Unofficial ISC Class 12 Mathematics Answer Key 2023 (Short Answer Questions Only)

Q1

Answers: -

I. a
II. c
III. c
IV. b
V. c
VI. d
VII. b
VIII. d
IX. a
X. d
XI.
XI) $\frac{dy}{dx} = \operatorname{cosec} y$

$\Rightarrow \frac{dy}{\operatorname{cosec} y} = dx$

$\Rightarrow \int \sin y \, dy = \int dx$

$\Rightarrow -\cos y = x + c$

XII. K = 6

XIII.
XIII) Ans 2

$\int_0^1 (2x+1)\, dx$

$\left| x^2 + x \right|_0^1$

$= 2$

XIV.
XIV) $\int \frac{1+\cos x}{\sin^2 x} dx$

$\Rightarrow \int (\operatorname{cosec}^2 x + \cot x \operatorname{cosec} x) dx$

$\Rightarrow -\cot x - \operatorname{cosec} x + c$

XV.

xv) 1, 2, 3, 4 19

There are 9 even no's

$$P = \frac{9}{19} \times \frac{9}{19} = \frac{81}{361}$$

Q2

Answers: -

I.

Q2) i) $f(x) = [4-(x-7)^3]^{1/5}$

$$y = [4-(x-7)^3]^{1/5}$$

$$y^5 = 4-(x-7)^3$$

$$(x-7)^3 = 4-y^5$$

$$x-7 = \sqrt[3]{4-y^5}$$

$$x = \sqrt[3]{4-y^5} + 7$$

II.

2 ii) One – one

$$f(x_1) = f(x_2)$$

$$\frac{x_1-1}{x_1-2} = \frac{x_2-1}{x_2-2}$$

$$(x_1-1)(x_2-2) = (x_2-1)(x_1-2)$$

$$\cancel{x_1x_2} - 2x_1 - x_2 + \cancel{2} = \cancel{x_1x_2} - 2x_2 - x_1 + \cancel{2}$$

$$\Rightarrow x_1 = x_2$$

Q3

Answer: -

Q3) $\begin{vmatrix} 5 & 5 & 5 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix}$

$$\xrightarrow{R_2' = R_2 + R_3} \begin{vmatrix} 5 & 5 & 5 \\ a+b+c & a+b+c & a+b+c \\ b+c & c+a & a+b \end{vmatrix}$$

$$5(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ b+c & c+a & a+b \end{vmatrix}$$

$= 0$ [Two rows same]

Q4

Answer: -

Q4) $P(A) = 1/3 \quad P(B) = 1/2$

$P(A \cap B) = 1/3 \times 1/2 = 1/6$

$P(A \cup B) = 1/2 + 1/3 - 1/6$

$P(A \cup B) = \frac{3+2-1}{6} = 4/6 = 2/3$

$P(A \cup B)' = 1 - 2/3 = 1/3$ Ans

Q5

Answer: -

Q5) $5\tan^{-1}x + 3\cot^{-1}x = 2\pi$

$3\tan^{-1}x + 3\cot^{-1}x + 2\tan^{-1}x = 2\pi$

$3(\tan^{-1}x + \cot^{-1}x) + 2\tan^{-1}x = 2\pi$

$3 \times \pi/2 + 2\tan^{-1}x = 2\pi$

$2\tan^{-1}x = 2\pi - 3\pi/2 = \pi/2$

$\tan^{-1}x = \pi/4$

$x = 1$

Q6

Answers: -

i)

Q6) i) $\int \cos^{-1}(\sin x)\,dx$

$\int \cos^{-1}(\cos(\pi/2 - x))\,dx$

$\int (\pi/2 - x)\,dx$

$\frac{\pi}{2}x - x^2/2 + C$

ii)

ii) $\int x^5 \cos(x^6)\,dx = k \sin x^6 + C$

$\int x^5 \cos(x^6)\,dx$ Let $x^6 = z$

$6x^5\,dx = dz$

$x^5\,dx = \frac{dz}{6}$

$\int \cos z \frac{dz}{6}$

$\frac{\sin z}{6} + C$

$\frac{1}{6} \sin x^6 + C$

$\therefore k = 1/6$ (Ans)

