JEE-Main-29-07-2022-Shift-1 (Memory Based)

Physics

Question: Position of a particle x at time t are related as $t=\sqrt{2x+4}$. The velocity of the particle at $t=4s$ is equal to (in S.I. units)

Options:

(a) 4

(b) 2

(c) 1

(d) 5

Answer: (a)

Solution:

$$t=\sqrt{2x+4} \Rightarrow x=\frac{1}{2}\left(t^2-4\right)$$

$$\Rightarrow \frac{dx}{dt}=v=t$$

$$\text{At } t=4s, v=4m/s$$

Question: Two rods of identical lengths and cross-sectional area are connected in series. If σ_1 and σ_2 is the thermal conductivity of material of two rods then equivalent conductivity of combination is equal to

Options:

(a) $\frac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$

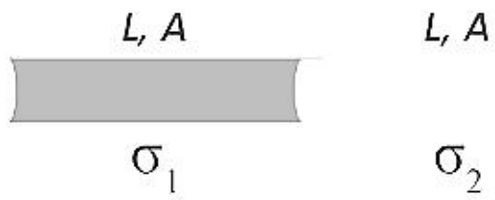
(b) $\frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}$

(c) $\frac{\sigma_1\sigma_2}{\sigma_1-\sigma_2}$

(d) $\frac{2\sigma_1\sigma_2}{\sigma_1-\sigma_2}$

Answer: (a)

Solution:



$$R_1=\frac{L}{\sigma_1 A}, R_2=\frac{L}{\sigma_2 A}$$

$$R_{net} = R_1 + R_2 = \frac{L}{A}\left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right)$$

Must be equivalent to $R_{net} = R_1 + R_2 = \frac{2L}{\sigma A}$

So, $\frac{2L}{\sigma A} = \frac{L}{A}\left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right)$

$$\sigma = \frac{2\sigma_1\sigma_2}{\sigma_1 + \sigma_2}$$

Question: A travelling microscope has vernier scale with 9MSD = 10 VSD. If one main scale division (MSD) is equal to 1 mm, then least count of travelling microscope is

Options:

(a) 0.005 m

(b) 0.002 m

(c) 0.0001 m

(d) 0.0005 m

Answer: (c)

Solution:

Least count, $LC = 1MSD - 1VSD$

$$\Rightarrow LC = 1MSD - \frac{9}{10}MSD$$

$$\Rightarrow LC = \frac{1}{10}MSD = \frac{1}{10} \times 0.001m$$

$$\Rightarrow LC = 0.0001m$$

Question: Find the ratio of energy of electron when it transitions from second to first energy state in comparison to highest state to first energy state of hydrogen atom

Options:

(a) $\frac{1}{4}$

(b) $\frac{5}{36}$

(c) $\frac{8}{9}$

(d) $\frac{3}{4}$

Answer: (d)

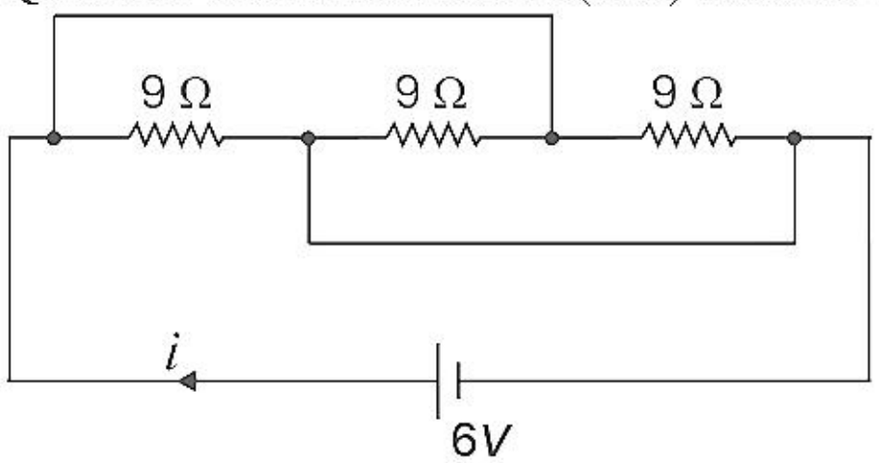
Solution:

Energy of photon is given as $E = h\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

So, $\frac{(hv)_{2\to1}}{(hv)_{\infty\to1}} = \frac{\left(\frac{1}{1^2} - \frac{1}{2^2}\right)}{\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)} = \frac{\left(\frac{3}{4}\right)}{1}$

Ratio = 3 : 4

Question: The value of current (in A) as shown is _____.



Options:

(a) 2A

(b) 3A

(c) 4A

(d) 5A

Answer: (a)

Solution:

All the resistance are in parallel.

