मोशन है, तो भरोसा है

MOTION
18 YEARS OF LEGACY

JEE ADVANCED 2025
QUESTION PAPER
WITH SOLUTIONS

MATHS [PAPER – 2]

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

1. Let x_0 be the real number such that $e^{x_0} + x_0 = 0$. For a given real number α, define

$$g(x) = \frac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x + 1)}$$

for all real numbers x.

Then which one of the following statements is TRUE?

(A) For $\alpha = 2$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

(B) For $\alpha = 2$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 1$

(C) For $\alpha = 3$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

(D) For $\alpha = 3$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = \frac{2}{3}$

Sol. **C**

$e^{x_0} + x_0 = 0$

$$g(x) = \frac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x + 1)}$$

(A) $\alpha = 2$

$$g(x) = \frac{3xe^x + 3x - 2e^x - 2x}{3(e^x + 1)} = g(x) = \frac{3xe^x - 2e^x + x}{3(e^x + 1)}$$

$$\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right|$$

$$\Rightarrow \lim_{x \to x_0} \left| \frac{\frac{3xe^x - 2e^x + x}{3(e^x + 1)} + e^{x_0}}{x - x_0} \right|$$

$$\Rightarrow \lim_{x \to x_0} \left| \frac{3xe^x - 2e^x + x + 3e^x \cdot e^{x_0} + 3e^{x_0}}{3(e^x + 1)(x - x_0)} \right|$$

$$\Rightarrow \lim_{x \to x_0} \left| \frac{3xe^x - 2e^x + x + 3e^x(-x_0) + 2e^{x_0} - x_0}{3(e^x + 1)(x - x_0)} \right|$$

$$\Rightarrow \lim_{x \to x_0} \left| \frac{3e^x(x - x_0) - 2(e^x - e^{x_0}) + (x - x_0)}{3(e^x + 1)(x - x_0)} \right|$$

$$\Rightarrow \lim_{x \to x_0} \left| \frac{(3e^x + 1)(x - x_0)}{3(x - x_0)(e^x + 1)} - \frac{2e^x(e^{x - x_0} - 1)}{3(e^x + 1)(x - x_0)} \right|$$

$$\lim_{x\to x_0}\left|\frac{3e^x+1}{3\left(e^x+1\right)}-\frac{2e^{x_0}}{3\left(e^x+1\right)}\right|$$

$$\Rightarrow\left|\frac{3e^{x_0}+1-2e^{x_0}}{3\left(e^{x_0}+1\right)}\right|$$

$$\Rightarrow\left|\frac{e^{x_0}+1}{3\left(e^{x_0}+1\right)}\right|=1/3$$

(C) $\alpha = 3$

$$g(x)=\frac{3xe^x+3x-3e^x-3x}{3\left(e^x+1\right)}$$

$$=\frac{3e^x(x-1)}{3\left(e^x+1\right)}=\frac{e^x(x-1)}{\left(e^x+1\right)}$$

$$\lim_{x\to x_0}\left|\frac{g(x)+e^{x_0}}{x-x_0}\right|$$

$$\Rightarrow\lim_{x\to x_0}\left|\frac{\frac{e^x(x-1)}{e^x+1}+e^{x_0}}{x-x_0}\right|$$

$$\Rightarrow\lim_{x\to x_0}\left|\frac{xe^x-e^x+e^{x_0}\cdot e^x+e^{x_0}}{\left(x-x_0\right)\left(e^x+1\right)}\right|$$

$$\Rightarrow\lim_{x\to x_0}\left|\frac{xe^x-e^x-x_0e^x+e^{x_0}}{\left(x-x_0\right)\left(e^x+1\right)}\right|$$

$$\Rightarrow\lim_{x\to x_0}\left|\frac{e^x\left(x-x_0\right)+e^{x_0}-e^x}{\left(x-x_0\right)\left(e^x+1\right)}\right|$$

$$\Rightarrow\lim_{x\to x_0}\left|\frac{e^x\left(x-x_0\right)}{\left(x-x_0\right)\left(e^x+1\right)}-\frac{e^{x_0}\left(e^{x-x_0}-1\right)}{\left(x-x_0\right)\left(e^x+1\right)}\right|$$

$$\Rightarrow\lim_{x\to x_0}\left|\frac{e^x}{e^x+1}-\frac{e^{x_0}}{e^x+1}\right|$$

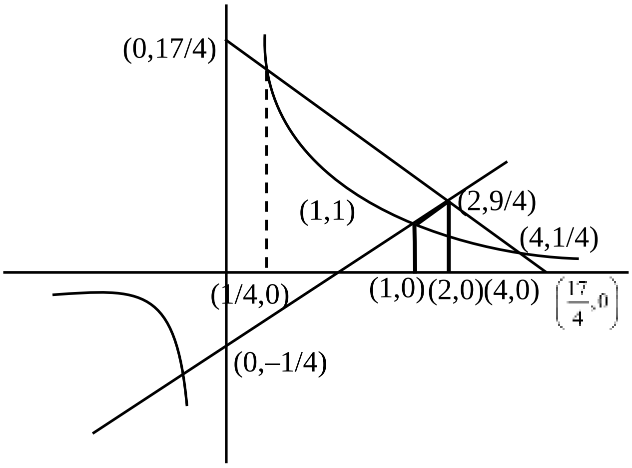
$$\Rightarrow\lim_{x\to x_0}\left|\frac{e^x-e^{x_0}}{e^x+1}\right|=0$$