Q7

Answer: -

Q7)

If $\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\left(\frac{23}{36}\right)$

$\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\left(\frac{23}{36}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\frac{x-1}{x+1} + \frac{2x-1}{2x+1}}{1 - \frac{x-1}{x+1} \times \frac{2x-1}{2x+1}}\right) = \tan^{-1}\left(\frac{23}{36}\right)$

$\Rightarrow \frac{(x-1)(2x+1) + (2x-1)(x+1)}{(x+1)(2x+1) - (x-1)(2x-1)} = \frac{23}{36}$

$\Rightarrow \frac{4x^2 - 2}{6x} = \frac{23}{36}$

$\Rightarrow 24x^2 - 12 = 23x$

$\Rightarrow 24x^2 - 23x - 12 = 0$

Q8

Answer: -

Q8) $y = e^{ax}\cos bx$

$$\frac{dy}{dx} = e^{ax}(-\sin bx)b + \cos bx \cdot e^{ax} \cdot a$$

$$\frac{dy}{dx} = e^{ax}(a\cos bx - b\sin bx)$$

$$\frac{d^2y}{dx^2} = e^{ax}(-a\sin bx \cdot b - b\cos bx \cdot b) + (a\cos bx - b\sin bx)e^{ax} \cdot a$$

$$\frac{d^2y}{dx^2} = e^{ax}\left((a^2-b^2)\cos bx - 2ab\sin bx\right)$$

$$\therefore \frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2+b^2)y$$

$$= e^{ax}((a^2-b^2)(\cos bx - 2ab\sin bx)$$

$$- 2ae^{ax}(a\cos bx - b\sin bx) + (a^2+b^2)e^{ax}\cos bx$$

$$= e^{ax}[(a^2-b^2-2a^2+a^2+b^2)\cos bx + (2ab-2ab)\sin bx]$$

$= 0$ proved.

Q9

Answer: -

i) Graduate (15%) $\longrightarrow$ 80% (Adm)

Non Graduate (85%) $\longrightarrow$ 10% (Adm)

$$P = \frac{15/100 \times 80/100}{15/100 \times 80/100 + 85/100 \times 10/100} = \frac{24}{41} \text{ Ans}$$

ii) $P(A) = 1/2$ $\quad P(\bar{A}) = 1 - 1/2 = 1/2$

$P(B) = 1/3$ $\quad P(\bar{B}) = 1 - 1/3 = 2/3$

$P(C) = 1/4$ $\quad P(\bar{C}) = 1 - 1/4 = 3/4$

i) $P(\text{exactly two solve}) = P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C)$

$$= 1/2 \times 1/3 \times 3/4 + 1/2 \times 2/3 \times 1/4 + 1/2 \times 1/3 \times 1/4$$

$$= 1/8 + 1/12 + 1/24 = \frac{3+2+1}{24} = 6/24 = 1/4$$

ii) $P(\text{atleast two}) = P(A \cap B \cap C) + P(\text{exactly two solve})$

$$= 1/2 \times 1/3 \times 1/4 + 1/4$$

Q10

Answer: -

i)

Q10 i) $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

$$\frac{dx}{dy} = \frac{\tan^{-1}y}{1+y^2} - \frac{x}{1+y^2}$$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

$$I.F = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

Soln

$$e^{\tan^{-1}y}(x) = \int \frac{e^{\tan^{-1}y}\cdot \tan^{-1}y}{1+y^2}\,dy$$

Let $e^{\tan^{-1}y} = z$

$$e^{\tan^{-1}y} \times \frac{1}{1+y^2}\,dy = dz$$

$$xe^{\tan^{-1}y} = \int \log z\,dz$$

$$xe^{\tan^{-1}y} = (z\log z - z) + C$$

$$xe^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y) - e^{\tan^{-1}y} + C$$

