Part 2

1. DIFFERENTIATION

I. MCQ (2 Marks each)

1) If y = sec ($\tan^{-1} x$) then $\frac{dy}{dx}$ at $x = 1$ is ______

(a) $\frac{1}{2}$ (b) 1 (c) $1/\sqrt{2}$ (d) $\sqrt{2}$

2) If f(x) = $\log_x (\log x)$ then $f'(e)$ is ______

(a)1 (b) e (c) $\frac{1}{e}$ (d) 0

3) If $y = 25^{\log_5 \sin x} + 16^{\log_4 \cos x}$ then $\frac{dy}{dx} =$ ____

(a)1 (b) 0 (c) 9 (d) cos x – sin x

4) If $f'(4) = 5, f(4) = 3, g'(6) = 7$ and R(x) = g[3 + f(x)] then $R'(4) =$ ______

(a)35 (b) 12 (c) $7/5$ (d) 105

5) If $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, $x \in (-1,1)$ then $\frac{dy}{dx} =$ ______ .

(a) $\frac{-2}{1+x^2}$ (b) 1 (c) $\frac{2}{1+x^2}$ (d) $\frac{1}{1+x^2}$

6) If g is the inverse of f and $f'(x) = \frac{1}{1+x^4}$ then $g'(x) =$ ______

(a) $\frac{1}{1+[g(x)]^4}$ (b) $\frac{4x^3}{1+x^4}$ (c) $\frac{1}{1+[g(x)]^3}$ (d) $1+[g(x)]^4$

7) If $\sin^{-1}(x^3 + y^3) = a$ then $\frac{dy}{dx} =$ ______

(a) $\frac{-x}{\cos a}$ (b) $\frac{-x^2}{y^2}$ (c) y^2/x^2 (d) $\frac{\sin a}{y}$

8) If x = $\cos^{-1}(t)$, $y = \sqrt{1-t^2}$ then $\frac{dy}{dx} =$ ______

(a) t (b) – t (c) $-1/t$ (d) $1/t$

9) If $x^2 + y^2 = 1$ then $\frac{d^2x}{dy^2} =$ ______ .

(a) x^3 (b) y^3 (c) $-y^3$ (d) $-1/x^3$

10) If $x^2 + y^2 = t + 1/t$ and $x^4 + y^4 = t^2 + 1/t^2$ then $\frac{dy}{dx} =$ ______

(a) $\frac{x}{2y}$ (b) $-y/x$ (c) $-x/2y$ (d) y/x

11) If $x = a t^4$ $y = 2a t^2$ then $\frac{dy}{dx} =$ ______

(a) $1/t$ (b) $-1/t$ (c) $1/t^2$ (d) $-1/t^2$

II. Very Short answer questions (1 mark each)

1) Differentiate $y = \sqrt{x^2 + 5}$ w.r. to x
2) Differentiate $y = e^{\tan x}$ w.r. to x
3) If $y = \sin^{-1}(2^x)$, find $\frac{dy}{dx}$.
4) If f(x) is odd and differentiable, then $f'(x)$ is
5) If $y = e^{1+\log x}$ then find $\frac{dy}{dx}$
6) Find the nth order derivative of the function$y = log\ x$
7) Find the nth order derivative of the function$y = cos\ x$
8) Find the nth order derivative of the function$y = \frac{1}{x}$

III. Short answer questions (2 mark each)

1) If $y = \log[\cos(x^5)]$ then find $\frac{dy}{dx}$
2) If $y = \sqrt{\tan\sqrt{x}}$, find $\frac{dy}{dx}$
3) Find the derivative of the inverse of function $y = 2x^3 - 6x$ and calculate its value at x = - 2
4) Let $f(x) = x^5 + 2x - 3$ find $(f^{-1})'(-3)$
5) If $y = \cos^{-1}[\sin(4^x)]$, find $\frac{dy}{dx}$

6) If $y=\tan^{-1}\left(\sqrt{\frac{1+\cos x}{1-\cos x}}\right)$, find $\frac{dy}{dx}$

7) If x= $\sin\theta$, y = $\tan\,\theta$ then find $\frac{dy}{dx}$

8) Differentiate $\sin^2(\sin^{-1}(x^2))$ w.r. to x

IV. Short answer questions (3 mark each)

1) If $y=\log\left[\sqrt{\frac{1-\cos\left(3x/2\right)}{1+\cos\left(3x/2\right)}}\right]$, find $\frac{dy}{dx}$

2) If $y=\log\left[4^{2x}\left(\frac{x^2+5}{\sqrt{2x^3-4}}\right)^{3/2}\right]$, find $\frac{dy}{dx}$

3) Differentiate $\cot^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ w.r. to x

4) Differentiate $\sin^{-1}\left(\frac{2\cos x+3\sin x}{\sqrt{13}}\right)$ w.r. to x

5) Differentiate $\tan^{-1}\left(\frac{8x}{1-15x^2}\right)$ w.r. to x

6) If $\log_5\left(\frac{x^4+y^4}{x^4-y^4}\right)=2$, show that $\frac{dy}{dx}=\frac{12x^3}{13y^2}$

7) If $y=\sqrt{\cos x+\sqrt{\cos x+\sqrt{\cos x+.....\infty}}}$, show that $\frac{dy}{dx}=\frac{\sin x}{1-2y}$

8) Find the derivative of $\cos^{-1}x$ w.r. to $\sqrt{1-x^2}$

9) If x sin(a + y) + sin a cos (a + y) = 0 then show that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

10) If $y = 5^x \cdot x^5 \cdot x^x \cdot 5^5$, find $\frac{dy}{dx}$

V. Long answer questions (4 mark each)

1) If $y=e^{m\tan^{-1}x}$, show that $(1+x^2)\frac{d^2y}{dx^2}+(2x-m)\frac{dy}{dx}=0$

2) If $x^7 \cdot y^5 = (x+y)^{12}$, show that $\frac{dy}{dx} = \frac{y}{x}$

3) Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r. to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

4) If $y = \sin^{-1}\left(\frac{a\cos x - b\sin x}{\sqrt{a^2+b^2}}\right)$ then find $\frac{dy}{dx}$

5) If $y = \cos(m\cos^{-1} x)$ then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2 y = 0$

6) Find the nth order derivative of the function $y = \sin(ax+b)$

7) Find the nth order derivative of the function $y = \log(ax+b)$

8) Find the nth order derivative of the function $y = \frac{1}{3x-5}$

:: Theorems ::

1) If y = f(u) is a differentiable function of u and u = g(x) is a differentiable function of x such that the composite function y = f[g(x)] is a differentiable function of x then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ Hence find $\frac{dy}{dx}$ if $y = \sin^2 x$

2) Suppose y = f(x) is a differentiable function of x on an interval I and y is one-one, onto and $\frac{dy}{dx} \neq 0$ on I. Also if $f'(y)$ is differentiable on f(I) then $\frac{dx}{dy} = \frac{1}{dy/dx}$, $dy/dx \neq 0$

3) If x = f(t) and y = g(t) are differentiable functions of t so that y is a differentiable function of x and it $dx/dt \neq 0$ then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$.