2. Let $\mathbb{R}$ denote the set of all real numbers. Then the area of the region

$$\left\{(x,y)\in\mathbb{R}\times\mathbb{R}:x>0,y>\frac{1}{x},5x-4y-1>0,4x+4y-17<0\right\}\text{ is}$$

(A) $\frac{17}{16}-\log_e 4$ (B) $\frac{33}{8}-\log_e 4$ (C) $\frac{57}{8}-\log_e 4$ (D) $\frac{17}{2}-\log_e 4$

Sol. **B**



$$A=\left[\frac{1}{2}\left[1+\frac{9}{4}\right](1)-\int_1^2\frac{1}{x}dx\right]$$

$$+\frac{1}{2}\left[\frac{1}{4}+\frac{9}{4}\right][2]-\int_2^4\frac{1}{x}dx$$

$$=\frac{13}{8}+\frac{10}{4}-(\ln\alpha)_1^4$$

$$=\frac{13+20}{8}-(\ln 4)$$

$$=\frac{33}{8}-\ln 4$$

3. The total number of real solutions of the equation

$\theta=\tan^{-1}(2\tan\theta)-\frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$ is

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\tan^{-1}x$ assume values in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, respectively.)

(A) 1 (B) 2 (C) 3 (D) 5

Sol. **C**

$$\theta=\tan^{-1}(2\tan\theta)-\frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$$

$$\theta=\tan^{-1}x \quad \theta\in(-\pi/2,\pi/2)$$

$$\tan^{-1}x-\tan^{-1}2x=-1/2\sin^{-1}\left(\frac{6x}{9+x^2}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{2x-x}{1+2x^2}\right)=\frac{1}{2}\sin^{-1}\left(\frac{6x}{9+x^2}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{x}{1+2x^2}\right)=\frac{1}{2}\sin^{-1}\left(\frac{6x}{9+x^2}\right)$$

$$2\tan^{-1}\left(\frac{x}{1+2x^2}\right)=\sin^{-1}\left(\frac{6x}{9+x^2}\right)$$

taking sine both side

$$\frac{2\left(\frac{x}{1+2x^2}\right)}{1+\left(\frac{x}{1+2x^2}\right)^2}=\frac{6x}{9+x^2}$$

$$\frac{2x\left(1+2x^2\right)}{\left(1+2x^2\right)^2+x^2}=\frac{6x}{9+x^2}$$

$x = 0$ or $(9 + x^2)(1 + 2x^2) = 3(4x^4 + 5x^2 + 1)$

$2x^4 + 19x^2 + 9 = 12x^4 + 15x^2 + 3$

$10x^4 - 4x^2 - 6 = 0$

$5x^4 - 2x^2 - 3 = 0$

$(5x^2 + 3)(x^2 - 1) = 0$

$x = 1$ or -1

by cross checking

no of solutions is 3

4. Let S denote the locus of the point of intersection of the pair of lines

$4x - 3y = 12\alpha$,

$4\alpha x + 3\alpha y = 12$

where α varies over the set of non-zero real numbers. Let T be the tangent to S passing through the points (p, 0) and (0, q), $q > 0$, and parallel to the line $4x-\frac{3}{\sqrt{2}}y=0$.

Then the value of pq is

(A) $-6\sqrt{2}$ (B) $-3\sqrt{2}$ (C) $-9\sqrt{2}$ (D) $-12\sqrt{2}$

Sol. **A**

$$4x-3y=12\alpha\}\times 3\alpha$$

$$4\alpha x+3\alpha y=12\}\times 3$$

$$12\alpha x-5\alpha y=36\alpha^2$$

$$12\alpha x+9\alpha y=36$$

$$24\alpha x=36\left(\alpha^2+1\right)$$

$$x=\frac{3}{2}(\alpha+1/\alpha) \qquad \ldots\ldots(1)$$

Now

$$4\left[\frac{3}{2}(\alpha+1/\alpha)\right]-3y=12\alpha$$

$$6\alpha+6/\alpha-3y=12\alpha$$

$$3y=6/\alpha-6\alpha$$

$$y=\frac{6[1/\alpha-\alpha]}{3}$$

$$y=2\left[\frac{1}{\alpha}-\alpha\right] \qquad \ldots\ldots(2)$$

from (1)

$$\frac{2x}{3} = \alpha + \frac{1}{\alpha} \qquad ...(3)$$

$$\frac{y}{2} = \frac{1}{\alpha} - \alpha \qquad ...(4)$$

$(3)^2 - (4)^2$

$$\frac{4}{9}x^2 - \frac{y^2}{4} = 4$$

$\frac{x^2}{9} - \frac{y^2}{16} = 1$ (Hyperbola)

point (3 secθ, 4 tanθ)

