zeros of f(x) are the zeros of the polynomial $2x^2 - x - 1$

We have,

$$2x^2 - x - 1 = 2x^2 - 2x + x - 1$$

$$= 2x(x-1) + 1(x-1)$$

$= (2x+1)(x-1)$

$f(x) = (x-2-\sqrt{3})(x-2+\sqrt{3})(2x+1)(x-1)$

Hence, the other two zeros are $-\frac{1}{2}$ and 1.

15. Find *HCF* and *LCM* of 404 and 96 and verify that *HCF* × *LCM* = Product of the two given numbers.

Ans. Using the factor tree for the prime factorization of
404 and 96, we have
$404 = 2^2 \times 101$ and $96 = 2^5 \times 3$
To find the HCF, we list common prime factors and their smallest exponent in 404 and 96 as under :
Common prime factor = 2, Least exponent = 2

$\therefore$ HCF $= 2^2 = 4$

To find the LCM, we list all prime factors of 404 and 96 and their greatest exponent as follows :

Prime factors of 404 and 96	**Greatest Exponent**
2	5
3	1
101	1

$\therefore$ LCM $= 2^5 \times 3^1 \times 101^1$

$= 2^5 \times 3^1 \times 101^1$

$= 9696$

Now,
HCF × LCM = 9696 × 4 = 38784
Product of two numbers = 404 × 96 = 38784
Therefore HCF × LCM = Product of two numbers.

16. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Ans. Given AP and AQ are two tangents from a point A to a circle C (O, r)

To prove AP = AQ

Construction join OP, OQ and OA

Proof In order to prove that $AP = AQ$, we shall first prove that $\Delta OPA = \Delta OQA$

since a tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore OP \perp AP$ and $OQ \perp AQ$

$\Rightarrow \angle OPA = \angle OQA = 90^0$ ....(i)

Now, in right triangles OPA and OQA, we have

$OP = OQ$ [Radii of a circle]

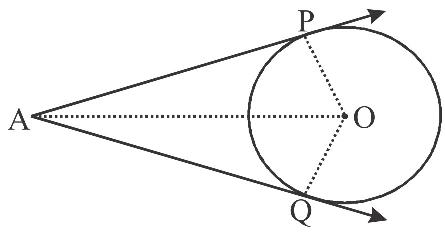
$\angle OPA = \angle OQA$ [from (i)]

and OA = OA

so, by RHS – criterion of congruence, we get

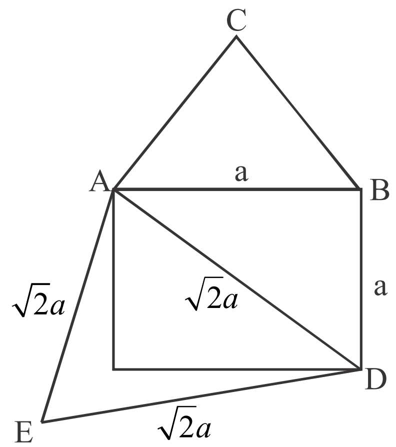
$\Delta OPA \cong \Delta OQA$

$\Rightarrow AP = AQ$



17. Prove that the area of an equilateral traingle described on one side of the square is equal to half the area of the equilateral triangle described on one of its diagonal.

Ans. Let a be the side of square.



$$A(\Delta ABC) = \frac{\sqrt{3}}{4} \times side^2 - \frac{\sqrt{3}}{4} \times a^2 .....(1)$$

using pythagoras theorem

$$AD^2 = AB^2 + BD^2 = a^2 + a^2 = 2a^2$$

$$AD = \sqrt{2}a$$

$$\therefore A(\Delta ADE) = \frac{\sqrt{3}}{4} \times \left(\sqrt{2}a\right)^2 = \frac{\sqrt{3}}{4} \times 2a^2 .....(2)$$

$$\frac{A(\Delta ABC)}{A(\Delta ADE)} = \frac{\sqrt{3}/4 \times a^2}{\sqrt{3}/4 \times 2a^2}$$

$$A(\Delta ABC) = \frac{1}{2} A(\Delta ADE)$$

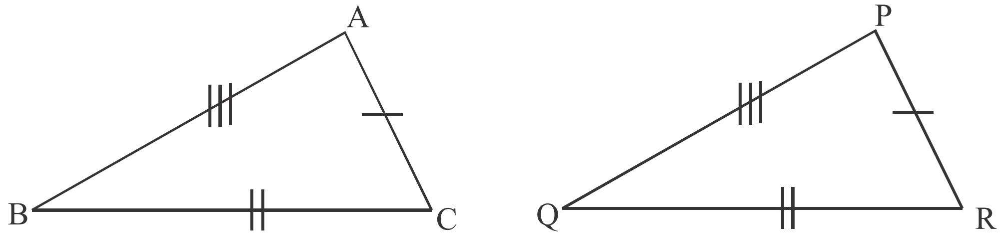
Area of equivalent triangle describes on

$$= \frac{1}{2}(\textit{area of equilateral } \Delta \textit{ described an one of its diagonal})$$

OR

If the area of two similar triangles are equal, prove that they are congruent.

Ans.



Let ΔABC is ΔPQR

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

Given that $(\Delta ABC) = A(\Delta PQR)$

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = 1$$

$$1 = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

$\therefore AB = PQ$

$BC = QR$

$AC = PR$

Hence corresponding sides are equal.

$\therefore \Delta ABC \cong \Delta PQR$ (SSS rule)

hence proved.

18. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.

Ans. Let the usual speed of the plane be x km/hr

Time taken to cover 1500 km with usual speed $= \frac{1500}{x} hrs$

Time taken to cover 1500 km with speed of $(x+100)\,km/hr = \frac{1500}{x+100}$ hrs.

$$\therefore \frac{1500}{x} = \frac{1500}{x+100} + \frac{1}{2}$$

$$\frac{1500}{x} - \frac{1500}{x+100} = \frac{1}{2}$$

$$1500\left(\frac{x+100-x}{x(x+100)}\right) = \frac{1}{2}$$

$$150000 \times 2 = x(x+100)$$

$x^2 + 100x - 300000 = 0$

$x^2 + 100x - 300000 = 0$

$x = -600$ or $x = 500$

But speed can't be negative

Hence usual speed 500 *km/hr*.

19. The table below shown the salaries of 280 persons:

Salary (In thousand □)	No. of Person
$5-10$	49
$10-15$	133
$15-20$	63
$20-25$	15
$25-30$	6
$30-35$	7
$35-40$	4
$40-45$	2
$45-50$	1

Calculate the median salary of the data.

Ans.

Class	Frequency	Cumulative Frequency
5 - 10	49	49
10 - 15	133	182
15 - 20	63	245
20 - 25	15	260
25 - 30	6	266
30 – 35	7	273
35 - 40	4	277
40 – 45	2	279
45 – 50	1	280

Let $N =$ total frequency

$\therefore$ we have $N = 280$

$$\therefore \frac{N}{2} = \frac{280}{2} = 140$$

The cumulative frequency just greater than $\frac{N}{2}$ is 182 and the corresponding class is $10-15$

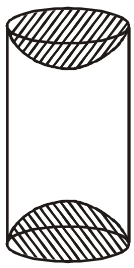
Thus, $10-15$ is the median class such that

$l=10, f=133, F=49$ and $h=5$

$$\text{Median} = l+\frac{\frac{N}{2}-F}{f}\times h = 10+\left(\frac{140-49}{133}\right)\times 5$$

$=13.42$

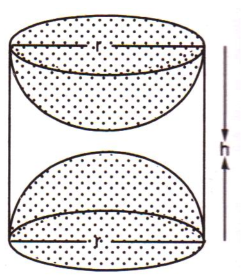
20. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 2. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article.



Ans. Let r be the radius of the base of the cylinder and h be its height. Then, total surface area of the article

= Curved surface area of the cylinder + 2 (surface area of a hemisphere)

$= 2\pi rh + 2\left(2\pi r^2\right)$



$= 2\pi r\left(h+2r\right)$

$= 2\times\frac{22}{7}\times 3.5\left(10+2\times 3.5\right) cm^2$

$=22\times 17\, cm^2$ $=374\, cm^2$

OR

A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

Ans. Given

Base diameter = 24 m

Base radius = 12 m

Height = 3.5 m

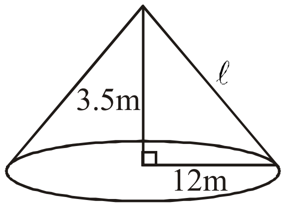
$\text{Volume} = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5$$

$$= 22 \times 4 \times 12 \times 0.5$$

$$= 264 \times 2$$

$$= 528 \text{ cubic meter}$$



$$\therefore\ \ell^2 = 12^2 + 3.5^2 = 144 + 12.25$$

$$\ell^2 = 156.25$$

$$\ell = \sqrt{156.25} = 12.5\,m$$

Curved surface area $= \pi r \ell$

$$= \frac{22}{7} \times 12 \times 12.5 = \frac{150 \times 22}{7} = 471.428 \text{ sq.meter}$$

21. Find the area of the shaded region in Fig. 3, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square of side 12 cm, [Use $\pi = 3.14$]

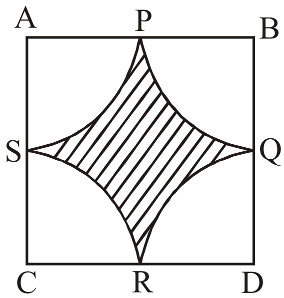


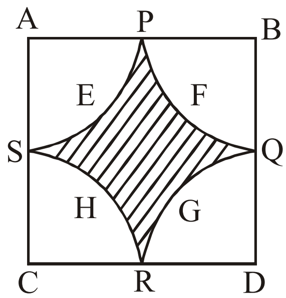
Fig.–3

Ans. Given that ABCD is a square & P,Q,R & S are the mid points of AB,BC,CD & DA respectively

& $AB = 12\,cm$

$\Rightarrow AP = 6cm$ $\quad\quad \{P \text{ bisects } AB\}$

area of the shaded region = Area of square ABCD – (Area of sector APEC + Area of sector PFQB + ... Area of sector RGQC + Area of sector RHSD)



$$= 12^2 - \left(\frac{\pi(6^2)}{4} + \frac{\pi 6^2}{4} + \frac{\pi 6^2}{4} + \frac{\pi 6^2}{4} \right)$$

$$= 12^2 - \pi \times 36$$

$=144-113.04$

$=30.96\,cm^2$

22. If $4\tan\theta=3$, evaluate $\left(\frac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}\right)$

Ans. Given that,

$\tan\theta=\frac{3}{4}$ $\quad\quad\therefore\ \tan^2\theta=\frac{9}{16}$

we know that,

$\sec^2\theta=1+\tan^2\theta$

$\therefore\ \sec^2\theta=1+\frac{9}{16}=\frac{25}{16}$

$\therefore\ \cos^2\theta=\frac{16}{25}$

$\therefore\ \cos\theta=\frac{4}{5}$

we know that,

$\sin^2\theta=1-\cos^2\theta$

$\therefore\ \sin^2\theta=1-\frac{16}{25}=\frac{9}{25}$

$\Rightarrow \sin\theta=\frac{3}{5}$

Now,

$$\left(\frac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}\right)=\left(\frac{4\times\left(\frac{3}{5}\right)-\frac{4}{5}+1}{4\times\left(\frac{3}{5}\right)+\left(\frac{4}{5}\right)-1}\right)$$

$$=\frac{12-4+5}{12+4-5}$$

$$=\frac{13}{11}$$

OR

If $\tan 2A = \cot(A - 18°)$, where $2A$ is an angle, find the value of A.

Ans. Given that,

$$\tan 2A = \cot\left(A - 18^0\right)$$

Now,

we know that,

$$\tan\theta = \cot\left(90^0 - \theta\right)$$

$$\therefore \cot\left(90^0 - 2A\right) = \cot\left(A - 18^0\right)$$

$$\therefore 90^0 - 2A = A - 18^0$$

$$\therefore 3A = 108^0$$

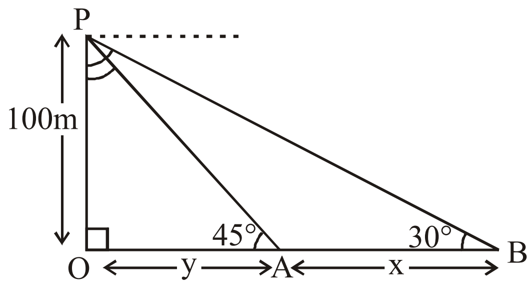
$$\therefore A = \frac{108^0}{3} = 36^0$$

$$\therefore A = 36^0$$

SECTION D

23. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behing the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3} = 1.732$]

Ans. Let ships are at distance x from each other



In ΔAPO

$$\tan 45° = \frac{100}{y} = 1 \qquad \therefore\ y = 100\,m \qquad ...(i)$$

In ΔPOB

$$\tan 30° = \frac{OP}{OB} = \frac{100}{x+y} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3} = \frac{x+y}{100}$$

$$x + y = 100\sqrt{3} \qquad ...(ii)$$

$$x = 100\sqrt{3} - y = 100\sqrt{3} - 100 = 100\left(\sqrt{3} - 1\right)$$

$\therefore\ x = 100(1.732 - 1)$

$= 100 \times 0.732$

$= 73.2\,m$

$\therefore$ Ships are 73.2 meters apart.

24. The diameters of the lower and upper ends of a bucket in the form of a frustum f a cone are 10 cm and 30 cm respectively. If its height is 24 cm, find:

(i) The area of the metal sheet used to make the bucket.

(ii) Why we should avoid the bucket made by ordinary plastic? [Use $\pi = 3.14$]

Ans. Let $r_1 = 5\,cm$ and $r_2 = 15\,cm$ are radii of lower and upper circular faces.

Metal sheet required = Area of curved surface + Area of Base

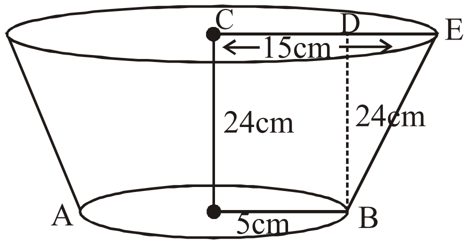
$$= \pi\,(r_1 + r_2)\,\ell + \pi r_1^2 \qquad \text{...(i)}$$

From diagram

$AB = CD = 5$ cm

$DE = 15 - 5 = 10$ cm

and $BD = 24$ cm



$\therefore\ BE^2 = BD^2 + DE^2$

$= 576 + 100$

$BE^2 = 676$

$BE = 26\,cm = \ell$

Metal required $= \pi(5+15)26 + \pi(5)^2$

$= \pi \times 20 \times 26 + \pi \times 25$

$= 5\pi(4 \times 26 + 5)$

$= 5\pi(109)$

$= 5 \times \dfrac{22}{7} \times 109$

$= 1712.85\,cm^2$

There is a chance of breakdown due to stress an ordinary plastic.

25. Prove that $\frac{\sin A-2\sin^3 A}{2\cos^3 A-\cos A}=\tan A.$

Ans. To prove

$$\frac{\sin A-2\sin^3 A}{2\cos^3 A-\cos A}=\tan A$$

$$L.H.S=\frac{\sin A\left(1-2\sin^2 A\right)}{\cos A\left(2\cos^2 A-1\right)}$$

We know that, $\sin^2 A+\cos^2 A=1$

$$=\frac{\sin A}{\cos A}\left(\frac{\left(\sin^2 A+\cos^2 A-2\sin^2 A\right)}{\left(2\cos^2 A-\sin^2 A-\cos^2 A\right)}\right)$$

$$=\tan A\left(\frac{\cos^2 A-\sin^2 A}{\cos^2 A-\sin^2 A}\right)$$

$=\tan A$

$=R.H.S.$ hence proved.

26. The mean of the following distribution is 18. Find the frequency f of the class 19-21.

Class	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

Ans.

Class	Mid values x_i	Frequence f_i	$d_i=x_i-18$	$u_i=\frac{x_i-18}{2}$	f_iu_i
11 – 13	12	3	-6	-3	-9
13 - 15	14	6	-4	-2	-12
15 - 17	16	9	-2	-1	-9
17 - 19	18	13	0	0	0
19 - 21	20	f	2	1	f
21 - 23	22	5	4	2	10
23 - 25	24	4	6	3	12
		$\sum f_i=40+f$			

$\sum f_iu_i=f-8$

we have

$h=2; A=18, N=40+f, \sum f_i u_i = f-8 \quad \overline{X}=18$

$\therefore\ Mean = A + h\left\{\frac{1}{N}\sum f_i u_i\right\}$

$18 = 18 + 2\left\{\frac{1}{40+f}(f-8)\right\}$

$\frac{2(f-8)}{40+f} = 0$

$f-8=0$

$f=8$

OR

The following distribution gives the daily income of 50 workers of a factory :

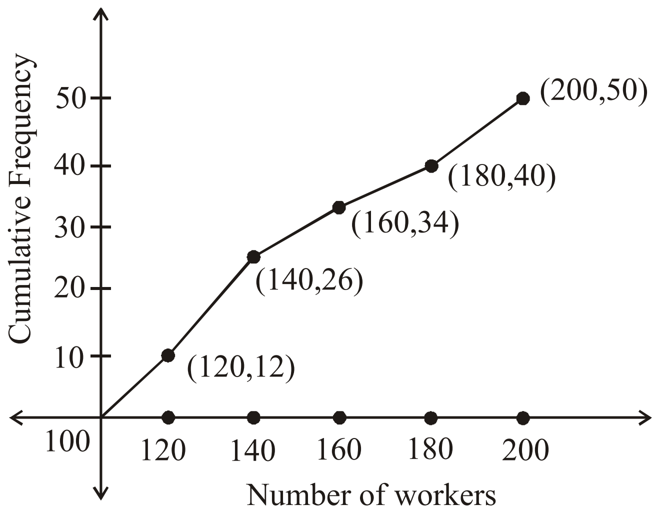
Daily Income(In)	100-120	120-140	140-160	160-180	180-200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

Ans.

Daily income	Frequency	Income less than	Cumulative frequency
100-120	12	120	12
120-140	14	140	26
140-160	8	160	34
160-180	6	180	40
180-200	10	200	50

Other than the given class intervals, we assum a class interval 80-100 with zero frequency.



27. A motor boat whose speed is 18 km/hr in still water 1 hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans. Let the speed of stream be $x\ km/hr$

Now, for upstream: speed $=(18-x)\,km/hr$

$\therefore$ time taken $=\left(\dfrac{24}{18-x}\right)hr$

Now, for downstream: speed $=(18+x)\,km/hr$

$\therefore$ time taken $=\left(\dfrac{24}{18+x}\right)hr$

Given that,

$$\frac{24}{18-x}=\frac{24}{18+x}+1$$

$$-1=\frac{24}{18+x}-\frac{24}{18-x}$$

$$-1=\frac{24\left[(18-x)-(18+x)\right]}{(18)^2-x^2}$$

$$-1=\frac{24\left[-2x\right]}{324-x^2}$$

$$-324+x^2=-48x$$

$$x^2+48x-324=0$$

$$x^2+54x-6x-324=0$$

$$(x+54)(x-6)=0$$

$$x=-54 \quad \text{or} \quad x=6$$

$x=-54\,km/hr$ (not possible)

Therefore, speed of the stream = 6 km/hr.

OR

A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. It it takes 3 hours to complete total journey, what is the original average speed ?

Ans. Let x be the original average speed of the train for 63 km.

Then, (x + 6) will be the new average speed for remaining 72 km.

Total time taken to complete the journey is 3 hrs.

$$\therefore \frac{63}{x}+\frac{72}{(x+6)}=3$$

$$\left(\because \text{ Time}=\frac{\text{Distance}}{\text{Speed}}\right)$$

$$\therefore \frac{63x+378+72x}{x(x+6)}=3$$

$$\Rightarrow 135x+378=3x^2+18x$$

$$\Rightarrow x^2-39x-126=0$$

$$\Rightarrow (x-42)(x+3)=0$$

$$\Rightarrow \boxed{x=42} \text{ OR } \boxed{x=-3}$$

Since speed can not be negative.
Therefore x = 42 km/hr.

28. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7 : 15. Find the numbers.

Ans. Let the numbers be (a, –3d), (a – d), (a + d) and (a + 3d)

$$\therefore (a-3d)+(a-d)+(a+d)+(a+3d)=32$$

$$\Rightarrow 4a=32$$

$$a=8$$

$$\text{Also, } \frac{(a-3d)(a+3d)}{(a-d)(a+d)}=\frac{7}{15}$$

$$\Rightarrow 15a^2-135d^2=7a^2-7d^2$$

$$\Rightarrow 8a^2=128d^2$$

$$d^2=\frac{8a^2}{128}=\frac{8\times8\times8}{128}$$

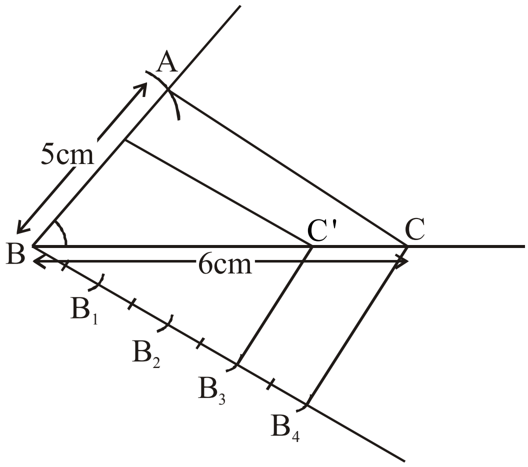
$$d^2=4$$

$$d=\pm2$$

If d =2 numbers are : 2, 6,10, 14
If $d=-2$ numbers are 14,10,16,2

29. Draw a triangle ABC with $BC = 6$ cm, $AB = 5$ cm and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the ΔABC.

Ans. STEPS OF CONSTRUCTION :



(i) Draw a line segment $BC = 6$ cm, draw a ray BX making 60° with BC.

(ii) Draw an arc with radius 5 cm from B so that it cuts BX at A.

(iii) Now join AC to form ΔABC.

(iv) Draw a ray by making an acute angle with NC opposite to vertex A.

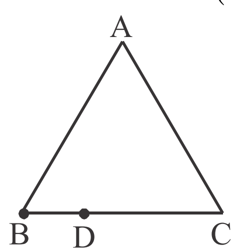
(v) Locate 4 points B_1, B_2, B_3, B_4 on by such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

(vi) Join B_4C and now draw a line from B_3 parallel to B_4C so that it cuts BC at C'.

(vii) From C' draw a line parallel to AC and cuts AB at A'.

(viii) $\Delta A'BC'$ is the required triangle.

30. In an equilateral ΔABC, is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9(AD)^2 = 7(AB)^2$.



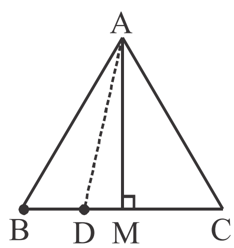
Ans. Let the each side of ΔABC be 'a' unit

$\therefore$ $BD = \frac{a}{3}$

to prove : $9(AD)^2 = 7(AB)^2$

construction : Draw $AM \perp BC$:

$DM = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$



$\therefore$ In ΔABM

$AB^2 = BM^2 + AM^2$ ....(1)

and in ΔADM

$$AD^2 = AM^2 + DM^2 \qquad ....(2)$$

In ΔABM, $\sin 60^0 = \dfrac{AM}{AB}$

$\Rightarrow AM = AB \sin 60^0$

$= a\dfrac{\sqrt{3}}{2}$

Now, taking $9(AD)^2$

$9(AM^2 + DM^2)$

$9\left(\left(\dfrac{a\sqrt{3}}{2}\right)^2 + \left(\dfrac{a}{6}\right)^2\right)$

$9\left[\dfrac{3a^2}{4} + \dfrac{a^2}{36}\right] = 9 \times \dfrac{28a^2}{36}$

$7(AB)^2 = 7a^2$

or

$\therefore 9(AD^2 =)7(AB^2)$ Hence proved.

OR

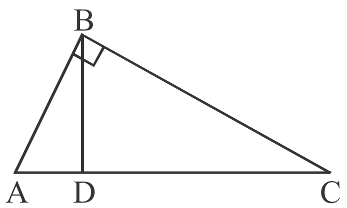
Prove that, in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Ans. Given : A right - angled triangle ABC in which $\angle B = 90^0$

To Prove : $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

i.e., $AC^2 = AB^2 + BC^2$

Construction from B draw $BD \perp AC$.



Proof : In triangle ADB and ABC, we have

$\angle ADB = \angle ABC$ [Each equal to 90^o]

and, $\angle A = \angle A$ [Common]

So, by AA - similarity criterian, we have

$\Delta ADB \sim \Delta ABC$

$\Rightarrow \dfrac{AD}{AB} = \dfrac{AB}{AC}$ [$\because$ In similar triangles corresponding sides are proportional]

$\Rightarrow \quad AB^2 = AD \times AC$ ....(1)

In triangles BDC and ABC, we have

$\angle CDB = \angle ABC$ [Each equal to 90^o]

and, $\angle C = \angle C$ [Common]

So, by AA-similarity criterian, we have

$\Delta BDC \sim \Delta ABC$

$\Rightarrow \frac{DC}{BC} = \frac{BC}{AC}$ [$\because$ In similar triangles corresponding sides are proportional]

$\Rightarrow BC^2 = AC \times DC$ ......(2)

Adding equation (1) and (2), we get

$AB^2 + BC^2 = AD \times AC + AC \times DC$

$\Rightarrow AB^2 + BC^2 = AC(AD + DC)$

$\Rightarrow AB^2 + BC^2 = AC \times AC$

$\Rightarrow AB^2 + BC^2 = AC^2$

Hence, $AC^2 = AB^2 + BC^2$

CBSE-X-2017 EXAMINATION

MATHEMATICS

Paper & Solution

Time: 3 Hrs. Max. Marks: 90

General Instructions :

*(i) **All** questions are compulsory.*

*(ii) The question paper consists of **31** questions divided into four sections - A, B, C and D.*

*(iii) Section A contains **4** questions of **1** mark each. Section B contains **6** questions of **2** marks each, Section C contains **10** questions of **3** marks each and Section D contains **11** questions of **4** marks each.*

*(iv) Use of calculators is **not** permitted.*

SECTION – A

Question numbers 1 to 4 carry 1 mark each.

1. What is the common difference of an A.P. in which $a_{21} - a_7 = 84$?

Solution:

Given

$a_{21} - a_7 = 84$ ....(1)

In an A.P a_1, a_2, a_3, a_4 ....

$a_n = a_1 + (n-1)d$ d = common difference

$a_{21} = a_1 + 20d$ ....(2)

$a_7 = a_1 + 6d$ ....(3)

substituting (2) & (3) in (1)

$a_1 + 20d - a_1 - 6d = 84$

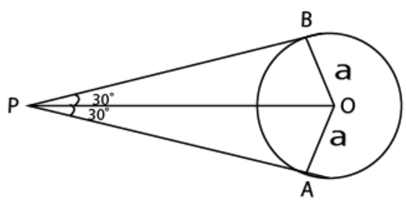
$14d = 84$

$d = 6$

$\therefore$ common difference = 6

2. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60 , then find the length of OP.

Solution:



Given that $\angle BPA = 60^\circ$

OB = OA = a [radii]

PA = PB [length of tangents Equal]

OP = OP

$\therefore \Delta PBO$ and ΔPAO are congruent. [By SSS test of congruency]

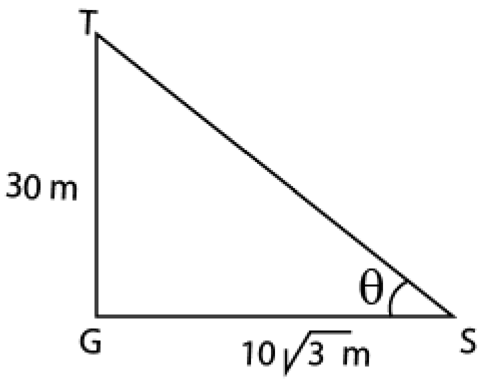
$\therefore \angle BPO = \angle OPA = \dfrac{60^\circ}{2} = 30^\circ$

In a $\Delta PBO \sin 30^\circ = \dfrac{a}{OP}$

OP = 2a units

3. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun ?

Solution:



Angle of equation of sun = $\angle GST = \theta$

Height of lower TG = 30m

Length of shadow GS = $10\sqrt{3}$ m

ΔTGS is a right angled triangle

$\therefore \tan\theta = \dfrac{30}{10\sqrt{3}}$

$\tan\theta = \sqrt{3}$

$\theta = 60^\circ$

4. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0·18. What is the number of rotten apples in the heap ?

Solution:

Probability of selecting rotten apple = $\frac{\text{Number of rotten apples}}{\text{Total number of apples}}$

$\therefore 0.18 = \frac{\text{No. of rotten apples}}{900}$

No. of rotten apples = $900 \times 0.18 = 162$

SECTION B

Question numbers 5 to 10 carry 2 marks each.

5. Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.

Solution:

Given Quadriatic Equation $Px^2 – 14x + 8 = 0$

Also, one root is 6 times the other

Lets say one root = x

Second root = 6x

From the equation : Sum of the roots = $+\frac{14}{P}$

Product of roots = $\frac{8}{P}$

$\therefore x + 6x = \frac{14}{P}.$

$x = \frac{2}{P}$

$\Rightarrow 6x^2 = \frac{8}{P}$

$\Rightarrow 6\left(\frac{2}{P}\right)^2 = \frac{8}{P}$

$\frac{6 \times 4}{P^2} = \frac{8}{P}$

$P = 3$

6. Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ...$ is the first negative term ?

Solution:

Given progression 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ...$

This is an Arithmetic progression because

Common difference (d) = $19\frac{1}{4} - 20 = 18\frac{1}{2} - 19\frac{1}{4} = ...$

$d = \frac{-3}{4}$

Any n^{th} term a_n = 20 + $(n-1)\left(\frac{-3}{4}\right) = \frac{83-3n}{4}$

Any term an < 0 when 83 < 3n

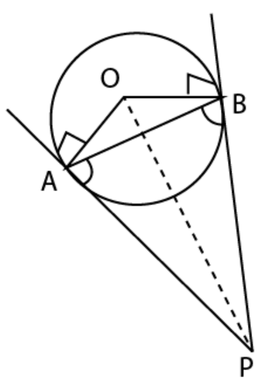
This is valid for n = 28 and 28^{th} term will be the first negative term.

7. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

Solution:

Need to prove that

$\angle BAP = \angle ABP$



AB is the chord

We know that OA = OB (radii)

$\angle OBP = \angle OAP = 90^\circ$

Join OP and OP = OP

By RHS congruency

$\Delta OBP \cong \Delta OAP$

$\therefore By\ CPCT \quad BP = AP$

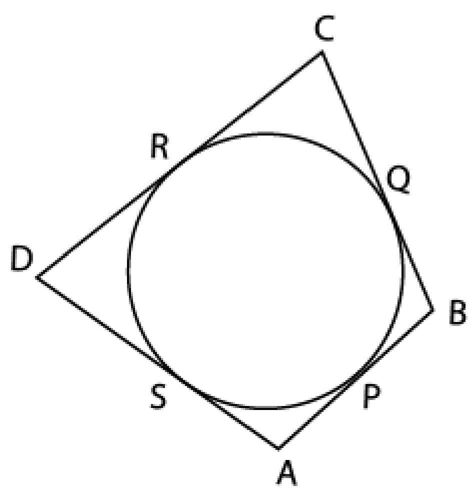
In $\Delta ABP \quad BP = AP$

Angles opposite to equal sides are equal

$\therefore \angle BAP = \angle ABP$

Hence proved.

8. A circle touches all the four sides of a quadrilateral ABCD. Prove that AB + CD = BC + DA
Solution:



ABCD is the Quadrilateral

Circle touches the sides at P, Q, R,S points

For the circle AS & AP are tangents

$\therefore AS = AP$...............(1)

In the similar way

BP = BQ........(2)

CQ = CR............(3)

RD = DS ....(4)

Now AB + CD = AP + PB + CR + RD

BC + AD = BQ + QC + DS + AS

Using (1), (2), (3), (4) in above equation BC + AD = BP + CR + RD + AP

$\therefore$ AB + CD = BC + AD

Hence proved

9. A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, -5) is the mid-point of PQ, then find the coordinates of P and Q.

Solution:

Equation of a line: $\frac{x}{a}+\frac{y}{b}=1$

Where a =x-intercept

b = y-intercept

Given that line intersects y-axis at P

$\therefore$ P lies on y-axis and P = (0, b)

Line intersects x-axis at Q

$\therefore$ Q lies on x-axis and Q = (a, 0)

Midpoint of PQ = (2, -5)

$\left(\frac{a}{2},\frac{b}{2}\right)=(2,-5)$

$\frac{a}{2}=2,\frac{b}{2}=-5$

$a=4\ \&\ b=-10$

$\therefore P=(0,-10)$

$Q=(4,0)$

10. If the distances of P(x, y) from A(5, 1) and B(-1, 5) are equal, then prove that 3x = 2y.

Solution:

Given that

PA = PB

P(x, y), A(5, 1), B(-1, 5)

$PA=\sqrt{(x-5)^2+(y-1)^2}$

$PB=\sqrt{(x+1)^2+(y-5)^2}$

$\sqrt{(x-5)^2+(y-1)^2}=\sqrt{(x+1)^2+(y-5)^2}$

Squaring on both sides

$x^2+25-10x+y^2-2y+1=x^2+2x+1+y^2-10y+25$

$-10x-2y=2x-10y$

$8y=12x$

$\therefore 3x=2y$

SECTION C

Question numbers 11 to 20 carry 3 marks each.

11. If ad $\neq$ bc, then prove that the equation
$(a^2 + b^2) x^2 + 2 (ac + bd) x + (c^2 + d^2) = 0$ has no real roots.

Solution:

Given $ad \neq bc$ for the equation $(a^2+b^2)x^2+2(ac+bd)x+(c^2+d^2)=0$

For this equation not to have real roots its discriminant < 0

$D = 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2)$

$D = 4a^2c^2 + 4b^2d^2 + 8acbd - 4a^2c^2 - 4b^2d^2 - 4b^2c^2 - 4a^2d^2$

$D=-4(a^2d^2+b^2c^2-2acbd)$

$D=-4(ad-bc)^2$

Given $ad \neq bc$

$\therefore D<0$

Quadratic equation has no real roots

12. The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P.

Solution:

First term (a) = 5

Last term (l) = 45

Sum of all the terms = 400

$400=\frac{n}{2}(a+l)$

$\frac{800}{50}=n$

$n=16$

No. of terms = 16

l = 45 (16^{th} term)

$a+(n-1)d=45$

$5+15d=45$

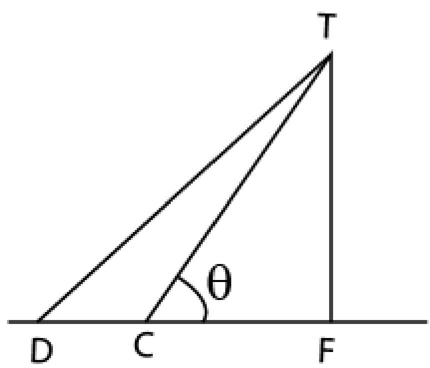
$d=\frac{40}{15}$

$d=\frac{8}{3}$

Common difference $=\frac{8}{3}$

13. On a straight line passing through the foot of a tower, two points C and D are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower.

Solution:



Given CF = 4m

DF = 16m

$\angle TCF+\angle TDF=90^\circ$

Lets say $\angle TCF=\theta$

$\angle TDF=90-\theta$

In a right angled triangle TCF

$\tan\theta=\frac{TF}{CF}=\frac{TF}{4}$

$TF=4\tan\theta$ ....(1)

In ΔTDF

$\tan(90-\theta)=\frac{TF}{16}$

$TF=16\cot\theta$..............(2)

Multiply (1) & (2)

$(TF)^2 = 64 \Rightarrow TF = 8mt$

$\Rightarrow$ Height of tower = 8mt

14. A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag.
Solution:

Bag contains 15 white balls

Lets say there are x black balls

Probability of drawing a black ball

P(B) = $\dfrac{x}{15+x}$

Probability of drawing a white ball

P(W) = $\dfrac{15}{15+x}$

Given that P(B) = 3P(W)

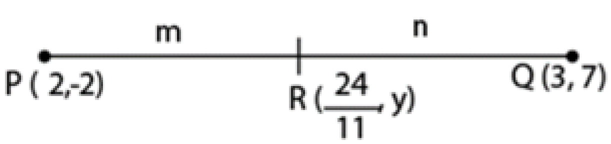
$\therefore \dfrac{x}{15+x} = \dfrac{3\times 15}{15+x}$

$x = 45$

No. of black balls = 45

15. In what ratio does the point $\left(\dfrac{24}{11}, y\right)$ divide the line segment joining the points P(2, -2) and Q(3, 7) ?

