# TRIGONOMETRY

## *Trigonometric Ratios of Compound Angles:*

- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$
- $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
- $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$
- $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A.$
- $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A.$
- $\cot(A + B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$
- $\cot(A - B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

## *Trigonometric Ratios of Multiples of an Angle:*

- $\sin 2A = 2\sin A \cos A = \dfrac{2\tan A}{1 + \tan^2 A}$
- $\sin A = 2\sin\dfrac{A}{2}\cos\dfrac{A}{2} = \dfrac{2\tan\frac{A}{2}}{1 + \tan^2\frac{A}{2}}$
- $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$
- $\cos A = \cos^2\dfrac{A}{2} - \sin^2\dfrac{A}{2} = 1 - 2\sin^2\dfrac{A}{2} = 2\cos^2\dfrac{A}{2} - 1 = \dfrac{1 - \tan^2\frac{A}{2}}{1 + \tan^2\frac{A}{2}}$
- $\cos^2 A = \dfrac{1 + \cos 2A}{2}$; $\sin^2 A = \dfrac{1 - \cos 2A}{2}$

- $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$ $\qquad \tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$
- $\cot 2A = \frac{\cot^2 A - 1}{2\cot A}$ $\qquad \cot A = \frac{\cot^2\frac{A}{2} - 1}{2\cot\frac{A}{2}}$
- $\sin 3A = 3\sin A - 4\sin^3 A$
- $\cos 3A = 4\cos^3 A - 3\cos A$
- $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$
- $\cot 3A = \frac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}$

*Sum of sines/cosines in terms of Products:*

- $2\sin A\cos B = \sin(A + B) + \sin(A - B)$
- $2\cos A\sin B = \sin(A + B) - \sin(A - B)$
- $2\cos A\cos B = \cos(A + B) + \cos(A - B)$
- $2\sin A\sin B = \cos(A - B) - \cos(A + B)$

**Conversely:**

- $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$
- $\sin C - \sin D = 2\sin\frac{C-D}{2}\cos\frac{C+D}{2}$
- $\cos C + \cos D = 2\cos\frac{C-D}{2}\cos\frac{C+D}{2}$
- $\cos C - \cos D = -2\sin\frac{C-D}{2}\sin\frac{C+D}{2}$
- $\tan A + \tan B = \frac{\sin(A+B)}{\cos A . \cos B}$

## Inverse Trigonometrical functions

***Some Important Results:***

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, \qquad |x| \le 1$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \qquad x \in R$$

$$\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}, \quad |x| \ge 1$$

$$\sin^{-1}(-x) = -\sin^{-1}(x)$$

$$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x)$$

$$\tan^{-1}(-x) = -\tan^{-1}(x)$$

$$\cos^{-1}(-x) = \pi - \cos^{-1}(x)$$

$$\sec^{-1}(-x) = \pi - \sec^{-1}(x)$$

$$\cot^{-1}(-x) = \pi - \cot^{-1}(x)$$

$$\sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1-x^2}} = \operatorname{cosec}^{-1} \frac{1}{x}, \ x > 0$$

$$\sin^{-1} x = \cos^{-1} \sqrt{1-x^2} = \cot^{-1} \frac{\sqrt{1-x^2}}{x} = \sec^{-1} \frac{1}{\sqrt{1-x^2}}, \quad x > 0$$

$$\cos^{-1} x = \sin^{-1} \sqrt{1-x^2} = \tan^{-1} \frac{\sqrt{1-x^2}}{x} = \operatorname{cosec}^{-1} \frac{1}{\sqrt{1-x^2}}, \quad x > 0$$

$$\tan^{-1} x = \sin^{-1} \frac{x}{\sqrt{1+x^2}} = \operatorname{cosec}^{-1} \frac{\sqrt{1+x^2}}{x}, \ x \in R \sim \{0\}$$

$$\cot^{-1} x = \cos^{-1} \frac{x}{\sqrt{1+x^2}} = \sec^{-1} \frac{\sqrt{1+x^2}}{x}, \ x \in R \sim \{0\}$$

$$\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right], \quad x \ge 0, \quad y \ge 0$$

$$\cos^{-1}x - \cos^{-1}y = \begin{cases} \cos^{-1}\left[xy + \sqrt{1-x^2}\sqrt{1-y^2}\right], & x \ge 0, \ y \ge 0, \ x \le y \\ -\cos^{-1}\left[xy + \sqrt{1-x^2}\sqrt{1-y^2}\right], & x \ge 0, \ y \ge 0, \ x > y \end{cases}$$

$$\sin^{-1}x + \sin^{-1}y = \begin{cases} \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right], & x \ge 0, \ y \ge 0, \ x^2 + y^2 \le 1 \\ \pi - \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right], & x \ge 0, \ y \ge 0, \ x^2 + y^2 > 1 \end{cases}$$

## Trigonometric Equations

- $\sin^2\theta = \sin^2\alpha$ , $\cos^2\theta = \cos^2\alpha$, $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha$,
- $\sin\theta = 1 \quad \Rightarrow \theta = (4n+1)\frac{\pi}{2}$,
- $\sin\theta = -1 \quad \Rightarrow \theta = (4n-1)\frac{\pi}{2}$,
- $\cos\theta = 1 \quad \Rightarrow \theta = 2n\pi$ ,
- $\cos\theta = -1 \quad \Rightarrow \theta = (2n+1)\pi$,

## SOLUTION OF TRIANGLE

In any triangle three sides and three angles are called the elements of the triangle. the three sides $BC = a$, $CA = b$, $AB = c$ and the three angles $A$, $B$, $C$.

(i) $A + B + C = \pi^c = 180°$

(ii) Area of the $\Delta ABC$, $\Delta = \frac{1}{2}$ (base) (height)

*SINE FORMULA*

In a $\Delta ABC$, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

*i.e.*, sides are proportional to sines of opposite angles.

## COSINE FORMULA

In a $\Delta ABC$, $a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = c^2 + a^2 - 2ca \cos B$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

## PROJECTION FORMULA

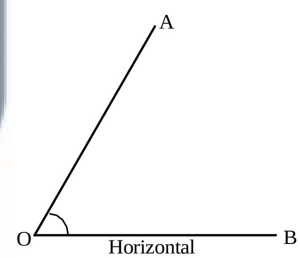
In any $\Delta ABC$, $\begin{cases} a = b\cos C + c\cos B \\ b = c\cos A + a\cos C \\ c = a\cos B + b\cos A \end{cases}$

## HALF-ANGLE FORMULA

$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ and $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

## Height and Distance

1. **Angle of Elevation:** Consider a point A being observed from a point O (usually called observer) at a lower horizontal level. Draw a horizontal line OB through O in the direction of A. Then, OA is called the line of sight or observation and ∠AOB is called the angle of elevation of point A as seen from O.



**2. Angle of Depression:**

Consider a point A being observed from a point O at a higher horizontal level ∠AOB here is called the angle of depression of the point A as seen from O. This is also measured w.r.t the horizontal at O.