Equation to tangent

$$\frac{x(\sec\theta)}{3} - \frac{y\tan\theta}{4} = 1$$

$||^{rl}$ to line

$$y = \frac{4x}{3}(\sqrt{2})$$

$$y = \frac{4\sqrt{2}}{3}x$$

passes through (p, 0), (0, q)

$$p\frac{\sec\theta}{3} = 1 \Rightarrow \sec\theta = 3/p$$

$$-\frac{2\tan\theta}{4} = 1 \Rightarrow \tan\theta = -\frac{4}{2}$$

$$-\left[\frac{\frac{\sec\theta}{3}}{-\frac{\tan\theta}{4}}\right] = \frac{4\sqrt{2}}{3}$$

$$\frac{4}{3}\cdot\frac{1}{\sin\theta} = \frac{4\sqrt{2}}{3}$$

$$\Rightarrow \quad \sin 0 = 1/\sqrt{2}$$

$$\theta = \pi/4; \qquad \frac{3\pi}{4}$$

$$\theta = \frac{\pi}{4} \qquad \sqrt{2} = 3/p \quad \therefore P = 3/\sqrt{2}$$

$$1 = \frac{-4}{q} \Rightarrow q = -4$$

$$pq = -6\sqrt{2}$$

$\theta = \dfrac{3\pi}{4}$ $\quad -\sqrt{2} = \dfrac{3}{p} \Rightarrow p = -3/\sqrt{2}$

$-1 = \dfrac{-4}{q}, q = 4$

$pq = -6\sqrt{2}$

Ans. $pq = -6\sqrt{2}$

SECTION 2 (Maximum Marks: 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get -2 marks.

5. Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $P = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. Let $Q = \begin{pmatrix} x & y \\ z & 4 \end{pmatrix}$ for some non-zero real numbers x, y, and z, for which there is a 2 × 2 matrix R with all entries being non-zero real numbers, such that QR = RP.
Then which of the following statements is (are) TRUE?

(A) The determinant of Q – 2I is zero
(B) The determinant of Q – 6I is 12
(C) The determinant of Q – 3I is 15
(D) yz = 2

Sol. **(A), (B)**

$$R = \begin{bmatrix} a & b \\ c & d \end{bmatrix}. \quad (a, b, c, d \neq 0)$$

$$\begin{bmatrix} x & y \\ z & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} ax+cy & bx+dy \\ az+4c & bz+4d \end{bmatrix} = \begin{bmatrix} 2a & 3b \\ 2c & 3d \end{bmatrix}$$

$ax + cy = 2a$ ...(1)

bx + dy = 3b ...(2)

az + 4c = 2c ...(3)

bz + 4d = 3d ...(4)

az = – 2c

bz = – d

$\frac{a}{b} = \frac{2c}{d}$

ad = 2bc

$$\begin{aligned} ax + cy = 2a \} \times d \\ bx + dy = 3b \} \times c \end{aligned}$$

$(ad - bc)x = 2ad - 3bc$

$$x = \frac{2ad - 3bc}{ad - bc}$$

$$x = \frac{4bc - 3bc}{2bc - bc}$$

x = 1

$$y = \frac{a}{c}$$

$$z = -\frac{2c}{a} \text{or} -\frac{d}{b}$$

cy = a

$$y = \frac{a}{c}$$

$$|Q - 2I| = \left| \begin{bmatrix} i & i/c \\ -\frac{2c}{a} & 4 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right|$$

$$= \begin{vmatrix} -1 & a/c \\ -\frac{2c}{a} & 2 \end{vmatrix}$$

= –2 + 2 = 0

$$|Q - 6I| = \left| \begin{bmatrix} 1 & a/c \\ -\frac{2c}{a} & 4 \end{bmatrix} - \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \right|$$

$$= \begin{vmatrix} -5 & a/c \\ -\frac{2c}{a} & -2 \end{vmatrix}$$

= 10 + 2 = 12

$$|\theta - 3I| = \left| \begin{bmatrix} 1 & a/c \\ -\frac{2c}{a} & 4 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \right|$$

$$= \begin{vmatrix} -2 & a/c \\ -\frac{2c}{a} & 1 \end{vmatrix}$$

$= -2 + 2 = 0$

$$yz = \left(\frac{a}{c}\right)\left(-\frac{2c}{a}\right) = -2$$

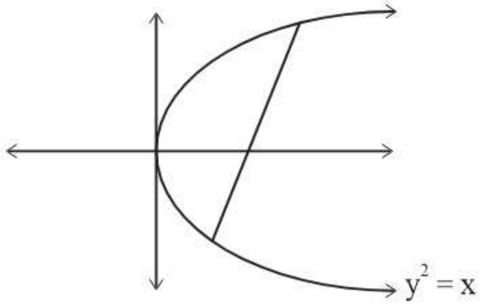
6. Let S denote the locus of the mid-points of those chords of the parabola $y^2 = x$, such that the area of the region enclosed between the parabola and the chord is $\frac{4}{3}$. Let R denote the region lying in the first quadrant, enclosed by the parabola $y^2 = x$, the curve S, and the lines $x = 1$ and $x = 4$.

Then which of the following statements is (are) TRUE?