Also find the value of y.
Solution:



Lets say ratio is m + n

Then

$$\left(\frac{24}{11}, y\right) = \left(\frac{3m+2n}{m+n}, \frac{7m-2n}{m+n}\right)$$

$$\frac{24}{11} = \frac{3m+2n}{m+n}, y = \frac{7m-2n}{m+n}$$

$$\therefore 24(m+n) = 11(3m+2n)$$

$$24m + 24n = 33m + 22n$$

$$2n = 9m$$

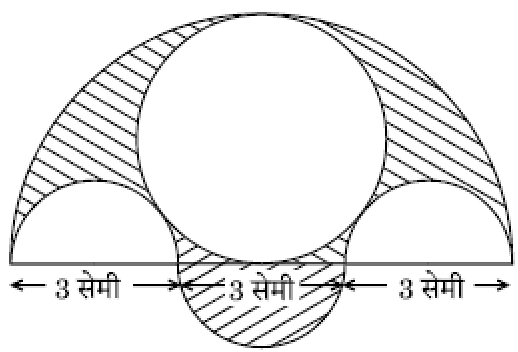
$$\therefore \frac{m}{n} = \frac{2}{9} \Rightarrow \text{ratio} = 2:9$$

$$m = 2, n = 9$$

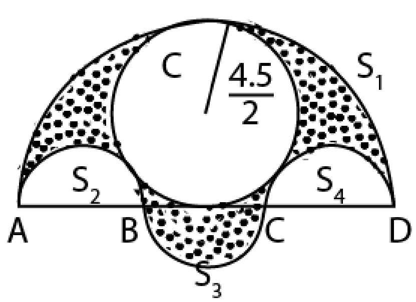
$$y = \frac{7\times 2 - 2\times 9}{11}$$

$$y = \frac{-4}{11}$$

16. Three semicircles each of diameter 3 cm, a circle of diameter 4·5 cm and a semicircle of radius 4·5 cm are drawn in the given figure. Find the area of the shaded region.



Solution:



Given that AB = BC = CD = 3 cm

Circle c has diameter = 4.5 cm

Semicircle s_1 has diameter = 9cm

Area of shaded region

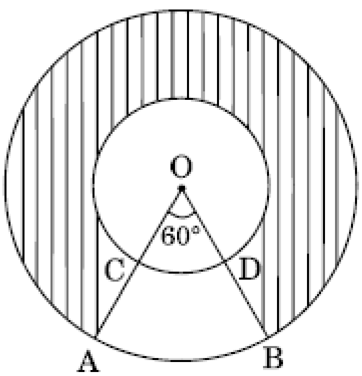
= Area of s_1 – Area of (s_2+s_4) – Area of c + Area of s_3

Area of shaded region

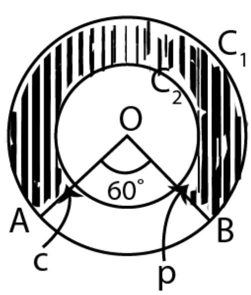
$$=\frac{\pi}{2}\left(\frac{9}{2}\right)^2-\frac{\pi}{2}\left(\frac{3}{2}\right)^2-\frac{\pi}{2}\left(\frac{3}{2}\right)^2-\pi\left(\frac{4.5}{2}\right)^2+\frac{\pi}{2}\left(\frac{3}{2}\right)^2$$
$$=\frac{\pi\times 81}{16}-\frac{\pi\times 9}{8}$$
$$=12.36\,cm^2$$

17. In the given figure, two concentric circles with centre O have radii 21 cm and 42 cm. If $\angle$AOB = 60 , find the area of the shaded region.

$$\left[\text{Use } \pi=\frac{22}{7}\right]$$



Solution:



Given OC = OD = 21 cm

OA = OB = 42 cm

Area of ACDB region

= Area of sector OAB – Area sector OCD

$$=\frac{60^\circ}{360^\circ}\times\pi(42)^2-\frac{60^\circ}{360^\circ}\times\pi\times(21)^2$$

Area of ACDB region = $\frac{1}{6}\times\frac{22}{7}\times 21\times 63$

$= 11 \times 63 = 693\ cm^2$

Area of shaded region= area of c_1 – Area of c_2 – Area of ACDB region

$= \pi(42)^2 - \pi(21)^2 - 693$

$= \frac{22}{7} \times 21 \times 63 - 693$

$= 3,465\ cm^2$

18. Water in a canal, 5·4 m wide and 1·8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation ?

Solution:

Given canal width = 5.4 mt

Depth = 1.8 mt

Water flow speed = 25 km/hr

Distance covered by water in 40 minutes

$= \frac{25 \times 40}{60}$

$= \frac{50}{3} km$

Volume of water flows through pipe = $\frac{50}{3} \times 5.4 \times 1.8 \times 1000$

$= 162 \times 10^3 m^3$

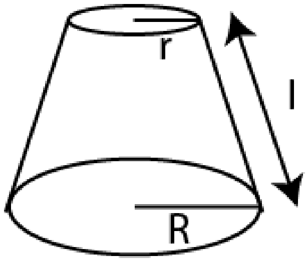
Area irrigate with 10 cm of water standing

$= \frac{162 \times 10^3}{10 \times 10^{-2}}$

$= 162 \times 10^4 m^2$

19. The slant height of a frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Solution:



Given:

$2\pi r = 6cm$

$2\pi R = 18cm$

$l = 4cm$

Curved surface area of frustum of a cone

$= \frac{1}{2}(2\pi R + 2\pi r) \times l$

$= \frac{1}{2}(6 + 18)4$

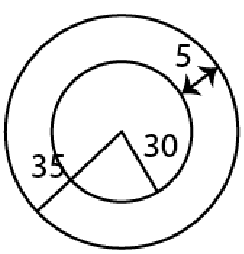
$= 48cm^2$

20. The dimensions of a solid iron cuboid are 4·4 m × 2·6 m × 1·0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.

Solution:

Volume of cuboid = $4.4 \times 2.6 \times 1$

$11.44m^3$



length = l

Inner radius = 30 cm

outer radius = 35 cm

Volume of cuboid = volume of cylindrical pipe

$11.44=\frac{\pi\times l\times(35^2-30^2)}{100\times100\times100}$

$l=10.205\times10^4\ cm$

$l=102.05\ km$

SECTION D

Question numbers 21 to 31 carry 4 marks each.

21. Solve for x :

$\frac{1}{x+1}+\frac{3}{5x+1}=\frac{5}{x+4}, x\neq-1,-\frac{1}{5},-4$

Solution:

$\frac{1}{x+1}+\frac{3}{5x+1}=\frac{5}{x+4}; x\neq-1,\frac{-1}{5},-4$

Take L.C.M. on the left hand side of equation

$\frac{5x+1+3(x+1)}{(x+1)(5x+1)}=\frac{5}{x+4}$

$(x+4)(8x+4)=5(5x+1)(x+1)$

$8x^2+4x+32x+16=25x^2+5+5x+25x$

$17x^2-6x-11=0$

$17x^2-17x+11x-11=0$

$17x(x-1)+11(x-1)=0$

$(x-1)(17x+11)=0$

$\therefore x=\frac{-11}{17},1$

22. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank ?

Solution:

Two taps when run together fill the tank in $3\frac{1}{13}$hrs

Say taps are A, B and

A fills the tank by itself in x hrs

B fills tank in (x+3) hrs

Portion of tank filled by A (in 1hr) = $\frac{1}{x}$

Portion of tank filled by B (in 1hr) = $\frac{1}{x+3}$

Portion of tank filled by A & B (both in 1hr) = $\frac{13}{40}$

$$\therefore \frac{1}{x}+\frac{1}{x+3}=\frac{13}{40}$$

$$(x+3+x)40=13(x)(x+3)$$

$$80x+120=13x^2+39x$$

$$13x^2-41x-120=0$$

$$x=5$$

$\therefore$ A fills tank in 5hrs

B fills tank in 8hrs

23. If the ratio of the sum of the first n terms of two A.Ps is (7n + 1) : (4n + 27), then find the ratio of their 9th terms.

Solution:

Given two A.P's with n terms each

A.P $_I$ = first term = a_1

A.P$_{II}$ = first term = a_2

Common difference = d_2

Sum of first n terms for A.P$_I$ = S_1

$$S_1=\frac{n}{2}\left[2a_1+(n-1)d_1\right]$$

Similarly $S_2=\frac{n}{2}\left[2a_2+(n-1)d_2\right]$

$$\frac{S_1}{S_2}=\frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{a_1+\frac{(n-1)}{2}d_1}{a_2+\frac{(n-1)}{2}d_2}=\frac{7n+1}{4n+27}$$

Ratio of their 9th terms $=\frac{a_1+8d_1}{a_2+8d_2}$

Comparing

$$\frac{a_1+\frac{(n-1)}{2}d_1}{a_2+\frac{(n-1)}{2}d_2} \& \frac{a_1+8d_1}{a_2+8d_2}$$

Upon comparing

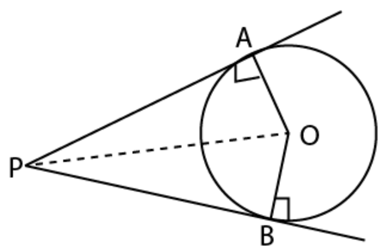
$$\frac{n-1}{2}=8$$

$\Rightarrow (n=17)$ substituting n value

$$\therefore \frac{a_1+8d_1}{a_2+8d_2}=\frac{a_1+\frac{(n-1)}{2}d_1}{a_2+\frac{(n-1)}{2}d_2}=\frac{7n+1}{4n+27}=\frac{7(17)+1}{4(17)+27}=\frac{120}{95}$$

ratio $=24:19$

24. Prove that the lengths of two tangents drawn from an external point to a circle are equal.
Solution:



PA & PB are the length of the tangents drawn from an external point P to circle C with radius r

$OA=OB=r$

$OA\perp PA$

$OB\perp PB$

Join O & P

In the triangles OAP & OBP

OA = OB (radii)

OP = OP (common side)

$\angle OAP=\angle OBP=90^\circ$ (Right angle)

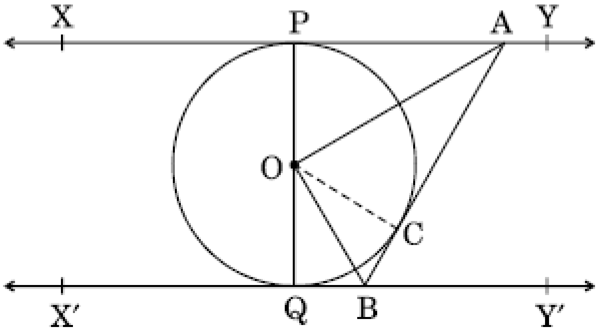
By RHS congruency

$\Delta OAP\cong\Delta OBP$

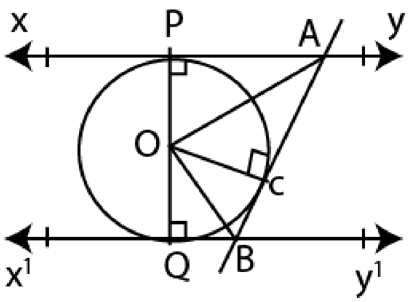
$\therefore$ By CPCT

PA = PB

25. In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C, is intersecting XY at A and X'Y' at B. Prove that ∠AOB = 90 .



Solution:



Prove that $\angle AOB = 90^\circ$

In ΔAOC and ΔAOP

OA = OA (hypotenuse)

OP = OC (radii)

$\angle ACO = \angle APO$ (right angle)

$\therefore \Delta AOC \cong \Delta AOP$ By RHS congruency

By CPCT $\angle AOC = \angle AOP$ .....(1)

Similarly In $\Delta BOC \;\&\; \Delta BOQ$

OC = OQ

OB = OB

$\angle BCO = \angle BQO = 90^\circ$

By RHS congruency $\Delta BOC \cong \Delta BOQ$

By CPCT $\angle BOC = \angle BOQ$ .....(2)

PQ is a straight line

$\therefore \angle AOP + \angle AOC + \angle BOC + \angle BOQ = 180^\circ$

From equations (1) and (2)

$2(\angle AOC + \angle BOC) = 180°$

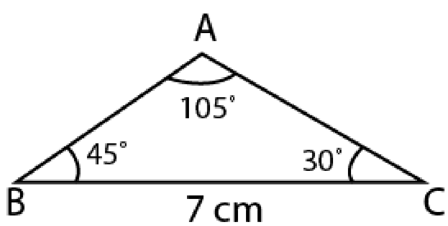
$\angle AOB = \frac{180°}{2}$

$\therefore \angle AOB = 90°$

26. Construct a triangle ABC with side BC = 7 cm, ∠B = 45 , ∠A = 105 .

Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the ΔABC.

Solution:



In the $\Delta ABC, \angle A + \angle B + \angle C = 180°$

$\therefore \angle C = 30°$

To construct the similar triangle first we need to construct ΔABC

For ΔABC

1). Draw $\overline{BC} = 7cm$ with help of a ruler

2) Take a protractor measure angle 45 from point B and draw a ray $\overrightarrow{BX}$

3) From point c, mark 30 with help of protractor & draw a ray $\overrightarrow{CY}$

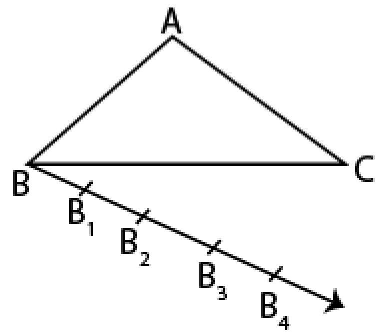
4) Now both $\overrightarrow{BX}$ and $\overrightarrow{BY}$ intersect at a point and this point is A

Now we have ΔABC

To construct similar triangle with corresponding sides $\frac{3}{4}$ of the sides of ΔABC

Step 1: Draw any raw making an acute angle with BC

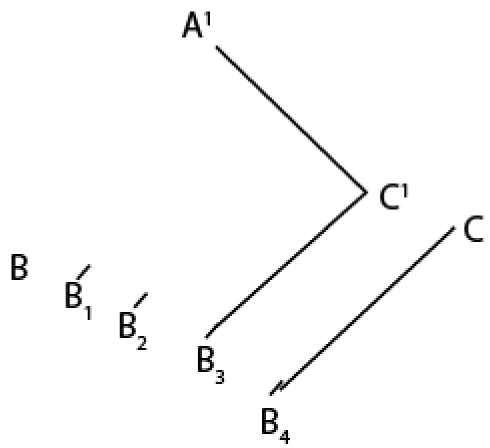
Step 2: Along the ray BZ mark 4 points B_1, B_2, B_3, B_4 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$



Step 3: Now join B_4 to C and draw a line parallel to B_4C from B_3 intersecting the line BC line BC at C'

Step 4: Draw a line through C' parallel to CA which intersects BA at A'

A'BC' is the required tri



Justification:

$\because C'A' || CA$ By construction

$\therefore \Delta A'BC' \sim \Delta ABC$ [using AA similarity]

$\therefore \frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC}$ [corresponding sides ratio will be proportional

$B_4C || B_3C'$ [By construction]

$\therefore \Delta BB_4C \sim \Delta BB_3C'$ [By AA similarity]

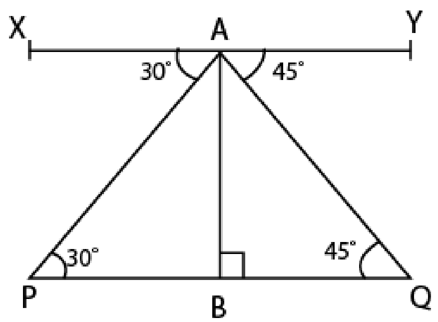
$\frac{BC'}{BC} = \frac{BB_3}{BB_4}$ [By BPT]

But we know $\frac{BB_3}{BB_4} = \frac{3}{4}$

$\therefore \frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'.C'}{AC} = \frac{3}{4}$

27. An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45 and 30 respectively. Find the width of the river. [Use $\sqrt{3}$ = 1·732]

Solution:



Given aeroplane is at height of 300m

$\therefore AB = 300m$ and $XY \,||\, PQ$

Angles of depression of the two points P & Q are 30 and 45

$\angle XAP = 30° \;\&\; \angle YAQ = 45°$

$\angle XAP = \angle APB = 30°$
$\angle YAQ = \angle AQB = 45°$ [Alternative Interior angles]

In ΔPAB

$\tan 30° = \dfrac{AB}{PB}$

$PB = 300\sqrt{3}\; mt$

In ΔBAQ

$\tan 45° = \dfrac{AB}{BQ}$

$BQ = 300m$

$\therefore$ Width of the river = PB + BQ

$= 300(1+\sqrt{3})mt$

28. If the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k - 1, 5k) are collinear, then find the value of k.

Solution:

Given $A(k+1, 2k), B(3k, 2k+3), C(5k-1, 5k)$ are collinear.

If three points are collinear then the area of the triangle will be zero. For any 3 points (x_1, y_1), (x_2, y_2), (x_3, y_3) Area will be

$\Rightarrow A = \dfrac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$

$\therefore O = \dfrac{1}{2}\left|(k+1)(2k+3-5k) + 3k(5k-2k) + (5k-1)(2k-2k-3)\right|$

$O = \left|(k+1)(3-3k) + 3k(3k) - 15k + 3\right|$

$\left|-3k^2+3+9k^2+3-15k\right|=0$

$\left|6k^2-15k+6\right|=0$

$k=2,\dfrac{1}{2}$

29. Two different dice are thrown together. Find the probability that the numbers obtained have
(i) even sum, and
(ii) even product.
Solution:

Two dice are through together total possible outcomes $=6\times6=36$
(i) Sum of outcomes is even
This can be possible when
$\Rightarrow$Both outcomes are even
$\Rightarrow$Both outcomes are odd
For both outcomes to be Even number of cases = 3×3 = 9
Similarly
Both outcomes odd = 9 cases
Total favourable cases = 9 + 9 =18

Probability that $=\dfrac{18}{36}$

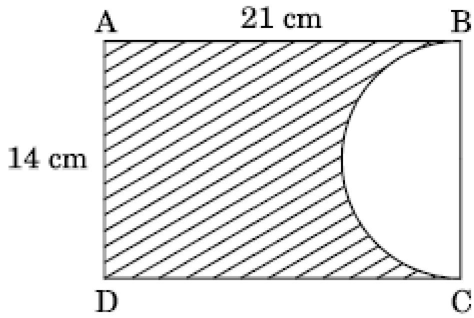
Sum of the even outcomes is $\dfrac{1}{2}$

(ii) Product of outcomes is even
This is possible when
$\Rightarrow$Both outcomes are even
$\Rightarrow$first outcome even & the other odd
$\Rightarrow$ first outcome odd & the other even
Number of cases where both outcomes are even = 9
Number of cases for first outcome odd = 9
and the other Even
No. of cases for first outcome odd & the other even = 9
Total favourable cases = 9 + 9 + 9 = 27

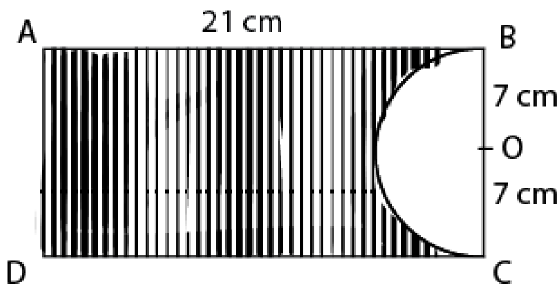
Probability = $\dfrac{27}{36}$

$=\dfrac{3}{4}$

30. In the given figure, ABCD is a rectangle of dimensions 21 cm $\times$ 14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure.



Solution:



Area of shaded region = Area of rectangle – Area of semicircle

$=21\times14-\frac{\pi(7)^2}{2}$

$=217\ \text{cm}^2$

Perimeter of shaded region

= AB + AD + CD + length of arc BC

$=21+14+21+\frac{180^\circ}{360^\circ}\times2\times\frac{22}{7}\times7$

$=78\ \text{cm}$

31. In a rain-water harvesting system, the rain-water from a roof of 22 m $\times$ 20 m drains into a cylindrical tank having diameter of base 2 m and height 3·5 m. If the tank is full, find the rainfall in cm. Write your views on water conservation.

Solution:

Water from the roof drains into cylindrical tank

Volume of water from roof flows into the tank of the rainfall is x cm and given the tank is full we can write volume of water collected on roof = volume of the tank

$\frac{22\times20\times x}{100}=\pi\left(\frac{2}{2}\right)^2\times3.5$

$x=2.5\,cm$

$\therefore$ .rainfall is of 2.5 cm

CBSE – X – 2016 EXAMINATION

MATHEMATICS

SET-1

Paper & Solution

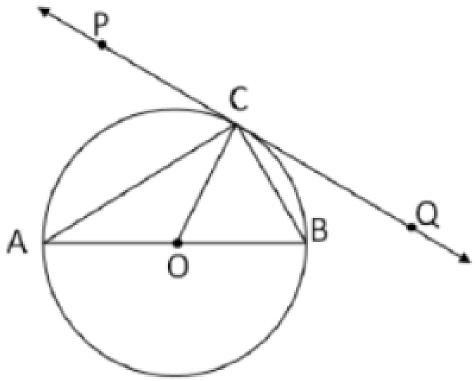
Time: 3 Hrs. Max. Marks: 90

General Instructions:

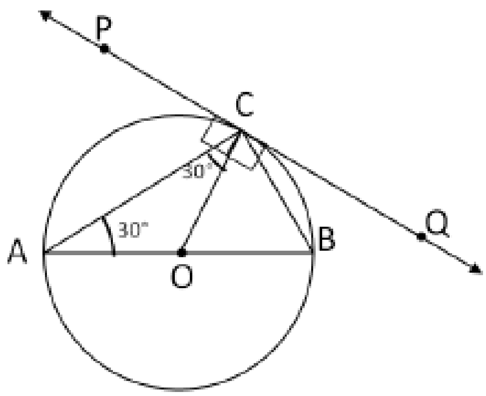
1. All questions are compulsory.
2. The question paper consists of 31 questions divided into four sections – A, B, C and D.
3. Section A contains 4 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
4. Use of calculators is not permitted.

SECTION A

Question 1. In the figure, PQ is a tangent at a point C to a circle with centre O. if AB is a diameter and ∠CAB = 30°, find ∠PCA.



Solution: In the given figure,



In ΔACO,

OA = OC …(Radii of the same circle)

∴ ΔACO is an isosceles triangle.

∠CAB = 30° …(Given)

∴ ∠CAO = ∠ACO = 30° …. (angles opposite to equal sides of an isosceles triangle are equal)

∠PCO = 90° …(radius drawn at the point of contact is perpendicular to the tangent)

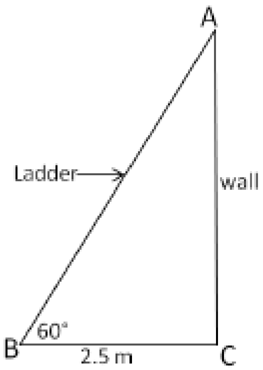
Now ∠PCA = ∠PCO – ∠CAO

∴ ∠PCA = 90° – 30° = 60°

Marks: 1

Question 2. For what value of k will k + 9, 2k – 1 and 2k + 7 are the consecutive terms of an A.P?
Solution: If k + 9, 2k – 1 and 2k + 7 are the consecutive terms of A.P., then the common difference will be the same.
$\therefore (2k - 1) - (k + 9) = (2k + 7) - (2k - 1)$
$\therefore k - 10 = 8$
$\therefore k = 18$
Marks: 1

Question 3. A ladder leaning against a wall makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.
Solution:



Let AB be the ladder and CA be the wall.
The ladder makes an angle of 60° with the horizontal.
∴ ΔABC is a 30°-60°-90°, right triangle.
Given: BC = 2.5 m, ∠ABC = 60°
∴ AB = 5 m
Hence, length of the ladder is AB = 5 m.
Marks: 1

Question 4. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability of getting neither a red card nor a queen.
Solution: There are 26 red cards including 2 red queens.
Two more queens along with 26 red cards will be 26 + 2 = 28
$\therefore$ P(getting a red card or a queen) = $\frac{28}{52}$
$\therefore$ P(getting neither a red card nor a queen) = $1 - \frac{28}{52} = \frac{24}{52} = \frac{6}{13}$
Marks: 1

SECTION B

Question 5. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x)k = 0$ has equal roots, find the value of k.

Solution: Given –5 is a root of the quadratic equation $2x^2 + px - 15 = 0$.

$\therefore$ –5 satisfies the given equation.

$\therefore 2(-5)^2 + p(-5) - 15 = 0$

$\therefore 50 - 5p - 15 = 0$

$\therefore 35 - 5p = 0$

$\therefore 5p = 35 \Rightarrow p = 7$

Substituting p = 7 in $p(x^2 + x) + k = 0$, we get

$7(x^2 + x) + k = 0$

$\therefore 7x^2 + 7x + k = 0$

The roots of the equation are equal.

$\therefore$ Discriminant = $b^2 - 4ac = 0$

Here, a = 7, b = 7, c = k

$b^2 - 4ac = 0$

$\therefore (7)^2 - 4(7)(k) = 0$

$\therefore 49 - 28k = 0$

$\therefore 28k = 49$

$$\therefore\ k = \frac{49}{28} = \frac{7}{4}$$

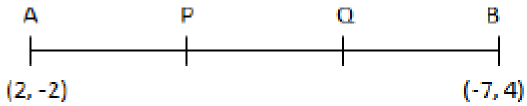
Marks: 2

Question 6. Let P and Q be the points of trisection of the line segment joining the points A(2, -2) and B(-7, 4) such that P is nearer to A. Find the coordinates of P and Q.

Solution: Since P and Q are the points of trisection of AB, AP = PQ = QB

Thus, P divides AB internally in the ratio 1 : 2

and Q divides AB internally in the ratio 2 : 1.



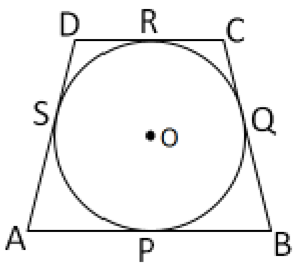
$\therefore$ By section formula,

$$P = \left(\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}\right) = \left(\frac{-7+4}{3}, \frac{4-4}{3}\right) = \left(\frac{-3}{3}, 0\right) = (-1, 0)$$

$$Q = \left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(-2)}{2+1}\right) = \left(\frac{-14+2}{3}, \frac{8-2}{3}\right) = \left(\frac{-12}{3}, \frac{6}{3}\right) = (-4, 2)$$

Marks: 2

Question 7. In figure, a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that AB + CD = BC + DA.



Solution: Since tangents drawn from an exterior point to a circle are equal in length,

AP = AS ….(1)

BP = BQ ….(2)

CR = CQ ….(3)

DR = DS ….(4)

Adding equations (1), (2), (3) and (4), we get

AP + BP + CR + DS = AS + BQ + CQ + DS

∴ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

∴ AB + CD = AD + BC

∴ AB + CD = BC + DA …..(proved)

Marks: 2

Question 8. Prove that the points (3, 0), (6, 4) and (-1, 3) are the vertices of a right angled isosceles triangle.

Solution: Let A(3, 0), B(6, 4) and C(–1, 3) be the given points.

Now,

$$AB = \sqrt{(6-3)^2 + (4-0)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25}$$

$$BC = \sqrt{(-1-6)^2 + (3-4)^2} = \sqrt{(-7)^2 + (-1)^2} = \sqrt{49+1} = \sqrt{50}$$

$$AC = \sqrt{(-1-3)^2 + (3-0)^2} = \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25}$$

∴ AB = AC

$AB^2 = (\sqrt{25})^2 = 25$

$BC^2 = (\sqrt{50})^2 = 50$

$AC^2 = (\sqrt{25})^2 = 25$

∴ $AB^2 = AC^2 = BC^2$

Thus, ΔABC is a right-angled isosceles triangle.

Marks: 2

Question 9. The 4^{th} term of an A.P. is zero. Prove that the 25^{th} term of the A.P. is three times its 11^{th} term.

Solution: 4th term of an A.P. = a_4 = 0

∴ a + (4 – 1)d = 0

∴ a + 3d = 0

∴ a = –3d ….(1)

25^{th} term of an A.P. = a_{25}

= a + (25 – 1)d

= –3d + 24d ….[From (1)]

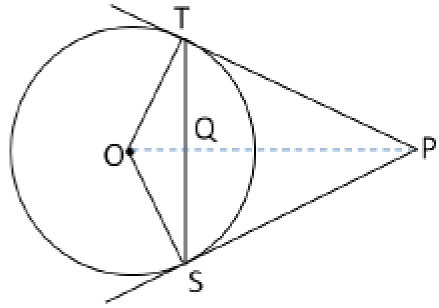
= 21d

3 times 11^{th} term of an A.P. = $3a_{11}$

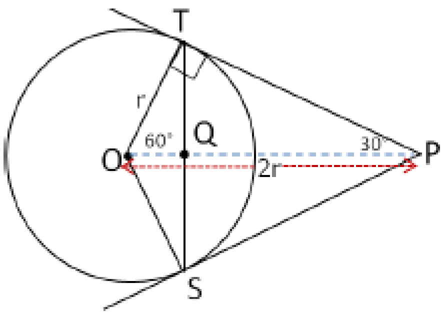
= 3[a + (11 – 1)d]

= 3[a + 10d]
= 3[–3d + 10d]
= 3 × 7d
= 21d
∴ $a_{25} = 3a_{11}$
i.e., the 25^{th} term of the A.P. is three times its 11^{th} term.
Marks: 2

Question 10. In figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If OP = 2r, show that ∠ OTS = ∠ OST = 30°.



Solution:



In the given figure,
OP = 2r … (Given)
∠OTP = 90° … (radius drawn at the point of contact is perpendicular to the tangent)
In ΔOTP,

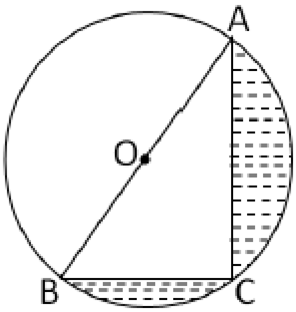
$$\sin \angle OPT = \frac{OT}{OP} = \frac{1}{2} = \sin 30°$$

∠OPT = 30°
∴ ∠TOP = 60°
∴ ΔOTP is a 30°-60°-90°, right triangle.
In ΔOTS,
OT = OS … (Radii of the same circle)
∴ ΔOTS is an isosceles triangle.
∴ ∠OTS = ∠OST … (Angles opposite to equal sides of an isosceles triangle are equal)
In ΔOTQ and ΔOSQ
OS = OT … (Radii of the same circle)
OQ = OQ ... (side common to both triangles)
∠OTQ = ∠OSQ … (angles opposite to equal sides of an isosceles triangle are equal)
∴ ΔOTQ ≅ ΔOSQ … (By S.A.S)
∴ ∠TOQ = ∠SOQ = 60° … (C.A.C.T)

∴ ∠TOS = 120° … (∠TOS = ∠TOQ + ∠SOQ = 60° + 60° = 120°)
∴ ∠OTS + ∠OST = 180° – 120° = 60°
∴ ∠OTS = ∠OST = 60° ÷ 2 = 30°
Marks: 2

SECTION C

Question 11. In figure, O is the centre of a circle such that diameter AB = 13 cm and AC = 12 cm. BC is joined. Find the area of the shaded region. (Take π = 3.14)



Solution: Diameter, AB = 13 cm

∴ Radius of the circle, $r = \frac{13}{2} = 6.5$ cm

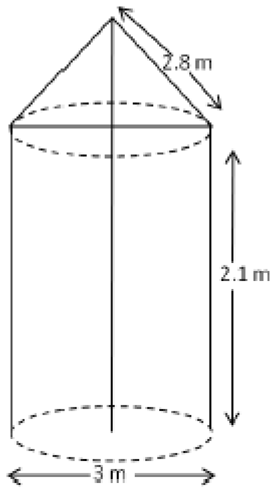
∠ACB is the angle in the semi-circle.
∴ ∠ACB = 90°
Now, in ΔACB, using Pythagoras theorem, we have
$AB^2 = AC^2 + BC^2$
$(13)^2 = (12)^2 + (BC)^2$
$(BC)^2 = (13)^2 – (12)^2 = 169 – 144 = 25$
$BC = \sqrt{25} = 5$ cm
Now, area of shaded region = Area of semi-circle – Area of ACB

$= \frac{1}{2}\pi r^2 - \frac{1}{2}\times BC\times AC$

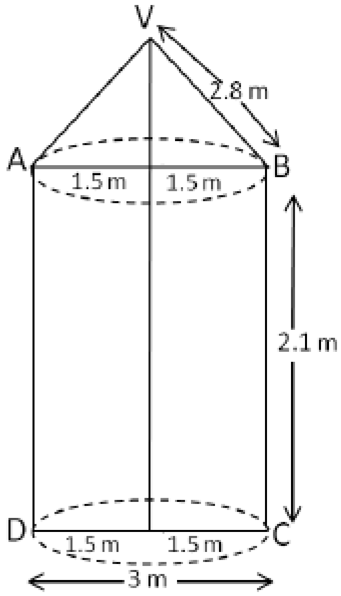
$= \frac{1}{2}\times 3.14\times (6.5)^2 - \frac{1}{2}\times 5\times 12$

= 66.33 – 30
= 36.33 cm^2
Thus, the area of the shaded region is 36.33 cm^2.
Marks: 3

Question 12. In figure, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs. 500/sq. metre. $\left(\text{Use } \pi = \frac{22}{7}\right)$



Solution:



For conical portion, we have

r = 1.5 m and l = 2.8 m

∴ S_1 = Curved surface area of conical portion

∴ $S_1 = \pi rl$

$= \pi \times 1.5 \times 2.8$

$= 4.2\pi$ m^2

For cylindrical portion, we have

r = 1.5 m and h = 2.1 m

∴ S_2 = Curved surface area of cylindrical portion

∴ $S_2 = 2\pi rh$

$= 2 \times \pi \times 1.5 \times 2.1$

$= 6.3\pi$ m^2

Area of canvas used for making the tent = $S_1 + S_2$

$= 4.2\pi + 6.3\pi$

$= 10.5\pi$

$= 10.5 \times \frac{22}{7}$

$= 33$ m^2

Total cost of the canvas at the rate of Rs. 500 per m^2 = Rs. (500 × 33) = Rs. 16500.
Marks: 3

Question 13. If the point P(x, y) is equidistant from the points A(a + b, b – a) and B(a – b, a + b). Prove that bx = ay.
Solution: P(x, y) is equidistant from the points A(a + b, b – a) and B(a – b, a + b).
∴ AP = BP

$$\sqrt{[x-(a+b)]^2+[y-(b-a)]^2}=\sqrt{[x-(a-b)]^2+[y-(a+b)]^2}$$

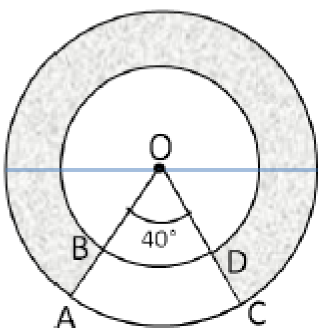
$$[x-(a+b)]^2+[y-(b-a)]^2=[x-(a-b)]^2+[y-(a+b)]^2$$

$$x^2-2x(a+b)+(a+b)^2+y^2-2y(b-a)+(b-a)^2=x^2-2x(a-b)+(a-b)^2+y^2-2y(a+b)+(a+b)^2$$

$$-2x(a+b)-2y(b-a)=-2x(a-b)-2y(a+b)$$

ax + bx + by – ay = ax – bx + ay + by
2bx = 2ay
∴ bx = ay ....(proved)
Marks: 3

Question 14. In figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where ∠AOC = 40°. $\left(\text{Use } \pi = \frac{22}{7}\right)$



Solution: Area of the region ABDC = Area of sector AOC – Area of sector BOD

$$= \frac{40°}{360°}\times\frac{22}{7}\times14\times14-\frac{40°}{360°}\times\frac{22}{7}\times7\times7$$

$$= \frac{1}{9}\times22\times14\times2-\frac{1}{9}\times22\times7\times1$$

$$= \frac{22}{9}\times(28-7)$$

$$= \frac{22}{9}\times21$$

$$= \frac{154}{3}$$

= 51.33 cm^2

Area of circular ring = $\frac{22}{7}\times14\times14-\frac{22}{7}\times7\times7$
= 22 × 14 × 2 – 22 × 7 × 1
= 22 × (28 – 7)
= 22 × 21

= 462 cm^2

∴ Required shaded region = Area of circular ring – Area of region ABDC

= 462 – 51.33

= 410.67 cm^2

Thus, the area of shaded region is 410.67 cm^2

Marks: 3

Question 15. If the ratio of the sum of first n terms of two A.P's is (7n +1): (4n + 27), find the ratio of their m^{th} terms.

Solution: Let a_1, a_2 be the first terms and d_1, d_2 the common differences of the two given A.P's.

Thus, we have $S_n = \frac{n}{2}[2a_1 + (n-1)d_1]$ and $S_n' = \frac{n}{2}[2a_2 + (n-1)d_2]$

$$\therefore \frac{S_n}{S_n'} = \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2}$$

It is given that $\frac{S_n}{S_n'} = \frac{7n+1}{4n+27}$

$$\therefore \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{7n+1}{4n+27} \quad ....(1)$$

To find the ratio of the m^{th} terms of the two given A.P's, replace n by (2m – 1) in equation (1).

$$\therefore \frac{2a_1 + (2m-1-1)d_1}{2a_2 + (2m-1-1)d_2} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$$\therefore \frac{2a_1 + (2m-2)d_1}{2a_2 + (2m-2)d_2} = \frac{14m-7+1}{8m-4+27}$$

$$\therefore \frac{a_1 + (m-1)d_1}{a_2 + (m-1)d_2} = \frac{14m-6}{8m+23}$$

Hence, the ratio of the m^{th} terms of the two A.P's is 14m – 6 : 8m 23.