ii)

ii) $(x^2-y^2)\,dx + 2xy\,dy = 0$

$$\frac{dy}{dx} = \frac{y^2-x^2}{2xy}$$

Let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2x\,vx}$$

$$v + x\frac{dv}{dx} = \frac{v^2-1}{2v}$$

$$x\frac{dv}{dx} = \frac{v^2-1}{2v} - v = \frac{v^2-1-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{-1-v^2}{2v}$$

$$\frac{2v}{1+v^2}\,dv = -\frac{dx}{x}$$

$$\log|1+v^2| = -\log x + C$$

Q11

Answers: -

Q11)
$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$
$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$
$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

$2a + 3b + 10c = 4$
$4a - 6b + 5c = 1$
$6a + 9b - 20c = 2$

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$$

$$X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$|A| = 1200$

$$\text{Adj } A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$a = 1/2 \quad b = 1/3 \quad c = 1/5$

$\therefore x = 2 \quad y = 3 \quad z = 5$

Q12

Answers: -

12 i)

$l^2 = h^2 + r^2$

$V = \frac{1}{3}\pi r^2 h$

$h = \frac{3V}{\pi r^2}$

α = semi vertical angle

$S = \pi r l$

$S^2 = \pi^2 r^2 l^2$

$S^2 = \pi^2 r^2 (h^2 + r^2)$

$S^2 = \pi^2 r^2\left(\left(\frac{3V}{\pi r^2}\right)^2 + r^2\right)$

$S^2 = \frac{9V^2}{r^2} + \pi^2 r^4$

$f(r) = \frac{9V^2}{r^2} + \pi^2 r^4$

$f'(r) = 9V^2\left(\frac{-2}{r^3}\right) + \pi^2 \cdot 4r^3$

$f''(r) = -18V^2\left(\frac{-3}{r^4}\right) + 4\pi^2 \cdot 3r^2$

$f''(r) = \frac{54V^2}{r^4} + 12\pi^2 r^2$

Now $f'(r) = 0 \Rightarrow \frac{-18V^2}{r^3} + 4\pi^2 r^3 = 0$

$r^6 = \frac{9V^2}{2\pi^2}$

Also $f''(r) > 0$ for every +ve value of r

S^2 is least

we set $V^2 = \frac{2\pi^2}{9} r^6$

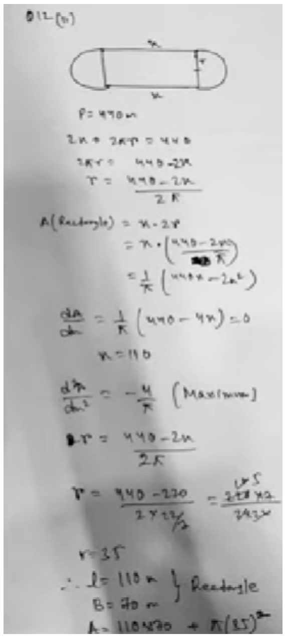
$V^2 = \frac{1}{9}\pi^2 r^4 h^2$

$\frac{2\pi^2}{9} r^6 = \frac{1}{9}\pi^2 r^4 h^2 \Rightarrow 2r^2 = h^2 \Rightarrow h = \sqrt{2}r$

$\cot\alpha = \frac{h}{r} = \frac{\sqrt{2}r}{r} = \sqrt{2}$

$\alpha = \cot^{-1}\sqrt{2}$

i)

ii) 

Q12 (ii)

$P = 440\,m$

$2x + 2\pi r = 440$

$2\pi r = 440 - 2x$

$r = \frac{440 - 2x}{2\pi}$

$A(\text{Rectangle}) = x \cdot 2r$

$= x \cdot \left(\frac{440 - 2x}{\pi}\right)$

$= \frac{1}{\pi}\left(440x - 2x^2\right)$

$\frac{dA}{dx} = \frac{1}{\pi}\left(440 - 4x\right) = 0$

$x = 110$

$\frac{d^2A}{dx^2} = -\frac{4}{\pi}$ (Maximum)