(A) $(4, \sqrt{3}) \in S$ (B) $(5, \sqrt{2}) \in S$

(C) Area of R is $\frac{14}{3} - 2\sqrt{3}$ (D) Area of R is $\frac{14}{3} - \sqrt{3}$

Sol. **AC**



Area enclosed between $y^2 = x$ and $y = mx + c$ is

$$A = \frac{(1-4mc)^{3/2}}{6m^3} = \frac{4}{3}$$

$$\frac{(1-4mc)^3}{36m^6} = \frac{16}{9}$$

$$(1-4mc)^3 = \left(4m^2\right)^3$$

$$1 - 4mc = 4m^2$$

equation of chord

$$yy_1 - \frac{(x+x_1)}{2} = y_1^2 - x_1$$

$$yy_1 = \frac{(x+x_1)}{2} + y_1^2 - x_1$$

$$yy_1 = \frac{x}{2} + y_1^2 - \frac{x_1}{2}$$

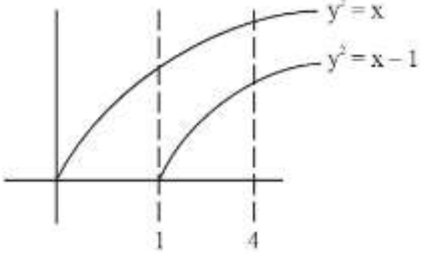
$$m = \frac{1}{2y_1}, \quad c = \frac{y_1^2 - x_1/2}{y_1}$$

$$1-4\frac{1}{2y_1}\left(\frac{y_1^2-x_1/2}{y_1}\right)=4\frac{1}{4y_1^2}$$

$$1-\frac{2\left(y^2-\frac{x}{2}\right)}{y^2}=\frac{1}{y^2}$$

$$y^2-2y^2+x=1$$

$y^2=x-1\rightarrow$ Curve (S)



$$R=\int_1^4(\sqrt{x}-\sqrt{x-1})dx=\left[\frac{x^{3/2}}{3/2}-\frac{(x-1)^{3/2}}{3/2}\right]_1^4$$

$$R=\frac{2}{3}[(8-3\sqrt{3})-(1-0)]$$

$$R=\frac{2}{3}[7-3\sqrt{3}]$$

7. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two distinct points on the ellipse

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

such that $y_1 > 0$, and $y_2 > 0$. Let C denote the circle $x^2 + y^2 = 9$, and M be the point (3, 0).
Suppose the line $x = x_1$ intersects C at R, and the line $x = x_2$ intersects C at S, such that the y-coordinates of R and S are positive. Let $\angle ROM=\frac{\pi}{6}$ and $\angle SOM=\frac{\pi}{3}$, where O denotes the origin (0, 0). Let |XY| denote the length of the line segment XY.
Then which of the following statements is (are) TRUE?

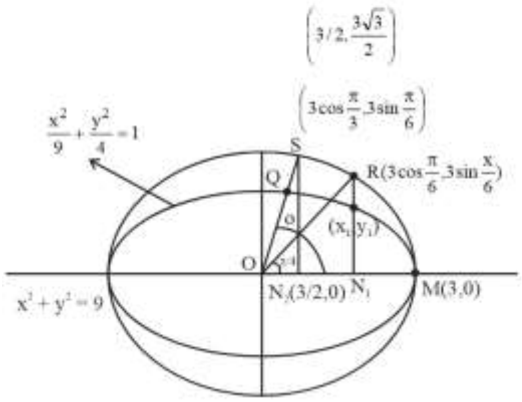
(A) The equation of the line joining P and Q is $2x + 3y = 3(1+\sqrt{3})$

(B) The equation of the line joining P and Q is $2x + y = 3(1+\sqrt{3})$

(C) If $N_2 = (x_2, 0)$, then $3|N_2Q| = 2|N_2S|$

(D) If $N_1 = (x_1, 0)$, then $9|N_1P| = 4|N_1R|$

Sol. **AC**



From diagram

$$P\left(3\cos\frac{\pi}{6}, 2\sin\frac{\pi}{6}\right) = P\left(\frac{3\sqrt{3}}{2}, 1\right)$$

$$Q(3\cos\pi/3, 2\sin\pi/3) = Q\left(\frac{3}{2}, \frac{2\sqrt{3}}{2}\right)$$

$$N_1\left(\frac{3\sqrt{3}}{2}, 0\right) \quad R\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$$

$$N_2\left(\frac{3}{2}, 0\right) \quad S\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$$

Equation of line PQ

$$y - 1 = \frac{\sqrt{3} - 1}{\frac{3}{2} - \frac{3}{2}\sqrt{3}}\left(x - \frac{3\sqrt{3}}{2}\right)$$

(A) $2x + 3y = 3 + 3\sqrt{3}$

$N_1P = 1$

$N_1R = \frac{3}{2}$

(D) $9(N_1\ P) = 9$ D is not correct

$4(N_1R) = 6$

$9(N_1P) \neq 4N_1R$

(C) $N_2\ Q = \sqrt{3}$

$N_2S = \frac{3\sqrt{3}}{2}$

$2|N_2S| = 3\sqrt{3}$

$3|N_2Q| = 3\sqrt{3}$

$2(N_2S) = 3(N_2Q)$

8. Let $\mathbb{R}$ denote the set of all real numbers. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \dfrac{6x + \sin x}{2x + \sin x} & \text{if } x \neq 0, \\ \dfrac{7}{3} & \text{if } x = 0. \end{cases}$$

Then which of the following statements is (are) TRUE?