Marks: 3

Question 16. Solve for *x*: $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3},\ x \neq 1, 2, 3$

Solution:

$$\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{(x-3)+(x-1)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{x-3+x-1}{(x^2-3x+2)(x-3)} = \frac{2}{3}$$

$$\frac{2x-4}{x^3-3x^2-3x^2+9x+2x-6} = \frac{2}{3}$$

$$\frac{2x-4}{x^3-6x^2+11x-6} = \frac{2}{3}$$

$$6x-12=2x^3-12x^2+22x-12$$
$$2x^3-12x^2+16x=0$$
$$2x(x^2-6x+8)=0$$
$$x^2-6x+8=0$$
$$x^2-4x-2x+8=0$$
$$x(x-4)-2(x-4)=0$$
$$(x-4)(x-2)=0$$
$$x-4=0 \text{ or } x-2=0$$
$$x=4 \text{ or } x=2$$

Marks: 3

Question 17. A conical vessel, with base radius 5 cm and height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Solution: Let the radius of the conical vessel = r_1 = 5 cm

Height of the conical vessel = h_1 = 24 cm

Radius of the cylindrical vessel = r_2

Let the water rise upto the height of h_2 cm in the cylindrical vessel.

Now, volume of water in conical vessel = volume of water in cylindrical vessel

$\frac{1}{3}\pi r_1^2 h_1 = r_2^2 h_2$

$r_1^2 h_1 = 3r_2^2 h_2$

$5 \times 5 \times 24 = 3 \times 10 \times 10 \times h_2$

$h_2 = \frac{5\times5\times24}{3\times10\times10} = 2$ cm

Thus, the water will rise upto the height of 2 cm in the cylindrical vessel.

Marks: 3

Question 18. A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

Solution: Radius of sphere = r = 6 cm

Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (6)^3 = 288\pi$ cm^3

Let R be the radius of cylindrical vessel.

Rise in the water level of cylindrical vessel = h = $3\frac{5}{9}$ cm = $\frac{32}{9}$ cm

Increase in volume of cylindrical vessel = $\pi R^2 h = \pi R^2 \times \frac{32}{9} = \frac{32}{9}\pi R^2$

Now, volume of water displaced by the sphere is equal to volume of sphere.

$\therefore \frac{32}{9}\pi R^2 = 288\pi$

$\therefore R^2 = \frac{288 \times 9}{32} = 81$

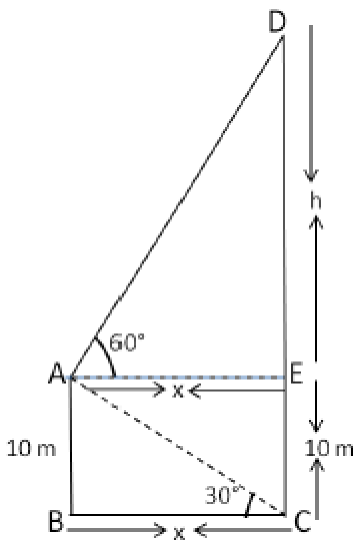
∴ R = 9 cm

∴ Diameter of the cylindrical vessel = 2 × R = 2 × 9 = 18 cm

Marks: 3

Question 19. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of a hill as 30°. Find the distance of the hill from the ship and the height of the hill.

Solution:



Let CD be the hill and suppose the man is standing on the deck of a ship at point A.

The angle of depression of the base C of the hill CD observed from A is 30° and the angle of elevation of the top D of the hill CD observed from A is 60°.

∴ ∠EAD = 60° and ∠BCA = 30°

In ΔAED,

$$\tan 60° = \frac{DE}{EA}$$

$$\sqrt{3} = \frac{h}{x}$$

$h = \sqrt{3}\,x$ ....(1)

In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{x}$$

$x = 10\sqrt{3}$ ....(2)

Substituting $x = 10\sqrt{3}$ in equation (1), we get

$h = \sqrt{3} \times 10\sqrt{3} = 10 \times 3 = 30$

DE = 30 m

CD = CE + ED = 10 + 30 = 40 m

Thus, the distance of the hill from the ship is $10\sqrt{3}$ m and the height of the hill is 40 m.

Marks: 3

Question 20. Three different coins are tossed together. Find the probability of getting

(i) exactly two heads

(ii) at least two heads

(iii) at least two tails.

Solution: When three coins are tossed together, the possible outcomes are

HHH, HTH, HHT, THH, THT, TTH, HTT, TTT

∴ Total number of possible outcomes = 8

(i) Favourable outcomes of exactly two heads are HTH, HHT, THH

∴ Total number of favourable outcomes = 3

∴ P(exactly two heads) = $\frac{3}{8}$

(ii) Favourable outcomes of at least two heads are HHH, HTH, HHT, THH

∴ Total number of favourable outcomes = 4

∴ P(at least two heads) = $\frac{4}{8} = \frac{1}{2}$

(iii)Favourable outcomes of at least two tails are THT, TTH, HTT, TTT

∴ Total number of favourable outcomes = 4

∴ P(at least two tails)= $\frac{4}{8} = \frac{1}{2}$

Marks: 3

SECTION D

Question 21. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the governments and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 cm and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs Rs. 120 per sq. m, find the amount shared by each school to set up the tents. What value is generated by the above problem? $\left(\text{Use } \pi = \frac{22}{7}\right)$

Solution: Height of conical upper part = 3.5 m, and radius = 2.8 m

(Slant height of cone)^2 = $2.1^2 + 2.8^2 = 4.41 + 7.84$

Slant height of cone = $\sqrt{12.25}$ = 3.5 m

The canvas used for each tent

= curved surface area of cylindrical base + curved surface area of conical upper part

= 2πrh + πrl

= πr(2h + l)

= $\frac{22}{7} \times 2.8(7 + 3.5)$

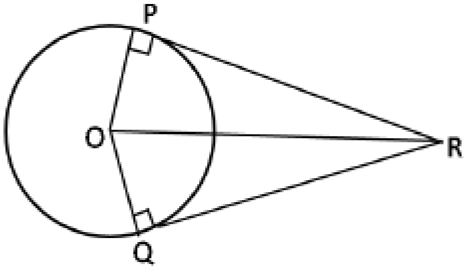
= $\frac{22}{7} \times 2.8 \times 10.5$

$= 92.4\ m^2$
So, the canvas used for one tent is $92.4\ m^2$.
Thus, the canvas used for 1500 tents = $(92.4 \times 1500)\ m^2$.
Canvas used to make the tents cost Rs. 120 per sq. m.
So, canvas used to make 1500 tents will cost Rs. 92.4 × 1500 × 120.
The amount shared by each school to set up the tents

$$= \frac{92.4 \times 1500 \times 120}{50} = \text{Rs. } 332640$$

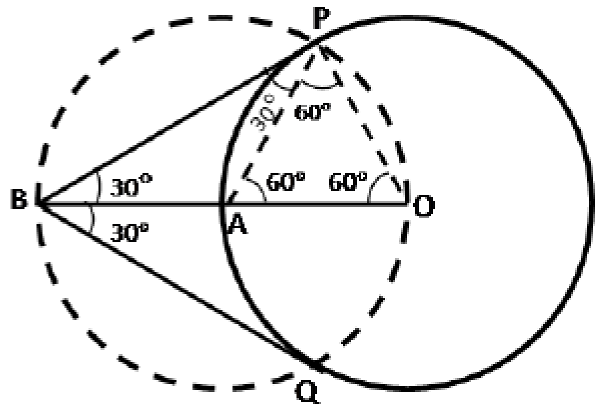
The amount shared by each school to set up the tents is Rs. 332640.
The value to help others in times of troubles is generated from the problem.
Marks: 4

Question 22. Prove that the lengths of the tangents drawn from an external point to a circle are equal.
Solution: Consider a circle centered at O.
Let PR and QR are tangents drawn from an external point R to the circle touching at points P and Q respectively.
Join OR.



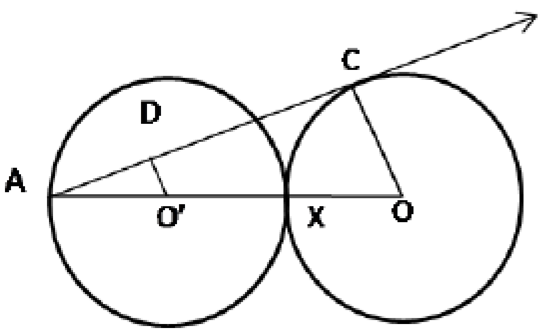
Proof:
In ΔOPR and ΔOQR,
OP = OQ ... (Radii of the same circle)
∠OPR = ∠OQR …. (Since PR and QR are tangents to the circle)
OR = OR ... (Common side)
∴ ΔOPR ≅ ΔOQ R ….(By R.H.S)
∴ PR = QR ….(c.p.c.t)
Thus, tangents drawn from an external point to a circle are equal.
Marks: 4

Question 23. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.
Solution: Steps of construction:
(i) Take a point O on the plane of the paper and draw a circle of radius OA = 4 cm.
(ii) Produce OA to B such that OA = AB = 4 cm.
(iii) Draw a circle with centre at A and radius AB.
(iv) Suppose it cuts the circle drawn in step (i) at P and Q.
(v) Join BP and BQ to get the desired tangents.



Justification:
In ΔOAP, OA = OP = 4 cm ... (radii of the same circle)
Also, AP = 4 cm ….(Radius of the circle with centre A)
∴ ΔOAP is equilateral.
∴ ∠PAO = 60°
∴ ∠BAP = 120°
In ΔBAP, we have BA = AP and ∠ BAP = 120°
∴ ∠ABP = ∠APB = 30°
Similarly we can get ∠ABQ = 30°
∴ ∠PBQ = 60°
Marks: 4

Question 24. In figure, two equal circles, with centres O and O', touch each other at X. OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre O, at the point C. O'D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$.



Solution: AO' = O'X = XO = OC …..(Since the two circles are equal.)
So, OA = AO' + O'X + XO …..(A-O'-X-O)
∴ OA = 3O'A
In ΔAO'D and ΔAOC,
∠DAO' = ∠CAO .... (Common angle)
∠ADO' = ∠ACO .... (both measure $90°$)
ΔADO' ~ ΔACO .... (By AA test of similarity)
$$\frac{DO'}{CO} = \frac{O'A}{OA} = \frac{O'A}{3O'A} = \frac{1}{3}$$
Marks: 4

Question 25. Solve for x: $\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4};\ X \neq -1,-2,-4$

Solution: $\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4}$

L.C.M. of all the denominators is (x + 1)(x + 2)(x + 4)

Multiply throughout by the L.C.M., we get

(x + 2)(x + 4) + 2(x + 1)(x + 4) = 4(x + 1)(x + 2)

(x + 4)(x + 2 + 2x +2) = 4(x^2 + 3x + 2)

(x + 4)(3x + 4) = 4x^2 + 12x + 8

3x^2 + 16x + 16 = 4x^2 + 12x + 8

∴ x^2 – 4x – 8 = 0

Now, a = 1, b = –4, c = –8

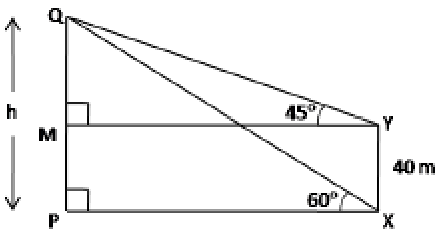
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{4\pm\sqrt{16+32}}{2}=\frac{4\pm\sqrt{48}}{2}=\frac{4\pm4\sqrt{3}}{2}$$

∴ x = $2\pm2\sqrt{3}$

Marks: 4

Question 26. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the tower PQ and the distance PX. (Use $\sqrt{3}$ = 1.73)

Solution:



MP = YX = 40 m

∴ QM = h – 40

In right angled ΔQMY,

$$\tan 45° = \frac{\text{QM}}{\text{MY}} \Rightarrow 1 = \frac{h-40}{\text{PX}} \quad ....(\text{MY = PX})$$

∴ PX = h – 40 ....(1)

In right angled ΔQPX,

$$\tan 60° = \frac{QP}{PX} \Rightarrow \sqrt{3} = \frac{QP}{PX}$$

PX = $\frac{h}{\sqrt{3}}$ ....(2)

From (1) and (2), h – 40 = $\frac{h}{\sqrt{3}}$

∴ $\sqrt{3}$h – 40$\sqrt{3}$ = h

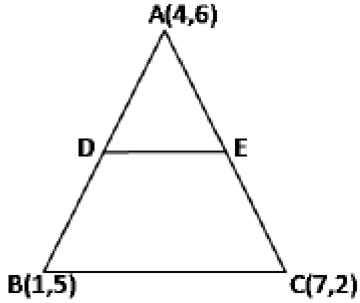
∴ $\sqrt{3}$h – h = 40$\sqrt{3}$

∴ 1.73h – h = 40(1.73) ⇒ h = 94.79 m

Thus, PQ is 94.79 m.
Marks: 4

Question 27. The houses in a row numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses preceding the house numbered X is equal to sum of the numbers of houses following X.
Solution: Let there be a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it.
That is, 1 + 2 + 3 + ... + (x – 1) = (x + 1) + (x + 2) + ... + 49
∴ 1 + 2 + 3 + ... + (x – 1)
= [1 + 2 + ... + x + (x – 1) + ... + 49] – (1 + 2 + 3 +. . . . + x)
$\therefore \frac{x-1}{2}[1+x-1]=\frac{49}{2}[1+49]-\frac{x}{2}[1+x]$
∴ x(x – 1) = 49 × 50 – x(1 + x)
∴ x(x – 1) + x(1 + x) = 49 × 50
∴ $x^2 - x + x + x^2 = 49 \times 50$
∴ $x^2 = 49 \times 50$
∴ $x^2 = 49 \times 25$
∴ x = 7 × 5 = 35
Since x is not a fraction, the value of x satisfying the given condition exists and is equal to 35.
Marks: 4

Question 28. In figure, the vertices of ΔABC are A(4, 6), B(1, 5) and C(7, 2). A line-segment DE is drawn to intersect the sides AB and AC at D and E respectively such that $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{3}$. Calculate the area of ΔADE and compare it with area of ΔABC.



Solution:
$\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{3}$
$\therefore \frac{AB}{AD}=\frac{AC}{AE}=3$
$\therefore \frac{AD+DB}{AD}=\frac{AE+EC}{AE}=3$
$\therefore 1+\frac{DB}{AD}=1+\frac{EC}{AE}=3$

$$\therefore \frac{DB}{AD} = \frac{EC}{AE} = 2$$

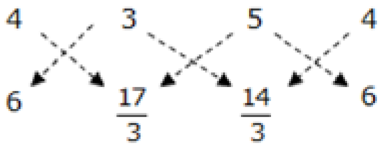
$$\therefore \frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{2}$$

$$\therefore AD:DB = AE:EC = 1:2$$

So, D and E divide AB and AC respectively in the ratio 1:2.

So the coordinates of D and E are

$\left(\frac{1+8}{1+2}, \frac{5+12}{1+2}\right) \equiv \left(3, \frac{17}{3}\right)$ and $\left(\frac{7+8}{1+2}, \frac{2+12}{1+2}\right) \equiv \left(5, \frac{14}{3}\right)$ respectively.



Area of ΔADE

$$= \frac{1}{2}\left|\left(4\times\frac{17}{3} + 3\times\frac{14}{3} + 5\times 6\right) - \left(3\times 6 + 5\times\frac{17}{3} + 4\times\frac{14}{3}\right)\right|$$

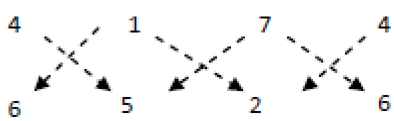
$$= \frac{1}{2}\left|\left(\frac{68}{3} + 14 + 30\right) - \left(18 + \frac{85}{3} + \frac{56}{3}\right)\right|$$

$$= \frac{1}{2}\left|\left(\frac{68+42+90}{3}\right) - \left(\frac{54+85+56}{3}\right)\right|$$

$$= \frac{1}{2}\left|\left(\frac{200}{3}\right) - \left(\frac{195}{3}\right)\right|$$

$$= \frac{1}{2}\times\frac{5}{3}$$

$= \frac{5}{6}$ sq. units



Area of ΔABC

$$= \frac{1}{2}\left|(4\times 5 + 1\times 2 + 7\times 6) - (1\times 6 + 7\times 5 + 4\times 2)\right|$$

$$= \frac{1}{2}\left|(20 + 2 + 42) - (6 + 35 + 8)\right|$$

$$= \frac{1}{2}\left|(64) - (49)\right|$$

$= \frac{15}{2}$ sq. units

$$\therefore \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \frac{5/6}{15/2} = \frac{1}{9}$$

Marks: 4

Question 29. A number x is selected at random from the numbers 1, 2, 3, and 4. Another number y is selected at random from the numbers 1, 4, 9 and 16. Find the probability that product of x and y is less than 16.

Solution: x is selected from 1, 2, 3 and 4

1, 2, 3, 4

y is selected from 1, 4, 9 and 16

Let A = {1, 4, 9, 16, 2, 8, 18, 32, 3, 12, 27, 48, 36, 64} which consists of elements that are product of x and y

P(product of x and y is less than 16) = $\dfrac{\text{Number of outcomes less than 16}}{\text{Total number of outcomes}}$

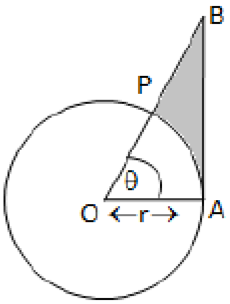
$= \dfrac{7}{14}$

$= \dfrac{1}{2}$

Marks: 4

Question 30. In figure, is shown a sector OAP of a circle with centre O, containing ∠θ. AB is perpendicular to the radius OQ and meets OP produced at B. Prove that the perimeter of shaded region is

$r\left[\tan\theta + \sec\theta + \dfrac{\pi\theta}{180} - 1\right]$.



Solution: Perimeter of shaded region = AB + PB + arc length AP ...(1)

Arc length AP = $\dfrac{\theta}{360} \times 2\pi r = \dfrac{\pi\theta r}{180}$ ....(2)

In right angled ΔOAB,

tan θ = $\dfrac{AB}{r} \Rightarrow$ AB = r tan θ ....(3)

sec θ = $\dfrac{OB}{r} \Rightarrow$ OB = r sec θ

OB = OP + PB

∴ r sec θ = r + PB

∴ PB = r sec θ – r .....(4)

Substitute (2), (3) and (4) in (1), we get

Perimeter of shaded region = AB + PB + arc length AP

$= r\tan\theta + r\sec\theta - r + \dfrac{\pi\theta r}{180}$

$= r\left[\tan\theta + \sec\theta + \dfrac{\pi\theta}{180} - 1\right]$

Marks: 4

Question 31. A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

Solution: Let the speed of the stream be s km/h.

Speed of the motor boat 24 km/h

Speed of the motor boat upstream 24 – s

Speed of the motor boat downstream 24 + s

According to the given condition,

$$\frac{32}{24-s}-\frac{32}{24+s}=1$$

$$\therefore 32\left(\frac{1}{24-s}-\frac{1}{24+s}\right)=1$$

$$\therefore 32\left(\frac{24+s-24+s}{576-s^2}\right)=1$$

$\therefore$ 32 × 2s = 576 – s^2

$\therefore$ s^2 + 64s – 576 = 0

$\therefore$ (s + 72)(s – 8) = 0

$\therefore$ s = –72 or s = 8

Since, speed of the stream cannot be negative, the speed of the stream is 8 km/h.

Marks: 4

CBSE-X-2015 EXAMINATION

MATHEMATICS

Paper & Solution

Time: 3 Hrs. Max. Marks: 90

General Instructions:
(i) All questions are compulsory.
(ii) The question paper consists of 31 questions divided into four sections – A, B, C and D.
(iii) Section A contains 4 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
(iv) Use of calculators is not permitted.

SECTION – A

Question numbers 1 to 4 carry 1 mark each.

1. If the quadratic equation $px^2 - 2\sqrt{5}px + 15 = 0$ has two equal roots then find the value of p.

Solution:

Given quadratic equation is,

$px^2 - 2\sqrt{5}px + 15 = 0$

Here, a = p, b = $2\sqrt{5}$ p, c= 15

For real equal roots, discriminant = 0

$\therefore b^2 - 4ac = 0$

$\therefore \left(2\sqrt{5}p\right)^2 - 4p(15) = 0$

$\therefore 20p^2 - 60p = 0$

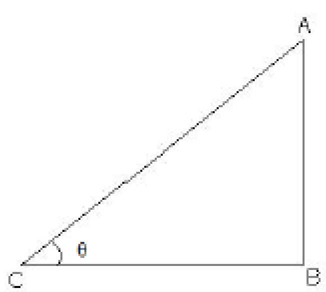
$\therefore 20p(p-3) = 0$

$\therefore p = 3$ or $p = 0$

But, p = 0 is not possible.

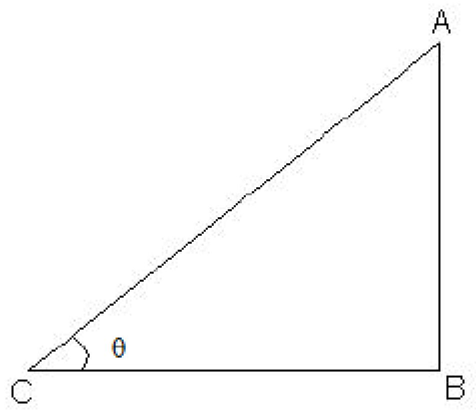
$\therefore$ p = 3

2. In the following figure, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. Find the Sun's altitude.



Solution:

Let AB be the tower and BC be its shadow.



AB = 20, BC = $20\sqrt{3}$

In ΔABC,

$$\tan\theta = \frac{AB}{BC}$$

$$\tan\theta = \frac{20}{20\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\text{but, } \tan\theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30$$

The Sun is at an altitude of 30°.

3. Two different dice are tossed together. Find the probability that the product of the two numbers on the top of the dice is 6.

Solution:

Two dice are tossed

S = [(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),
(5,1),(5,2),(5,3),(5,4), (5,5),(5,6),
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)]

Total number of outcomes when two dice are tossed = $6 \times 6 = 36$

Favourable events of getting product as 6 are:

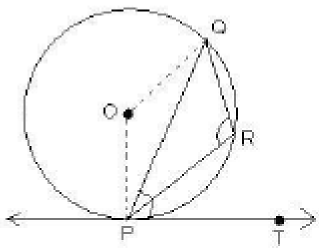
($1\times 6= 6$), ($6 \times 1 =6$),($2\times 3= 6$),($3\times 2 =6$)

i.e.(1,6), (6,1), (2,3), (3,2)

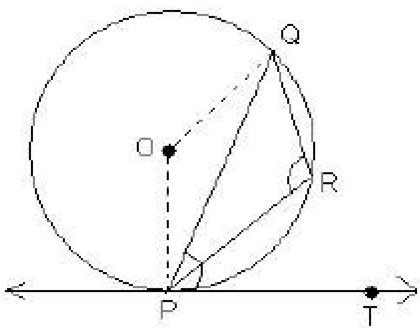
Favourable events of getting product as 6 = 4

$\therefore$ P(getting product as 6) $= \frac{4}{36} = \frac{1}{9}$

4. In the following figure, PQ is a chord of a circle with centre O and PT is a tangent. If ∠QPT = 60˚, find ∠PRQ



Solution:



m$\angle$OPT 90° (radius is perpendicular to the tangent)
So, $\angle$OPQ = $\angle$OPT - $\angle$QPT
= 90° - 60°
= 30°
m$\angle$POQ = 2$\angle$QPT 2 × 60° =120°
reflex m$\angle$POQ = 360° - 120° =240°

$$m\angle PRQ = \frac{1}{2}\text{reflex}\angle POQ$$

$$= \frac{1}{2} \times 240°$$

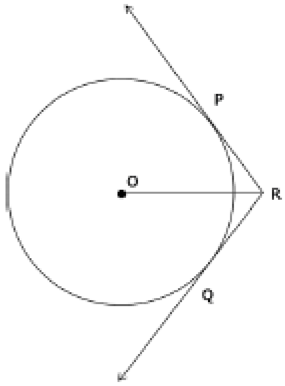
$$= 120°$$

$\therefore \angle$PRQ = 120°

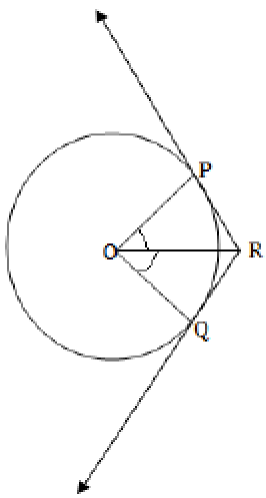
SECTION B

Question numbers 5 to 10 carry 2 marks each.

5. In the following figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O, If ∠PRQ = 120˚, then prove that OR = PR + RQ.



Solution:



Given that m $\angle$ PRQ = 120°

We know that the line joining the centre and the external point is the angle bisector between the tangents.

Thus, $m\angle PRO = m\angle QRO = \frac{120^\circ}{2} = 60^\circ$

Also we know that lengths of tangents from an external point are equal.

Thus, PR = RQ.

Join OP and OQ.

Since OP and OQ are the radii from the centre O,

OP $\perp$ PR and OQ $\perp$ RQ.

Thus, Δ OPR and Δ OQR are right angled congruent triangles.

Hence, $\angle POR = 90^\circ - \angle PRO = 90^\circ - 60^\circ = 30^\circ$

$\angle QOR = 90^\circ - \angle QRO = 90^\circ - 60^\circ = 30^\circ$

$\sin\angle QRO = \sin 30^\circ = \frac{1}{2}$

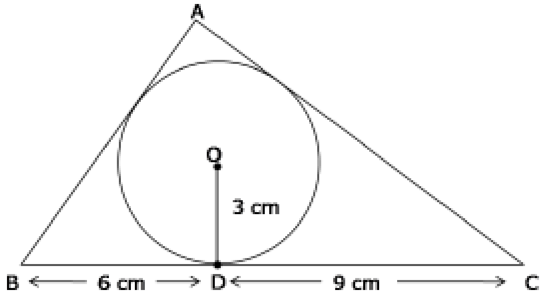
$\frac{PR}{OR} = \frac{1}{2}$

Thus, $\Rightarrow OR = 2PR$

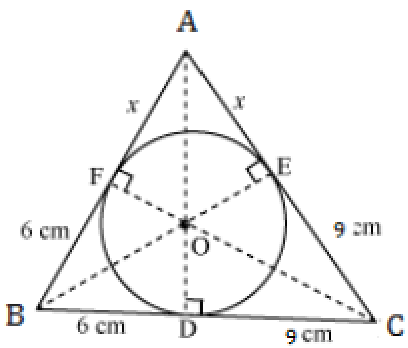
$\Rightarrow OR = PR + PR$

$\Rightarrow OR = PR + QR$

6. In Figure 4, a ΔABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of Δ ABC is 54 cm^2, then find the lengths of sides AB and AC.



Solution:



Let the given circle touch the sides AB and AC of the triangle at points F and E respectively and let the length of line segment AF be *x.*

Now, it can be observed that:

BF = BD = 6 cm (tangents from point B)

CE = CD = 9 cm (tangents from point C)

AE = AF = x (tangents from point A)

AB = AF + FB = x + 6

BC = BD + DC = 6 + 9 = 15

CA = CE + EA = 9 + x

2s = AB + BC + CA = x + 6 + 15 + 9 + x = 30 + 2x

s = 15 + x

s – a = 15 + x – 15 = x

s – b = 15 + x – (x + 9) = 6

s – c = 15 + x – (6 + x) = 9

Area of Δ ABC $\sqrt{s(s-a)(s-b)(s-c)}$

$$54=\sqrt{(15+x)(x)(6)(9)}$$

$$54=3\sqrt{6(15x+x^2)}$$

$$18=\sqrt{6(15x+x^2)}$$

$$324=6(15x+x^2)$$

$$54=15x+x^2$$

$x^2+15x-54=0$

$x^2+18x-3x-54=0$

$x(x+18)-3(x+18)$

$(x+18)(x-3)=0$

$x=-18$ and $x=3$

As distance cannot be negative, x = 3

AC = 3 + 9 = 12

AB = AF + FB = 6 + x = 6 + 3 = 9

7. Solve the following quadratic equation for x:

$4x^2 + 4bx - (a^2 - b^2) = 0$

Solution:

$$4x^2+4bx-(a^2-b^2)=0$$

$$\Rightarrow x^2+bx-\left(\frac{a^2-b^2}{4}\right)=0$$

$$\Rightarrow x^2+2\left(\frac{b}{2}\right)x=\frac{a^2-b^2}{4}$$

$$\Rightarrow x^2+2\left(\frac{b}{2}\right)x+\left(\frac{b}{2}\right)^2=\frac{a^2-b^2}{4}+\left(\frac{b}{2}\right)^2$$

$$\Rightarrow \left(x+\frac{b}{2}\right)^2=\frac{a^2}{4}$$

$$\Rightarrow x+\frac{b}{2}=\pm\frac{a}{2}$$

$$\Rightarrow x=\frac{-b}{2}\pm\frac{a}{2}$$

$$\Rightarrow x=\frac{-b-a}{2},\frac{-b+a}{2}$$

Hence, the roots are $-\left(\frac{a+b}{2}\right)$ and $\left(\frac{a-b}{2}\right)$.

8. In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the A.P., where S_n denotes the sum of its first n terms

Solution:

$S_5+S_7=167$ and $S_{10}=235$

Now, $S_n=\frac{n}{2}\{2a+(n-1)d\}$

$\therefore S_5+S_7=167$

$$\Rightarrow \frac{5}{2}\{2a+4d\}+\frac{7}{2}\{2a+6d\}=167$$

$$\Rightarrow 5a+10d+7a+21d=167$$

$\Rightarrow 12a+31d=167$ ....(1)

Also, $S_{10} = 235$

$\therefore \frac{10}{2}\{2a+9d\}=235$

$\Rightarrow 10a+45d=235$

$\Rightarrow 2a+9d=47$ ....(2)

Multiplying equation (2) by 6, we get

12a + 54d = 282 ....(3)

Subtracting (1) from (3), we get

$$\begin{array}{r} 12a+54d=282 \\ (-)12a+31d=167 \\ - \quad - \quad - \\ \hline 23d=115 \end{array}$$

$\therefore d=5$

Substituting value of d in (2), we have

2a + 9(5)= 47

$\Rightarrow$ 2a + 45= 47

$\Rightarrow$ 2a = 2

$\Rightarrow$ a = 1

Thus, the given A.P. is 1, 6, 11, 16,..........

9. The points A(4, 7), B(p, 3) and C(7, 3) are the vertices of a right triangle, right-angled at B, Find the values of P.

Solution:

Δ ABC is right angled at B.

$\therefore AC^2=AB^2+BC^2$ ....(1)

Also, $A\equiv(4,7), B=(p,3)$ and $C\equiv(7,3)$

Now, $AC^2=(7-4)^2+(3-7)^2=(3)^2+(-4)^2=9+16=25$

$AB^2=(p-4)^2+(3-7)^2=p^2-8p+16+(-4)^2$

$=p^2-8p+16+16$

$=p^2-8p+32$

$BC^2=(7-p)^2+(3-3)^2=49-14p+p^2+0$

$=p^2-14p+49$

From (1), we have

25= (p^2 - 8p + 32)+ (p^2 -14p +49)

$\Rightarrow$ 25 = 2p^2- 22p +81

$\Rightarrow$ 2p^2 - 22p +56= 0

$\Rightarrow$ p^2 - 11p + 28 = 0

$\Rightarrow$ p^2 - 7p - 4p+ 28 = 0

$\Rightarrow$ p(p -7) – 4(p – 7) = 0

$\Rightarrow$ (p – 7) (p – 4) = 0

$\Rightarrow$ p = 7 and p = 4

10. Find the relation between x and y if the points A(x, y), B(-5, 7) and C(-4, 5) are collinear.

Solution:

Given, the points A(x,y), B(-5,7) and C(-4,5) are collinear.
So, the area formed by these vertices is 0.

$$\therefore \frac{1}{2}\left[x(7-5)+(-5)(5-y)+(-4)(y-7)\right]=0$$

$$\Rightarrow \frac{1}{2}\left[2x-25+5y-4y+28\right]=0$$

$$\Rightarrow \frac{1}{2}\left[2x+y+3\right]=0$$

$$\Rightarrow 2x+y+3=0$$

$$\Rightarrow y=-2x-3$$

SECTION C

Question numbers 11 to 20 carry 3 marks each.

11. The 14th term of an A.P. is twice its 8th term. If its 6th term is -8, then find the sum of its first 20 terms.

Solution:

Here it is given that,

$T_{14} = 2(T_8)$

$\Rightarrow a + (14 – 1)d = 2[a + (8 – 1)d]$

$\Rightarrow a + 13d = 2[a + 7d]$

$\Rightarrow a + 13d = 2a + 14d$

$\Rightarrow 13d – 14d = 2a – a$

$\Rightarrow – d = a$ .... (1)

Now, it is given that its 6^{th} term is –8.

$T_6 = –8$

$\Rightarrow a + (6 – 1)d = – 8$

$\Rightarrow a + 5d = –8$

$\Rightarrow –d + 5d = –8$ [$\because$ Using (1)]

$\Rightarrow 4d = –8$

$\Rightarrow d = –2$

Subs. this in eq. (1), we get a = 2

Now, the sum of 20 terms,

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{20}=\frac{20}{2}\left[2a+(20-1)d\right]$$

= 10 [2(2) + 19(-2)]

= 10[4 - 38]

= -340

12. Solve for x:

$$\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$$

Solution:

For the given equation, $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

Comparing this equation with $ax^2 +bx + c = 0$, we obtain

$a=\sqrt{3}, b=-2\sqrt{2}, c=-2\sqrt{3}$

Now, $D=\sqrt{b^2-4ac}$

$=\sqrt{(-2\sqrt{2})^2-4\left(4\sqrt{3}\right)\left(-2\sqrt{3}\right)}$

$=\sqrt{8+24}=\sqrt{32}=4\sqrt{2}$

Using quadratic formula, we obtain

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\Rightarrow x=\dfrac{-(-2\sqrt{2})\pm 4\sqrt{2}}{2\sqrt{3}}$

$\Rightarrow x=\dfrac{2\sqrt{2}+4\sqrt{2}}{2\sqrt{3}}$ or $x=\dfrac{2\sqrt{2}-4\sqrt{2}}{2\sqrt{3}}$

$\Rightarrow x=\dfrac{\sqrt{2}+2\sqrt{2}}{\sqrt{3}}$ or $x=\dfrac{\sqrt{2}-2\sqrt{2}}{\sqrt{3}}$

$\Rightarrow x=\dfrac{3\sqrt{2}}{\sqrt{3}}$ or $x=\dfrac{-\sqrt{2}}{\sqrt{3}}$

$\Rightarrow x=\sqrt{3}\sqrt{2}$ or $x=\dfrac{-\sqrt{2}}{\sqrt{3}}$

$\therefore x=\sqrt{6}$ or $x=\dfrac{-\sqrt{2}}{\sqrt{3}}$

13. The angle of elevation of an aeroplane from point A on the ground is 60°. After flight of 15 seconds, the angle of elevation changes to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the plane in km/hr.

Solution:

Let BC be the height at watch the aeroplane is observed from point A.

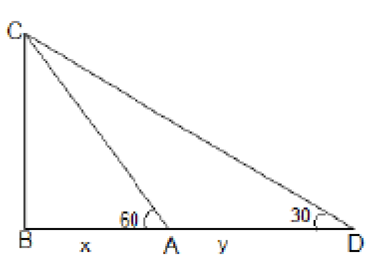
Then, BC = 1500 $\sqrt{3}$

In 15 seconds, the aeroplane moves from point A to D.

A and D are the points where the angles of elevation 60° and 30° are formed respectively.

Let BA = x metres and AD = y metres

BC = x + y



In ΔCBA,

$\tan 60^\circ = \frac{BC}{BA}$

$\sqrt{3} = \frac{1500\sqrt{3}}{x}$

$\therefore x = 1500\ m$ ....(1)

In ΔCBD,

$\tan 30^\circ = \frac{BC}{BD}$

$\frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{x+y}$

$\therefore x + y = 1500(3) = 4500$

$\therefore 1500 + y = 4500$

$\therefore y = 3000\ m$ ....(2)

We know that, the aeroplane moves from point A to D in 15 seconds and the distance covered is 3000 metres. (by 2)

Speed $= \frac{\text{distance}}{\text{time}}$

Speed $= \frac{3000}{15}$

Speed 200m/s

Converting it to km/hr $= 200 \times \frac{18}{5} = 720 km/hr$

14. If the coordinates of points A and B are (-2, -2) and (2, -4) respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB, where P lies on the line segment AB.

Solution:

Here, P(x,y) divides line segment AB, such that

$AP = \frac{3}{7}AB$

$\frac{AP}{AB} = \frac{3}{7}$

$\frac{AB}{AP} = \frac{7}{3}$

$\frac{AB}{AP} - 1 = \frac{7}{3} - 1$

$\frac{AB - AP}{AP} = \frac{7-3}{3}$

$\frac{BP}{AP} = \frac{4}{3}$

$\frac{AP}{BP} = \frac{3}{4}$

$\therefore$ P divides AB in the ratio 3: 4

$x = \frac{3\times 2+4(-2)}{3+4}; y = \frac{3\times(-4)+4(-2)}{3+4}$

$x = \frac{6-8}{7}; y = \frac{-12-8}{7}$

$x = \frac{-2}{7}; y = \frac{-20}{7}$

$\therefore$ The coordinates of P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$

15. The probability of selecting a red ball at random from a jar that contains only red, blue and orange balls is $\frac{1}{4}$. The probability of selecting a blue ball at random from the same jar $\frac{1}{3}$. If the jar contains 10 orange balls, find the total number of balls in the jar.

