INDIAN MARITIME UNIVERSITY
(A Central University, Government of India)

B.Tech. (Marine Engineering)
Semester – I – December 2015 End Semester Examinations

Mathematics - I
Subject Code: UG11T2102/ UG11T1102

Time: 3 Hours — Max Marks: 100
Date: 12.12.2015 — Pass Marks: 50

Part - A (10 X 3 Marks = 30 marks)
Compulsory Question

1(a) If $f(x) = x^3 + 8x^2 + 15x - 24$, calculate the value of $f(11/10)$ by the application of Taylor's series.

(b) Find the asymptote of the spiral $r = \frac{a}{\theta}$.

(c) If $u = \frac{x-y}{1+xy}, v = \tan^{-1} x - \tan^{-1} y$, then verify that they are functionally dependent and if so find it.

(d) Find the saddle point of the function $x^3 + y^3 - 3x - 12y + 10$.

(e) Show that the value of $\Gamma(1/2) = 1.772$

(f) Find the equation of the tangent plane to the surface $2x^2 + y^2 + 2z - 3$ at (2,2,-3).

(g) The orthonormal vector triad (I, J, K) is self reciprocal where I, J and K are unit vectors along x, y and z axis respectively.

(h) If (a,b,c) and $(a^/, b^/, c^/)$ are reciprocal triads of vectors, show that $[abc][a^/ b^/ c^/] = 1$.

(i) Using Cayley Hamilton theorem, find A^8 if A= $\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$.

(j) Evaluate $\int_C \frac{z-3}{z^2+2z+5} dz$, where c is the circle |z| =1

Part - B (5×14 Marks=70Marks)

Answer **any five** of the following

2. (a) Show that the n th derivative of $\frac{\log x}{x}$ is

$$\frac{(-1)^n n!}{x^{n+1}}[\log x - 1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots - \frac{1}{n}]$$ (7)

(b) If ρ_1 and ρ_2 be the radii of curvature at the ends of a focal chord of the parabola $y^2 = 4ax$, then show that $\rho_1^{-2/3} + \rho_2^{-2/3} = (2a)^{-2/3}$. (7)

3(a) If $u = x\phi(y/x) + \psi(y/x)$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0 .$$ (7)

(b) Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum. (7)

4(a) Trace the curve $a^2 y^2 = x^2 (a^2 - x^2)$. (6)

(b) Find the volume of the solid formed by revolving about x-axis, the area enclosed by the parabola $Y^2 = 4ax$, its evolute $27ay^2 = 4(x-2a)^3$ and the x-axis. (8)

5(a) Prove that the shortest distance between the two points in a plane is a straight line. (7)

(b) Find the curves on which the functional $\int_0^{\Pi/2} (y'^2 - y^2 + 2xy)dy$ with y(0)=0 and $Y(\Pi/2) = 0$ can be extremized. (7)

6(a) Show that the vector $\vec{F} = (2x - yz)I + (2y - zx)J + (2z - xy)K$ is irrotational. For this F , find a scalar function ϕ, such that $F = grad\phi$. (7)

(b) Find div $\vec{F}$ and curl $\vec{F}$, where $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$ (7)

7(a) Reduce the following matrix into its normal form and hence find its rank. (8)

$$\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

(b) Given that $A = \begin{pmatrix} 0 & 1+2i \\ -1+2i & 0 \end{pmatrix}$, show that $(I - A)(I + A)^{-1}$ is a unitary matrix, Where I is the unit matrix. (6)

8(a) Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2 (z+2)}$ and the residue at each pole. Hence evaluate $\int_C f(z)dz$, where C is the circle $|z| = 2.5$ (7)

.(b) Evaluate $\int_c \tan z dz$, where C is the circle $|z| = 2$ (7)

****