$r = \frac{440 - 2x}{2\pi}$

$r = \frac{440 - 220}{2 \times 22/7} = \frac{220 \times 7}{22 \times 2}$

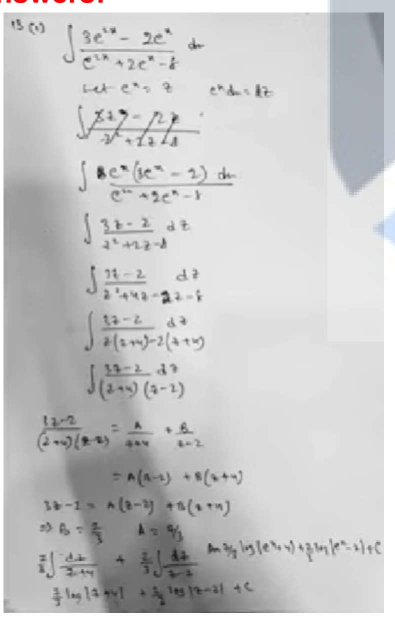
$r = 35$

$\therefore\ l = 110\,m$, $B = 70\,m$ } Rectangle

$A = 110 \times 70 + \pi(35)^2$

Q13

Answers: -

i) 

13 (i) $\int \frac{3e^{2x} - 2e^x}{e^{2x} + 2e^x - 8}\,dx$

Let $e^x = z$ $\quad e^x dx = dz$

$\int \frac{e^x(3e^x - 2)\,dx}{e^{2x} + 2e^x - 8}$

$\int \frac{3z - 2}{z^2 + 2z - 8}\,dz$

$\int \frac{3z - 2}{z^2 + 4z - 2z - 8}\,dz$

$\int \frac{3z - 2}{z(z+4) - 2(z+4)}\,dz$

$\int \frac{3z - 2}{(z+4)(z-2)}\,dz$

$\frac{3z-2}{(z+4)(z-2)} = \frac{A}{z+4} + \frac{B}{z-2}$

$= A(z-2) + B(z+4)$

$3z - 2 = A(z-2) + B(z+4)$

$\Rightarrow B = \frac{2}{3}$ $\quad A = \frac{7}{3}$

$\frac{7}{3}\int \frac{dz}{z+4} + \frac{2}{3}\int \frac{dz}{z-2}$

Ans $\frac{7}{3}\log|e^x + 4| + \frac{2}{3}\log|e^x - 2| + C$

$\frac{7}{3}\log|z+4| + \frac{2}{3}\log|z-2| + C$

13) ii) $\int \frac{2}{(1-x)(1+x^2)}\,dx$

$$\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$$

$$2 = A(1+x^2) + (Bx+C)(1-x)$$

$A=1,\ B=1;\ C=1$

$$\int \left(\frac{1}{1-x}\right)dx + \int \left(\frac{x+1}{1+x^2}\right)dx$$

$$= \log|1-x| + \frac{\log|1+x^2|}{2} + \tan^{-1}x + C$$

ii)

Q14

14/ Total = 30

Rotten 10

unspoiled = 20

Two fruits can be drawn in $^{30}C_2$

$$P(X=0) = \frac{^{10}C_2}{^{30}C_2}$$

$$P(X=1) = \frac{^{10}C_1 \times {}^{20}C_1}{^{30}C_2}$$

$$P(X=2) = \frac{^{20}C_2}{^{30}C_2}$$

Mean = $\Sigma P_i x_i$

x_i	P_i
0	$\rightarrow {}^{10}C_2 / {}^{30}C_2$
1	$\rightarrow \frac{^{10}C_1 \times {}^{20}C_1}{^{30}C_2}$
2	$\rightarrow \frac{^{20}C_2}{^{30}C_2}$

Calculate $P_i x_i$ and then mean

Answers: -