Solution:

Here the jar contains red, blue and orange balls.
Let the number of red balls be x.
Let the number of blue balls be y.
Number of orange balls = 10
Total number of balls = x + y + 10
Now, let P be the probability of drawing a ball from the jar

P(a red ball) $= \frac{x}{x+y+10}$

$\Rightarrow \frac{1}{4} = \frac{x}{x+y+10}$

$\Rightarrow 4x = x+y+10$

$\Rightarrow 3x - y = 10$ ....(i)

Next,

P (a blue ball) $= \frac{y}{x+y+10}$

$\Rightarrow \frac{1}{3} = \frac{y}{x+y+10}$

$\Rightarrow 3y = x+y+10$

$\Rightarrow 2y - x = 10$ ....(ii)

Multiplying eq. (i) by 2 and adding to eq. (ii), we get

$$\begin{array}{r} 6x - 2y = 20 \\ \underline{-x + 2y = 10} \\ 5x = 30 \end{array}$$

$\Rightarrow x = 6$

Subs. x = 6 in eq. (i), we get y = 8

Total number of balls = x + y + 10 = 6 + 8 + 10 = 24

Hence, total number of balls in the jar is 24.

16. Find the area of the minor segment of a circle of radius 14 cm, when its central angle is 60˚. Also find the area of the corresponding major segment. [Use $\pi = \frac{22}{7}$]

Solution:

Radius of the circle =14 cm

Central Angle, θ = 60 ,

Area of the minor segment

$$= \frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

$$= \frac{60^\circ}{360^\circ} \times \pi \times 14^2 - \frac{1}{2} \times 14^2 \times \sin 60^\circ$$

$$= \frac{1}{6} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{2} \times 14 \times 14 \times \frac{\sqrt{3}}{2}$$

$$= \frac{22 \times 14}{3} - 49\sqrt{3}$$

$$= \frac{22 \times 14}{3} - \frac{147\sqrt{3}}{3}$$

$$= \frac{308 - 147\sqrt{3}}{3} \text{cm}^2$$

Area of the minor segment $= \frac{308 - 147\sqrt{3}}{3} cm^2$

17. Due to sudden floods, some welfare associations jointly requested the government to get 100 tents fixed immediately and offered to contribute 50% of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2 m and height 4 m with the conical upper part of same diameter but height 2.8 m, and the canvas to be used costs Rs. 100 per sq. m, find the amount, the associations will have to pay. What values are shown by these associations? [Use $\pi = \frac{22}{7}$]

Solution:

Diameter of the tent = 4.2 m

Radius of the tent, r = 2.1 m

Height of the cylindrical part of tent, $h_{cylinder}$ = 4 m

Height of the conical part, h_{cone} = 2.8 m

Slant height of the conical part, l

$$= \sqrt{h_{cone}{}^2 + r^2}$$

$$= \sqrt{2.8^2 + 2.1^2}$$

$$= \sqrt{2.8^2 + 2.1^2}$$

$$= \sqrt{12.25} = 3.5\, m$$

Curved surface area of the cylinder = 2 π r $h_{cylinder}$

$$= 2 \times \frac{22}{7} \times 2.1 \times 4$$

$$= 22 \times 0.3 \times 8 = 52.8\, m^2$$

Curved surface area of the conical tent $= \pi rl = \frac{22}{7} \times 2.1 \times 3.5 = 23.1\ m^2$

Total area of cloth required for building one tent
= Curved surface area of the cylinder + Curved surface area of the conical tent
= 52.8 + 23.1
= 75.9 m^2

Cost of building one tent = 75.9 × 100 = Rs 7590
Total cost of 100 tents = 7590 × 100 = Rs 7,59,000

Cost to be borne by the associations $= \frac{759000}{2}$ = Rs. 3,79,500

It shows the helping nature, unity and cooperativeness of the associations.

18. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of each bottle, if 10% liquid is wasted in this transfer.

Solution:

Internal diameter of the bowl = 36 cm
Internal radius of the bowl, r = 18 cm

Volume of the liquid, $V = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \pi \times 18^3$

Let the height of the small bottle be 'h'.
Diameter of a small cylindrical bottle = 6 cm
Radius of a small bottle, R = 3 cm
Volume of a single bottle = $\pi R^2 h = \pi \times 3^2 \times h$
No. of small bottles, n = 72

Volume wasted in the transfer $= \frac{10}{100} \times \frac{2}{3} \times \pi \times 18^3$

Volume of liquid to be transferred in the bottles

$$= \frac{2}{3} \times \pi \times 18^3 - \frac{10}{100} \times \frac{2}{3} \times \pi \times 18^3$$

$$= \frac{2}{3} \times \pi \times 18^3 \left(1 - \frac{10}{100}\right)$$

$$= \frac{2}{3} \times \pi \times 18^3 \times \frac{90}{100}$$

Number of small cylindrical bottles $= \frac{\text{Volume of the liquid to be transferred}}{\text{Volume of a single bottle}}$

$$\Rightarrow 72 = \frac{\frac{2}{3} \times \pi \times 18^3 \times \frac{90}{100}}{\pi \times 3^2 \times h}$$

$$\Rightarrow 72 = \frac{\frac{2}{3} \times 18^3 \times \frac{9}{10}}{3^2 \times h}$$

$$\Rightarrow h = \frac{\frac{2}{3} \times 18 \times 18 \times 18 \times \frac{9}{10}}{3^2 \times 72}$$

$$\therefore h = 5.4\ cm$$

Height of the small cylindrical bottle = 10.8 cm

19. A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have? Find the cost of painting the total surface area of the solid so formed, at the rate of Rs. 5 per sq. cm. [Use π = 3.14]

Solution:

Side of the cubical block, a = 10 cm

Longest diagonal of the cubical block = $a\sqrt{3} = 10\sqrt{3}$ cm

Since the cube is surmounted by a hemisphere, therefore the side of the cube should be equal to the diameter of the hemisphere.

Diameter of the sphere = 10 cm

Radius of the sphere, r = 5 cm

Total surface area of the solid = Total surface area of the cube – Inner cross-section area of the hemisphere + Curved surface area of the hemisphere

$= 6a^2 - \pi r^2 + 2 + \pi r^2$

$= 6a^2 + \pi r^2$

$= 6 \times (10)^2 + 3.14 \times 5^2$

$= 600 + 78.5 = 678.5\ cm^2$

Total surface area of the solid = 678.5 cm^2

20. 504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere, Find the diameter of the sphere and hence find its surface area. [Use $\pi = \frac{22}{7}$]

Solution:

No. of cones = 504

Diameter of a cone = 3.5 cm

Radius of the cone, r = 1.75 cm

Height of the cone, h = 3 cm

Volume of a cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times \left(\frac{3.5}{2}\right)^2 \times 3$

$= \frac{1}{3} \times \pi \times \frac{3.5}{2} \times \frac{3.5}{2} \times 3\ cm^3$

Volume of 504 cones

$= 504 \times \frac{1}{4} \times \pi \times \frac{3.5}{2} \times \frac{3.5}{2} \times 3\ cm^3$

Let the radius of the new sphere be 'R'.

Volume of the sphere $= \frac{4}{3}\pi R^3$

Volume of 504 cones = Volume of the sphere

$$504\times\frac{1}{3}\times\pi\times\frac{3.5}{2}\times\frac{3.5}{2}\times 3=\frac{4}{3}\pi R^3$$

$$\Rightarrow \frac{504\times 1\times\pi\times 3.5\times 3.5\times 3\times 3}{3\times 2\times 2\times 4\times\pi}=R^3$$

$$\Rightarrow R^3=\frac{504\times 3\times 49}{64}$$

$$\Rightarrow R^3=\frac{7\times 8\times 9\times 3\times 7^2}{64}$$

$$\Rightarrow R^3=\frac{8\times 27\times 7^3}{64}$$

$$\Rightarrow R=\frac{2\times 3\times 7}{4}$$

$$\therefore R=\frac{21}{2}=10.5\,cm$$

Radius of the new sphere = 10.5 cm

SECTION D

Question numbers 21 to 31 carry 4 marks each.

21. The diagonal of a rectangular field is 16 metres more than the shorter side. If the longer side is 14 metres more than the shorter side, then find the lengths of the sides of the field.

Solution:

Let l be the length of the longer side and b be the length of the shorter side.

Given that the length of the diagonal of the rectangular field is 16 metres more than the shorter side.

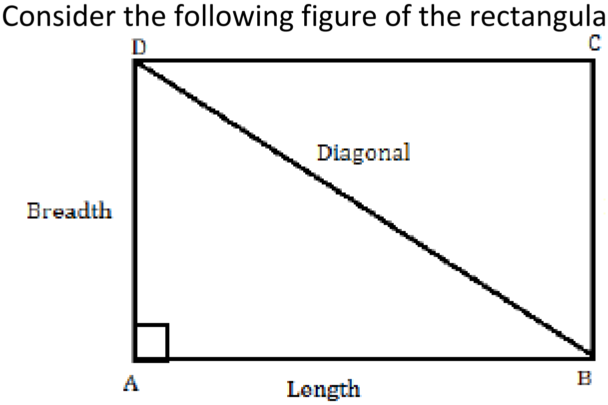
Thus, diagonal = 16 + b

Since longer side is 14 metres more than shorter side, we have,

l = 14 + b

Diagonal is the hypotenuse of the triangle.

Consider the following figure of the rectangular field.



By applying Pythagoras Theorem in ΔABD, we have,

Diagonal^2 = Length^2 = Breadth^2

$$\Rightarrow (16+b)^2=(14+b)^2+b^2$$

$$\Rightarrow 256+b^2+32b=196+b^2+28b+b^2$$

$$\Rightarrow 256+32b=196+28b+b^2$$

$$\Rightarrow 60+32b=28b+b^2$$

$\Rightarrow b^2 - 4b - 60 = 0$

$\Rightarrow b^2 - 10b + 6b - 60 = 0$

$\Rightarrow b(b-10) + 6(b-10) = 0$

$\Rightarrow (b+6)(b-10) = 0$

$\Rightarrow (b+6) = 0 \; or \; (b-10) = 0$

$\Rightarrow b = -6 \; or \; b = 10$

As breadth cannot be negative, breadth = 10 m
Thus, length of the rectangular field = 14 + 10 = 24 m

22. Find the 60th term of the AP 8, 10, 12, ……., if it has a total of 60 terms and hence find the sum of its last 10 terms.

Solution:

Consider the given A.P. 8, 10, 12, …
Here the initial term is 8 and the common difference is 10 - 8 = 2 and 12 - 10 = 2
General term of an A.P. is t_n and formula to find out t_n is

$t_n = a + (n-1)d$

$\Rightarrow t_{60} = 8 + (60-1) \times 2$

$\Rightarrow t_{60} = 8 + 59 \times 2$

$\Rightarrow t_{60} = 8 + 118$

$\Rightarrow t_{60} = 126$

We need to find the sum of the last 10 terms.
Thus,
Sum of last 10 terms = Sum of first 60 terms - Sum of first 50 terms

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\Rightarrow S_{60} = \frac{60}{2}\left[2 \times 8 + (60-1) \times 2\right]$

$\Rightarrow S_{60} = 30\left[16 + 59 \times 2\right]$

$\Rightarrow S_{60} = 30\left[134\right]$

$\Rightarrow S_{60} = 4020$

Similarly,

$\Rightarrow S_{50} = \frac{50}{2}\left[2 \times 8 + (50-1) \times 2\right]$

$\Rightarrow S_{50} = 25\left[16 + 49 \times 2\right]$

$\Rightarrow S_{50} = 25\left[114\right]$

$\Rightarrow S_{50} = 2850$

Thus the sum of last 10 terms $= S_{60} - S_{50}$ = 4020 – 2850 = 1170
Therefore,
Sum of last 10 terms = Sum of first 60 terms - Sum of first 50 terms

23. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed?

Solution:

Let x be the first speed of the train.

We know that $\frac{\text{Distance}}{\text{Speed}}$ = time

Thus, we have,

$$\frac{54}{x}+\frac{63}{x+6}=3\,\text{hours}$$

$$\Rightarrow \frac{54(x+6)+63x}{x(x+6)}=3$$

$$\Rightarrow 54(x+6)+63x=3x(x+6)$$

$$\Rightarrow 54x+324+63x=3x^2+18x$$

$$\Rightarrow 117x+324=3x^2+18x$$

$$\Rightarrow 3x^2-117x-324+18x=0$$

$$\Rightarrow 3x^2-99x-324=0$$

$$\Rightarrow x^2-33x-108=0$$

$$\Rightarrow x^2-36x+3x-108=0$$

$$\Rightarrow x(x-36)+3(x-36)=0$$

$$\Rightarrow (x+3)(x-36)=0$$

$$\Rightarrow (x+3)=0 \text{ or } (x-36)=0$$

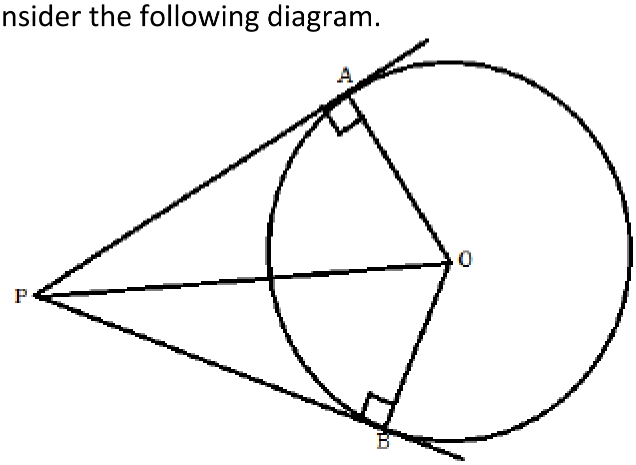
$$\Rightarrow x=-3 \text{ or } x=36$$

Speed cannot be negative and hence initial speed of the train is 36 km/hour.

24. Prove that the lengths of the tangents drawn from an external point to a circle are equal.

Solution:

Consider the following diagram.



Let P be an external point and PA and PB be tangents to the circle.

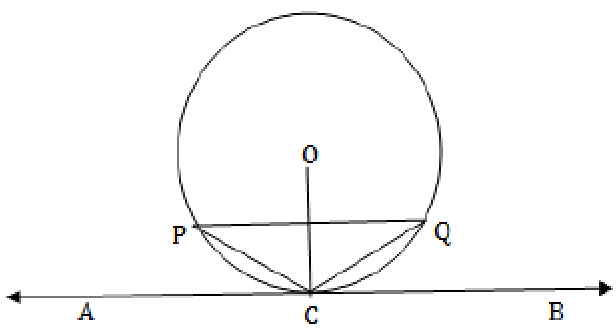
We need to prove that PA = PB

Now consider the triangles Δ OAP and Δ OBP

m ∠ A = m ∠ B = 90

OP = OP [common]
OA = OB = radii of the circle
Thus, by Right Angle-Hypotenuse-Side criterion of congruence we have,
$\Delta OAP \cong \Delta OBP$
The corresponding parts of the congruent triangles are congruent.
Thus,
PA = PB

25. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.
Solution:
In the figure, C is the midpoint of the minor arc PQ, O is the centre of the circle and
AB is tangent to the circle through point C.
We have to show the tangent drawn at the midpoint of the arc PQ of a circle is parallel to the chord joining the end points of the arc PQ.
We will show PQ || AB.
It is given that C is the midpoint point of the arc PQ.
So, arc PC = arc CQ.
$\Rightarrow$ PC = CQ



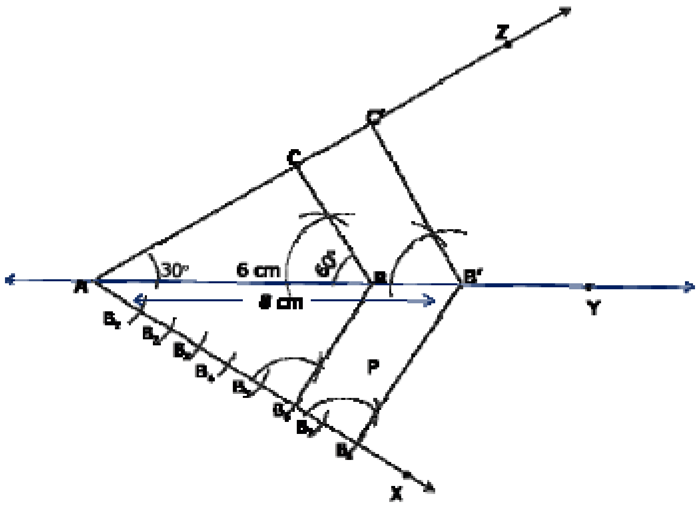
This shows that Δ PQC is an isosceles triangle.
Thus, the perpendicular bisector of the side PQ of Δ PQC passes through vertex C.
The perpendicular bisector of a chord passes through the centre of the circle.
So the perpendicular bisector of PQ passes through the centre O of the circle.
Thus perpendicular bisector of PQ passes through the points O and C.
$\Rightarrow PQ \perp OC$
AB is the tangent to the circle through the point C on the circle.
$\Rightarrow AB \perp OC$
The chord PQ and the tangent PQ of the circle are perpendicular to the same line OC.
PQ || AB.

26. Construct a Δ ABC in which AB = 6 cm, $\angle A = 30°$ and $\angle B = 60°$, Construct another $\Delta AB'C'$ similar to ΔABC with base AB' = 8 cm.
Solution:
Construct the Δ ABC as per given measurements.
In the half plane of $\overline{AB}$ which does not contain C, draw $\overrightarrow{AX}$ such that $\angle BAX$ is an acute angle.
3) With some appropriate radius and centre A, Draw an arc to intersect $\overrightarrow{AX}$ at B_1. Similarly, with centre B_1 and the same radius, draw an arc to intersect $\overrightarrow{BX}$ at B_2 such that $B_1B_2 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7 = B_7B_8$

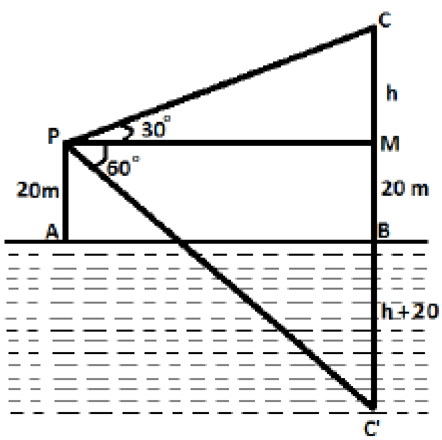
4) Draw $\overline{B_6B}$.

5) Through B_8 draw a ray parallel to $\overline{B_6B}$. to intersect $\overline{AY}$ at B'.

6) Through B' draw a ray parallel to $\overline{BC}$ to intersect $\overline{AZ}$ at C'.
Thus, Δ AB'C' is the required triangle.



27. At a point A, 20 metres above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at a A is 60°. Find the distance of the cloud from A.
Solution:



Let AB be the surface of the lake and P be the point of observation such that
AP = 20 metres. Let C be the position of the cloud and C' be its reflection in the lake.
Then CB = C'B. Let PM be perpendicular from P on CB.
Then m$\angle$CPM=30 and m$\angle$C'PM = 60
Let CM = h. Then CB = h + 20 and C'B = h + 20.
In CMP we have,

$\tan 30° = \frac{CM}{PM}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{PM}$

$\Rightarrow PM = \sqrt{3}h$..................(i)

In Δ PMC' we have,

$\tan 60° = \frac{C'M}{PM}$

$\Rightarrow \sqrt{3} = \frac{C'B + BM}{PM}$

$\Rightarrow \sqrt{3} = \frac{h + 20 + 20}{PM}$

$\Rightarrow PM = \frac{h + 20 + 20}{\sqrt{3}}$..................(ii)

From equation (i) and (ii), we get

$\sqrt{3}h = \frac{h + 20 + 20}{\sqrt{3}}$

$\Rightarrow 3h = h + 40$

$\Rightarrow 2h = 40$

$\Rightarrow h = 20\text{ m}$

Now, CB = CM + MB = h + 20 + 20 + 20 = 40.

Hence, the height of the cloud from the surface of the lake is 40 metres.

28. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is

i. a card of spade or an ace.

ii. a black king.

iii. neither a jack nor a king

iv. either a king or a queen.

Solution:

Let S be the sample space of drawing a card from a well-shuffled deck.

$n(S) = {}^{52}C_1 = 52$

(i)There are 13 spade cards and 4 ace's in a deck As ace of spade is included in 13 spade cards, so there are 13 spade cards and 3 ace's

a card of spade or an ace can be drawn in ${}^{13}C_1 + {}^{3}C_1 = 13 + 3 = 16$

Probability of drawing a card of spade or an ace $= \frac{16}{52} = \frac{4}{13}$

(ii)There are 2 black King cards in a deck a card of black King can be drawn in ${}^{2}C_1 = 2$

Probability of drawing a black king $= \frac{2}{52} = \frac{1}{26}$

(iii) There are 4 Jack and 4 King cards in a deck.

So there are 52 - 8 = 44 cards which are neither Jacks nor Kings. a card which is neither a Jack nor a King can be drawn in $^{44}C_1 = 44$

Probability of drawing a card which is neither a Jack nor a King = $\frac{44}{52}=\frac{11}{13}$

(iv)There are 4 King and 4 Queen cards in a deck.
So there are 4 - 4 = 8 cards which are either King or Queen.
a card which is either a King or a Queen can be drawn in $^{8}C_1 = 8$

Probability of drawing a card which is either a King or a Queen = $\frac{8}{52}=\frac{2}{13}$

29. Find the values of k so that the area of the triangle with vertices (1, -1), (-4, 2k) and (-k, -5) is 24 sq. units.

Solution:

Take $(x_1, y_1) = (1, -1), (-4, 2k)$ and $(-k, -5)$

It is given that the area of the triangle is 24 sq. unit

Area of the triangle having vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is given by

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\therefore 24 = \frac{1}{2}[1(2k - (-5)) + (-4)((-5) - (-1)) + (-k)((-1) - 2k)]$$

$$48 = [(2k + 5) + 16 + (k + 2k^2)]$$

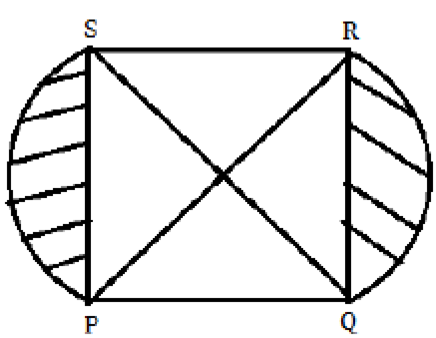
$$\therefore 2k^2 + 3k - 27 = 0$$

$$\therefore (2k + 9)(k - 3) = 0$$

$$\therefore k = -\frac{9}{2} \text{ or } k = 3$$

The values of k are $-\frac{9}{2}$ and 3.

30. In the following figure, PQRS is square lawn with side PQ = 42 metres. Two circular flower beds are there on the sides PS and QR with centre at O, the intersections of its diagonals. Find the total area of the two flower beds (shaded parts).



Solution:

PQRS is a square.

So each side is equal and angle between the adjacent sides is a right angle.

Also the diagonals perpendicularly bisect each other.

In Δ PQR using pythagoras theorem,
$PR^2 = PQ^2 + QR^2$
$PR^2 = (42)^2 + (42)^2$
$PR = \sqrt{2}(42)$
$OR = \frac{1}{2}PR = \frac{42}{\sqrt{2}} = OQ$
From the figure we can see that the radius of flower bed ORQ is OR.
Area of sector ORQ = $\frac{1}{4}\pi r^2$
$= \frac{1}{4}\pi\left(\frac{42}{\sqrt{2}}\right)^2$
Area of the Δ ROQ $= \frac{1}{2} \times RO \times OQ$
$= \frac{1}{2} \times \frac{42}{\sqrt{2}} \times \frac{42}{\sqrt{2}}$
$= \left(\frac{42}{2}\right)^2$
Area of the flower bed ORQ
= Area of sector ORQ - Area of the ROQ
$= \frac{1}{4}\pi\left(\frac{42}{\sqrt{2}}\right)^2 - \left(\frac{42}{2}\right)^2$
$= \left(\frac{42}{2}\right)^2\left[\frac{\pi}{2} - 1\right]$
= (441)[0.57]
= 251.37 cm^2
Area of the flower bed ORQ = Area of the flower bed OPS
= 251.37 cm^2
Total area of the two flower beds
= Area of the flower bed ORQ + Area of the flower bed OPS
= 251.37 + 251.37
= 502.74 cm^2

31. From each end of a solid metal cylinder, metal was scooped out in hemispherical from of same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire. [Use $\pi = \frac{22}{7}$]

Solution:
Height of the cylinder (h) = 10 cm
Radius of the base of the cylinder = 4.2 cm
Volume of original cylinder $= \pi r^2 h$
$= \frac{22}{7} \times (4.2)^2 \times 10$
$= 554.4\ cm^3$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (4.2)^3$

$= 155.232\ cm^3$

Volume of the remaining cylinder after scooping out hemisphere from each end

= Volume of original cylinder - 2 $\times$ Volume of hemisphere

$= 554.4 - 2 \times 155.232$

$= 243.936\ cm^3$

The remaining cylinder is melted and converted to

a new cylindrical wire of 1.4 cm thickness.

So they have same volume and radius of new cylindrical wire is 0.7 cm.

Volume of the remaining cylinder = Volume of the new cylindrical wire

$243.936 = \pi r^2 h$

$243.936 = \frac{22}{7}(0.7)^2 h$

$h = 158.4\ cm$

$\therefore$ The length of the new cylindrical wire of 1.4 cm thickness is 158.4 cm.

CBSE-X-2014 EXAMINATION

MATHEMATICS

Paper & Solution

Time: 3 Hrs. Max. Marks: 90

***General Instuctions** :*

1. All questions are **compulsory**.

2. The question paper consists of **34** questions divided into **four sections** A, B, C, and D.

3. Section A contains of **8** questions of 1 mark each, which are multiple choice type question, **Section B** contains of **6** questions of 2 marks each, **Section C** contains of **10** questions of 3 marks each and **Section D** contains of **10** questions of 4 marks each.

4. Use of calculator is **not** permitted.

SECTION – A

Q-1 The first three terms of an AP respectively are 3y – 1, 3y +5 and 5y +1. Then y equals

(A) -3

(B) 4

(C) 5

(D) 2

Solution:

The first three terms of an AP are 3y-1, 3y+5 and 5y+1, respectively.

We need to find the value of y.

We know that if a, b and c are in AP, then:

$b - a = c - b \Rightarrow 2b = a + c$

$\therefore 2\ (3y+5) = 3y - 1 + 5y + 1$

$\Rightarrow 6y + 10 = 8y$

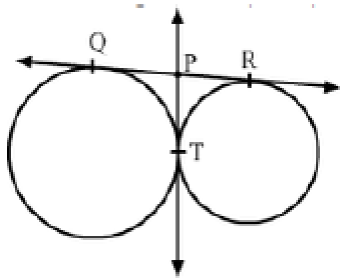
$\Rightarrow 10 = 8y - 6y$

$\Rightarrow 2y = 10$

$\Rightarrow y = 5$

Hence the correct option is C.

Q-2 In Fig. 1, QR is a common tangent to the given circles, touching externally at the point T. The tangent at T meets QR at P. If PT = 3.8 cm, then the length of QR (in cm) is:



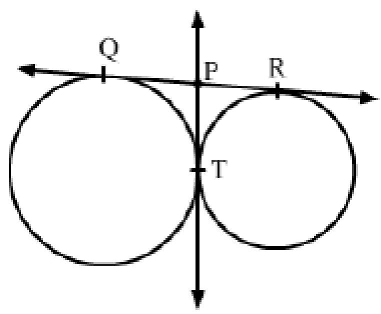
(A) 3.8

(B) 7.6

(C) 5.7

(D) 1.9

Solution:



It is known that the length of the tangents drawn from an external point to a circle is equal.

∴ QP = PT = 3.8 cm ... (1)

PR = PT = 3.8 cm ... (2)

From equations (1) and (2), we get:

QP = PR = 3.8 cm

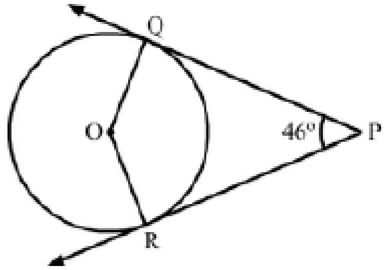
Now, QR = QP + PR

= 3.8 cm + 3.8 cm

= 7.6 cm

Hence, the correct option is B.

3. In Fig. 2, PQ and PR are two tangents to a circle with centre O. If ∠QPR = 46° , then ∠QOR equals:



(A) 67°

(B) 134°

(C) 44°

(D) 46°

Solution:
Given: ∠QPR = 46°

PQ and PR are tangents.

Therefore, the radius drawn to these tangents will be perpendicular to the tangents.

So, we have OQ ⊥ PQ and OR ⊥ RP.

⇒ ∠OQP = ∠ORP = 90°

So, in quadrilateral PQOR, we have

∠OQP +∠QPR + ∠PRO + ∠ROQ = 360°

⇒ 90° + 46° + 90° + ∠ROQ = 360°

⇒ ∠ROQ = 360° − 226° = 134°

Hence, the correct option is B.

4. A Ladder makes an angle of 60° with the ground when placed against a wall. If the foot of the ladder is 2 m away from the wall, then the length (in meters) is:

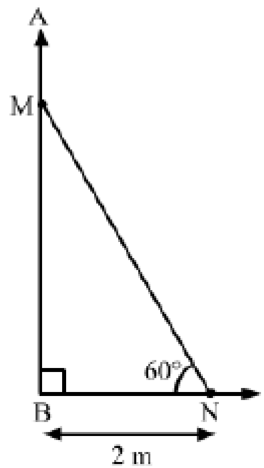
(A) $\frac{4}{\sqrt{3}}$

(B) $4\sqrt{3}$

(C) $2\sqrt{2}$

(D) 4

Solution:



In the figure, MN is the length of the ladder, which is placed against the wall AB and makes an angle of 60° with the ground.

The foot of the ladder is at N, which is 2 m away from the wall.

∴ BN = 2 m

In right-angled triangle MNB:

$$\cos 60° = \frac{BN}{MN} = \frac{2m}{MN}$$
$$\Rightarrow \frac{1}{2} = \frac{2m}{MN}$$
$$\Rightarrow MN = 4m$$

Therefore, the length of the ladder is 4 m.

Hence, the correct option is D

Q5. If two different dice are rolled together, the probability of getting an even number on both dice, is:

(A) 1/36

(B) 1/2

(C) 1/6

(D) 1/4

Solution:
Possible outcomes on rolling the two dice are given below:

{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Total number of outcomes = 36

Favourable outcomes are given below:

{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)}

Total number of favourable outcomes = 9

∴ Probability of getting an even number on both dice =

$$\frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{9}{36} = \frac{1}{4}$$

Hence, the correct option is D.

Q6. A number is selected at random from the numbers 1 to 30. The probability that it is a prime number

(A) 2/3

(B) 1/6

(C) 1/3

(D) 11/30

Solution:
Total number of possible outcomes = 30

Prime numbers between 1 to 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

Total number of favourable outcomes = 10

∴ Probability of selecting a prime number from 1 to 30

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{10}{30} = \frac{1}{3}$$

Hence, the correct option is C.

Q7 If the points A(x, 2), B (-3,-4) and C (7, -5) are collinear, then the value of x is:

(A) -63

(B) 63

(C) 60

(D) -60

Solution:
It is given that the three points A(*x*, 2), B(−3, −4) and C(7, −5) are collinear.

∴ Area of ΔABC = 0

$\Rightarrow \frac{1}{2}\ [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\ ] = 0$

Here, $x_1 = x$, $y_1 = 2$, $x_2 = -3$, $y_2 = -4$, and $x_3 = 7$, $y_3 = -5$

$\Rightarrow x[-4 - (-5)] - 3(-5 - 2) + 7[2 - (-4)\ ] = 0$

$\Rightarrow x(-4 + 5) - 3(-5 - 2) + 7(2 + 4) = 0$

$\Rightarrow x - 3 \times (-7) + 7 \times 6 = 0$

$\Rightarrow x + 21 + 42 = 0 \Rightarrow x + 63 = 0$

$\Rightarrow x = -63$

Thus, the value of *x* is − 63.

Hence, the correct option is A.

Q8 The number of solid of solid spheres, each of diameter 6cm that can be made by melting a solid metal cylinder of height 45 cm and diameter 4 cm is:

(A) 3

(B) 5

(C) 4

(D) 6

Solution:
Let *r* and *h* be the radius and the height of the cylinder, respectively.

Given: Diameter of the cylinder = 4 cm

∴ Radius of the cylinder, *r* = 2 cm

Height of the cylinder, *h* = 45 cm

Volume of the solid cylinder = $\pi r^2 h = \pi \times (2)^2 \times 45\ cm^3 = 180\pi\ cm^3$

Suppose the radius of each sphere be *R* cm.

Diameter of the sphere = 6 cm

∴ Radius of the sphere, *R* = 3 cm

Let *n* be the number of solids formed by melting the solid metallic cylinder.

∴ *n* × Volume of the solid spheres = Volume of the solid cylinder

$$\Rightarrow n\times\frac{4}{3}\pi R^3 = 180\pi$$

$$\Rightarrow n\times\frac{4}{3}\pi R^3 = 180\pi \text{ k}$$

$$\Rightarrow \text{n} = \frac{180\times 3}{4\times 27} = 5$$

Thus, the number of solid spheres that can be formed is 5.

Hence, the correct option is B.

SECTION-B

Q9 Solve the quadratic equation $2x^2 + ax - a^2 = 0$ for x.

Solution:
We have: $2x^2 + ax - a^2 = 0$

Comparing the given equation with the standard quadratic equation ($ax^2 + bx + c = 0$), we get a =2, b = a and c =-a^2

Using the quadratic formula, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, we get:

$$x = \frac{-a\pm\sqrt{a^2-4\times 2\times(-a)^2}}{2\times 2}$$

$$= \frac{-a\pm\sqrt{9a^2}}{4}$$

$$= \frac{-a\pm 3a}{4}$$

$$\Rightarrow x = \frac{-a+3a}{4} = \frac{a}{2} \text{ or } x = \frac{a-3a}{4} = -a$$

So, the solutions of the given quadratic equation are $x = \frac{a}{2}$ or $x = -a$.

Q10. The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, Find its common difference.

Solution:
Let a be the first term and d be the common difference.

Given: a = 5

$T_n = 45$
$S_n = 400$

We know:

$T_n = a + (n-1)d$
$\Rightarrow 45 = 5 + (n-1)d$
$\Rightarrow 40 = (n-1)d$ ....(1)

And $S_n = \frac{n}{2}a + T_n$

$\Rightarrow 400 = \frac{n}{2}(5+45)$

$\Rightarrow \frac{n}{2} = \frac{400}{50}$
$\Rightarrow n = 2\times 8 = 16$

On substituting n = 16 in (1), we get:

40 =(16-1)d

⇒ 40 = (15)d

$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$

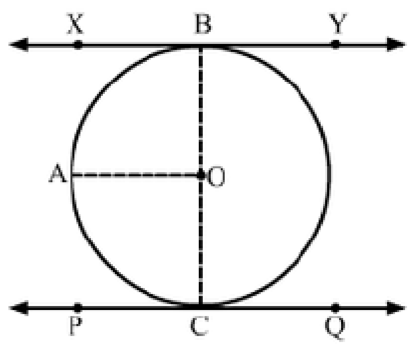
Thus, the common difference is 83.

Q11. Prove that the line segment joining the point of contact of two parallel tangles of a circle passes through its centre.

Solution:
Let XBY and PCQ be two parallel tangents to a circle with centre O.

Construction: Join OB and OC.

Draw OA||XY



Now, XB | |AO

⇒ ∠XBO +∠AOB = 180° (sum of adjacent interior angles is 180°)

Now, ∠XBO = 90° (A tangent to a circle is perpendicular to the radius through the point of contact)

⇒90° +∠AOB =180°

⇒∠AOB = 180° - 90° =180°

Similarly , ∠AOC = 90^0

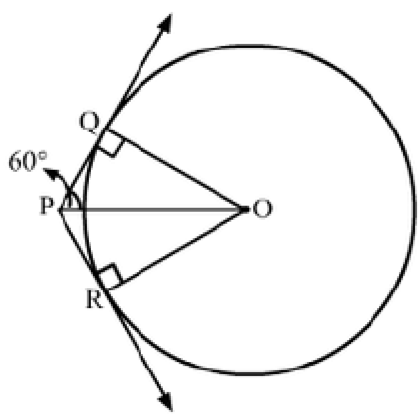
∠AOB + ∠AOC = 90^0 + 90^0= 180°

Hence, BOC is a straight line passing through O.

Thus, the line segment joining the points of contact of two parallel tangents of a circle passes through its centre.

Q12 If from an external point P of a circle with centre O, two tangents PQ and PR are drawn such that ∠QPR = 120° , prove that 2PQ = PO.

Solution:
Let us draw the circle with extent point P and two tangents PQ and PR.



We know that the radius is perpendicular to the tangent at the point of contact.

∴∠OQP = 90°

We also know that the tangents drawn to a circle from an external point are equally inclined to the joining the centre to that point.

