3. TRIGONOMETRIC FUNCTIONS

I. MCQ (2 marks each)

1) The principal solutions of $\sqrt{3}\sec x - 2 = 0$ are _____.

a) $\frac{\pi}{3}, \frac{11\pi}{6}$ b) $\frac{\pi}{6}, \frac{11\pi}{6}$ c) $\frac{\pi}{4}, \frac{11\pi}{4}$ d) $\frac{\pi}{6}, \frac{11\pi}{3}$

2) In ΔABC, if $\cos A = \frac{\sin B}{2\sin C}$, then ΔABC is _____.

a) an equilateral triangle. b) a right angled triangle.

c) an isosceles triangle. d) an isosceles right angled triangle.

3) $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then $x =$ _____.

a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $-\frac{1}{2}$ d) $-\frac{\sqrt{3}}{2}$

4) The principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ is _____.

a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{2\pi}{3}$ d) $\frac{3\pi}{2}$

5) The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is _____.

b) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{2\pi}{3}$ d) $\frac{3\pi}{2}$

6) In ΔABC, if $\angle A = 30°$, $\angle B = 60°$, then the ratio of sides is _____.

a) 1:√3:2 b) 2:√3:1 c) √3:1:2 d) √3:2:1

7) In ΔABC, if $b^2 + c^2 - a^2 = bc$, then $\angle A =$ _____.

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{6}$

8) If polar co-ordinates of a point are $\left(\frac{3}{4}, \frac{3\pi}{4}\right)$, then its Cartesian co-ordinate are _____.

a) $\left(\frac{3}{4\sqrt{2}}, -\frac{3}{4\sqrt{2}}\right)$ b) $\left(\frac{3}{4\sqrt{2}}, \frac{3}{4\sqrt{2}}\right)$ c) $\left(-\frac{3}{4\sqrt{2}}, \frac{3}{4\sqrt{2}}\right)$ d) $\left(-\frac{3}{4\sqrt{2}}, -\frac{3}{4\sqrt{2}}\right)$

9) $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) =$ _____.

a) $-\frac{\pi}{6}$ b) $\frac{\pi}{6}$ c) $\frac{13\pi}{6}$ d) $\frac{5\pi}{6}$

10) If $\sin(\sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1}(x)) = 1$, then $x =$ _____.

a) $\frac{1}{5}$ b) $-\frac{1}{5}$ c) 5 d) -5

11) If in ΔABC , $\sin\frac{A}{2} . \sin\frac{C}{2} = \sin\frac{B}{2}$ and $2s$ is the perimeter of the Δ , then $s =$

A) $2b$ B) b C) $3b$ D) $4b$

II. Very Short Answers (1 mark)

1) Evaluate $\cot(\tan^{-1}(2x) + \cot^{-1}(2x))$.

2) In ΔABC, prove that $ac\cos B - bc\cos A = a^2 - b^2$.

3) In ΔABC, if $\sin^2 A + \sin^2 B = \sin^2 C$, then show that $a^2 + b^2 = c^2$.

4) Find the polar co-ordinates of point whose Cartesian co-ordinates are $(1,\sqrt{3})$.

5) Prove that $2\tan^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\left(\frac{24}{7}\right)$.

6) Evaluate $\sin[\cos^{-1}\left(\frac{3}{5}\right)]$.

7) In ΔABC, $a = 3, b = 4$ and $\sin A = \frac{3}{4}$, find $\angle B$.

8) Find the principal solutions of $\operatorname{cosec} x = 2$.

9) Find the principal solutions of $\sin x - 1 = 0$.

10) Find the Cartesian co-ordinates of point whose polar co-ordinates are $\left(4,\frac{\pi}{3}\right)$.

III. Short Answer Questions (*2 marks each*):

1) With usual notations, prove that $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2+b^2+c^2}{2abc}$.

2) Find the principal solutions of $\cos 2x = 1$.

3) In ΔABC, prove that $(b-c)^2 \cos^2\left(\frac{A}{2}\right) + (b+c)^2 \sin^2\left(\frac{A}{2}\right) = a^2$.

4) Find the principal solutions of $\sin x = -\frac{1}{2}$.

5) Find the value of $\cos^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

6) In ΔABC, if $a = 13, b = 14, c = 15$, then find the value of $\cos B$.

7) In ΔABC, if $\frac{\cos A}{a} = \frac{\cos B}{b}$, then show that it is an isosceles triangle.