(A) The point x = 0 is a point of local maxima of f

(B) The point x = 0 is a point of local minima of f

(C) Number of points of local maxima of f in the interval $[\pi, 6\pi]$ is 3

(D) Number of points of local minima of f in the interval $[2\pi, 4\pi]$ is 1

Sol. **BCD**

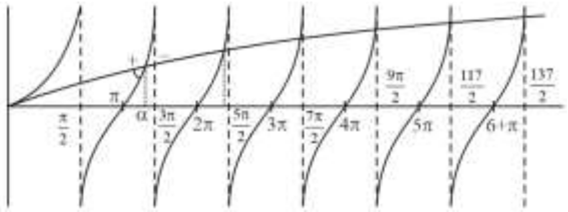
$$f'(x) = \frac{(2x + \sin x)(6 + \cos x) - (6x + \sin x)(2 + \cos x)}{(2x + \sin x)^2}$$

$$\Rightarrow \frac{(12x + 6\sin x + 2x\cos x + \sin x\cos x) - (12x + 2\sin x + 6x\cos x + \sin x\cos x)}{(2x + \sin x)^2}$$

$\Rightarrow \dfrac{4\sin x - 4x\cos x}{(2x+\sin x)^2}$

$\Rightarrow \dfrac{4(\sin x - x\cos x)}{(2x+\sin x)^2}$

$\Rightarrow \dfrac{4\cos x(\tan x - x)}{(2x+\sin x)^2}$



$\left(\pi, \frac{3\pi}{2}\right)$	maxima
$\left(3\pi, \frac{7\pi}{2}\right)$	maxima
$\left(5\pi, \frac{11\pi}{2}\right)$	maxima
$(2\pi, 4\pi)$	one point of minima

SECTION 3 (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

9. Let y(x) be the solution of the differential equation

$$x^2\frac{dy}{dx} + xy = x^2 + y^2, \quad x > \frac{1}{e},$$

satisfying y(1) = 0. Then the value of $2\dfrac{(y(e))^2}{y\left(e^2\right)}$ is _________.

Sol. **0.75**

$$x^2\frac{dy}{dx} + xy = x^2 + y^2$$

$$\frac{dy}{dx} + \frac{y}{x} = 1 + \left(\frac{y}{x}\right)^2$$

put $\dfrac{y}{x} = v \quad \Rightarrow y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v+\frac{xdv}{dx}+v=1+v^2$$

$$\frac{xdv}{dx}=v^2-2v+1$$

$$x\frac{dv}{dx}=(v-1)^2$$

$$\frac{dv}{(v-1)^2}=\frac{dx}{x}$$

$$\int\frac{dv}{(v-1)^2}=\frac{dx}{x}$$

$$-\frac{1}{(v-1)}=\ln x+c$$

$$-\frac{x}{y-x}=\ln x+c$$

$$\frac{x}{x-y}=\ln x+c\Rightarrow x-y=\frac{x}{\ln x+c}\Rightarrow y=x-\frac{x}{\ln x+c}\quad ---(1)$$

$\because\ y(1)=0$

$$0=1-\frac{x}{\ln 1+c}\Rightarrow 1=\frac{1}{c}\Rightarrow c=1$$

$$y=x-\frac{x}{\ln x+1}\quad\Rightarrow y(e)=e-\frac{e}{\ln e+1}\quad=e-\frac{e}{2}=\frac{e}{2}$$

$$y\left(e^2\right)=e^2-\frac{e^2}{\ln e^2+1}=e^2-\frac{e^2}{2+1}=e^2-\frac{e^2}{3}=\frac{2e^2}{3}$$

Now, $2\frac{\left(y(e)\right)^2}{y\left(e^2\right)}=\frac{3}{8}\times 2=0.75$

10. Let $a_0, a_1, \ldots, a_{23}$ be real numbers such that

$$\left(1+\frac{2}{5}x\right)^{23}=\sum_{i=0}^{23}a_ix^i$$

for every real number x. Let a_r be the largest among the numbers a_j for $0\le j\le 23$. Then the value of r is ________.

Sol. **6**

$a_0, a_1, \ldots.\ a_{23}\in R$

$$\left(1+\frac{2}{5}x\right)^{23}=\sum_{i=0}^{23}a_ix^i$$

$$\left(1+\frac{2}{5}x\right)^{23}=a_0x^0+a_1x^1+a_2x^2+a_3x^3+\ldots.+a_{23}x^{23}$$

$$\sum_{r=0}^{23}{}^{23}C_r\left(\frac{2}{5}x\right)^r=a_0x^0+a_1x^1+a_2x^2\ \ldots.+a_{23}x^{23}$$

$$a_r={}^{23}C_r\left(\frac{2}{5}\right)^r$$

$$\frac{23+1}{1+\frac{5}{2}}-1 \le r \le \frac{23+1}{1+\frac{5}{2}}$$

$5.8 \le r \le 6.8$

$r = 6$

11. A factory has a total of three manufacturing units, M_1, M_2, and M_3, which produce bulbs independent of each other. The units M_1, M_2, and M_3 produce bulbs in the proportions of 2 : 2 : 1, respectively. It is known that 20% of the bulbs produced in the factory are defective. It is also known that, of all the bulbs produced by M_1, 15% are defective. Suppose that, if a randomly chosen bulb produced in the factory is found to be defective, the probability that it was produced by M_2 is $\frac{2}{5}$.

If a bulb is chosen randomly from the bulbs produced by M_3, then the probability that it is defective is _______.