∴∠QPO = 60°

Now, in ΔQPO:

Cos 60°= $\frac{PQ}{PO}$

$\Rightarrow \frac{1}{2} = \frac{PQ}{PO}$

$\Rightarrow 2PQ = PO$

Q13 Rahim tosses two different coins simultaneously. Find the probability of getting at least one tail.

Solution:
Rahim tosses two coins simultaneously. The sample space of the experiment is {HH, HT, TH, and TT}.

Total number of outcomes = 4

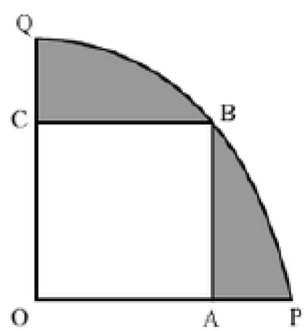
Outcomes in favour of getting at least one tail on tossing the two coins = {HT, TH, TT}

Number of outcomes in favour of getting at least one tail = 3

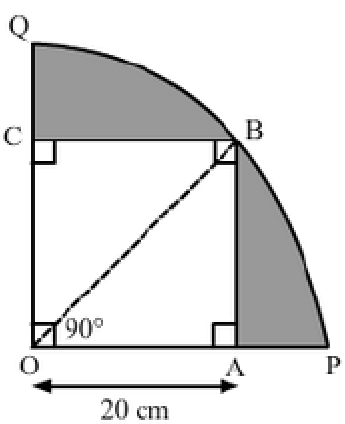
∴ Probability of getting at least one tail on tossing the two coins

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{3}{4}$

Q14 In fig. 3, a square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 20 cm, find the area of the shaded region (Use π= 3.14)



Let us join OB.



In ΔOAB: $OB^2 = OA^2 + AB^2 = (20)^2 + (20)^2 = 2 \times (20)^2$

$\Rightarrow OB = 20\sqrt{2}$

Radius of the circle, $r = 20\sqrt{2} cm$

Area of quadrant OPBQ $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{90°}{360°} \times 3.14 \times (20\sqrt{2})^2 cm^2$

$= \frac{1}{4} \times 3.14 \times 800\ cm^2$

$= 628\ cm^2$

Area of square OABC = $(\text{Side})^2 = (20)^2\ cm^2 = 400\ cm^2$

∴ Area of the shaded region = Area of quadrant OPBQ – Area of square OABC

= (628 - 400) cm

$= 228\ cm^2$

SECTION-C

Q15 Solve the equation $\frac{4}{x} - 3 = \frac{5}{2x+3}; x \neq 0, -3/2$, for x.

Solution:
Given equation:

$\frac{4}{x} - 3 = \frac{5}{2x+3}; x \neq 0, -\frac{3}{2}$

$\frac{4}{x} - 3 = \frac{5}{2x+3}$

$\Rightarrow \frac{4-3x}{x} = \frac{5}{2x+3}$

$\Rightarrow (4-3x)(2x+3) = 5x$

$\Rightarrow -6x^2 + 8x - 9x + 12 = 5x$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow (x+2)(x-1) = 0$

$\Rightarrow (x+2) = 0\ (x-1) = 0$

$\Rightarrow$ x = -2 or x = 1

Thus, the solution of the given equation is -2 or 1.

Q16. If the seventh term of an AP is 1/9 and its ninth term is 1/7, find its 63rd term.

Solution:
Let a be the first term and d be the common difference of the given A.P.

Given:

$$a_7 = \frac{1}{9}$$

$$a_9 = \frac{1}{7}$$

$$a7 = a + (7-1)d = \frac{1}{9}$$

$$\Rightarrow a + 6d = \frac{1}{9} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

$$a_9 = a + (9-1)d = \frac{1}{7}$$

$$\Rightarrow a + 8d = \frac{1}{7} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Subtracting equation (1) from (2), we get:

$$2d = \frac{2}{63}$$

$$\Rightarrow d = \frac{1}{63}$$

Putting $d = \frac{1}{63}$ in equation (1), we get:

$$a + \left(6 \times \frac{1}{63}\right) = \frac{1}{9}$$

$$\Rightarrow a = \frac{1}{63}$$

$$\therefore a_{63} = a + (63-1)d = \frac{1}{63} + 62\left(\frac{1}{63}\right) = \frac{63}{63} = 1$$

Thus, the 63rd term of the given A.P. is 1.

Q17. Draw a right triangle ABC is which AB = 6 cm, BC = 8 cm and ∠B = 90°. Draw BD perpendicular from B on AC and draw a circle passing through the points B, C and D. Construct tangents from A to this circle.

Solution:

Follow the given steps to construct the figure.

Step 1

Draw a line BC of 8 cm length.

Step 2

Draw BX perpendicular to BC.

Step 3

Mark an arc at the distance of 6 cm on BX. Mark it as A.

Step 4

Join A and C. Thus, ΔABC is the required triangle.

Step 5

With B as the centre, draw an arc on AC.

Step 6

Draw the bisector of this arc and join it with B. Thus, BD is perpendicular to AC.

Step 7

Now, draw the perpendicular bisector of BD and CD. Take the point of intersection as O.

Step 8

With O as the centre and OB as the radius, draw a circle passing through points B, C and D.

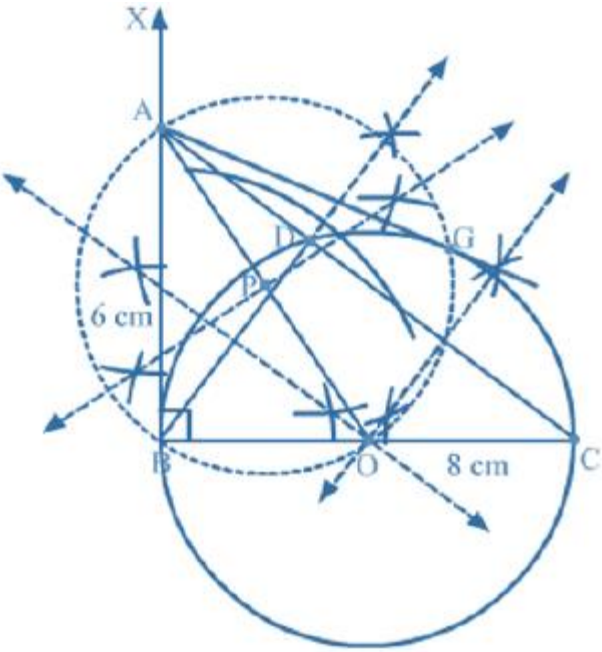
Step 9

Join A and O and bisect it Let P be the midpoint of AO.

Step 10

Taking P as the centre and PO as its radius, draw a circle which will intersect the circle at point B and G. Join A and G.

Here, AB and AG are the required tangents to the circle from A.



Q18. If the point A (0,2) is equidistant from the points B(3, p) and C(p, 5), find P. Also find the the length of AB.

Solution:
The given points are A (0, 2), B (3, p) and C (p, 5).

It is given that A is equidistant from B and C.

∴ AB = AC

$\Rightarrow AB^2 = AC^2$

$\Rightarrow (3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$

$\Rightarrow 9 + p^2 + 4 - 4p = p^2 + 9$

$\Rightarrow 4 - 4p = 0$

$\Rightarrow 4p = 4$

$\Rightarrow p = 1$

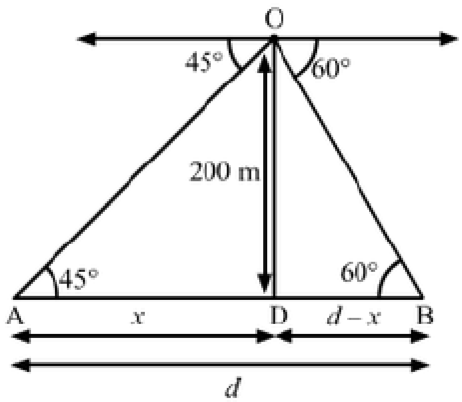
Thus, the value of p is 1

Length of AB $= \sqrt{(3-0)^2 + (1-2)^2} = \sqrt{3^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$ units.

Q19. Two ships are there in the sea on either side of a light house in such a way that the ships and the light house are in the same straight line. The angles of depression of two ships as observed from the top of the light house are 60° and 45°. If the height of the light house is 200 m, find the distance between the two ships. [Use $\sqrt{3} = 1.73$]

Solution:

Let d be the distance between the two ships. Suppose the distance of one of the ships from the light house is X meters, then the distance of the other ship from the light house is (d-x) meter.



In right –angled ΔADO, we have.

$$\tan 45° = \frac{OD}{AD} = \frac{200}{X}$$
$$\Rightarrow 1 = \frac{200}{X}$$
$$\Rightarrow X = 200 \quad ....(1)$$

In right-angled Δ BDO, we have

$$\tan 60° = \frac{OD}{BD} = \frac{200}{d = x}$$
$$\Rightarrow \sqrt{3} = \frac{200}{d - x}$$
$$\Rightarrow d - x = \frac{200}{\sqrt{3}}$$

Putting x=200. We have:

$$d - 200 = \frac{200}{\sqrt{3}}$$
$$d = \frac{200}{\sqrt{3}} + 200$$
$$\Rightarrow d = 200\left(\frac{\sqrt{3+1}}{\sqrt{3}}\right)$$
$$\Rightarrow d = 200 \times 1.58$$
$$\Rightarrow d = 316 \quad \text{(approx.)}$$

Thus, the distance between two ships is approximately 316 m.

Q20 If the points A(-2. 1), B (a, b) and C (4, -1) are collinear and a-b = 1, find the values of a and b.

Solution:
The given points are A (-2, 1), B (a, b) and C (4-1).

Since the given points are collinear, the area of the triangle ABC is 0.

$$\Rightarrow \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x3(y_1-y_2)\right]=0$$

Here, x_1=-2,y_1 =1,x_2=a, y_2 =b and x_3=4, y_3=-1

$$\therefore \frac{1}{2}\left[-2(b+1)+a(-1-1)+4(1-b)\right]=0$$
$$\Rightarrow -2b-2-2a+4-4b=0$$
$$\Rightarrow 2a+6b=2$$
$$\Rightarrow a+3b=1 \quad ....(1)$$

Given :

a-b= 1 ....(2)

Subtracting equation (1) from (2) we get:

4b =0

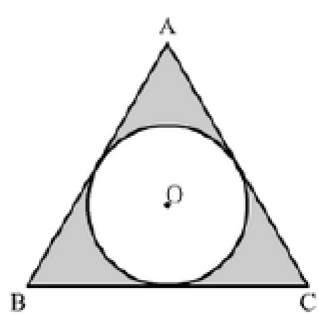
⇒b=0

Subtracting b= 0 in (2), we get:

a= 1

Thus, the values of a and b are 1 and 0, respectively.

Q21 In Fig 4, a circle is inscribed in an equilateral triangle ABC of side 12 cm. Find the radius of inscribed circle and the area of the shaded region. [Use π =3.14 and $\sqrt{3}=1.73$]



Solution:
It is given that ABC is an equilateral triangle of side 12 cm.

Construction:

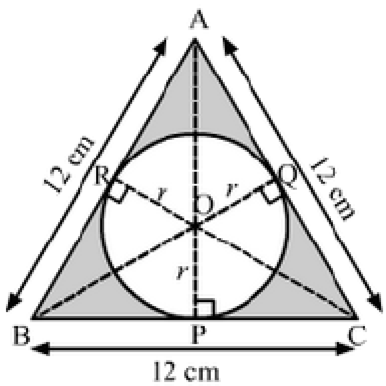
Join OA, OB and OC.

Draw.

OP ⊥ BC

OQ ⊥ AC

OR ⊥ AB



Let the radius of the circle be r cm.

Area of ΔAOB +Area of ΔBOC + Area of ΔAOC = Area of ΔABC

$$\Rightarrow \frac{1}{2}\times AB\times OR+\frac{1}{2}\times BC\times OP+\frac{1}{2}\times AC\times OQ=\frac{\sqrt{3}}{4}\times(\text{Side})^2$$
$$\Rightarrow \frac{1}{2}\times 12\times r+\frac{1}{2}\times 12\times r+\frac{1}{2}\times 12\times r=\frac{\sqrt{3}}{4}\times(12)^2$$
$$\Rightarrow 3\times\frac{1}{2}\times 12\times r=\frac{\sqrt{3}}{4}\times 12\times 12$$
$$\Rightarrow r=2\sqrt{3}=2\times 1.73=3.46$$

Therefore the radius of the inscribed circle is 3.46 cm.

Now, area of the shaded region = Area of ΔABC – Area of the inscribed circle

$$=\left[\frac{\sqrt{3}}{4}\times(12)^2-\pi(2\sqrt{3})^2\right]cm^2$$
$$=\left[36\sqrt{3}-12\pi\right]cm^2$$

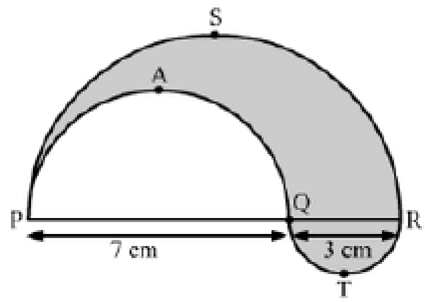
= [36 ×1.73 – 12 × 3.14] cm^2

= [62.28 – 37.68] cm^2

= 24.6 cm^2

Therefore, the area of the shaded region is 24.6 cm^2.

Q22. In Fig.5. PSR, RTQ and PAQ are three semicircles of diameters 10cm, 3cm and 7 cm respectively. Find the perimeter of the shaded region. [Use π = 3.14]



Solution:

Radius of Semicircle PSR = $\frac{1}{2}$ × 10 cm =5 cm

Radius of Semicircle RTQ = $\frac{1}{2}$ × 3 = 1.5 cm

Radius of semicircle PAQ = $\frac{1}{2}$ ×7 cm = 3.5 cm

Perimeter of the shaded region = Circumference of semicircle PSR + Circumference of semicircle RTQ + Circumference of semicircle PAQ

$$= \left[\frac{1}{2}\times 2\pi(5) + \frac{1}{2}\times 2\pi(1.5) + \frac{1}{2}\times 2\pi(3.5)\right] cm$$
$$= \pi(5+1.5+3.5) cm$$
$$= 3.14 \times 10 cm$$
$$= 31.4\ cm$$

Q23 A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank which is 10 m in diameter and 2 m deep? If the water flows through the pipe at the rate of 4 km per hour, in how much time will the tank be filled completely?

Solution:

For the given tank.

Diameter = 10 m

Radius, R = 5m

Depth, H =2m

Internal radius of the pipe = r = $\frac{20}{2}$ cm =10 cm = $\frac{1}{10}$ m

Rate of flow of water = v = 4 km/h = 4000 m/h

Let t be the time taken to fill the tank.

So, the water flown through the pipe in t hours will equal to the volume of the tank

$$\therefore \pi r^2 \times v \times t = \Pi R^2 H$$

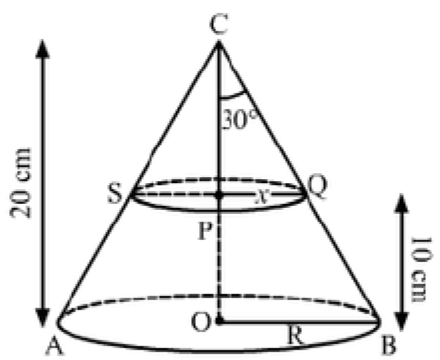
$$\Rightarrow \left(\frac{1}{10}\right)^2 \times 4000 \times t = (5)^2 \times 2$$

$$\Rightarrow t = \frac{25 \times 2 \times 100}{4000} = 1\frac{1}{4}$$

Hence, the time taken is $1\frac{1}{4}$ hours.

Q24. A solid metallic right circular cone 20 cm high and whose vertical angle is 60°, is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{12}$ cm, find the length of the wire.

Solution:



Let ACB be the cone whose vertical angle ∠ACB = 60°. Let R and x be the radii of the lower and upper end of the frustum.

Here, height of the cone, OC = 20 cm = H

Height CP = h = 10 cm

Let us consider P as the mid-Point of OC.

After cutting the cone into two parts through P.

OP = $\frac{20}{2}$ = 10 cm

Also, ∠ACO and ∠OCB = $\frac{1}{2}$ × 60° = 30°

After cutting cone CQS from cone CBA, the remaining solid obtained is a frustum.

Now, in triangle CPQ:

$$\tan 30° = \frac{x}{10}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{10}$$
$$\Rightarrow x = \frac{10}{\sqrt{3}} cm$$

In triangle COB:

$$\text{Tan}30° = \frac{R}{CO}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{R}{20}$$
$$\Rightarrow R = \frac{20}{\sqrt{3}} cm$$

Volume of the frustum, $V = \frac{1}{3}\pi(R^2H - x^2h)$

$$\Rightarrow V = \frac{1}{3}\pi\left(\left(\frac{20}{\sqrt{3}}\right)^2 \cdot 20 - \left(\frac{10}{\sqrt{3}}\right)^2 \cdot 10\right)$$
$$= \frac{1}{3}\pi\left(\frac{8000}{3} - \frac{1000}{3}\right)$$
$$= \frac{1}{3}\pi\left(\frac{7000}{3}\right)$$
$$= \frac{1}{9}\pi \times 7000$$
$$= \frac{7000}{9}\pi$$

The volumes of the frustum and the wire formed are equal.

$$\pi \times \left(\frac{1}{24}\right)^2 \times l = \frac{7000}{9} \times [\text{Volume of wire} = \pi r^2 h]$$

$$\Rightarrow l = \frac{7000}{9} \times 24 \times 24$$
$$\Rightarrow l = 448000 cm = 4480 m$$

Hence, the length of the wire is 4480 m.

SECTION –D

Q 25. The difference of two natural number is 5 and the difference of their reciprocals is 110. Find the numbers.

Solution:
Let the two natural numbers be X and Y such that x > y.

Given:

Difference between the natural numbers = 5

∴ X - Y =5 .....(1)

Difference of their reciprocals $= \frac{1}{10}$ (given)

$$\frac{1}{y} - \frac{1}{x} = \frac{1}{10}$$

$$\Rightarrow \frac{x-y}{xy} = \frac{1}{10}$$

$$\Rightarrow \frac{5}{xy} = \frac{1}{10}$$

$$\Rightarrow xy = 50 \quad ....(ii)$$

Putting the value of x from equation (i) in equation (ii), we get

(y+5) y =50

$\Rightarrow y^2 + 5y - 50 = 0$

$\Rightarrow y^2 + 10y - 5y - 50 = 0$

⇒ y (y+10) – 5 (y+10) = 0

⇒ (y -5) (y + 10) = 0

⇒ y = 5 or -10

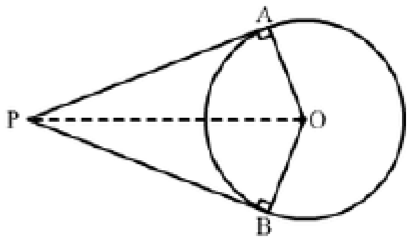
As y is a natural number, therefore y = 5

Other natural number = y + 5 = 5 + 5 = 10

Thus, the two natural numbers are 5 and 10.

Q26. Prove that the length of the tangents drawn from an external point to a circle are equal.

Solution:
Let AP and BP be the two tangents to the circle with centre O.



To Prove : AP = BP

Proof:

In ΔAOP and ΔBOP

OA =OB (radii of the same circle)

∠OAP = ∠OBP = 90° (since tangent at any point of a circle is perpendicular to the radius through the point of contact)

OP = OP (common)

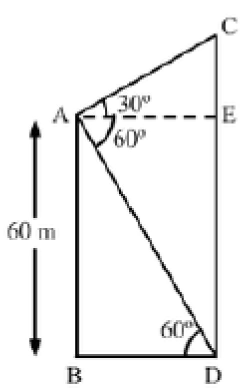
∴ ΔAOP ≅ Δ OBP (by R.H.S. congruence criterion)

∴ AP = BP (corresponding parts of congruent triangles)

Hence the length of the tangents drawn from an external point to a circle are equal.

Q27. The angles of elevation and depression of the top and the bottom of a tower from the top Of a building, 60 m high, are 30° and 60° respectively. Find the difference between the heights of the building and the tower and the distance between them.

Solution:
Let AB be the building and CD be the tower.



In right ΔABD.

$\frac{AB}{BD} = \tan 60^\circ$

$\Rightarrow \frac{60}{BD} = \sqrt{3}$

$\Rightarrow BD = \frac{60}{\sqrt{3}}$

$\Rightarrow BD = 20\sqrt{3}$

In right ΔACE:

$\frac{CE}{AE} = \tan 30^\circ$

$\Rightarrow \frac{CE}{BD} = \frac{1}{\sqrt{3}}$ $(\because AE = BD)$

$\Rightarrow CE = \frac{20\sqrt{3}}{\sqrt{3}} = 20$

Height of the tower = CE + ED = CE + AB = 20 m + 60 m = 80 m

Difference between the heights of the tower and the building = 80 m – 60m = 20 m

Distance between the tower and the building = BD = $20\sqrt{3}$ m.

Q28. A bag contains cards numbers from 1 to 49. A card is drawn from the bag at random, after mixing the cards thoroughly. Find the probability that the number on the drawn card is:

(1) An odd number

(2) A multiple of 5

(3) A perfect Square

(4) An even prime number.

Solution:
Total number of cards = 49

(1)

Total number of outcomes = 49

The odd numbers form 1 to 49 are 1, 3,5,7,9,11,13,15,17, 19,21, 23, 2527,29,31,33,35, 37, 39, 41, 43, 45, 47 and 49.

Total number of favourable outcomes = 25

$\therefore$ Required probability $= \frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{25}{49}$

(ii) Total number of outcomes = 49

The number 5,10,15,20,25,30,35,40 and 45 multiples of 5.

The number of favourable outcomes = 9

$\therefore$ Required probability $= \dfrac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{9}{49}$

(iii) Total number of outcomes = 49

The number 1, 4, 9, 16, 25, 36 and 49 are perfect squares.

Total number of favourable outcomes = 7

$\therefore$ Required probability $= \dfrac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{7}{49} = \dfrac{1}{7}$

(iv)

Total number of outcomes = 49

We know that there is only one even prime number which is 2

Total number of favourable outcomes = 1

$\therefore$ Required probability $= \dfrac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{1}{49}$

Q29. Find the ratio in which the point P (X, 2) divides the line segment joining the points A (12, 5)and B (4, -3). Also find the value of X

Solution:

Let the Point P (x, 2) divide the line segment joining the points A (12, 5) and B (4, -3) in the ratio k: 1

Then, the coordinates of P are $\left(\dfrac{4k+12}{k+1}, \dfrac{-3k+5}{k+1}\right)$

Now, the coordinates of P are (x,2)

$\therefore \dfrac{4k+12}{k+1} = x$ and $\dfrac{-3k+5}{k+1} = 2$

$\dfrac{-3k+5}{k+1} = 2$

$\Rightarrow -3k+5 = 2k+2$

$\Rightarrow 5k = 3$

$\Rightarrow k = \dfrac{3}{5}$

Substituting $k = \dfrac{3}{5}$ in $\dfrac{4k+12}{k+1} = x$, we get

$$x = \frac{4 \times \frac{3}{5} + 12}{\frac{3}{5} + 1}$$

$$\Rightarrow x = \frac{12 + 60}{3 + 5}$$

$$\Rightarrow x = \frac{72}{8}$$

$$\Rightarrow x = 9$$

Thus, the value of x is 9

Also, the point P divides the line segment joining the points A (12, 5) and (4, -3) in the ratio $\frac{3}{5} : 1$, i.e. 3:5.

Q30. Find the values of k for which the quadratic equation $(k + 4) x^2 + (k + 1)x + 1 = 0$ has equal roots. Also find these roots.

Solution:

Given quadratic equation:

$(k + 4)x^2 + (k + 1)x + 1=0$

Since the given quadratic equation has equal roots, Its discriminant should be zero.

$\therefore D = 0$

$\Rightarrow (k+1)^2 - 4 \times (k+4) \times 1 = 0$

$\Rightarrow k^2+2k+1- 4k - 16=0$

$\Rightarrow k^2- 2k -15 =0$

$\Rightarrow k^2 - 5k+ 3k -15 = 0$

$\Rightarrow (k-5) (k+3) =0$

$\Rightarrow k-5 =0$ or $k+3 =0$

$\Rightarrow k = 5$ or -3

Thus, the values of k are 5 and -3

For k = 5 $(k+4)x^2 +(k+1)x +1= 0$

$\Rightarrow 9x^2 +6x +1 = 0$

$\Rightarrow (3x)^2 +2(3x) +1 =0$

$\Rightarrow (3x +1)^2 =0$

$\Rightarrow x = -\frac{1}{3}, -\frac{1}{3}$

For k = -3 $(k+4)x^2 + (k+1)x + 1 = 0$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x-1)^2 = 0$

$\Rightarrow x = 1, 1$

Thus, the equal root of the given quadratic equation is either 1 or $-\frac{1}{3}$

Q31. In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the A.P.

Solution:
Let a and d be the first term and the common difference of an A. P. respectively.

n^{th} term of an A. P, $a_n = a + (n-1)d$

Sum of n terms of an A. P , $S_n = \frac{n}{2}[2a + (n-1)d]$

We have:

Sum of the first 10 terms = $\frac{10}{2}[2a + 9d]$

$\Rightarrow 210 = 5[2a+9d]$

$\Rightarrow 42 = 2a+9d$ .... (1)

15^{th} term from the last = $(50-15+1)^{th}$ = 36^{th} term from the beginning

Now, $a_{36} = a + 35d$

$\therefore$ Sum of the last 15 terms = $\frac{15}{2}(2a_{36} + (15-1)d)$

$= \frac{15}{2}[2(a+35d) + 14d]$

$= 15[a + 35d + 7d]$

$\Rightarrow 2565 = 15[a+42d]$

$\Rightarrow 171 = a+42d$ ....(2)

From (1) and (2), we get,

d = 4

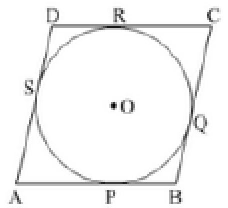
a = 3

So, the A. P. formed is 3, 7, 11, 15 …. and 199.

Q32 . Prove that a parallelogram circumscribing a circle is a rhombus.

Solution:
Given ABCD be a parallelogram circumscribing a circle with centre O.

To Prove: ABCD is a rhombus.



We know that the tangents drawn to a circle from an exterior point are equal is length.

∴ AP = AS, BP = BQ, CR =CQ AND DR = DS.

AP +BP +CR +DR = AS +BQ +CQ +DS

(AP +BP) + (CR+DR) = (AS+DS) +BQ +CQ)

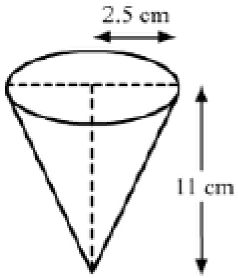
∴AB +CD =AD +BC OR 2AB = 2BC   (since AB =DC and AD = BC)

∴ AB = BC = DC =AD

Therefore, ABCD is a rhombus.

Q33. Sushant has a vessel, of the form of an inverted cone, open at the top, of height 11 cm and Radius of top as 2.5 cm and is full of water. Metallic spherical balls each of diameter 0.5 cm are put In the vessel due to which $\frac{2}{5}$th of the water in the vessel flows out. Find how many balls were put in the vessel. Sushant made the arrangement so that the water that flows out irrigates the Flower beds. What value has been shown by Sushant?

Solution:



Height (h) of the conical vessel = 11 cm

Radius (r_1) of the conical Vessel = 2.5 cm

Radius (r_2) of the metallic spherical balls = $\frac{0.5}{2} = 0.25$ cm

Let n be the number of spherical balls = that were dropped in the the vessel.

Volume of the water spilled = Volume of the spherical balls dropped

$\frac{2}{5} \times$ Volume of cone = n × Volume of one spherical ball

$$\Rightarrow \frac{2}{5} \times \frac{1}{3} \pi r \frac{2}{1} h = n \times \frac{4}{3} \pi r \frac{3}{2}$$

$$\Rightarrow r \frac{2}{1} h = n \times 10 r \frac{3}{2}$$

$$\Rightarrow (2.5)^2 \times 11 = n \times 10 \times (0.25)^3$$

⇒ 68.75 = 0.15625n

⇒ n = 440

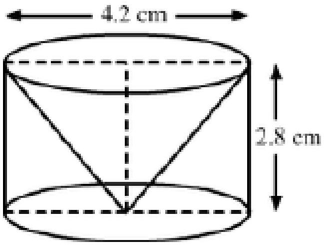
Hence, the number of spherical balls that were dropped in the vessel is 440.

Sushant made the arrangement so that the water that flows out, irrigates the flower beds.

This shows the judicious usage of water.

Q34. From a solid cylinder of height 2.8 cm and diameter 4.2 cm. a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. [Take π=22/7]

Solution:
The following figure shows the required cylinder and the conical cavity.



Given Height (b) of the conical Part = Height (h) of the cylindrical part = 2.8 cm

Diameter of the cylindrical part = Diameter of the conical part = 4.2 cm

$\therefore$ Radius $\rightarrow$ of the cylindrical part = Radius $\rightarrow$ of the conical part = 2.1 cm

Slant height (l) of the conical part $=\sqrt{r^2+h^2}$

$$=\sqrt{(2.1)^2+(2.8)^2}\,cm$$
$$=\sqrt{4.41+7.81}\,cm$$
$$=\sqrt{12.25}\,cm$$
$$=3.5\,cm$$

Total surface area of the remaining solid = Curved surface area of the cylindrical part +Curved surface area of the conical part + Area of the cylindrical base

$$=2\pi rh+\pi rl+\pi r^2$$
$$=\left(2\times\frac{22}{7}\times 2.1\times 2.8+\frac{22}{7}\times 2.1\times 3.5+\frac{22}{7}\times 2.1\times 2.1\right)cm^2$$

= (36.96 + 23.1 + 13.86) cm^2

=73.92 cm^2

Thus, the total surface area of the remaining solid is 73.92 cm^2

MATHEMATICS

Paper & Solution

Time: 3 Hrs. Max. Marks: 90

General Instructions :

1. All questions are **compulsory**.

2. The question paper consists of **34** questions divided into **four sections** A, B, C, and D.

3. Section A contains of **8** questions of 1 mark each, which are multiple choice type question, **Section B** contains of **6** questions of 2 marks each, **Section C** contains of **10** questions of 3 marks each and **Section D** contains of **10** questions of 4 marks each.

4. Use of calculator is **not** permitted.

SECTION – A

1. The angle of depression of a car, standing on the ground, from the top of a 75 m high tower, is 30° The distance of the car from the base of the tower (in m.) is:

(A) $25\sqrt{3}$

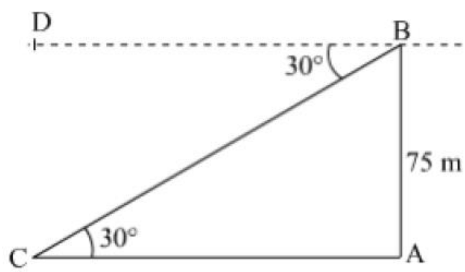
(B) $50\sqrt{3}$

(C) $75\sqrt{3}$

(D) 150

Solution:

Correct answer: C



Let AB be the tower of height 75 m and C be the position of the car

In Δ ABC,

$$\cot 30^\circ = \frac{AC}{AB}$$

$$\Rightarrow AC = AB\cot 30^\circ$$

$$\Rightarrow AC = 75m \times \sqrt{3}$$

$$\Rightarrow AC = 75\sqrt{3m}$$

Thus, the distance of the car from the base of the tower is $75\sqrt{3\,m}$.

2. The probability of getting an even number, when a die is thrown once, is:

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{6}$

(D) $\frac{5}{6}$

Solution:

Correct answer: A

S = {1,2,3,4,5,6}

Let event E be defined as 'getting an even number'.

n(E) = {1,4,6}

$$\therefore P\,E = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}} = \frac{3}{6} = \frac{1}{2}$$

3. A box contains 90 discs, numbered from 1 to 90. If one disc is drawn at random from the box, the probability that it bears a prime-number less than 23, is:

(A) $\frac{7}{90}$

(B) $\frac{10}{90}$

(C) $\frac{4}{45}$

(D) $\frac{9}{89}$

Solution:

Correct answer: C

S = {1, 2, 3,..90}

n(S) = 90

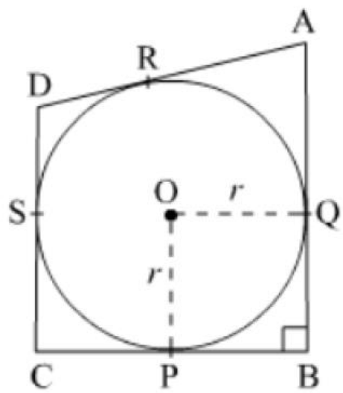
The prime number less than 23 are 2, 3, 5, 7, 11, 13, 17, and 19.

Let event E be defined as 'getting a prime number less than 23'.

n(E) = 8

$$\therefore P\,E = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}} = \frac{8}{90} = \frac{4}{45}$$

4. In fig., a circle with centre O is inscribed in a quadrilateral ABCD such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively, If AB = 29 cm, AD = 23 cm, $\angle B = 90^{\circ}$ and DS = 5 cm, then the radius of the circle (in cm) is:



(A)11

(B)18

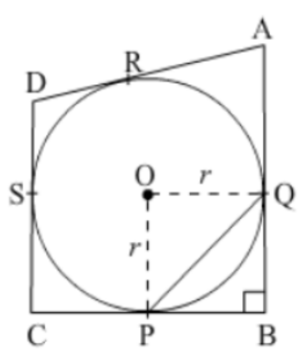
(C)6

(D)15

Solution:

Correct answer: A

Given: AB, BC, CD and AD are tangents to the circle with centre O at Q, P, S and R respectively. AB = 29 cm, AD = 23, DS = 5 cm and $\angle B = 90^{\circ}$ Construction: Join PQ.



We know that, the lengths of the tangents drawn from an external point to a circle are equal.

$DS = DR = 5\,cm$

$\therefore AR = AD - DR = 23\,cm - 5\,cm = 18\,cm$

$AQ = AR = 18\,cm$

$\therefore QB = AB - AQ = 29\,cm - 18\,cm = 11\,cm$

$QB = BP = 11\,cm$

$In\ \Delta\ PQB,$

$PQ^2 = QB^2 + BP^2 = (11\,cm)^2 + (11\,cm)^2 = 2 \times (11\,cm)^2$

$PQ = 11\sqrt{2}\,cm \qquad ...(1)$

$In\ \Delta\ OPQ,$

$PQ^2 = OQ^2 + OP^2 = r^2 + r^2 = 2r^2$

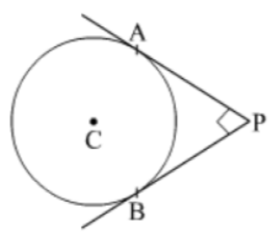
$(11\sqrt{2})^2 = 2r^2$

$121 = r^2$

$r = 11$

Thus, the radius of the circle is 11 cm.

5. In fig., PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA $\perp$ PB, then the length of each tangent is:



(A) 3cm

(B) 4cm

(C) 5cm

(D) 6cm

Solution:

Correct answer: B

$AP \perp PB$ (Given)

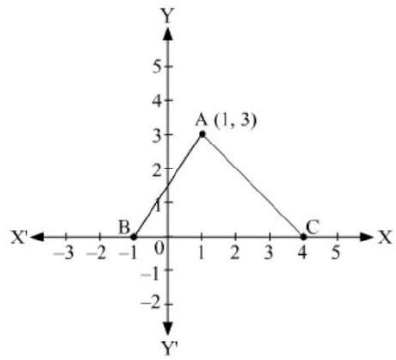
$CA \perp AP, CB \perp BP$ (Since radius is perpendicular to tangent)

AC = CB = radius of the circle

Therefore, APBC is a square having side equal to 4 cm.

Therefore, length of each tangent is 4 cm.

6. In fig., the area of triangle ABC (in sq. units) is:



(A) 15

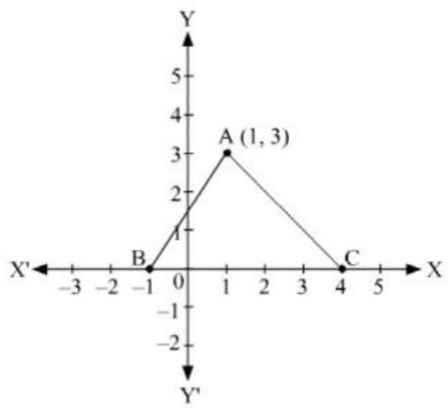
(B) 10

(C) 7.5

(D) 2.5

Solution:

Correct answer: C



From the figure, the coordinates of A, B, and C are (1, 3), (-1, 0) and (4, 0) respectively.

Area of ΔABC

$$=\frac{1}{2}\left|1(0-0)+(-1)(0-3)+4(3-0)\right|$$

$$=\frac{1}{2}\left|0+3+12\right|$$

$$=\frac{1}{2}\left|15\right|$$

$$=7.5\ sq\ units$$

7. If the difference between the circumference and the radius of a circle is 37 cm, then using $\pi = \frac{22}{7}$, the circumference (in cm) of the circle is:

(A) 154

(B) 44

(C) 14

(D) 7

Solution:

Correct answer: B

Let r be the radius of the circle.

