Answer any 6 questions from 1 to 8. Each carries 3 scores. **(6 × 3 = 18)**

1. Let R be a relation on a set A = {1, 2, 3, 4, 5, 6} defined as R = {(x, y) : y = 2x – 1}.

   (i) Write R in roster form and find its domain and range. **(2)**

   (ii) Is R is an equivalence relation ? Justify. **(1)**

2. $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that

   $$A^2 - 5A + 7I = O$$

   (Where I is the identity matrix)

   0 0
   0 0

3. (i) Check the continuity of the function $f(x) = 2x + 3$ at $x = 1$. **(1)**

   (ii) Determine the value of k so that the function **(2)**

   $$f(x) = \begin{cases} kx + 1 & \text{if} \quad x \le 5 \\ 3x - 5 & \text{if} \quad x > 5 \end{cases}$$

   1/5

   is continuous at $x = 5$.

4. (i) Find the principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ **(1)**

   (ii) Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ **(2)**

5. (i) Which of the following function is increasing in its domain :

   (A) $\sin x$

   (B) $\cos x$

   (C) $-2x$

   (D) $\log x$ **(1)**

(ii) Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is

(a) increasing (b) decreasing. $-\infty, 2$ dec **(2)**

3.

6. (i) If θ is the angle between two non zero vectors $\vec{a}$ and $\vec{b}$ and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ then $\theta = \underline{\pi/4}$. **(1)**

(ii) Find the projection of the vector $\vec{a} = \hat{i} - \hat{j}$ on the vector $\vec{b} = \hat{i} + \hat{j}$. **(2)**

7. Let A and B are independent events with $P(A) = 0.3$, $P(B) = 0.4$, find

(i) $P(A \cap B)$

(ii) $P(A \cup B)$

(iii) $P(A/B)$

$\sqrt{x} = \frac{4}{2\sqrt{x}}$

8. Evaluate $\int_{-1}^{1} 5x^4 \sqrt{x^5 + 1}\, dx$.

Answer any 6 questions from 9 to 16. Each carries 4 scores. **(6 × 4 = 24)**

9. (i) What is the minimum number of ordered pairs to form a reflexive relation on a set of 4 elements ? **(1)**

(ii) Let $A = R - \{3\}$, $B = R - \{1\}$

Consider the function $f : A \to B$ defined by $f(x) = \frac{x-2}{x-3}$.

Check whether f is one-one and onto. **(3)**

$x_1 = x_2$

$y = \frac{x-2}{x-3}$

10. (i) If A is a skew symmetric matrix then A' = ______. (1)

(ii) Express the matrix $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

as the sum of a symmetric and skew symmetric matrix. (3)

$\frac{1}{2}(A+A') + \frac{1}{2}(A-A')$

11. A wire of length 28 m is cut into two pieces, one of the pieces is to be made into a square and other into a circle. What should be the length of the two pieces so that the combined area of square and circle is minimum ?

12. (i) Write the order and degree of the differential equation

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - \sin\left(\frac{dy}{dx}\right) = 0.$$ (1)

(ii) Find the integrating factor of the differential equation

$$x\frac{dy}{dx} + 2y = x^2.$$ (2)

(iii) Solve the differential equation

$$x\frac{dy}{dx} + 2y = x^2.$$ (1)

13. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\frac{-2i}{\sqrt{24}}$

(i) Find $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$. (1)

(ii) Find a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$. $a \times b$ (3)

$-4/\sqrt{59}$

14. Find the shortest distance between the lines $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$.

$\left|\frac{b_1 \times b_2 \cdot a_2 - a_1}{|b_1 \times b_2|}\right|$

15. In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 45% of the bolt. Of their output 5%, 4% and 2% are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by machine B ?

16. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ using integration.

12π

Answer any 3 questions from 17 to 20. Each carries 6 scores. **(3 × 6 = 18)**

17. Solve the following system of equations by matrix method :

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

18. (i) $\sin x + \cos y = xy$ find $\frac{dy}{dx}$. **(2)**

$\frac{\cos x - y}{\sin y + x}$.

(ii) $x = a\cos^3 t$; $y = a\sin^3 t$ find $\frac{dy}{dx}$. **(2)**

(iii) If $y = (\sin^{-1} x)^2$ then show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$. **(2)**

19. (i) Find $\int \frac{x-1}{(x-2)(x-3)}\,dx$. **(3)**

(ii) Prove that $\int_0^{-\frac{\pi}{4}} \log(1 + \tan x)\,dx = \frac{\pi}{8}\log 2$. **(3)**

~~20.~~ 11. Solve the following linear programming problem graphically :

Maximise $Z = 60x + 15y$

Subject to the constraints

$x + y \leq 50$

$3x + y \leq 90$ 1800

$x \geq 0, y \geq 0$