Sol. **0.3**

$P(M_1)$ Probability of bulb is chosen form M_1

$P(M_2)$ Probability of bulb is chosen form M_2

$P(M_3)$ Probability of bulb is chosen form M_3

$$P(M_1)=\frac{2}{5}, P(M_2)=\frac{2}{5}, P(M_3)=\frac{1}{5}$$

$$P(D)=\frac{20}{100}, P\left(\frac{D}{M_1}\right)=\frac{15}{100}$$

$$P\left(\frac{M_2}{D}\right)=\frac{2}{5}$$

$$P\left(\frac{M_2}{D}\right)=\frac{P(D\cap M_2)}{P(D)}$$

$$\frac{2}{5}=\frac{P(M_2).P\left(\frac{D}{M_2}\right)}{P(D)}$$

$$\frac{2}{5}=\frac{\frac{2}{5}\times P\left(\frac{D}{M_2}\right)}{\frac{1}{5}}$$

$$P\left(\frac{D}{M_2}\right)=\frac{1}{5}$$

$$P\left(\frac{D}{M_3}\right)=?$$

$$P(D)=P(M_1)\cdot P\left(\frac{D}{M_1}\right)+P(M_2)\cdot P\left(\frac{D}{M_2}\right)+P(M_3)\cdot P\left(\frac{D}{M_3}\right)$$

$$\frac{1}{5}=\frac{2}{5}\cdot\frac{3}{20}+\frac{2}{5}\cdot\frac{1}{5}+\frac{1}{5}P\left(\frac{D}{M_3}\right)$$

$$P\left(\frac{D}{M_3}\right)=\frac{3}{10}=0.3$$

12. Consider the vectors $\vec{x} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{y} = 2\hat{i} + 3\hat{j} + \hat{k}$, and $\vec{z} = 3\hat{i} + \hat{j} + 2\hat{k}$

For two distinct positive real numbers α and β, define

$\vec{X} = \alpha\vec{x} + \beta\vec{y} - \vec{z}$, $\vec{Y} = \alpha\vec{y} + \beta\vec{z} - \vec{x}$, and $\vec{Z} = \alpha\vec{z} + \beta\vec{x} - \vec{y}$

If the vectors $\vec{X}, \vec{Y}$, and $\vec{Z}$ lie in a plane, then the value of $\alpha + \beta - 3$ is __________.

Sol. **–2**

$[\vec{x}\,\vec{y}\,\vec{z}] = 0$

$(\alpha\vec{x} + \beta\vec{y} - \vec{z}) \cdot ((\alpha\vec{y} + \beta\vec{z} - \vec{x}) \times (\alpha\vec{z} + \beta\vec{x} - \vec{y})) = 0$

$(\alpha\vec{x} + \beta\vec{y} - \vec{z}) \cdot (\alpha^2(\vec{y} \times \vec{z}) + \alpha\beta(\vec{y} \times \vec{x}) - \alpha(\vec{y} \times \vec{y})$

$+\alpha\beta(\vec{z} \times \vec{z}) + \beta^2(\vec{z} \times \vec{x}) - \beta(\vec{z} \times \vec{y})$

$-\alpha(\vec{x} \times \vec{z}) - \beta(\vec{x} \times \vec{x}) + \vec{x} \times \vec{y}) = 0$

$\alpha^3[\vec{x}\,\vec{y}\,\vec{z}] + \alpha\beta[\vec{x}\,\vec{y}\,\vec{z}] + \alpha\beta[\vec{x}\,\vec{y}\,\vec{z}] + \beta^3[\vec{x}\,\vec{y}\,\vec{z}] + \alpha\beta[\vec{x}\,\vec{y}\,\vec{z}] + \alpha\beta[\vec{x}\,\vec{y}\,\vec{z}] = 0$

$\alpha^3 + \beta^3 + 3\alpha\beta - 1 = 0$

$\alpha^3 + \beta^3 + (-1)^3 = 3(\alpha)(\beta)(-1)$

$\therefore\ \alpha + \beta - 1$ or $\alpha = \beta = -1$

Since α and β are distinct

Hence $\alpha + \beta = 1$

$\alpha + \beta - 3 = -2$

13. For a non-zero complex number z, let arg(z) denote the principal argument of z, with $-\pi < \arg(z) \le \pi$. Let ω be the cube root of unity for which $0 < \arg(\omega) < \pi$. Let

$$\alpha = \arg\left(\sum_{n=1}^{2025} (-\omega)^n\right)$$

Then the value of $\frac{3\alpha}{\pi}$ is _______.

Sol. **–2**

$$\alpha = \arg\left(\sum_{n=1}^{2025} (-\omega)^n\right)$$

$$\sum_{n=1}^{2025} (-\omega)^n = (-\omega) + (-\omega)^2 + (-\omega)^3 + \cdots + (-\omega)^{2025}$$

$$= (-\omega)\frac{\left(1 - (-\omega)^{2025}\right)}{1 + \omega}$$

$$= -\omega\frac{\left(1 + \left(\omega^3\right)^{675}\right)}{1 + \omega}$$

$$= \frac{-2\omega}{1 + \omega}$$

Since $\omega = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$

$$\therefore \quad = \frac{-2\left(\frac{-1}{2}+\frac{\sqrt{3}}{2}i\right)}{\frac{1}{2}+\frac{\sqrt{3}}{2}i} \times \frac{\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)}{\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)}$$

$$= \frac{-2\left(\frac{-1}{4}+\frac{\sqrt{3}}{4}i+\frac{\sqrt{3}}{4}i+\frac{3}{4}\right)}{\frac{1}{4}+\frac{3}{4}}$$

$$= -2\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$

$$= -1-\sqrt{3}i$$

$$\alpha = \arg\left(-1-\sqrt{3}i\right) = \frac{-2\pi}{3}$$

$$\therefore \frac{3\alpha}{\pi} = -2$$

14. Let $\mathbb{R}$ denote the set of all real numbers. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to (0, 4)$ be functions defined by

$f(x) = \log_e\left(x^2+2x+4\right)$, and $g(x) = \frac{4}{1+e^{-2x}}$

Define the composite function $f \circ g^{-1}$ by $\left(f \circ g^{-1}\right)(x) = f\left(g^{-1}(x)\right)$, where g^{-1} is the inverse of the function g.