From the given information, we have:

$$2\pi r - r = 37\ cm$$
$$\Rightarrow r\ 2\pi - 1 = 37cm$$
$$\Rightarrow r\left(2 \times \frac{22}{7} - 1\right) = 37cm$$
$$\Rightarrow r \times \frac{37}{7} = 37cm$$
$$\Rightarrow r = 7cm$$

$\therefore$ Circumference of the circle $= 2\pi r = 2 \times \frac{22}{7} \times 7cm = 44cm$

8. The common difference of AP $\frac{1}{3q}, \frac{1-6q}{3q}, \frac{1-12q}{3q}$, ...is:

(A) q

(B) –q

(C) -2

(D) 2

Solution:

Correct answer: C

Common difference =

$$\frac{1-6q}{3q} - \frac{1}{3q} = \frac{1-6q-1}{3q} = \frac{-6q}{3q} = -2$$

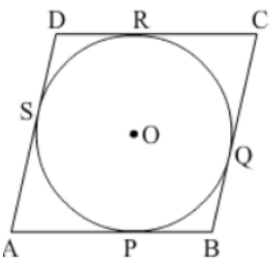
SECTION B

9. Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:

Given: ABCD be a parallelogram circumscribing a circle with centre O.

To prove: ABCD is a rhombus.



We know that the tangents drawn to a circle from an exterior point are equal in length. Therefore, AP = AS, BP = BQ, CR = CQ and DR = DS. Adding the above equations,

AP + BP + CR + DR = AS + BQ + CQ + DS

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

2AB = 2BC

(Since, ABCD is a parallelogram so AB = DC and AD = BC)

AB = BC

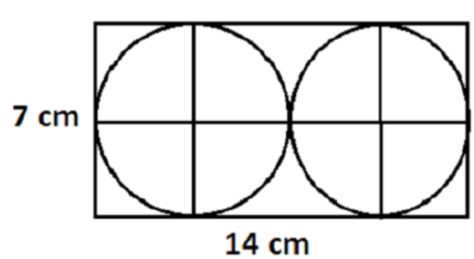
Therefore, AB = BC = DC = AD.

Hence, ABCD is a rhombus.

10. Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular card board of dimensions 14 cm $\times$7 cm. Find the area of the remaining card board. $\left[Use \pi = \frac{22}{7}\right]$

Solution:

Dimension of the rectangular card board = $14\,cm \times 7\,cm$ Since, two circular pieces of equal radii and maximum area touching each other are cut from the rectangular card board, therefore, the diameter of each of each circular piece is $\frac{14}{2} = 7\,cm$.



Radius of each circular piece = $=\frac{7}{2}cm.$

$\therefore$ Sum of area of two circular pieces $=2\times\pi\left(\frac{7}{2}\right)^2=2\times\frac{22}{7}\times\frac{49}{4}=77cm^2$

Area of the remaining card board

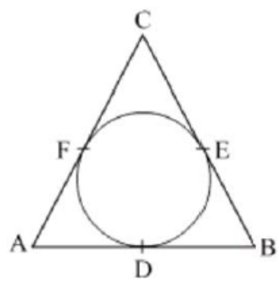
= Area of the card board - Area of two circular pieces

$=14\,cm\times 7\,cm-77\,cm^2$

$=98\,cm^2-77\,cm^2$

$=21\,cm^2$

11.In fig., a circle is inscribed in triangle ABC touches its sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, then find the length of AD, BE and CF.



Solution:
Given: AB = 12 cm, BC = 8 cm and AC = 10 cm.

Let, AD = AF = x cm, BD = BE = y cm and CE = CF = z cm

(Tangents drawn from an external point to the circle are equal in length)

$\Rightarrow$ 2(x + y + z) = AB + BC + AC = AD + DB + BE + EC + AF + FC = 30 cm

$\Rightarrow$ x + y + z = 15 cm

AB = AD + DB = x + y = 12 cm

$\therefore$ z = CF = 15 - 12 = 3 cm

AC = AF + FC = x + z = 10 cm

$\therefore$ y = BE = 15 - 10 = 5 cm

$\therefore$ x = AD = x + y + z - z - y = 15 - 3 - 5 = 7 cm

12. How many three-digit natural numbers are divisible by 7?
Solution:

Three digit numbers divisible by 7 are

105, 112, 119, … 994

This is an AP with first term (a) = 105 and common difference (d) = 7

Let a_n be the last term.

$$a_n = a + (n-1)d$$
$$994 = 105 + (n-1)(7)$$
$$7(n-1) = 889$$
$$n - 1 = 127$$
$$n = 128$$

Thus, there are 128 three-digit natural numbers that are divisible by 7.

13.Solve the following quadratic equation for x: $4\sqrt{3}\times^{2} + 5\times - 2\sqrt{3} = 0$

Solution:

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

$$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$
$$\Rightarrow 4x\ \sqrt{3}x + 2 - \sqrt{3}\ \sqrt{3}x + 2 = 0$$
$$\Rightarrow 4x - \sqrt{3}\ \sqrt{3}x + 2 = 0$$
$$\therefore x = \frac{\sqrt{3}}{4}\ or\ x = -\frac{2}{\sqrt{3}}$$

14.A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability that the drawn card is neither a king nor a queen.

Solution:

Let E be the event that the drawn card is neither a king nor a queen.

Total number of possible outcomes = 52

Total number of kings and queens = 4 + 4 = 8

Therefore, there are 52 - 8 = 44 cards that are neither king nor queen.

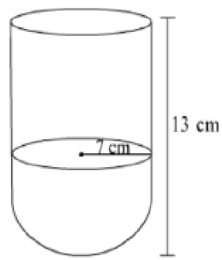
Total number of favourable outcomes = 44

$\therefore$ Required probability = P(E) = $\dfrac{\text{Favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{44}{52} = \dfrac{11}{13}$

SECTION C

15.A vessel is in the form of hemispherical bowl surmounted by a hollow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the total surface area of the vessel. $\left[use\pi=\frac{22}{7}\right]$

Solution:



Let the radius and height of cylinder be r cm and h cm respectively.

Diameter of the hemispherical bowl = 14 cm

$\therefore$ Radius of the hemispherical bowl = Radius of the cylinder

$=r=\frac{14}{2}cm=7cm$

Total height of the vessel = 13 cm

$\therefore$Height of the cylinder, h = 13 cm - 7 cm = 6 cm

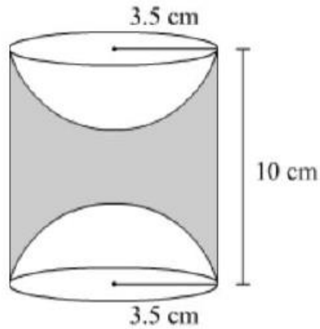
Total surface area of the vessel = 2 (curved surface area of the cylinder + curved surface area of the hemisphere) (Since, the vessel is hollow)

$=2\ 2\pi rh+2\pi r^2=4\pi r\ h+r=4\times\frac{22}{7}\times7\times6+7\ cm^2$

$=1144\ cm^2$

16. A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy. $\left[use\pi=\frac{22}{7}\right]$

Solution:



Height of the cylinder, h = 10 cm

Radius of the cylinder = Radius of each hemisphere = r = 3.5 cm

Volume of wood in the toy = Volume of the cylinder - 2$\times$ Volume of each

Hemisphere

$$= \pi r^2 h - 2 \times \frac{2}{3} \pi r^3$$
$$= \pi r^2 \left(h - \frac{4}{3} r \right)$$
$$= \frac{22}{7} \times (3.5)^2 \left(10 - \frac{4}{3} \times 3.5 \right)$$
$$= 38.5 \times 10 - 4.67$$
$$= 38.5 \times 5.33$$
$$= 205.205\, cm^3$$

Radius = 21 cm

17.In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find: (i) the length of the arc (ii) area of the sector formed by the arc. $\left[use \pi = \frac{22}{7} \right]$

Solution:

The arc subtends an angle of 60o at the centre.

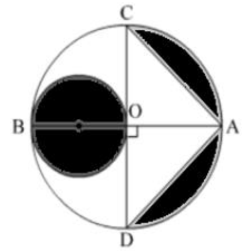
$$(i)\ l = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21 cm$$
$$= 22 cm$$

$$(ii)\ Area\ of\ the\ sector = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21 cm^2$$
$$= 231\, cm^2$$

18. In Fig., AB and CD are two diameters of a circle with centre O, which are perpendicular to each other. OB is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region. $\left[use \pi = \frac{22}{7}\right]$



Solution:

AB and CD are the diameters of a circle with centre O.

$\therefore$ OA = OB = OC = OD = 7 cm (Radius of the circle)

Area of the shaded region

= Area of the circle with diameter OB + (Area of the semi-circle ACDA – Area of Δ ACD)

$$= \pi\left(\frac{7}{2}\right)^2 + \left(\frac{1}{2} \times \pi \times 7^2 - \frac{1}{2} \times CD \times OA\right)$$
$$= \frac{22}{7} \times \frac{49}{4} + \frac{1}{2} \times \frac{22}{7} \times 49 - \frac{1}{2} \times 14 \times 7$$
$$= \frac{77}{2} + 77 - 49$$
$$= 66.5\, cm^2$$

19. Find the ratio in which the y-axis divides the line segment joining the points (-4, -6) and (10, 12). Also, find the coordinates of the point of division.

Solution:

. Let the y-axis divide the line segment joining the points (-4,-6) and (10,12) in the ratio k: 1 and the point of the intersection be (0,y). Using section formula, we have:

$$\left(\frac{10k+-4}{k+1}, \frac{12k+-6}{k+1}\right)=0, y$$

$$\therefore \frac{10k-4}{k+1}=0 \Rightarrow 10k-4=0$$

$$\Rightarrow k=\frac{4}{10}=\frac{2}{5}$$

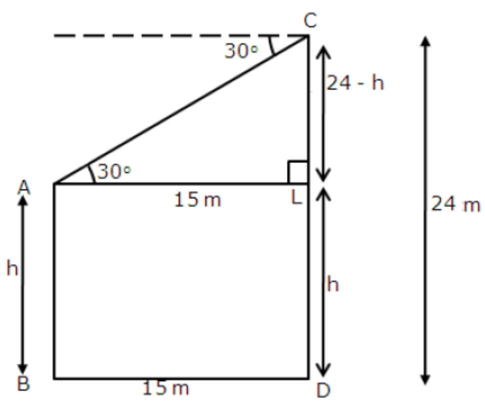
Thus, the y-axis divides the line segment joining the given points in the ratio 2:5

$$\therefore y=\frac{12k+-6}{k+1}=\frac{12\times\frac{2}{5}-6}{\frac{2}{5}+1}=\frac{\left(\frac{24-30}{5}\right)}{\left(\frac{2+5}{5}\right)}=-\frac{6}{7}$$

Thus, the coordinates of the point of division are $\left(0,-\frac{6}{7}\right)$.

20. The horizontal distance between two poles is 15 m. The angle of depression of the top of first pole as seen from the top of second pole is 30°. If the height of the second pole is 24 m, find the height of the first pole. $\left[Use\ \sqrt{3}=1.732\right]$

Solution:



Let AB and CD be the two poles, where CD (the second pole) = 24 m.

BD = 15 m

Let the height of pole AB be h m.

AL = BD = 15 m and AB = LD = h

So, CL = CD - LD = 24 – h

$In\ \Delta\ ACL,$

$$\tan 30^\circ = \frac{CL}{AL}$$

$$\Rightarrow \tan 30^\circ = \frac{24-h}{15}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{24-h}{15}$$

$$\Rightarrow 24-h = \frac{15}{\sqrt{3}} = 5\sqrt{3}$$

$$\Rightarrow h = 24 - 5\sqrt{3}$$

$$\Rightarrow h = 24 - 5 \times 1.732 \quad \left[Taking \sqrt{3} = 1.732\right]$$

$$\Rightarrow h = 15.34$$

Thus, height of the first pole is 15.34 m.

21. For what values of k, the roots of the quadratic equation $(k+4)\times^2 + (k+1)\times + 1 = 0$ are equal?

Solution:

$$k+4)\times^2 + (k+1)\times + 1 = 0$$
$$a = k+4, b = k+1, c = 1$$

For equal roots, discriminant, D = 0

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow (k+1)^2 - 4(k+4)\times 1 = 0$$

$$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0$$

$$\Rightarrow k^2 - 2k - 15 = 0$$

$$\Rightarrow k^2 - 5k + 3k - 15 = 0$$

$$\Rightarrow k(k-5) + 3(k-5) = 0$$

$$\Rightarrow (k-5)(k+3) = 0$$

$$\Rightarrow k = 5\ or\ k = -3$$

Thus, for k = 5 or k = -3, the given quadratic equation has equal roots.

22. The sum of first n terms of an AP is $3n^2 + 4n$. Find the 25^{th} term of this AP.

Solution:

$$S_n = 3n^2 + 4n$$

First term $(a_1) = S_1 = 3(1)^2 + 4(1) = 7$

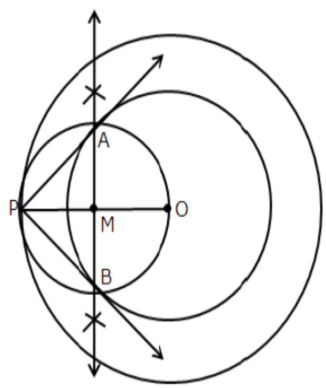
$S_2 = a_1 + a_2 = 3(2)^2 + 4(2) = 20$
$a_2 = 20 - a_1 = 20 - 7 = 13$

So, common difference (d) = $a_2 - a_1$ = 13 - 7 = 6

Now, $a_n = a + (n - 1)d$

$\therefore a_{25} = 7 + (25 - 1) \times 6 = 7 + 24 \times 6 = 7 + 144 = 151$

23. Construct a tangent of a circle of radius 4 cm from a point on the concentric circle of radius 6 cm.
Solution:



Steps of construction:

1. Draw two concentric circle with centre O and radii 4 cm and 6 cm. Take a point P on the outer circle and then join OP.
2. Draw the perpendicular bisector of OP. Let the bisector intersects OP at M.
3. With M as the centre and OM as the radius, draw a circle. Let it intersect the inner circle at A and B.
4. Join PA and PB. Therefore, $\overline{PA}$ and $\overline{PB}$ are the required tangents.

24. Show that the points (-2, 3), (8, 3) and (6, 7) are the vertices of a right triangle.
Solution:

The given points are A(-2,3) B(8,3) and C(6,7). Using distance formula, we have:

$AB^2 = 8 - -2\ ^2 + 3 - 3\ ^2$

$\Rightarrow AB^2 = 10^2 + 0$

$\Rightarrow AB^2 = 100$

$BC^2 = 6 - 8\ ^2 + 7 - 3\ ^2$

$\Rightarrow BC^2 = (-2)^2 + 4^2$

$\Rightarrow BC^2 = 4 + 16$

$\Rightarrow BC^2 = 20$

$CA^2 = -2 - 6^2 + 3 - 7^2$

$\Rightarrow CA = (-8)^2 + (-4)^2$

$\Rightarrow CA^2 = 64 + 16$

$\Rightarrow CA^2 = 80$

It can be observed that:

$BC^2 + CA^2 = 20 + 80 = 100 = AB^2$

So, by the converse of Pythagoras Theorem,

Δ ABC is a right triangle right angled at C.

SECTION D

25. Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in level of water in the tank in half an hour.
Solution:

Diameter of circular end of pipe = 2 cm

$\therefore$ Radius r_1 of circular end of pipe $= \frac{2}{200} m = 0.01\, m$

Area of cross-section $= \pi \times r_1^2 = \pi \times 0.01^2 = 0.0001\pi m^2$

Speed of water $= 0.4\, m/s = 0.4 \times 60 = 24\, metre/\text{min}$

Volume of water that flows in 1 minute from pipe = $24 \times 0.0001\pi m^3 = 0.0024\pi m^3$

Volume of water that flows in 30 minutes from pipe = $30 \times 0.0024\pi m^3 = 0.072\pi m^3$

Radius (r_2) of base of cylindrical tank = 40 cm = 0.4 m

Let the cylindrical tank be filled up to h m in 30 minutes. Volume of water filled in tank in 30 minutes is equal to the volume of water flowed out in 30 minutes from the pipe.

$\therefore \pi \times r_2^2 \times h = 0.072\pi$

$\Rightarrow 0.4^2 \times h = 0.072$

$\Rightarrow 0.16\,h = 0.072$

$\Rightarrow h = \dfrac{0.072}{0.16}$

$\Rightarrow h = 0.45\,m = 45\,cm$

Therefore, the rise in level of water in the tank in half an hour is 45 cm.

26. A Group consists of 12 persons, of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is (i) extremely patient (ii) extremely kind or honest. Which of the above values you prefer more?

Solution:

The group consists of 12 persons.

$\therefore$ Total number of possible outcomes = 12

Let A denote event of selecting persons who are extremely patient

$\therefore$ Number of outcomes favourable to A is 3.

Let B denote event of selecting persons who are extremely kind or honest. Number of persons who are extremely honest is 6. Number of persons who are extremely kind is 12 - (6 + 3) = 3 $\therefore$ Number of outcomes favourable to B = 6 + 3 = 9.

(i)

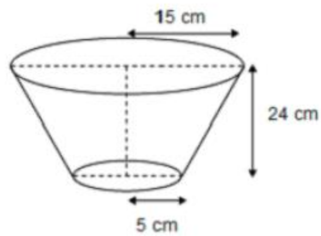
$$P\,A = \frac{\text{Number of outcomes favrouableto A}}{\text{Total number of possible outcomes}} = \frac{3}{12} = \frac{1}{4}$$

(ii)

$$P\,B = \frac{\text{Number of outcomes favrouableto B}}{\text{Total number of possible outcomes}} = \frac{9}{12} = \frac{3}{4}$$

Each of the three values, patience, honesty and kindness is important in one's life.

27.A bucket open at the top, and made up of a metal sheet is in the form of a frustum of a cone. The depth of the bucket is 24 cm and the diameters of its upper and lower circular ends are 30 cm and 10 cm respectively. Find the cost of metal sheet used in it at the rate of Rs 10 per 100 cm^2. $\left[Use\ \pi = 3.14\right]$



Solution:

Diameter of upper end of bucket = 30 cm

$\therefore$ Radius (r_1) of upper end of bucket = 15 cm

Diameter of lower end of bucket = 10 cm

$\therefore$ Radius (r_1) of lower end of bucket = 5 cm

Slant height (l) of frustum

$$=\sqrt{r_1-r_2\ {}^2+h^2}$$

$$=\sqrt{15-5^2+24\ {}^2}=\sqrt{10\ {}^2+24\ {}^2}=\sqrt{100+576}$$

$$\sqrt{676}=26cm$$

Area of metal sheet used to make the bucket

$$=\pi\ r_1+r_2\ l+\pi r_2^{\ 2}$$

$$=\pi\ 15+5\ 26+\pi\ 5\ {}^2$$

$$=520\pi+25\pi=545\pi cm^2$$

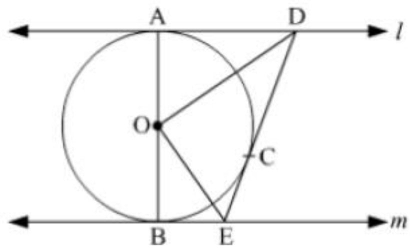
Cost of 100 cm^2 metal sheet = Rs 10

Cost of 545 $\pi\ cm^2$ metal sheet

$$=Rs.\frac{545\times 3.14\times 10}{100}=Rs.171.13$$

Therefore, cost of metal sheet used to make the bucket is Rs 171.13.

28. In fig., l and m are two parallel tangents to a circle with centre O, touching the circle at A and B respectively. Another tangent at C intersects the line l at D and m at E. Prove that $\angle DOE=90^\circ$



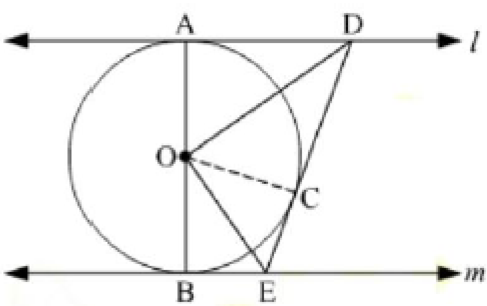
Solution:

Given: *l* and m are two parallel tangents to the circle with centre O touching the circle at A and B respectively. DE is a tangent at the point C, which intersects *l* at D and m at E.

To prove: $\angle$ DOE = 90

Construction: Join OC.

Proof:



In Δ ODA and Δ ODC,

OA = OC (Radii of the same circle)

AD = DC (Length of tangents drawn from an external point to a circle are equal)

DO = OD (Common side)

Δ ODA $\cong$ Δ ODC (SSS congruence criterion)

$\therefore \angle DOA = \angle COD$ ....(1)

Similarly, Δ OEB $\cong$ Δ OEC

$\therefore \angle EOB = \angle COE$ ....(2)

Now, AOB is a diameter of the circle. Hence, it is a straight line.

$\angle$ DOA + $\angle$ COD + $\angle$ COE + $\angle$ EOB = 180

From (1) and (2), we have:

2 $\angle$ COD + 2 $\angle$ COE = 180

$\Rightarrow$ $\angle$ COD + $\angle$ COE = 90

$\Rightarrow$ $\angle$ DOE = 90

Hence, proved.

29. Sum of the areas of two squares is 400 Cm^2. If the difference of their perimeters is 16 cm, find the sides of the two squares.

Solution:

Let the sides of the two squares be x cm and y cm where x > y.

Then, their areas are x^2 and y^2 and their perimeters are 4x and 4y.

By the given condition:

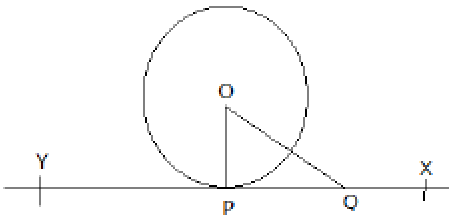
$x^2 + y^2 = 400$ ... (1)
and $4x - 4y = 16$
$\Rightarrow 4(x - y) = 16\ x - y = 4$
$\Rightarrow x = y + 4$ ... (2)
Substituting the value of x from (2) in (1), we get:
$(y + 4)^2 + y^2 = 400$
$\Rightarrow y^2 + 16 + 8y + y^2 = 400$
$\Rightarrow 2y^2 + 16 + 8y = 400$
$\Rightarrow y^2 + 4y - 192 = 0$
$\Rightarrow y^2 + 16y - 12y - 192 = 0$
$\Rightarrow y(y + 16) - 12(y + 16) = 0$
$\Rightarrow (y + 16)(y - 12) = 0$
$\Rightarrow y = -16$ or $y = 12$
Since, y cannot be negative, y = 12.
So, x = y + 4 = 12 + 4 = 16
Thus, the sides of the two squares are 16 cm and 12 cm.

30. Solve that following for x: $\frac{1}{2a+b+2\times} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2\times}$

Solution:

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$
$$\Rightarrow \frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$$
$$\Rightarrow \frac{2x-2a-b-2x}{2x\ 2a+b+2x} = \frac{b+2a}{2ab}$$
$$\Rightarrow \frac{-2a+b}{2x\ 2a+b+2x} = \frac{b+2a}{2ab}$$
$$\Rightarrow \frac{-1}{x\ 2a+b+2x} = \frac{1}{ab}$$
$$\Rightarrow 2x^2 + 2ax + bx + ab = 0$$
$$\Rightarrow 2x\ x+a+b\ x+a = 0$$
$$\Rightarrow x+a\ 2x+b = 0$$
$$\Rightarrow x+a = 0 \text{ or } 2x+b = 0$$
$$\Rightarrow x = -a, \text{ or } x = \frac{-b}{2}$$

31. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.
Solution:



Given: A circle with centre O and a tangent XY to the circle at a point P
To Prove: OP is perpendicular to XY.
Construction: Take a point Q on XY other than P and join OQ.
Proof: Here the point Q must lie outside the circle as if it lies inside the tangent XY will become secant to the circle.
Therefore, OQ is longer than the radius OP of the circle, That is, OQ > OP.
This happens for every point on the line XY except the point P.
So OP is the shortest of all the distances of the point O to the points on XY.
And hence OP is perpendicular to XY.
Hence, proved.

32. Find the number of terms of the AP -12, -9, -6,... 12. If 1 is added to each term of this AP, then find the sum of all terms of the AP thus obtained.
Solution:

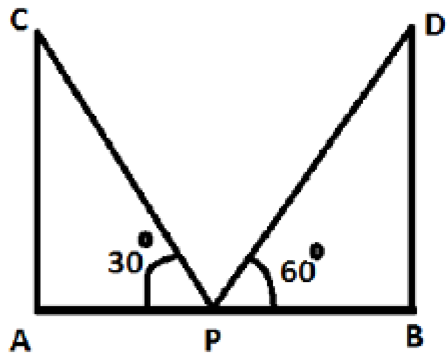
Given AP is -12, -9, -6, ..., 21
First term, a = -12
Common difference, d = 3
Let 21 be the n^{th} term of the A.P.
21 = a + (n - 1)d
$\Rightarrow$ 21 = -12 + (n - 1) $\times$ 3
$\Rightarrow$ 33 = (n - 1) $\times$ 3
$\Rightarrow$ n = 12
Sum of the terms of the AP = S_{12}

$$=\frac{n}{2}2a+n-1d=\frac{12}{2}-24+11\times3=54$$

If 1 is added to each term of the AP, the sum of all the terms of the new AP will increase by n, i.e., 12.
$\therefore$ Sum of all the terms of the new AP = 54 + 12 = 66

33. Two poles of equal heights are standing opposite each other on either side of the roads, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.
Solution:

Let AC and BD be the two poles of the same height h m.



Given AB = 80 m

Let AP = x m, therefore, PB = (80 - x) m

In Δ APC,

$$\tan^\circ = \frac{AC}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x} \qquad \text{......(1)}$$

In Δ BPD,

$$\tan 60^\circ = \frac{BD}{AB}$$

$$\sqrt{3} = \frac{h}{80-x} \qquad \text{....(2)}$$

Dividing (1) by (2),

$$\frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{\frac{h}{x}}{\frac{h}{80-x}}$$

$$\Rightarrow \frac{1}{3} = \frac{80-x}{x}$$

$$\Rightarrow x = 240 - 3x$$

$$\Rightarrow 4x - 240$$

$$\Rightarrow x = 60$$

From (1),

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow h = \frac{60}{\sqrt{3}} = 20\sqrt{3}m$$

Thus, the height of both the poles is $20\sqrt{3}$ m and the distances of the point from the poles are 60 m and 20 m.

34. If the area of triangle ABC formed by A(x,y), B(1,2) and C(2,1) is 6 square units, then prove that x + y = 15.

Solution:

The given vertices are A(x,y), B(1,2) and C(2,1).

It is know that the area of a triangle whose vertices are (x_1, y_1), (x_2,y_2) and (x_3, y_3) is given by

$$\frac{1}{2}| x_1\, y_2 - y_3 + x_2\, y_3 - y_1 + x_3\, y_1 - y_2 |$$

$\therefore$ Area of Δ ABC

$$= \frac{1}{2}| x\, 2 - 1 + 1 \times 1 - y + 2\, y - 2 |$$

$$= \frac{1}{2}| x + 1 - y + 2y - 4 |$$

$$= \frac{1}{2}| x + y - 3 |$$

The area of Δ ABC is given as 6 sq units.

$$\Rightarrow \frac{1}{2}\left[x + y - 3\right] = 6 \Rightarrow x + y - 3 = 12$$

$$\therefore x + y = 15$$

CBSE-X-2012 EXAMINATION

MATHEMATICS

Paper & Solution

Time: 3 Hrs. Max. Marks: 90

General Instuctions :

1. All questions are **compulsory**.

2. The question paper consists of **34** questions divided into **four sections** A, B, C, and D.

3. Section A contains of **10** questions of 1 mark each, which are multiple choice type question, **Section B** contains of **8** questions of 2 marks each, **Section C** contains of **10** questions of 3 marks each and **Section D** contains of **6** questions of 4 marks each.

4. Question numbers **1 to 8** in **Section A** are multiple choice questions where you are to select **one** correct option out of the given four.

5. There is no overall choice. However, internal choice has been provided in **one** question of **2 marks**, **three** questions of **3 marks** each and **two** questions of 4 **marks** each. You have to attempt only one of the alternatives in all such questions.

6. Use of calculator is **not** permitted.

SECTION – A

1. The length of shadow of a tower on the plane ground is 3 times the height of the tower. The angle of elevation of sun is:

(A) 45°

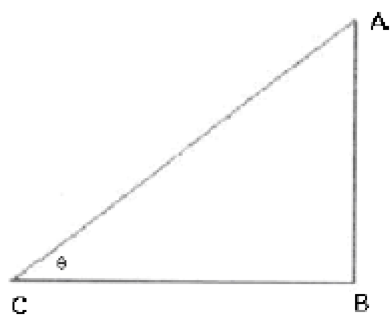
(B) 30°

(C) 60°

(D) 90°

Solution:

Correct answer: B



Let AB be the tower and BC be its shadow. Let θ be the angle of elevation of the sun.

According to the given information,

BC = $\sqrt{3}$ AB ... (1)

In ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{AB}{\sqrt{3}AB} = \frac{1}{\sqrt{3}} \text{ [Using (1)]}$$

We know that tan 30 = $\frac{1}{\sqrt{3}}$

$\therefore \theta = 30^\circ$

Hence, the angle of elevation of the sun is 30°.

2. If the area of a circle is equal to sum of the areas of two circles of diameters 10 cm and 24 cm, then the diameter of the larger circle (in cm) is:

(A) 34

(B) 26

(C) 17

(D) 14

Solution:

Correct answer: B

Diameters of two circles are given as 10 cm and 24 cm.

Radius of one circle = r_1 = 5 cm

Radius of one circle = r_2 = 12 cm

According to the given information,

Area of the larger circle = $\pi(r_1)^2 + \pi(r_2)^2$

$= \pi(5)^2 + \pi(12)^2$

$= \pi(25 + 144)$

$= 169\pi$

$= \pi(13)^2$

$\therefore$ Radius of larger circle = 13 cm

Hence, the diameter of larger circle = 26 cm

3. If the radius of the base of a right circular cylinder is halved, keeping the height the same, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is:

(A) 1 : 2

(B) 2 : 1

(C) 1 : 4

(D) 4 : 1

Solution:

Correct answer: C

Let the original radius and the height of the cylinder be r and h respectively.

Volume of the original cylinder = $\pi r^2 h$

Radius of the new cylinder = $\frac{r}{2}$

Height of the new cylinder = h

Volume of the new cylinder = $\pi\left(\frac{r}{2}\right)^2 h = \frac{\pi r^2 h}{4}$

Required ratio $\frac{\text{Volume of the new cylinder}}{\text{Volume of the original cylinder}} = \frac{\frac{\pi r^2 h}{4}}{\pi r^2 h} = \frac{1}{4} = 1:4$

4. Two dice are thrown together. The probability of getting the same number on both dice is:

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{6}$

(D) $\frac{1}{12}$

Solution:

Correct answer: C

When two dice are thrown together, the total number of outcomes is 36.

Favourable outcomes = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

$\therefore$ Required probability = $\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{6}{36} = \frac{1}{6}$

5. The coordinates of the point P dividing the line segment joining the points A(1,3) and B(4,6) in the ratio 2 : 1 are:

(A) (2,4)

(B) 3,5)

(C) (4,2)

(D) 5,3)

Solution:

Correct answer: B

It is given that the point P divides AB in the ratio 2: 1.

Using section formula, the coordinates of the point P are

$$\left(\frac{1\times 1+2\times 4}{2+1}, \frac{1\times 3+2\times 6}{2+1}\right) = \left(\frac{1+8}{3}, \frac{3+12}{3}\right) = (3,5)$$

Hence the coordinates of the point P are (3, 5).

6. If the coordinates of the one end of a diameter of a circle are (2,3) and the coordinates of its centre are (-2,5), then the coordinates of the other end of the diameter are:

(A) (-6,7)
(B) (6,-7)
(C) (6,7)
(D) (-6,-7)

Solution:
Correct answer: A
Let the coordinates of the other end of the diameter be (x, y).
We know that the centre is the mid-point of the diameter. So, O(-2, 5) is the mid-point of the diameter AB.
The coordinates of the point A and B are (2, 3) and (x, y) respectively.
Using mid-point formula, we have,

$$-2=\frac{2+x}{2}\Rightarrow -4=2+x\Rightarrow x=-6$$
$$5=\frac{3+y}{2}\Rightarrow 10=3+y\Rightarrow y=7$$

Hence, the coordinates of the other end of the diameter are (-6, 7).

7. The sum of first 20 odd natural number is :
(A) 100
(B) 210
(C) 400
(D) 420
Solution:
Correct answer: C
The first 20 odd numbers are 1, 3, 5, … … 39
This is an AP with first term 1 and the common difference 2.

Sum of 20 terms = S_{20}

$$S_{20}=\frac{20}{2}[2(1)+(20-1)(2)]=10[2+38]=400$$

Thus, the sum of first 20 odd natural numbers is 400.

8. If 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then ab equals:
(A) 3
(B) $-\frac{7}{2}$
(C) 6
(D) -3
Solution:

Correct answer: A

It is given that 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$.

Therefore, y = 1 will satisfy both the equations.

$\therefore\ a(1)^2 + a(1) + 3 = 0$

$\Rightarrow a + a + 3 = 0$

$\Rightarrow 2a + 3 = 0$

$\Rightarrow a = \frac{-3}{2}$

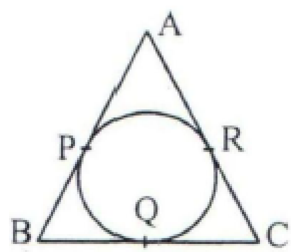
Also, $(1)^2 + (1) + b = 0$

$\Rightarrow 1 + 1 + b = 0$

$\Rightarrow b = -2$

$\therefore ab = \frac{-3}{2} \times -2 = 3$

9. In Fig., the sides AB, BC and CA of a triangle ABC, touch a circle at P, Q and R respectively. If PA = 4 cm, BP = 3 cm and AC = 11 cm, then the length of BC (in cm) is:



(A) 11

(B) 10

(C) 14

(D) 15

Solution:

Correct answer: B

It is known that the lengths of tangents drawn from a point outside a circle are equal in length.

Therefore, we have:

AP = AR … (1) (Tangents drawn from point A)

BP = BQ … (2) (Tangents drawn from point B)

CQ = CR … (3) (Tangents drawn from point C)

Using the above equations,

AR = 4 cm (AP = 4 cm, given)

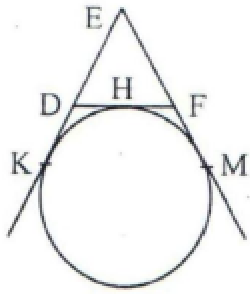
BQ = 3 cm (BP = 3 cm, given)

AC = 11 cm $\Rightarrow$ RC = 11 cm – 4 cm = 7 cm

$\Rightarrow$ CQ = 7 cm

Hence, BC = BQ + CQ = 3 cm + 7 cm = 10 cm

10. In Fig., a circle touches the side DF of Δ EDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then the perimeter of Δ EDF (in cm) is:



(A) 18
(B) 13.5
(C) 12
(D) 9

Solution:

Correct answer: A

It is known that the tangents from an external point to the circle are equal.

$\therefore$ EK = EM, DK = DH and FM = FH … (1)

Perimeter of ΔEDF = ED + DF + FE

= (EK – DK) + (DH + HF) + (EM – FM)

= (EK – DH) + (DH + HF) + (EM – FH) [Using (1)]

= EK + EM

= 2 EK = 2 (9 cm) = 18 cm

Hence, the perimeter of ?EDF is 18 cm.

SECTION – B

11. If a point A(0,2) is equidistant from the points B(3,p) and C(p,5) then find the value of p.

Solution:

Solution:

It is given that the point A (0, 2) is equidistant from the points B(3, p) and C(p, 5).

So, AB = AC $\Rightarrow$ $AB^2 = AC^2$

Using distance formula, we have:

$\Rightarrow (0-3)^2 + (2-p)^2 = (0-p)^2 + (2-5)^2$

$\Rightarrow 9 + 4 + p^2 - 4p = p^2 + 9$

$\Rightarrow 4 - 4p = 0$

$\Rightarrow 4p = 4$

$\Rightarrow p = 1$

Hence, the value of p = 1

12. A number is selected at random from first 50 natural numbers. Find the probability that it is a multiple of 3 and 4.

Solution:

Solution:

The total number of outcomes is 50.

Favourable outcomes = {12, 24, 36, 48}

$$\therefore \text{Required probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{4}{50} = \frac{2}{25}$$

13.The volume of a hemisphere is $2425\frac{1}{2}$ cm^3. Find its curved surface area.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution:

Solution:

Given volume of a hemisphere $= 2425\frac{1}{2} cm^2 = \frac{4851}{2} cm^3$

Now, let r be the radius of the hemisphere

Volume of a hemisphere $= \frac{2}{3}\pi r^3$

$$\therefore \frac{2}{3}\pi r^3 = \frac{4851}{2}$$

$$\Rightarrow \frac{2}{3} \times \frac{22}{7} \times r^3 = \frac{4851}{2}$$

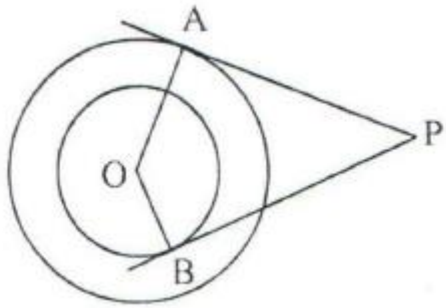
$$\Rightarrow r^3 = \frac{4851}{2} \times \frac{3}{2} \times \frac{7}{22} = \left(\frac{21}{2}\right)^3$$

$$\therefore r = \frac{21}{2} cm$$

So, Curved surface area of the hemisphere $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = 693\, sq.\, cm$$

14.Tangents PA and PB are drawn from an external point P to two concentric circle with centre O and radii 8 cm and 5 cm respectively, as shown in Fig., If AP = 15 cm, then find the length of BP.