8) Find the principal solution of $\tan x = -\sqrt{3}$.

9) Evaluate $\cos[\frac{\pi}{6} + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)]$.

10) Find the general solution of the equation $\sin 2x = \frac{\sqrt{3}}{2}$

11) Find the general solution of the equation $\cos 2\theta = \cos\theta$

12) Find the general solution of the equation $4\ tan^2 x = 3$

IV. Short Answer Questions (*3 marks each*):

1) In ΔABC, if $a\cos A = b\cos B$, then prove that ΔABC is either a right angled or an isosceles triangle.

2) In ΔABC, prove that $\frac{\cos 2A}{a^2} - \frac{\cos 2C}{c^2} = \frac{1}{a^2} - \frac{1}{c^2}$.

3) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then show that $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx} = 1$.

4) Prove that $\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]=1$.

5) In ΔABC, if $\frac{2\cos A}{a}+\frac{\cos B}{b}+\frac{2\cos C}{c}=\frac{a}{bc}+\frac{b}{ca}$, then show that the triangle is a right angled.

6) In ΔABC, prove that $\sin\left(\frac{A-B}{2}\right)=\left(\frac{a-b}{c}\right)\cos\left(\frac{C}{2}\right)$.

7) If the angles A, B, C of ΔABC are in A.P. and its sides a, b, c are in G.P., then show that a^2, b^2, c^2 are in A.P.

8) Prove that $\cot^{-1}(7)+2\cot^{-1}(3)=\frac{\pi}{4}$.

9) Find the general solution of the equation $\tan\theta\tan 2\theta=1$

10) Find the general solution of the equation $\frac{\sin 3x}{2\cos 2x+1}=\frac{1}{2}$

11) Find the general solution of the equation $2\cot\frac{\theta}{2}=(1+\cot\theta)^2$

12) Find the general solution of the equation $\cos 2\theta=\left(\sqrt{2}+1\right)\left(\cos\theta-\frac{1}{\sqrt{2}}\right)$

13) Find the general solution of the equation $4cos^2x\sin x-2sin^2x=3\sin x$

V. Long Answer Questions (*4 marks each*):

1) In ΔABC, prove that $\frac{\cos^2 A-\cos^2 B}{a+b}+\frac{\cos^2 B-\cos^2 C}{b+c}+\frac{\cos^2 C-\cos^2 A}{c+a}=0$.

2) Show that $\sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{8}{17}\right)=\cos^{-1}\left(\frac{36}{85}\right)$.

3) In ΔABC, prove that $\frac{a^2\sin(B-C)}{\sin A}+\frac{b^2\sin(C-A)}{\sin B}+\frac{c^2\sin(A-B)}{\sin C}=0$.

4) In ΔABC, prove that $\frac{b^2-c^2}{a}\cos A+\frac{c^2-a^2}{b}\cos B+\frac{a^2-b^2}{c}\cos C=0$.

5) Prove that $2\tan^{-1}\left(\frac{1}{8}\right)+\tan^{-1}\left(\frac{1}{7}\right)+2\tan^{-1}\left(\frac{1}{5}\right)=\frac{\pi}{4}$.

6) In ΔABC, if $\angle A=\frac{\pi}{2}$, then prove that $\sin(B-C)=\frac{b^2-c^2}{b^2+c^2}$.

7) If $\cos^{-1}x+\cos^{-1}y-\cos^{-1}z=0$, then show that

$x^2 + y^2 + z^2 - 2xyz = 1.$

8) Find the general solution of the equation $\cos\theta + \sin\theta = \frac{1}{\sqrt{2}}$

9) Find the general solution of $\tan x + \tan 2x + \sqrt{3}\tan x \tan 2x = \sqrt{3}$

10) In a ΔABC , if $a = 25$, $b = 52$, $c = 63$ then

Find the values of i) $\cos C$ ii) $\sin\frac{C}{2}$ iii) $\tan\frac{C}{2}$ iv) $A(\Delta ABC)$

11) In ΔABC , prove that $\left(\cot\frac{A}{2} + \cot\frac{B}{2}\right)\left(a.sin^2\frac{B}{2} + b.sin^2\frac{A}{2}\right) = c\cot\frac{C}{2}$

12) In ΔABC , prove that $\frac{1}{a}cos^2\frac{A}{2} + \frac{1}{b}cos^2\frac{B}{2} + \frac{1}{c}cos^2\frac{C}{2} = \frac{s^2}{abc}$