Then the value of the derivative of the composite function $f \circ g^{-1}$ at $x = 2$ is ________.

Sol. **0.25**

$f(x) = \ln\left(x^2+2x+4\right)$,

$$f'(x) = \frac{2x+2}{x^2+2x+4}$$

$$= \frac{2(x+1)}{(x+1)^2+3}$$

$$= \ g(x) = \frac{4}{1+e^{-2x}}$$

$$g'(x) = \frac{-4}{\left(1+e^{-2x}\right)^2}\left(-2e^{-2x}\right)$$

$y = f(g^{-1}(x))$

$$\frac{dy}{dx} = f'\left(g^{-1}(x)\right) \cdot \frac{d}{dx} g^{-1}(x)$$

Let h(x) is inverse of g(x).

$\therefore$ h (g(x)) = x

h' (g(x)).g'(x) = 1

$$h'(g(x)) = \frac{1}{g'(x)} = \frac{\left(1+e^{-2x}\right)^2}{8e^{-2x}}$$

$$h'(t)=\frac{\left(1+e^{-2g^{-1}(x)}\right)^2}{8\cdot e^{-2g^{-1}(x)}}$$

$$\left(g^{-1}(x)\right)'=h'(x)=\frac{1}{8}\left(e^{g^{-1}(x)}+e^{-g^{-1}(x)}\right)^2$$

$$g^{-1}(x)=\ln\sqrt{\frac{x}{4-x}}$$

$$\frac{dy}{dx}=\frac{2\left(g^{-1}(x)+1\right)}{\left(g^{-1}(x)+1\right)^2+3}\cdot\frac{1}{8}\left(e^{g^{-1}(x)}+e^{-g^{-1}(x)}\right)^2$$

$g^{-1}(2) = 0$

$$\left.\frac{dy}{dx}\right|_{x=2}=\frac{2}{4}\cdot\frac{1}{8}(4)$$

$$=\frac{1}{4}$$

15. Let $\alpha=\frac{1}{\sin 60^\circ \sin 61^\circ}+\frac{1}{\sin 62^\circ \sin 63^\circ}+\cdots+\frac{1}{\sin 118^\circ \sin 119^\circ}$.

Then the value of $\left(\frac{\operatorname{cosec}1^\circ}{\alpha}\right)^2$ is __________.

Sol. **3**

$$\alpha=\frac{1}{\sin 60^\circ \sin 61^\circ}+\frac{1}{\sin 62^\circ \sin 63^\circ}+\cdots\cdots+\frac{1}{\sin 118^\circ \sin 119^\circ}$$

in general $\alpha=\frac{1}{\sin n^\circ \sin(n+1)^\circ}$

First term n = 60

Last term n = 118

Here $\frac{1}{\sin n^\circ\cdot\sin(n+1)^\circ}=\frac{1}{\sin 1^\circ}\left(\cot n^\circ-\cot(n+1)^\circ\right)$

$$\alpha=\sum_{n=60,62\cdots118^\circ}\frac{1}{\sin 1^\circ}\left(\cot n^\circ-\cot(n+1)^\circ\right)$$

We can write

cot n° – cot (n + 1)° = (cot 60° + cot 62° + …. cot 118°) – (cot 61° + cot 63° + …. + cot 119°)

we know cot 119° = cot (180° – 61°) = – cot 61°

cot 118° = – cot 62°

similarly all calculate and put all

$$\alpha=\frac{1}{\sin 1^\circ}.\cot 60^\circ=\frac{1}{\sqrt{3}\sin 1^\circ}$$

$$\left(\frac{\operatorname{cosec}1^\circ}{\alpha}\right)^2=\left(\frac{\frac{1}{\sin 1^\circ}}{\frac{1}{\sqrt{3}}\sin 1^\circ}\right)^2=(\sqrt{3})^2=3$$

16. If $\alpha = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{2x^2 - 3x + 2} dx$, then the value of $\sqrt{7}\tan\left(\frac{2a\sqrt{7}}{\pi}\right)$ is ________.

(Here, the inverse trigonometric function $\tan^{-1}x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.)