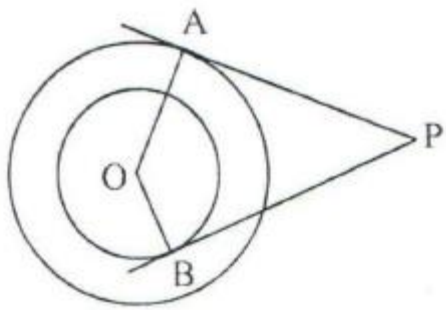


Solution:

Given: Tangents PA and PB are drawn from an external point P to two concentric circles with centre O and radii OA = 8 cm, OB = 5 cm respectively. Also, AP = 15 cm

To find: Length of BP

Construction: We join the points O and P.



Solution: $OA \perp AP$; $OB \perp BP$

[Using the property that radius is perpendicular to the tangent at the point of contact of a circle]

In right angled triangle OAP,

$OP^2 = OA^2 + AP^2$ [Using Pythagoras Theorem]

$= (8)^2 + (15)^2 = 64 + 225 = 289$

$\therefore OP = 17$ cm

In right angled triangle OBP,

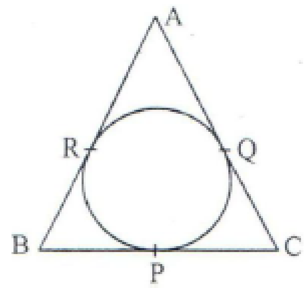
$OP^2 = OB^2 + BP^2$

$\Rightarrow BP^2 = OP^2 - OB^2$

$= (17)^2 - (5)^2 = 289 - 25 = 264$

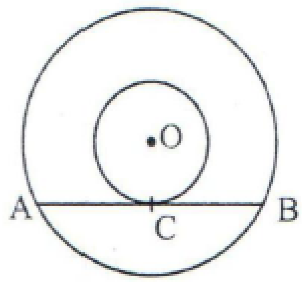
$\therefore BP = \sqrt{264} = 2\sqrt{66}\ cm.$

15.In fig., an isosceles triangle ABC, with AB = AC, circumscribes a circle. Prove that the point of contact P bisects the base BC.



OR

In fig., the chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C. Prove that AC = CB.



Solution:

Given: ABC is an isosceles triangle, where AB = AC, circumscribing a circle.

To prove: The point of contact P bisects the base BC.

i.e. BP = PC

Proof: It can be observed that

BP and BR ; CP and CQ; AR and AQ are pairs of tangents drawn to the circle from the external points B , C and A respectively.

So, applying the result that the tangents drawn from an external point to a circle, we get

BP = BR --- (i)

CP = CQ --- (ii)

AR = AQ --- (iii)

Given that AB = AC

⇒ AR + BR = AQ + CQ

⇒ BR = CQ [from (iii)]

⇒ BP = CP [from (i) and (ii)]

∴ P bisects BC.

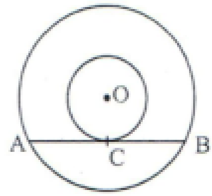
Hence proved.

OR

Given: The chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C.

To prove: AC = CB

Construction: Let us join OC.



Proof: In the smaller circle, AB is a tangent to the circle at the point of contact C.

∴ OC ⊥ AB ------ (i)

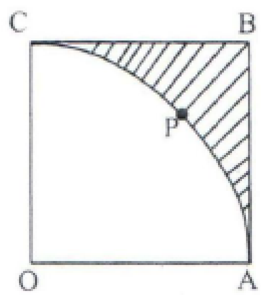
(Using the property that the radius of a circle is perpendicular to the tangent at the point of contact)

For the larger circle, AB is a chord and from (i) we have OC ⊥ AB

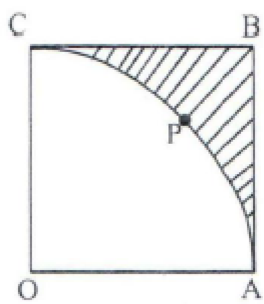
∴ OC bisects AB

(Using the property that the perpendicular drawn from the centre to a chord of a circle bisects the chord)

∴ AC = CB



16. In fig., OABC is a square of side 7 cm. If OAPC is a quadrant of a circle with centre O, then find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Solution:

Given, OABC is a square of side 7 cm

i.e. OA = AB = BC = OC = 7cm

∴ Area of square OABC = $(\text{side})^2 = 7^2 = 49$ sq.cm

Given, OAPC is a quadrant of a circle with centre O.

∴ Radius of the sector = OA = OC = 7 cm.

Sector angle = 90°

$\therefore$ Area of quadrant OAPC $\frac{90^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times (7)^2 = \frac{77}{2} sq.\,cm = 38.5\, sq.\,cm$

$\therefore$ Area of shaded portion = Area of Square - OABC Area of quadrant OAPC

$= (49 - 38.5) sq.\,cm = 10.5\, sq.cm$

17.Find the sum of all three digit natural numbers, which are multiples of 7.

Solution:

First three- digit number that is divisible by 7 = 105

Next number = 105 + 7 = 112

Therefore the series is 105, 112, 119,…

The maximum possible three digit number is 999.

When we divide by 7, the remainder will be 5.

Clearly, 999 – 5 = 994 is the maximum possible three – digit number divisible by 7.

The series is as follows:

105, 112, 119, …., 994

Here a = 105, d = 7

Let 994 be the nth term of this A.P.

$a_n = a + (n-1)d$

$\Rightarrow 994 = 105 + (n-1)7$

$\Rightarrow (n-1)7 = 889$

$\Rightarrow (n-1) = 127$

$\Rightarrow n = 128$

So, there are 128 terms in the A.P.

$\therefore$ Sum $= \frac{n}{2}$ {first term + last term}

$= \frac{128}{2}\{a_1 + a_{128}\}$

$= 64\{105 + 994\} = (64)(1099) = 70336$

18.Find the values (s) of k so that the quadratic equation $3x^2 – 2kx + 12 = 0$ has equal roots.

Solution:

Given quadratic equation is $3x^2 – 2kx + 12 = 0$

Here a = 3, b = -2k and c = 12

The quadratic equation will have equal roots if Δ = 0

$\therefore b^2 - 4ac = 0$

Putting the values of a,b and c we get

$(2k)^2 - 4(3)(12) = 0$

$\Rightarrow 4k^2 - 144 = 0$

$\Rightarrow 4k^2 = 144$

$\Rightarrow k^2 = \frac{144}{4} = 36$

Considering square root on both sides,

$k = \sqrt{36} = \pm 6$

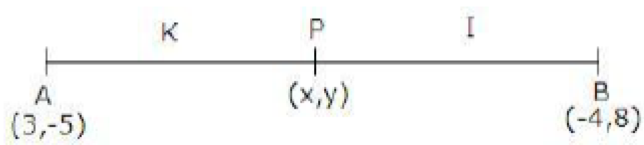
Therefore, the required values of k are 6 and -6.

SECTION – C

19.A point P divides the line segment joining the points A(3,-5) and B(-4,8) such that $\frac{AP}{PB} = \frac{K}{1}$. If P lies on the line x + y = 0, then find the value of K.

Solution:

Solution:



Let the co-ordinates of point P be (x, y)

Then using the section formula co-ordinates of P are.

$x = \frac{-4K+3}{K+1} \qquad y = \frac{8K-5}{K+1}$

Since P lies on x+y=0

$\therefore \frac{-4K+3}{K+1} + \frac{8K-5}{K+1} = 0$

$\Rightarrow 4K - 2 = 0$

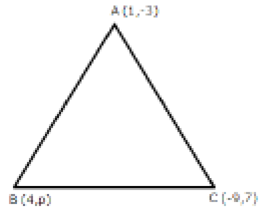
$\Rightarrow K = \frac{2}{4}$

$\Rightarrow K = \frac{1}{2}$

Hence the value of $K = \frac{1}{2}$.

20.If the vertices of a triangle are (1,-3), (4,p) and (-9,7) and its area is 15 sq. units, find the value (s) of p.

Solution:



The area of a Δ, whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$\Delta = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Substituting the given coordinates

Area of $\Delta = \frac{1}{2}|1(p-7) + 4(7+3) + (-9)(-3-p)|$

$$\Rightarrow \frac{1}{2}|(p-7) + 40 + 27 + 9p| = 15$$

$$\Rightarrow 10p + 60 = \pm 30$$

$$\Rightarrow 10p = -30 \quad or \quad 10p = -90$$

$$\Rightarrow p = -3. \quad or \quad p = -9$$

Ans hence the value of p=-3 or -9

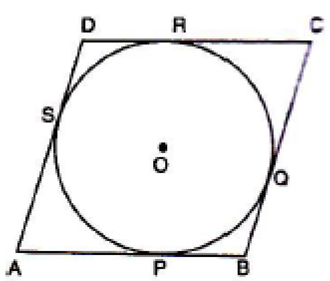
21.Prove that the parallelogram circumscribing a circle is a rhombus.

OR

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:

Let ABCD be a parallelogram such that its sides touching a circle with centre O. We know that the tangents to a circle from an exterior point are equal in length.



$\therefore$ AP = AS [From A] ...(i)

BP = BQ [From B] ...(ii)

CR = CQ [From C] ...(iii)

and, DR = DS [From D] ...(iv)

Adding (i), (ii), (iii) and (iv), we get

AP+BP+CR+DR=AS+BQ+CQ+DS

$\Rightarrow$ (AP+BP)+(CR+DR)=(AS+DS)+(BQ+CQ)

$\Rightarrow$ AB+CD = AD+BC

$\Rightarrow$ 2 AB = 2 BC [$\because$ ABCD is a parallelogram $\therefore$ AB=CD and BC = AD]

$\Rightarrow$ AB=BC

Thus, AB=BC=CD=AD

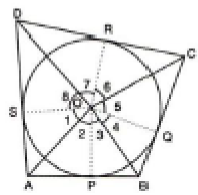
Hence, ABCD is a rhombus.

OR

A circle with centre O touches the sides AB, BC, CD, and DA of a quadrilateral ABCD at the points P, Q, R and S respectively.

TO PROVE : $\angle$AOB + $\angle$COD = 180°

and, $\angle$AOD + $\angle$BOC = 180°



CONSTRUCTION

Join OP, OQ, OR and OS.

PROOF Since the two tangents drawn from a external point to a circle subtend equal angles at the centre.

$\therefore$ $\angle 1 = \angle 2$, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$ and $\angle 7 = \angle 8$

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\left[\begin{array}{l}\text{Sum of all the angles} \\ \text{subtended at a point is } 360°\end{array}\right]$

$\Rightarrow$ $2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360$ and $2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360°$

$\Rightarrow$ $(\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180$ and $(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$

$\left[\begin{array}{l}\because \angle 2 + \angle 3 = \angle AOB, \angle 6 + \angle 7 = \angle COD \\ \angle 1 + \angle 8 = \angle AOD \text{ and } \angle 4 + \angle 5 = \angle BOC\end{array}\right]$

$\Rightarrow \angle AOB + \angle COD = 180°$

and $\angle AOD + \angle BOC = 180°$

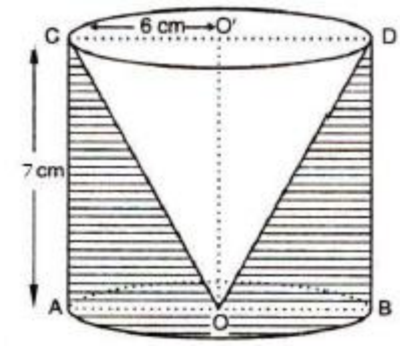
Hence Proved

22. From a solid cylinder of height 7 cm and base diameter 12 cm, a conical cavity of same height and same base diameter is hollowed out. Find the total surface area of the remaining solid. $\left[\text{Use } \pi = \frac{22}{7}\right]$

OR

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, then find the radius and slant height of the heap.

Solution:



Given: radius of cyl=radius of cone=r=6cm

Height of the cylinder=height of the cone=h=7cm

Slant height of the cone= l

$\sqrt{7^2+6^2}$

$=\sqrt{85}cm$

Total surface area of the remaining solid

= curved surface area of the cylinder + area of the base of the cylinder + curved surface area of the cone

$(2\pi rh+\pi r^2+\pi rl)$

$=2\times\frac{22}{7}\times6\times7+\frac{22}{7}\times6^2+\frac{22}{7}\times6\times\sqrt{85}$

$=264+\frac{792}{7}+\frac{132}{7}\sqrt{85}$

$=377.1+\frac{132}{7}\sqrt{85}cm^2$

OR

Volume of the conical heap=volume of the sand emptied from the bucket.

Volume of the conical heap=

$\frac{1}{3}\pi r^2h=\frac{1}{3}\pi r^2\times24cm^2$ (height of the coneis24)----------(1)

Volume of the sand in the bucket $\begin{aligned}&=\pi r^2h\\&=\pi(18)^2 32cm^2 ------- (2)\end{aligned}$

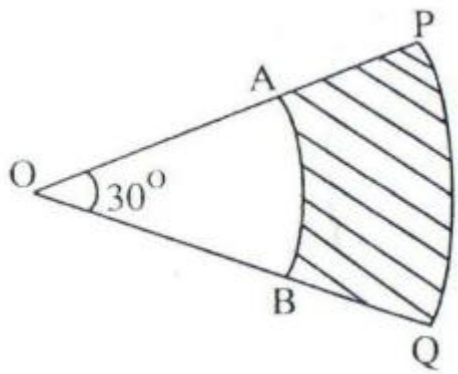
Equating 1 and 2

$\frac{1}{3}\pi r^2\times24=\pi(18)^2 32$

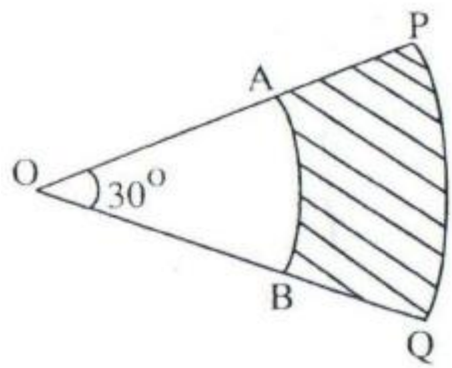
$\Rightarrow r^2=\frac{(18)^2\times32\times3}{24}$

$\Rightarrow r=36cm$

23. In fig., PQ and AB are respectively the arcs of two concentric circles of radii 7 cm and 3.5 cm and centre O. If $\angle$ POQ = 30°, then the area of the shaded region. $\left[\text{Use } \pi=\frac{22}{7}\right]$



Solution:



Area of the shaded region=

Area of sector POQ-Area of sector AOB

$$\left(\frac{\theta}{360}\pi R^2 - \frac{\theta}{360}\pi r^2\right)$$

Area of Shaded region $= \frac{30}{360} \times \frac{22}{7} \times (7^2 - 3.5^2)$

$= \frac{77}{8} cm^2$

24. Solve for x: $4x^2 - 4ax + (a^2 - b^2) = 0$

Or

Solve for x: $3x^2 - 6x + 2 = 0$

Solution:

$$4x^2 - 4ax + (a^2 - b^2) = 0$$
$$\Rightarrow (4x^2 - 4ax + a^2) - b^2 = 0$$
$$\Rightarrow \left[(2x^2) - 2.2x.a + a^2\right] - b^2 = 0$$
$$\Rightarrow \left[(2x - a)^2\right] - b^2 = 0$$
$$\Rightarrow \left[(2x - a)^2 - b\right]\left[(2x - a) + b\right] = 0$$
$$\Rightarrow \left[(2x - a) - b\right] = 0 \text{ or } \left[(2x - a) + b\right] = 0$$
$$\Rightarrow x = \frac{a + b}{2}; x = \frac{a - b}{2}$$

OR

$$3x^2 - 2\sqrt{6}x + 2 = 0$$
$$\Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$$
$$\Rightarrow \sqrt{3} \times \left[\sqrt{3}x - \sqrt{2}\right] - \sqrt{2}\left[\sqrt{3}x - \sqrt{2}\right] = 0$$
$$\Rightarrow \left(\sqrt{3}x - \sqrt{2}\right)\left(\sqrt{3}x - \sqrt{2}\right) = 0$$
$$\Rightarrow \left(\sqrt{3}x - \sqrt{2}\right)^2 = 0$$
$$\therefore \sqrt{3}x - \sqrt{2} = 0$$
$$\Rightarrow \sqrt{3}x = \sqrt{2}$$
$$\Rightarrow x = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2} \times \sqrt{3}}{(\sqrt{3})^2} = \frac{\sqrt{6}}{3}$$

25. A kite is flying at a height of 45 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60 . Find the length of the string assuming that there is slack in the string.

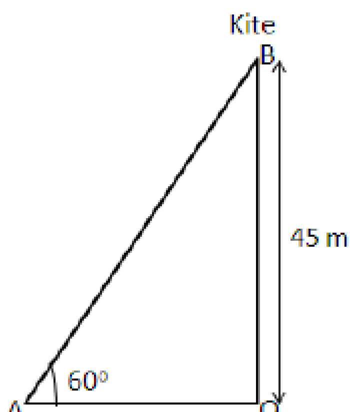
Solution:

Given: Position of kite is B.

Height of kite above ground= 45 m

Angle of inclination = 60°

Required length of string = AB



In right angled triangle AOB,

$$\sin A = \frac{OB}{AB}$$

$$\Rightarrow \sin 60^\circ = \frac{45}{AB}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{45}{AB}$$

$$\Rightarrow AB = \frac{45 \times 2}{\sqrt{3}} = \frac{90}{\sqrt{3}} = 30\sqrt{3}m$$

Hence, the length of the string is $30\sqrt{3}m$

26. Draw a triangle ABC with side BC = 6 cm, $\angle C = 30^\circ$ and $\angle A = 105^\circ$. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of ΔABC.

Solution:

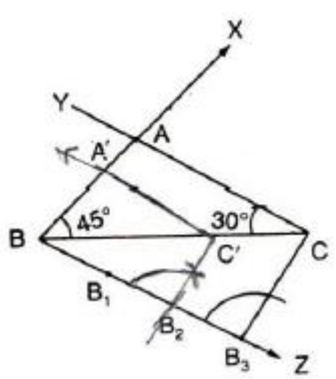
It is given that $\angle A = 105^\circ$, $\angle C = 30$.

Using angle sum property of triangle, we get, $\angle B = 45$

The steps of construction are as follows:

1. Draw a line segment BC = 6 cm.
2. At B, draw a ray making an angle of 45° with BC.
3. At C, draw a ray making an angle of 30° with BC. Let the two rays meet at point A.
4. Below BC, make an acute angle $\angle CBX$.
5. Along BX mark off three points B_1, B_2 , B_3 such that $BB_1 = B_1B_2 = B_2B_3$.
6. Join B_3C.
7. From B_2, draw $B_2C' \parallel B3C$.
8. From C′, draw $C'A' \parallel CA$, meeting BA at the point A′.

Then A′BC′ is the required triangle.



27. The 16th term of an AP is 1 more than twice its 8th term. If the 12th term of the AP is 47, then find its nth term.

Solution:

Let a and d respectively be the first term and the common difference of the AP.

We know that the n^{th} term of an AP is given by $a_n = a + (n - 1)d$

According to the given information,

$A_{16} = 1 + 2\ a_8$

$\Rightarrow$ a + (16 – 1)d = 1 + 2[a + (8 – 1)d]

$\Rightarrow$ a + 15d = 1 + 2a + 14d

$\Rightarrow$ a + 15d = 1 + 2a + 14d

$\Rightarrow$ –a + d = 1 ... (1)

Also, it is given that, $a_{12} = 47$

$\Rightarrow$ a + (12 – 1)d = 47

$\Rightarrow$ a + 11d = 47 ... (2)

Adding (1) and (2), we have:

12d = 48

$\Rightarrow$ d = 4

From (1),

–a + 4 = 1 $\Rightarrow$ a = 3

Hence, $a_n = a + (n - 1)d = 3 + (n - 1)(4) = 3 + 4n - 4 = 4n - 1$

Hence, the n^{th} term of the AP is 4n – 1.

28.A card is drawn from a well shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour (ii) a face card (iii) the queen of diamonds.

Solution:

Total number of outcomes=52

(i) Probability of getting a red king

Here the number of favourable outcomes=2

$$\text{probability} = \frac{\text{no.of favourable outcomes}}{\text{total number of outcome}}$$

$$= \frac{12}{52}$$

$$= \frac{3}{13}$$

(iii)Probability of queen of diamonds

number of queens of diamond=1,hence

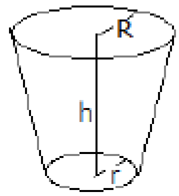
$$\text{probability} = \frac{\text{no.of favourable outcomes}}{\text{total number of outcome}}$$

$$\frac{1}{52}$$

SECTION – D

29.A bucket is in the form of a frustum of a cone and its can hold 28.49 litres of water. If the radii of its circular ends are 28 cm and 21 cm, find the height of the bucket. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution:



Here, R = 28 cm and r = 21 cm, we need to find h.

Volume of frustum = 28.49 L = 28.49×1000 cm^3 = 28490 cm^3

Now, Volume of frustum = $\frac{\pi h}{3}\left(R^2+Rr+r^2\right)$

$$\Rightarrow \frac{22h}{7\times 3}\left(28^2+28\times 21+21^2\right)=28490$$

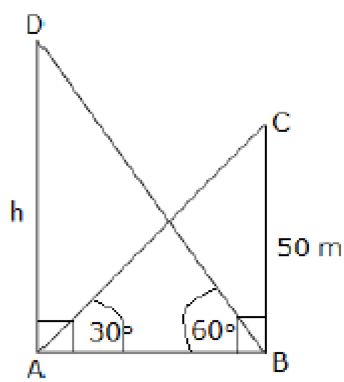
$$\Rightarrow \frac{22}{21}h\times 1813=28490$$

$$\Rightarrow h=\frac{28490\times 21}{22\times 1813}=15cm$$

Hence the height of bucket is 15 cm.

30.The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of depression from the top of the tower of the foot of the hill is 30°. If the tower is 50 m high, find the height of the hill.

Solution:



Let the height of hill is h.

In right triangle ABC,

$$\frac{50}{AB}=\tan 30^\circ \Rightarrow \frac{50}{AB}=\frac{1}{\sqrt{3}}\Rightarrow AB=50\sqrt{3}$$

In right triangle ABD,

$$\frac{h}{AB}=\tan 60^\circ \Rightarrow \frac{h}{AB}=\sqrt{3}\Rightarrow h=\sqrt{3}AB$$

$$\Rightarrow h=\sqrt{3}\left(50\sqrt{3}\right)=150m$$

Hence the height of hill is 150 m.

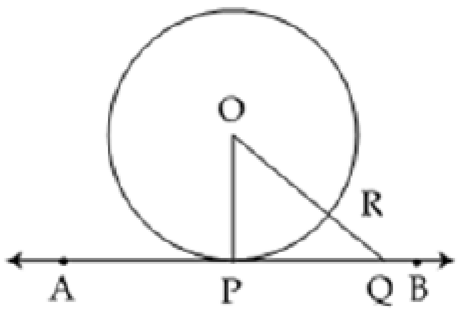
31. AB + CD = AD + BC

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

OR

A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.

Solution:



Given: AB is a tangent to a circle with centre O.

To prove: OP is perpendicular to AB.

Construction: Take a point Q on AB and join OQ.

Proof: Since Q is a point on the tangent AB, other than the point of contact P, so Q will be outside the circle.

Let OQ intersect the circle at R.

Now OQ = OR + RQ

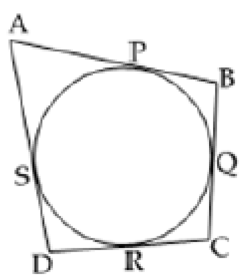
$\Rightarrow$ OQ > OR $\Rightarrow$ OQ > OP [as OR = OP]

$\Rightarrow$ OP < OQ

Thus OP is shorter than any other segment among all and the shortest length is the perpendicular from O on AB.

$\therefore$ OP $\perp$ AB. Hence proved.

OR



Let ABCD be a quadrilateral, circumscribing a circle.

Since the tangents drawn to the circle from an external point are equal,

we have

AP = AS ... (1)

PB = BQ ... (2)
RC = QC ... (3)
DR = DS ... (4)
Adding, (1), (2), (3) and (4), we get
AP + PB + RC + DR = AS + BQ + QC + DS
(AP + PB) + (DR + RC) = (AS + SD) + (BQ + QC)
AB + CD = AD + BC.
Hence, Proved.

32.A shopkeeper buys some books for ₹80. If he had bought 4 more books for the same amount, each book would have cost ₹1 less. Find the number of books he bought.

OR

The sum of two number is 9 and the sum of their reciprocals is $\frac{1}{2}$. Find the numbers.

Solution:
Total cost of books = Rs 80
Let the number of books = x

So the cost of each book = Rs $\frac{80}{x}$

Cost of each book if he buy 4 more book $= \text{Rs} \frac{80}{x+4}$

As per given in question:

$$\frac{80}{x} - \frac{80}{x+4} = 1$$
$$\Rightarrow \frac{80x + 320 - 80x}{x(x+4)} = 1$$
$$\Rightarrow \frac{320}{x^2 + 4x} = 1$$
$$\Rightarrow x^2 + 4x - 320 = 0$$
$$\Rightarrow (x+20)(x-16) = 0$$
$$\Rightarrow x = -20, 16$$

Since number of books cannot be negative,
So the number of books he brought is 16.

OR

Let the first number be x then the second number be 9 – x as the sum of both numbers is 9.

Now the sum of their reciprocal is $\frac{1}{2}$, therefore

$$\frac{1}{x}+\frac{1}{9-x}=\frac{1}{2}$$
$$\Rightarrow \frac{9-x+x}{x(9-x)}=\frac{1}{2}$$
$$\Rightarrow \frac{9}{9x-x^2}=\frac{1}{2}$$
$$\Rightarrow 18=9x-x^2$$
$$\Rightarrow x^2-9x+18=0$$
$$\Rightarrow (x-6)(x-3)=0$$
$$\Rightarrow x=6,3$$

If x = 6 then other number is 3.

and if x = 3 then other number is 6.

Hence numbers are 3 and 6.

33.Sum of the first 20 terms of an AP is -240, and its first term is 7. Find its 24th term.

Solution:

Given: $S_{20} = -240$ and a = 7

Consider, $S_{20} = -240$

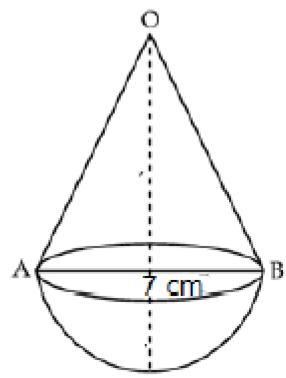
$$\Rightarrow \frac{20}{2}(2\times 7+19d)=-240 \quad \left[\because S_n=\frac{n}{2}[2a+(n-1)d]\right]$$
$$\Rightarrow 10(14+19d)=-240$$
$$\Rightarrow 14+19d=-24$$
$$\Rightarrow 19d=-38$$
$$\Rightarrow d=-2$$

Now, $a_{24}=a+23d=7+23\times -2=-39$

Hence, $a_{24}=-39$

34.A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid. $\left[\text{Use } \pi=\frac{22}{7}\right]$

Solution:



Radius of hemi-sphere = 7 cm

Radius of cone = 7 cm

Height of cone = diameter = 14 cm

Volume of solid = Volume of cone + Volume of Hemi-sphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 49(14 + 14)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 49 \times 28$$

$$= \frac{22 \times 7 \times 28}{3} = \frac{4312}{3} cm^3$$

CBSE Class 10

Mathematics

Previous Year Question Paper 2011

Series: RHB/1 **Code no. 30/1/1**

- Please check that this question paper contains **16** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **34** questions.
- **Please write down the serial number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer script during this period.

SUMMATIVE ASSESSMENT-II

MATHEMATICS

Time Allowed: **3** hours Maximum Marks: **80**

General Instructions

1. **All** questions are compulsory.
2. The question paper consists of **34** questions divided into four sections A, B, C and D.
3. Section A contains **30** questions of **1** mark each, which are multiple choice type questions, Section B contains **8** questions of **2** marks each, Section C contains **10** questions of **3** marks each, Section D contains **6** questions of **4** marks each.
4. There is no overall choice in the paper. However, internal choice is provided in one question of **2** marks. **3** questions of **3** marks each and two questions of **4** marks each.

5. Use of calculators is not permitted.

SECTION-A

Question Numbers 1 to 30 carry 1 mark each. For each of the questions 1 to 30, four alternative choices have been provided, of which only one is correct. Select the correct choice.

1. The roots of the equation $x^2+x-p(p+1)=0$, where p is a constant, are:

(A) p,p+1

(B) -p,p+1

(C) p,-(p+1)

(D) -p,-(p+1) **1 Mark**

Ans: The given equation is $x^2+x-p(p+1)=0$. Now, solving this equation using Quadratic formula, i.e. $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ Putting the values in the formula, we get :

$$\Rightarrow x=\frac{1\pm\sqrt{1-4(1)(-p(p+1))}}{2}$$

$$\Rightarrow x=\frac{1\pm\sqrt{1+4p^2+p}}{2}$$

$$\Rightarrow x=\frac{-1\pm\sqrt{(2p+1)^2}}{2}=-\frac{1}{2}\pm\frac{(2p+1)}{2}$$

Thus, the two roots are:

$$\alpha=-\frac{1}{2}+p+\frac{1}{2}=p$$

$$\beta=-\frac{1}{2}-p-\frac{1}{2}=-p-1$$

Therefore, option is the correct answer.

2. In an AP, if, d=-2, n=5 and an=0, then the value of a is : **1 Mark**

a) 10

(B) 5

(C) -8

(D) 8

Ans: It is given that

d=-2

n=5

an=0

The formula for an is given as an=a+(n-1)d . Putting the values, we get

0=a+(5-1)-2

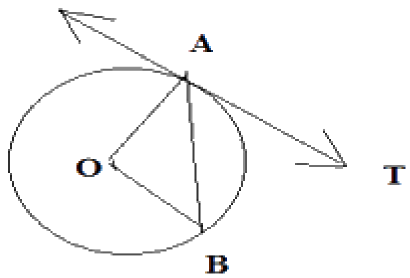
0=a+4(-2)

a=8

Hence, option (D) 8 is the correct answer.

3. In Fig. 1, O is the centre of a circle, AB is a chord and AT is the tangent at A. If

$< AOB = 100^o$ then $< BAT$ is equal to: 1 Mark



Ans: It is given in this question that AO and BO are radius therefore, triangle AOB is an isosceles triangle:

$< AOB + < OBA + < OAB = 180^\circ$

Putting values, we get

$100^\circ + < OAB + < OAB = 180^\circ$

$\Rightarrow 2 < AOB = 80^o$

$\Rightarrow < AOB=40^{o}$

We know that tangent and radius are perpendicular to each other,

$\therefore < OAT=40^{o}$

Simplifying,

$\therefore < OAB+<BAT=40^{o}$

$<BAT=90^{o}-40^{o}=50^{o}$

Hence, option (C) 50° is the correct answer.

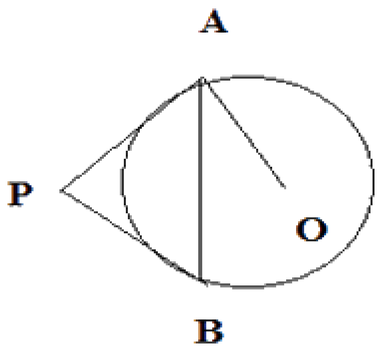
4. In Fig. 2, PA and PB are tangents to the circle with centre O. If <APB =60^{o}, then < OAB is : 1 Mark

a. 30^{o}

b. 60^{o}

c. 90^{o}

d. 15^{o}



Ans: It is given that PA and PB are two tangents. O is the centre of the circle and OA and OB are joined. Given: $< APB = 60^{\circ}$. We need to find < OAB PA and PB are tangents to the circle, so, PA = PB , < PAB = < PBA. But < APB $=60^{\circ}$. So, $< PAB + < PBA = 180^{\circ} - 60^{\circ} = 120^{\circ}$.

$\Rightarrow 2 < PAB = 120^{\circ}$

$\Rightarrow <PAB=60^{o}$

Now, OA is radius and PA is tangent, hence, $OA \perp PA$,

$\Rightarrow <OPA=90^{o}$

$\Rightarrow \angle OAB + \angle PAB = 90^{\circ}$

Putting the values, we get

$\Rightarrow \angle OAB + 60^{\circ} = 90^{\circ}$

$\Rightarrow \angle OAB = 30^{\circ}$

Hence, option (A) 30° is the correct answer.

5. The radii of two circles are 4 cm and 3 cm respectively. The diameter of the circle having area equal to the sum of the areas of the two circles (in cm) is: 1 Mark

a. 5

b. 7

c. 10

d. 14

Ans: The radius of one circle is $4\,\text{cm}$. Therefore, the area of the circle is :

$A = \pi \times (4)^2 = 16\pi$. The radius of the second circle is $3\,\text{cm}$. Therefore, the area of the circle is: $A = \pi \times (3)^2 = 9\pi$. Thus, the sum of the areas of two circles is:

$A' = 9\pi + 16\pi$

$\Rightarrow \pi r'^2 = 25\pi$

$\Rightarrow r' = 5\text{cm}$

Hence, the diameter of the circle is $10\,\text{cm}$. So, option (C) 10 is the correct answer.

6. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged, then the water level rises (in cm) by : 1 Mark

a) 3

(B) 4

(C) 5

(D) 6

Ans: It is given that: The diameter of the sphere is 18 cm. So, the radius is 9 cm. The diameter of the cylindrical vessel is 36 cm. So, the radius is 18 cm. Let us consider the height of water rise is H cm. Now, the sphere is completely submerged, Volume of the vessel is equal to the volume of the sphere. Putting the values, we get.

$$\pi R^2 H = \frac{4}{3}\pi r^3$$

$$\Rightarrow H = \frac{\frac{4}{3}r^3}{R^2} = \frac{4\times(9)^3}{3\times(18)^2} = 3\text{cm}$$

Hence, option (A)3 is the correct answer.

7. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 45°. The height of the tower (in metres) is:

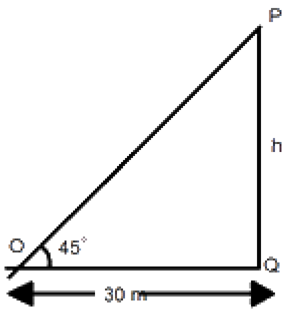
a) 15

(B) 30

(C) $30\sqrt{3}$

(D) $10\sqrt{3}$ **1 Mark**

Ans: Let us draw a figure for the given tower and the point in the question :



Let us consider the height of the tower to be h cm, now, in ΔOPQ

, $\tan 45^{\circ} = \frac{PQ}{OQ}$

Now the distance of the tower from the point is 30m, so,

$\tan 45 = \frac{h}{30}$

$\Rightarrow 1 = \frac{h}{30}$

$\Rightarrow h = 30m$

Hence, option (B)30 m is the correct answer.

8. The point P which divides the line segment joining the points A(2, - 5) and B(5, 2) in the ratio 2:3 lies in the quadrant: 1 Mark

a) I

(B) II

(C) III

(D) IV

Ans: Given the points A(2, - 5) and B(5, 2) are divided in the ratio 2:3. Thus, the point P is given by:

$$P = \left(\frac{2\times5+3\times2}{2+3}, \frac{2\times2-3\times5}{2+3} \right) = \left(3, -\frac{11}{5} \right)$$

Thus, the point clearly lies in the fourth quadrant. Hence option (D) IV is the correct answer.

9. The midpoint of segment AB is the point P(0, 4). If the coordinates of Bare (-2, 3) then the coordinates of A are :

(A) (2,5)

(B) (-2,-5)

(C) (2,9)

(D) (-2,11) **1 Mark**

Ans: It is given that P(0, 4) is the midpoint of segment AB .The point B is (-2, 3). Let the point A be (x, y), then,

$\frac{x+(-2)}{2}$. Solving this we get; x=2. For the point y.

$$\frac{y+3}{2}=4$$

$\Rightarrow y=8-3=5$. Hence, the point A is given by (2,5). Therefore, option (a) (2, 5) is the correct answer.

10. Which of the following cannot be the probability of an event? **1 Mark**

(A) 1.5

(B) $\frac{3}{5}$

(C) 25%

(D) 0.3

Ans: Out of these four options, 1.5 cannot be a probability of an event because it is greater than 1 and probability of any event cannot be greater than 1.

SECTION-B

Question Numbers 11 to 18 carry 2 marks each.

11. Find the value of p so that the quadratic equation px(x - 3) + 9 = 0 has two equal roots. **2 Marks**

Ans: Given the quadratic equation is $px(x-3)+9=0$. Simplifying this equation further, we get, $px^2-3px+9=0$.

Using the quadratic formula, we get $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Putting the values, we get: $x=\frac{3p\pm\sqrt{9p^2-4p(9)}}{2p}$. It is given that the equation has two equal roots, this implies, $9p^2-4p(9)=0$. Solving this to find p, we get

$9p^2-4p(9)=0$

$9p(p-4)=0$

$\Rightarrow p=0,4$

Therefore, the value of p is (0,4).

12. Find whether -150 is a term of the AP 17, 12, 7, 2,..........? 2 Marks

Ans: The given AP is 17,12,7,2........... Thus, we can see here that:

a=17

d=5

If -150 is a term, then, a_n =-150.Expanding the term to find the value of n,

$a_n = a+(n-1)d$

$\Rightarrow$ -150=17+(n-1)5

$\Rightarrow$ -150-17=(n-1)5

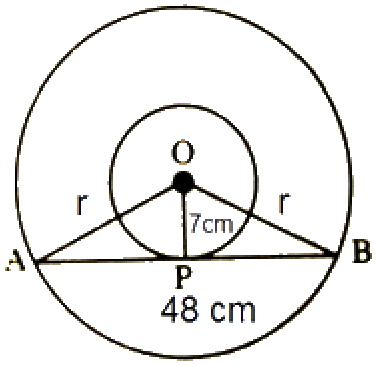
$\Rightarrow \frac{-167}{5}=n-1$

$\Rightarrow n=-\frac{162}{5}$

But this can't be an actual n^{th} term, thus, -150 does not belong to this AP.

13. Two concentric circles are of radii 7 cm and r cm respectively, where r >7. A chord of the larger circle, of length 48cm touches the smaller circle, find the value of r. 2 Marks

Ans: First of all, let us draw a figure of the two concentric circles,



We need to find the value of r. In the figure, we can clearly see that point P is the midpoint of AB. So, in ΔOPB, using Pythagoras theorem, we get:

$OB^2=OP^2+PB^2$

$\Rightarrow r^2=(7)^2+(24)^2$

$\Rightarrow r^2=49+576$

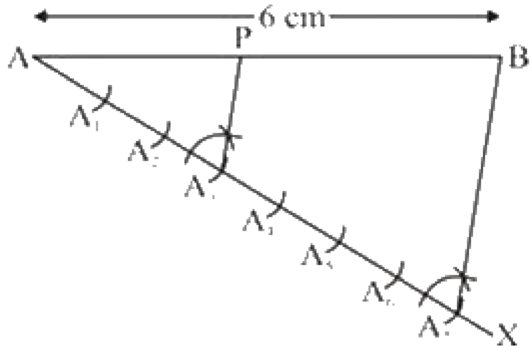
$\Rightarrow r^2=625$

$\Rightarrow r^2=25cm$

Hence, the value of r is 25cm.

14. Draw a line segment of length 6 cm. Using compasses and ruler, find a point P on it which divides it in the ratio 3:4. 2 Marks

Ans: In order to solve this question, we are first of all going to draw a line segment AB which is equal to length 6cm.

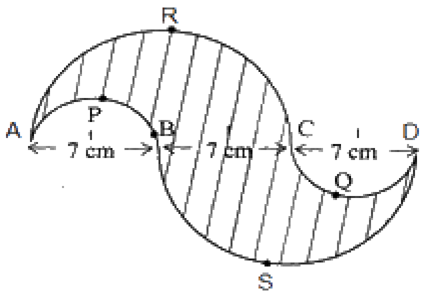


Next, you need to put the compass on points A and B and a perpendicular bisector of AB is drawn. The point at which the perpendicular bisector meets

AB is named O. The compass is put on points O and B and a perpendicular bisector of OB is drawn. Then, you should note that the Perpendicular bisector meets OB at a point which is named P which as given in the question divides the segment into 3:4.Thus,

$AP=\frac{3}{4}$ of AB

15. Fig. 3, APB and CQD are semi-circles of diameter 7 cm each, while ARC and BSD are semi-circles of diameter 14 cm each. Find the perimeter of the shaded region. [Use $\pi=\frac{22}{7}$]. **2 Marks**



Ans: Let us consider the semicircle ARC and BSD. They have diameters 14cm each, thus, the perimeter of ARC and BSD is equal to:

$p_1=p_2=2\pi r=2\pi(7)=14\pi$

The semicircles APB and CQD have diameters 7cm each.

Perimeter of the shaded region = Length of APB + Length of ARC + Length of CQD + Length of DSB . Now,

perimeter of APB $\frac{1}{2}\times 2\pi\left[\frac{7}{2}\text{cm}\right]=\frac{22}{7}\times\frac{7}{2}=11\text{cm}$.

Perimeter of ARC $=\frac{1}{2}\times 2\pi(7\text{cm})=\frac{22}{7}\times 7\text{cm}=22\text{cm}$

Perimeter of CQD $=\frac{1}{2}\times 2\pi\left[\frac{7}{2}\text{cm}\right]=\frac{22}{7}\times\frac{7}{2}\text{cm}=11\text{cm}$

Perimeter of DSB $=\frac{1}{2}\times 2\pi(7\text{cm})=\frac{22}{7}\times 7\text{cm}=22\text{cm}$

Thus, the perimeter of the shaded region = 11 cm + 22 cm + 11 cm + 22 cm = 66 cm.

Therefore, the perimeter of the shaded region is 66cm.

Or

Find the area of a quadrant of a circle, where the circumference of the circle is 44 cm. [Use $\pi=\frac{22}{7}$] 2 Marks

Ans: It is given that the circumference of the circle is equal to 44 cm. If we let the radius of the circle to be r cm. $2\pi r=44$.

Solving this to find the radius.

$$r=\frac{44}{2\pi}=\frac{44}{2\times\frac{22}{7}}=7\text{ cm}$$

Thus, the area of the quadrant is

$A=\frac{1}{4}\times\pi r^2=\frac{1}{4}\times\frac{22}{7}\times 7=\frac{77}{2}$. Thus, the area of the quadrant of the circle is equal to $\frac{77}{2}\text{ cm}^2$.

16. Two cubes, each 4 cm, are joined end to end. Find the surface area of the resulting cuboid. 2 Marks

Ans: It is given that the cubes of side 4 cm each are joined end to end. The joining of two cubes will result in a cuboid of length, breadth and height equal to 8cm, 4cm, 4cm respectively. Hence, the surface area of the resulting cuboid is:

$S= 2(lb+bh+hl)$

$S=(8\times4+4\times4+4\times8)$

$S=160\text{ cm}^2$

17. Find that value(s) of x for which the distance between the points P(x, 4) and Q(9, 10) is 10 units. 2 Marks

Ans: It is given that the distance between the points P(x, 4) and Q(9, 10) is 10 units. Finding the distance using the distance formula for two points, we get

$$PQ=\sqrt{(9-x)^2+(10-4)^2}$$

$$\Rightarrow PQ=\sqrt{(9-x)^2+36}$$

Putting the value of the distance, we get:

$$\Rightarrow 10=\sqrt{(9-x)^2+36}$$

$$\Rightarrow 100=(9-x)^2+36$$

$$\Rightarrow 64=(9-x)^2$$

$$\Rightarrow 9-x=\pm 8$$

Taking the two different cases, we get :

$$\Rightarrow x=9-8=1$$

And

$$\Rightarrow 9-x=-8$$

$$\Rightarrow x=9+9=17$$

Thus, $x=1,17$.

18. A coin is tossed two times. Find the probability of getting at least one head. **2 Marks**

Ans: When a coin is tossed twice, then the total number of outcomes is 4. The probability of getting no heads is: The possible outcome is (T,T) The probability is $\frac{1}{4}$. Thus, the probability of getting at least one head is:

$$1-\frac{1}{4}=\frac{3}{4}.$$

SECTION-C

Question numbers 19 to 28 carry 3 marks each.

19. Find the roots of the following quadratic equation: $2\sqrt{3}x^2-5x+\sqrt{3}=0$.

3 Marks

Ans: We are given the equation, $2\sqrt{3}x^2-5x+\sqrt{3}=0$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{5\pm\sqrt{(-5)^2-4\times2\sqrt{3}\times\sqrt{3}}}{2\times2\sqrt{2}}$$

$$x=\frac{5\pm\sqrt{25-24}}{4\sqrt{2}}$$

$$x=\frac{5\pm1}{4\sqrt{3}}$$

Further simplifying, we get

$$x=\frac{6}{4\sqrt{3}};\frac{4}{4\sqrt{3}}$$

$$x=\frac{\sqrt{3}}{2};\frac{1}{\sqrt{3}}$$

Thus, the roots of the equation are: $x=\frac{\sqrt{3}}{2};\frac{1}{\sqrt{3}}$

20. Find the value of the middle term of the following AP: - 6, -2, 2, ..., 58.
3 Marks

Ans: We are given the AP, - 6, -2, 2, ..., 58

For finding the middle term , we need to find the position of the middle term. Thus, we will first find the total number of terms in this AP.

That is,

$a=-6$

$d=4$

$l=58$

Thus, finding the value of n: $l=a+(n-1)d$

Putting the values in this, we get:

$58 = -6 + (n-1)4$

$\Rightarrow 58 + 6 = (n-1)4$

$\Rightarrow \frac{64}{4} = n - 1$

$\Rightarrow 16 + 1 = n$

$\Rightarrow n=17$

Hence, the middle term will be,

$n_m = \frac{n+1}{2}$.

$\Rightarrow n_m = \frac{17+1}{2} = 9$ So, we need to find the 9th term,

$a_9 = a + (n-1)d = -6 + (9-1)4$

$\Rightarrow a_9 = -6 + 8 \times 4 = -6 + 32$

$\Rightarrow a_9 = 26$

Thus, the middle term of the AP - 6, -2, 2, ..., 58 is 26.

Or

Determine the AP whose fourth term is 18 and the difference of the ninth term from the fifteenth term is 30. 3 Marks

Ans: In this question, we are given the fourth term and the difference between ninth and fifteenth term, i.e. $a_4 = 18$. Expanding the terms in terms of the formula to find the AP, $a_{15} - a_9 = 30$

$a + (4-1)d = 18$. Finding the other equation as well,

$\Rightarrow a + 3d = 18$

$a + 14d - a - 8d = 30$

$\Rightarrow 6d = 30$

$\Rightarrow d = 5$

So, the common difference is 5. Putting the above equation:

$a+(4-1)d=18$

$\Rightarrow a+3(5)=18$

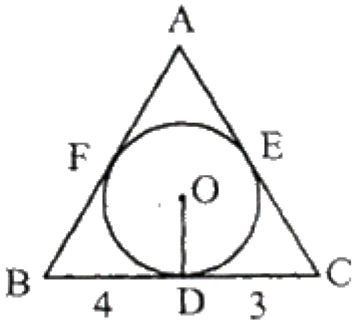
$\Rightarrow a=18-15=3$

Thus, the AP which is a,a+d,a+2d,18,…3+14d,…… becomes:

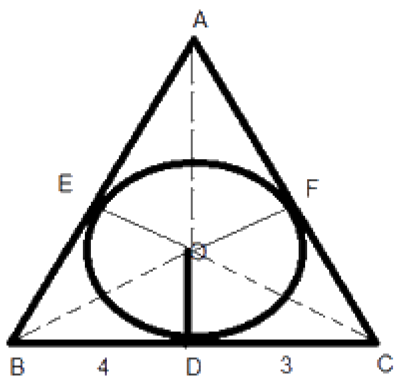
$3, 3+5, 3+10, 18, \ldots .. 3+70, \ldots .$

$3, 8, 13, 18, \ldots .., 73, \ldots .$

21. In Fig. 4, a triangle ABC is drawn to circumscribe a circle' of radius 2 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 4 cm and 3 cm respectively. If area of ABC= 21 cm2, then find the lengths of sides AB and AC. 3 Marks



Ans: In this question, we are given that the radius of the circle is 2cm and the area of triangle ABC is 21 cm^2, we need to find the lengths of the triangle. Construction: we need to join OA, OB, OC, OE ⊥ AB at E and OF ⊥ AC at F. The resulting figure will be:



Now we have, the following data:

AE = AF

BD = BE = 4 cm

CD = CF = 3 cm

Let the lengths of AE and Af be x. This implies, area of the triangle equals:

$$A=\text{area}(\Delta BOC)+\text{area}(\Delta AOB)+\text{area}(\Delta AOC)$$

$$\Rightarrow 21=\frac{1}{2}\times BC\times OD+\frac{1}{2}\times AB\times OE+\frac{1}{2}\times AC\times OF$$

$$\Rightarrow 42=7\times 4+(4+x)\times 2+(3+x)\times 2$$

Solving this further, we get:

$$\Rightarrow 21=7+(4+x)+(3+x)$$

$$\Rightarrow 21=14+2x$$

$$\Rightarrow x=3.5$$

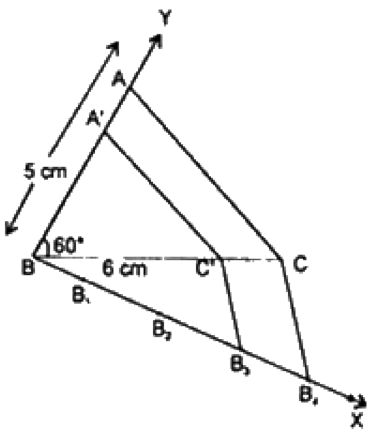
Thus, the lengths of the sides AB and AC are:

$AB=4+3.5=7.5$

$AC=3+3.5=6.5$.

22. Draw a triangle ABC in which AB = 5 cm, BC = 6 cm and $\angle ABC=60^{o}$. Then construct a triangle whose sides are times the corresponding sides of ABC. 3 Marks

Ans: Construction of the triangle:



1. A line segment AB of length equal to 5 cm is drawn.

2. An angle ABY equal to 60° is drawn from point B.

3. Now in order to measure angle at B:

3.1 With B as centre and with any radius, another arc are drawn cutting the line which is present at D.

3.2 Taking the point D as centre and taking the same radius, an arc cutting the first arc is drawn at point E.

3.3 A ray BY that passes through the point E is forming the angle equal to 60° with the line AB.

4. Taking point B as centre and radius equal to 6 cm, an arc intersecting the line BY is drawn at C.

5. A and C are joined, the required triangle is ΔABC.

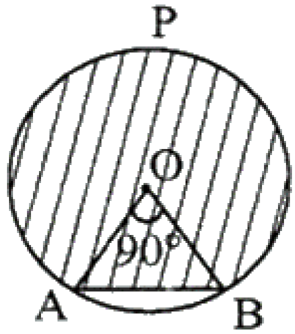
6. A ray from point A, named AX is drawn downwards which makes an acute angle.

7. Some points are marked named on the segment AX such that the lengths:

$AA_1=A_1A_2=A_2A_3=A_3A_4=A_4A_5=A_5A_6=A_6A_7$

8. The points A_7 and B are joined and from the point A_5, the segment A_5M is drawn parallel to A_7B that intersects AB at M. 9. Taking the point M, a segment MN is drawn parallel to BC intersecting AC at the point N. Then, ΔAMN is the required triangle, the sides of this triangle are equal to $\frac{5}{7}$ of the corresponding sides of ΔABC.

23. Find the area of the major segment APB, in Fig 5, of a circle of radius 35 cm and $\angle AOB=90^o$. [Use $\pi=\frac{22}{7}$] **3 Marks**



Ans: For finding the area of the major segment, firstly we will write the equation for area of minor segment. Area of minor segment = Area of sector AOBC- Area of AOB

Putting the values in this equation, we get :

$$A=\frac{90^o}{360^o}\times\pi\left(OA\right)^2-\frac{1}{2}\times OA\times OB$$

$$\Rightarrow A=\frac{1}{4}\times\frac{22}{7}\times\left(35\right)^2-\frac{1}{2}\times 35\times 35$$

$$\Rightarrow A=962.5-612.5$$

$$\Rightarrow A=350cm^2$$

Thus, the area of major segment is equal to Area of major segment= Area of circle – Area of minor segment. Therefore,

$$A'=\pi\left(OA\right)^2-350$$

$$\Rightarrow A'=\frac{22}{7}\times\left(35\right)^2-350$$

$$\Rightarrow A'=3850-350=3500cm^2$$

Hence, the area of the major segment is $3500cm^2$.

24. The radii of the circular ends of a bucket of height 15 cm are 14 cm and r cm (r < 14 cm). If the volume of the bucket is 5390 cm3, then find the value of r. [Use $\pi=\frac{22}{7}$]. **3 Marks**

Ans: In the question, we are given that: Height of bucket, $h=15cm$. Radius of outer end, R=14cm. Radius of inner end, =r. Volume of the bucket is, $V = 5390cm^3$. Putting the formula for the volume above, we get:

$$\frac{1}{3}\pi h\left[R^2+r^2+Rr\right]=5390cm^3$$

$$\Rightarrow \frac{1}{3}\times\frac{22}{7}\times 15\left[(14)^2+r^2+14r\right]=5390$$

$$\Rightarrow \frac{110}{7}\left[196+r^2+14r\right]=5390$$

$$\Rightarrow r^2+14r+196=343$$

$$\Rightarrow r^2+14r-147=0$$

Now solving this equation to find the value of r.

$$\Rightarrow r^2+21r-7r-147=0$$

$$\Rightarrow r(r+21)-7(r+21)=0$$

$$\Rightarrow r=7,-21$$

The value of a radius cannot be negative, so, the value of r is 7cm.

25. Two dice are rolled once. Find the probability of getting such numbers on two dice, whose product is a perfect square. 3 Marks

Ans: When two dice are rolled, the total number of possible outcomes is equal to 36

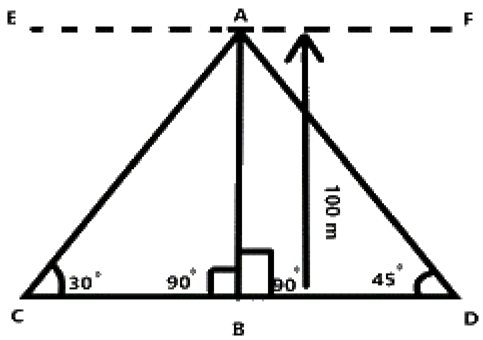
Now, the number pairs on the two dices whose product form perfect square have the possibilities, $(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(1,4),(4,1)$. Thus, the number of possible outcomes for this event is 8. Hence, the probability of getting such numbers on two dice, whose product is a perfect square is $\frac{8}{36}=\frac{2}{9}$.

Or

A game consists of tossing a coin 3 times and noting its outcome each time. Hanif wins if he gets three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game. 3 Marks

Ans: It is given that the coin is tossed thrice which means that the total number of outcomes is $2^3=8$. The probability that Hanif loses is equal to one minus the probability that he wins. So, we will find the probability of him winning the game first Now, the outcomes for getting three heads or three tails will be: $(\mathrm{H,H,H})$, $(\mathrm{T,T,T})$. Thus, the probability of Hanif winning is $\frac{2}{8}=\frac{1}{4}$. Thus, the probability that Hanif will lose the game is $1-\frac{1}{4}=\frac{3}{4}$.

26. From the top of a tower 100 m high, a man observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the cars. [Use $\sqrt{3}=1.73$] 3 Marks



Ans: First of all, we are going to construct a diagram for the given situation.

Now it is given that Height of tower, $\mathrm{H}=100\mathrm{m}$. Angle of depression of car 1, $\angle\mathrm{EAC}=30^{\circ}$. Angle of depression of car 2, $\angle\mathrm{FAD}=45^{\circ}$. Now, in the right angled triangle ABC, $\tan 30^{\circ}=\frac{\mathrm{AB}}{\mathrm{BC}}$. As $\mathrm{AB}=100\mathrm{m}$ and $\tan 30^{\circ}=\frac{1}{\sqrt{3}}$. Thus, putting these values in above equation, $\frac{1}{\sqrt{3}}=\frac{100}{\mathrm{BC}}$. In right angled triangle ABD, $\tan 45^{\circ}=\frac{\mathrm{AB}}{\mathrm{BD}}$

$\Rightarrow \mathrm{BC}=100\sqrt{3}$

Now, $\mathrm{AB}=100\mathrm{m}$ and

$\tan 45^{\circ}=1$. Substituting the values in above equation.

$1=\frac{100}{\mathrm{BD}}$.

$\Rightarrow$ BD=100m

Now, $\Rightarrow$ CD=CB+BD=$100\sqrt{3}+100=100\left(\sqrt{3}+1\right)$.

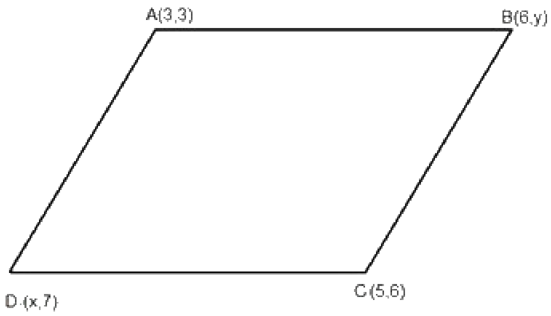
Putting $\sqrt{3}=1.732$. Thus, CD=$100\left(1.73+1\right)=273.2\text{m}$.

Therefore, the distance between the cars is 273.2m.

27. If (3, 3), (6, y), (x, 7) and (5, 6) are the vertices of a parallelogram taken in order, find the values of x and y. 3 Marks

Ans: It is given that the vertices of the parallelogram are (3, 3), (6, y), (x, 7) and (5, 6).

The figure is:



Now finding the midpoints of the diagonals AC and BD Coordinates of mid-point of diagonal BD are:

$$\left(\frac{5+6}{2},\frac{6+y}{2}\right).$$

$$\left(\frac{11}{2},\frac{6+y}{2}\right)$$

Coordinates of mid-point of diagonal AC are:

$$\left(\frac{3+x}{2},\frac{7+3}{2}\right).$$

$$\left(\frac{3+x}{2},\frac{10}{2}\right)$$

Now comparing the x and y coordinates of the midpoint as the mid points of both the diagonals coincide:

$\frac{3+x}{2}=\frac{11}{2}$. And the y-coordinate :

$\Rightarrow x=11-3=8$

$\frac{6+y}{2}=\frac{10}{2}$

$\Rightarrow y=10-6=4$

Thus, the values of x and y are 8 and 4 respectively.

28. If two vertices of an equilateral triangle are (3, 0) and (6, 0), find the third vertex. 3 Marks

Ans: Let the third vertex of the triangle be (x ,y). Thus, the vertices of the triangle are:

$A=(3,0)$

$B=(6,0)$

$C=(x,y)$

Finding the lengths of the sides of the triangle:

$AB=\sqrt{(6-3)^2}=3$

$BC=\sqrt{(x-6)^2+y^2}$

$CA=\sqrt{(x-3)^2+y^2}$

Equating CA and BC:

$(x-6)^2+y^2=(x-3)^2+y^2$

$(x-6)^2=(x-3)^2$

$\Rightarrow x-6=-(x-3)$

$\Rightarrow 2x=9$

$\Rightarrow x=\frac{9}{2}$

Using equation AB=BC and putting value of x:

$$(x-6)^2+y^2=9$$

$$\Rightarrow\left(\frac{9}{2}-6\right)^2+y^2=9$$

$$\Rightarrow\left(-\frac{3}{2}\right)^2-9=-y^2$$

$$\Rightarrow y^2=-\frac{9}{4}+9$$

$$\Rightarrow y^2=\frac{27}{4}$$

$$\Rightarrow y=\pm\frac{3\sqrt{3}}{2}$$

Thus, the values of x and y are $\frac{9}{2}$ and $\pm\frac{3\sqrt{3}}{2}$ respectively.

Or

Find the value of k, if the points P(5, 4), Q(7, k) and R(9, -2) are collinear.
3 Marks

Ans: The given points are collinear, hence taking two of them in pairs each, and finding the slope for them and equating the two slopes will give us the value of k. Thus, slope of PQ

$$\frac{k-4}{7-5}=\frac{k-4}{2}$$

Slope of QR:

$$\frac{-2-k}{9-7}=\frac{-k-2}{2}$$

Comparing both the slopes , we get:

$$\frac{k-4}{2}=\frac{-k-2}{2}$$

$$\Rightarrow 2k=-2+4=2$$

$\Rightarrow$ k=1

SECTION: D

Question Numbers 29 to 34 carry 4 marks each.

29. A motor boat whose speed is 20 km/h in still water, takes 1 hour more to go 48 km upstream than to return downstream to the same spot. Find the speed of the stream. 4 Marks

Ans: Before proceeding with the question, we must know the formulas that will be required to solve this question.

We have a formula, time = distance/speed ....(1). In the question, it is given that the motorboat takes 1 hour more to go 48 km upstream than to return downstream to the same spot. Also, it is given that the speed of the motor boat in still water is equal to 20 km/hr. We are required to find the speed of the stream.

Let us assume the speed of the stream = x km/hr. It is given that the speed of the motor boat in still water is equal to 20 km/hr. While going upstream, the direction of the stream will be against the direction of the motorboat. So, the speed of the motorboat while going upstream will be equal to (20-x) km/hr. It is given that the distance to be travelled is equal to 48 km. So, using formula (1), the time taken by the motorboat to go upstream is equal to

$\frac{48}{20\text{-x}}$..........(2) .

While going downstream, the direction of the stream will be along with the direction of the motorboat. So, the speed of the motorboat while going downstream will be equal to (20+x)km/hr . While going downstream, the direction of the stream will be along with the direction of the motorboat. So, the speed of the motorboat while going downstream will be equal to (20+x) km/hr. It is given that the distance to be travelled is equal to 48 km. So, using formula (1), the time taken by the motorboat to go downstream is equal to

$\frac{48}{20+x}$..........(3) . In the question, it is given that the motor boat takes 1 hour more to go 48 km upstream than to return downstream. So, we can say that,

(Time taken while going downstream) + 1 = Time taken while going upstream

Substituting these times from equation (1) and equation (3), we get, $\frac{48}{20+x}+1=\frac{48}{20-x}$.

$$\Rightarrow \frac{48}{20-x}-\frac{48}{20+x}=1$$

$$\Rightarrow (48)\left(\frac{(20+x)-(20-x)}{(20-x)(20+x)}\right)=1$$

$$\Rightarrow (48)(2x)=(20-x)(20+x)$$

$$\Rightarrow 96x=400+20x-20x-x^2$$

$$\Rightarrow x^2+96x-400=0$$

$$\Rightarrow x^2-4x+100x-400=0$$

$$\Rightarrow x(x-4)+100(x-4)=0$$

$$\Rightarrow (x+100)(x-4)=0$$

$$\Rightarrow x=4, x=(-100)$$

Since speed cannot be negative, hence, the speed of the stream is 4 km/hr.

Or

Find the roots of the equation: $\mathbf{\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}, x^1-4,7}$. **4 Marks**

Ans: By expressing the given equation $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}, x^1-4,7$ in the form of a general quadratic equation we get:

Taking a common denominator $(x+4)(x-7)$ in the LHS,

$$\frac{(x-7)-(x+4)}{(x+4)(x-7)}=\frac{11}{30}$$

$$\frac{-11}{x^2-7x+4x-28}=\frac{11}{30}$$

Cross multiplying the terms to express as a quadratic equation, cancelling out 11 and multiplying with (-1) on both sides, $x^2-3x-28=-30$. Expressing the equation in the simplest form possible, $x^2-3x+2=0$. Comparing the above equation with the general quadratic equation we get the values of a, b, c as, $a=1, b=-3, c=2$. We know the formula to find the roots of a general quadratic equation is given as $\frac{-b\pm\sqrt{D}}{2a}$ where $D=b^2-4ac$ is known as the discriminant. Applying the above formula by substituting the values of a, b, c to find the discriminant, for the roots to be real and unequal the discriminant must be greater than 0. $\left[(-3)^2-4(1)(2)\right]>0$. Squaring −3 and multiplying −4 with 2 we get, $(9-8)>0$. Subtracting 8 from 9 the resultant is greater than 0. $1>0$. Hence the roots are real and unequal. Applying the formula to find the roots of a general quadratic equation by substituting the values of a, b, c and the value of the discriminant b^2-4ac we get, $\frac{-(-3)\pm\sqrt{1}}{2}$. By taking square root of 1 and simplifying the expression for: $\frac{-b+\sqrt{D}}{2a}$ and $\frac{-b-\sqrt{D}}{2a}$ we get, $x=2$.

$x=1$

Hence the roots of the equation $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}$ are 2 and 1.

30. If the sum of first 4 terms of an AP is 40 and that of first 14 terms is 280, find the sum of its first n terms.

Ans: Given that S4 = 40 and S14 =280'

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_n=\frac{n}{2}\left[2a+(4-1)d\right]=40$$

$$\Rightarrow 2a+3d=20.......(i)$$

$$S_{14}=\frac{14}{2}\left[2a+(14-1)d\right]=280$$

$$\Rightarrow 2a+13d=40.....(ii)$$

$$(ii)-(i)$$

$$10d=20\Rightarrow d=2$$

Substituting the value of d in (i) we get

$2a+6=10 \Rightarrow a=7$

$$S_n=\frac{n}{2}\left[2a+\left(n-1\right)d\right]$$

$$=\frac{n}{2}\left[14+\left(n-1\right)2\right]$$

$= n(7+n-1)$

$= n(n+6)$

$=n^2+6n$

Therefore, $S_n = n^2 +6n$

Or

Find the sum of the first 30 positive integers divisible by 6

According to the question series is

6,12,18,24,….

Here,

a=6, d=6

$$S_n=\frac{n}{2}\left[2a+\left(n-1\right)d\right]$$

n = 30

$$S_{30}=\frac{30}{2}\left[2\times6+\left(30-1\right)\times6\right]$$

$$S_{30}=15\left(12+29\times6\right)$$

$$S_{30}=15\times186$$

$$S_{30}=2790$$

31. A train travels 180 km at a uniform speed. If the speed had been 9 km/hour more, it would have take 1 hour less for the same journey. Find the speed of the train.

Ans:

Let the speed of train be x km /h

Distance = 180 km

So, time = 180 / x

When speed is 9 km/h more, time taken = 180 / x+9

According to the given information:

$$\frac{180}{1}-\frac{180}{x+9}=1$$

180 (x+9-x) / x(x+9) = 1

180×9 = x(x+9)

1620 = x2 + 9x

x2+ 9x - 1620 = 0

x2 + 45x - 36x -1620 = 0

x(x+45) - 36(x+45) = 0

(x-36)(x+45) = 0

x=36 or-45

But x being speed cannot be negative.

So, x = 36

Hence, the speed of the train is 36 km/h

Or

Find the roots of the equation $\frac{1}{2x-3}+\frac{1}{x-5}=1;x^{1}\frac{3}{2},5$

The given equation is $\frac{1}{2x-3}+\frac{1}{x-5}=1;x^{1}\frac{3}{2},5$

Taking L.C.M

$$\frac{x-5+(2x-3)}{(x-5)(2x-3)}=1$$

$$\frac{3x-8}{2x^2-13x+15}=1$$

Cross-multiplying

$2x^2-13x + 15 = 3x-8$

$2x^2 - 16x + 23 = 0$

Which is a quadratic equation.

Comparing with

$ax^2 + bx + c = 0$, then $a = 2$, $b = -16$, $c = 23$

So, $D = b^2 - 4ac$

$= (-16)^2 - 4.2.\ 23$

$= 256 - 184$

$= 72 > 0$

Therefore, using quadratic formula

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{6 \pm \sqrt{72}}{2.6}$$

$$= \frac{16 + 6\sqrt{2}}{4}, \frac{16 - 6\sqrt{2}}{4}$$

$$= \frac{8 + 3\sqrt{2}}{2}, \frac{8 - 3\sqrt{2}}{2}$$

So the roots are $= \frac{8 + 3\sqrt{2}}{2}, \frac{8 - 3\sqrt{2}}{2}$

32. In Figure 6, three circles each of radius 3.5 cm are drawn in such a way that each of[It them touches the other two. Find the area enclosed. between these three circles (shaded region). $\left[\textbf{use} = \pi = \frac{22}{7}\right]$

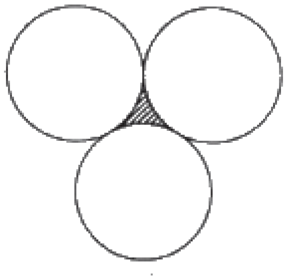


Figure 6

Given that, three circles are in such a way that each of them touches the other two.

Now, we join centre of all three circles to each other by a line segment. Since, radius of each circle is 3.5 cm.

So; AB = 2 × Radius of circle

= 2 × 3.5 = 7 cm.

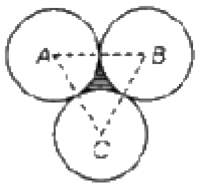
AC = BC = AB = 7cm

which shows that, Δ ABC is an equilateral triangle with side 7 cm.

We know that, each angle between two adjacent sides of an equilateral triangle is

60°

Area of sector with angle $\angle$A = 60°.



$$=\frac{\angle A}{360^\circ}\times\pi r^2=\frac{60^\circ}{360^\circ}\times\pi(3.5)^2$$

So, area of each sector = 3×Area of sector with angle A

$$=3\times\frac{60^\circ}{360^\circ}\times\pi\times(3.5)^2$$

$$=\frac{1}{2}\times\frac{22}{7}\times3.5\times3.5$$

$=11\times\frac{5}{10}\times\frac{35}{10}=\frac{11}{2}\times\frac{7}{2}$

$=\frac{77}{4}=19.25\,cm^2$

And Area of $\Delta ABC=\frac{\sqrt{3}}{4}\times(7)^2$

[area of an equilateral triangle $=\frac{\sqrt{3}}{4}$ (side)^2]

$=49\frac{\sqrt{3}}{4}cm^2$

Area of shaded region enclosed between these circles

=area of Δ ABC - Area of each sector

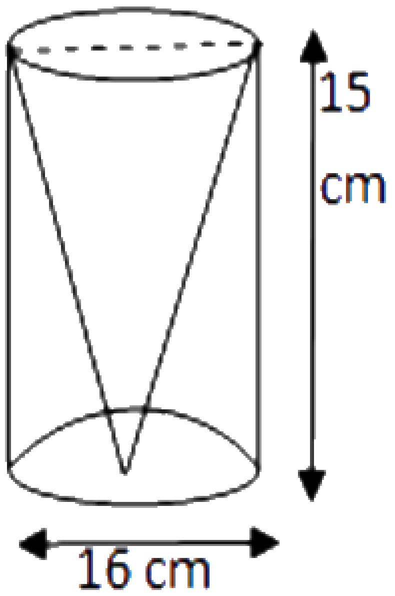
$= 49\frac{\sqrt{3}}{4}$-1925 -1225 × $\sqrt{3}$-1925

= 21.2176 - 1925=1 9676 cm^2?

Hence, the required area enclosed between these circles is 1.967 cm^2? (approx).

33. From a solid cylinder whose height is 15 cm and diameter 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. [Taken=3.14] 4 Marks

Ans:



In this question, we are given a solid cylinder with Height, $h=15cm$ .The diameter of the cylinder is, $d=16cm$. Thus, the radius, $R=8cm$. The slant height of the cone can be found as: $l=\sqrt{h^2+R^2}$. Putting the values above,

$l=\sqrt{(15)^2+(8)^2}=\sqrt{225+64}=\sqrt{289}$. Total surface of the solid is:

$\Rightarrow l=17$

T=Curved surface area of cone + Curved surface area of the cylinder
+Area of the bottom part

Putting the values, we get

$T=\pi Rl+2\pi Rh+\pi R^2$

$\Rightarrow T=3.14[8\times17+2\times8\times15+8\times8]$

$\Rightarrow T=3.14[440]$

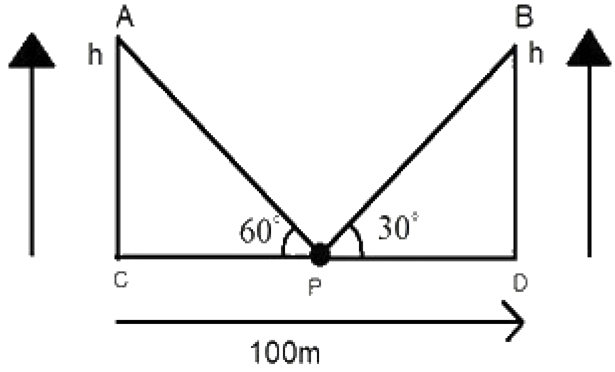
$\Rightarrow T=1381.6cm^2$

Thus, the total surface area of the remaining solid is $1381.6cm^2$.

34. Two poles of equal heights are standing opposite to each other on either side of the road, which is 100 m wide. From a point between them on the

road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles. 4 Marks

Ans: Let us firstly construct a figure reflecting the condition given in the question,



Considering the right-angled triangles, ABP and BPD, let the distance between the points C and P be x. Thus, in triangle ABP, $\tan 60^{\circ}=\frac{h}{x}$.

Putting here, the value of $\tan 60^{\circ}$

$\sqrt{3}=\frac{h}{x}$. Now, in triangle BPD,

$\Rightarrow h=\sqrt{3}x\text{---}(1)$

$\tan 30^{\circ}=\frac{BD}{PD}=\frac{h}{100-x}$. Using (1) in (2)

$\Rightarrow \frac{1}{\sqrt{3}}=\frac{h}{100-x}\text{---}(2)$

$\Rightarrow \frac{1}{\sqrt{3}}=\frac{\sqrt{3}x}{100-x}$

$\Rightarrow 3x=100-x$

$\Rightarrow 4x=100$

$\Rightarrow x=25$

Putting the value of x in (1); We get

$h=\sqrt{3}x=25\sqrt{3}$

The height of the poles is $25\sqrt{3}m$.