Sol. **21**

$$\alpha = \int_{1/2}^{2} \frac{\tan^{-1} x}{2x^2 - 3x + 2} dx. \quad \text{put } x = \frac{1}{x} dx = \frac{-1}{x^2} dx$$

$$\alpha = \int_{2}^{1/2} \frac{\tan^{-1}\left(\frac{1}{x}\right)}{2\left(\frac{1}{x}\right)^2 - 3\left(\frac{1}{x}\right) + 2}\left(\frac{-1}{x^2}\right) dx = \int_{1/2}^{2} \frac{\tan^{-1}\left(\frac{1}{x}\right)}{2x^2 - 3x + 2} dx$$

$$2\alpha = \int_{1/2}^{2} \frac{\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)}{2x^2 - 3x + 2} dx$$

$$\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2} \quad \text{for } x > 0$$

$$2\alpha = \frac{\pi}{2}\int_{1/2}^{2} \frac{1}{2x^2 - 3x + 2} dx$$

$$\alpha = \frac{\pi}{4}\int_{1/2}^{2} \frac{1}{2x^2 - 3x + 2} dx$$

$$\text{Let } I = \int \frac{1}{2x^2 - 3x + 2} dx = \frac{1}{2}\int \frac{1}{\left(x - \frac{3}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2}$$

$$= \frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{4x - 3}{\sqrt{7}}\right) + c$$

$$I = \int_{1/2}^{2} \frac{1}{2x^2 - 3x + 2} dx = \left(\frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{4x - 3}{\sqrt{7}}\right)\right)_{1/2}^{2}$$

$$I = \frac{2}{\sqrt{7}} \tan^{-1}\left(3\sqrt{7}\right)$$

$$\alpha = \frac{\pi}{4} \times \frac{2}{\sqrt{7}} \tan^{-1}\left(3\sqrt{7}\right) = \frac{\pi}{2\sqrt{7}} \tan^{-1}\left(3\sqrt{7}\right)$$

$$\sqrt{7}\tan\left(\frac{2\alpha}{\pi}\sqrt{7}\right) = \sqrt{7} \times 3\sqrt{7} = 3 \times 7 = 21$$

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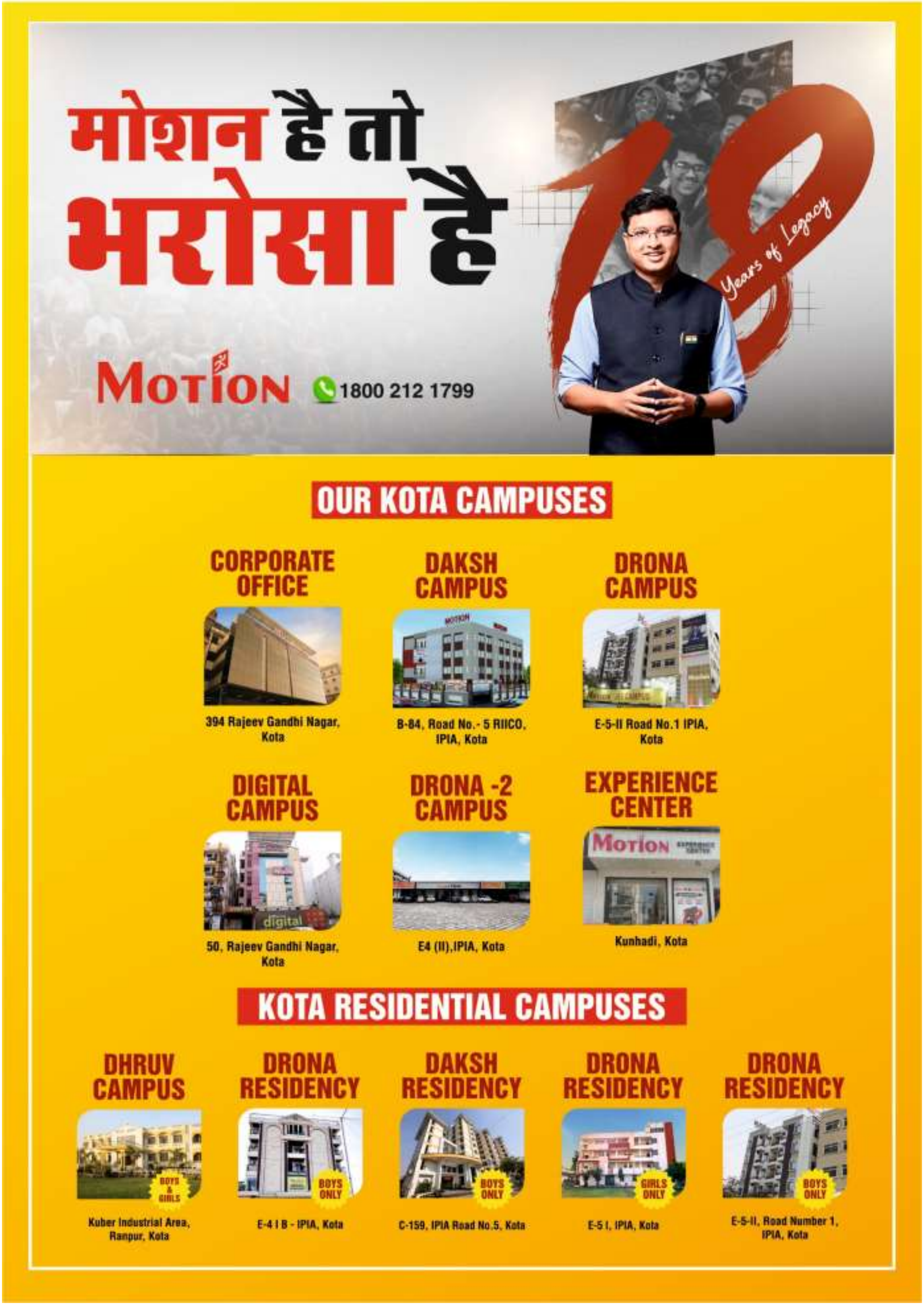
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