$\Rightarrow R_{net} = 3\Omega$

$\Rightarrow i = \frac{V}{R_{net}} = \frac{6}{3} = 2A$

Question: Find the value of electric field at depletion layer in p-n junction if width is $6\times10^{-6}\,m$ and potential difference is 0.6 V, is _____ $\times10^{5} V/m$

Options:

(a) 2×10^{-5} V/m

(b) 6×10^{-6} V/m

(c) 1×10^{5} V/m

(d) 3×10^{6} V/m

Answer: (c)

Solution:

$AV = E.D$

$E = \frac{(0.6)}{6\times10^{-6}}$

$E = 1\times10^{5} V/m$

Question: A projectile with kinetic energy E at point of projection is projected at angle 45°. Its kinetic energy at top most point is equal to

Options:

(a) $\frac{E}{2}$

(b) $\frac{3E}{2}$

(c) $\frac{E}{4}$

(d) $\frac{E}{3}$

Answer: (a)

Solution:

$$\Rightarrow K.E_i = \frac{1}{2}mv^2 = E$$

Speed at highest point $v'; v\cos 45° = \frac{v}{\sqrt{2}}$

$$\Rightarrow K.E_f = \frac{1}{2}mv'^2 = \frac{1}{4}mv^2$$

$$K.E_f = \frac{E}{2}$$

Question: A particle thrown at angle 45° with horizontal with speed u has its range equal to R. At what angle should it be thrown with same speed for its range to be half of its initial value.

Options:

(a) 60°

(b) 30°

(c) 15°

(d) 70°

Answer: (c)

Solution:

$$\Rightarrow R = \frac{u^2 \sin\left(2\times 45°\right)}{g} = \frac{u^2}{g}$$

For range $\frac{R}{2}$

$$\Rightarrow \frac{u^2}{2g} = \frac{u^2 \sin 2\theta}{g}$$

$$\sin 2\theta = \frac{1}{2}$$

$$\Rightarrow \theta = 15°$$

Question: A cart is moving down a smooth incline of inclination α. What is the time period of a bob hanging from the roof of the cart with a light string?

Options:

(a) $2\pi\sqrt{\frac{l}{g\cos\alpha}}$

(b) $2\pi\sqrt{\frac{l}{g}}$

(c) $2\pi\sqrt{\frac{l}{g\sin\alpha}}$

(d) $2\pi\sqrt{\frac{l}{g\cot\alpha}}$

Answer: (a)

Solution:

$g_{eff} = g\cos\alpha$

$T = 2\pi\sqrt{\frac{l}{g\cos\alpha}}$

Question: If one mole of monoatomic gas and three moles of diatomic gas are mixed, then the molar heat at constant volume is $\alpha^2 R/4$. The value of α is ----------

Options:

(a) 2

(b) 3

(c) 5

(d) 1

Answer: (b)

Solution:

$$C_{V_{mix}} = \frac{\left(n_1 C V_1 + n_2 C_{V_2}\right)}{n_1 + n_2}$$

$$C_{V_{mix}} = \frac{\left(1\times\frac{3}{2}R + 3\times\frac{5}{2}R\right)}{1+3}$$

$$C_{V_{mix}} = \frac{9}{4}R \qquad \text{So, } \alpha = 3$$

Question: A wire of length 314 cm is made into a circular coil. Find its magnetic moment (in Am^2 if I = 14 A. $(\pi = 3.14)$

Options:

(a) 10 Am^2

(b) 8 Am^2

(c) 6 Am^2

(d) 11 Am^2

Answer: (d)

Solution:

$$\mu = i\pi r^2$$

$$\mu = i\pi\left(\frac{l}{2\pi}\right)^2$$

$$\mu = 14\times\pi\left(\frac{3.14}{2\times 3.14}\right)^2$$

$$\mu = 11\,Am^2$$

Question: Assertion: Potential is constant on surface & inside of conductor.

Reason: E is perpendicular to surface of conductor.

Options:

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true, but the reason is not the correct explanation of the assertion.

(c) If assertion is true, but reason is false.

(d) If both the assertion and reason are false.

Answer: (a)

Solution:

Since E=0, therefore the potential V inside the surface is constant. Because there is no potential difference between any two points inside the conductor, the electrostatic potential is constant throughout the volume of the conductor.

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Chemistry

Question: Product for the given reaction is:

$Zn + NaOH \rightarrow$

Options:

(a) ZnO

(b) ZnO_2

(c) $[ZnO_3]^{4-}$

(d) $[Zn(OH)_4]^{2-}$

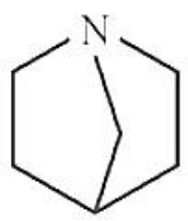
Answer: (d)

Solution: $Zn(s) + 2NaOH(aq) + 2H_2O(l) \rightarrow Na_2[Zn(OH)_4] + H_2(g)$

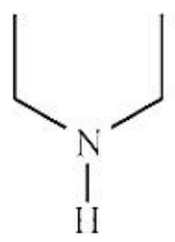
Question: Which of the following is the strongest Bronsted base?

Options:

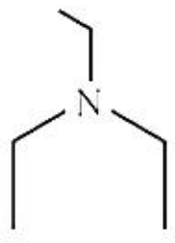
(a)



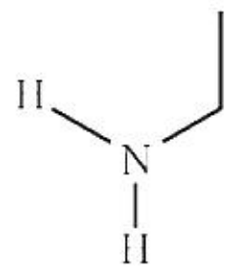
(b)



(c)



(d)



Answer: (a)

Solution: 3° aliphatic amines are strongest base among 3°, 2° and 1° amines. A is strongest base as it is 3° and lone pair is more available due to bridged alkyl group.

Question: Which of the following are examples of herbicides?

Options:

(a) Sodium arsinite, Sodium chlorate

(b) PAN, Sodium arsinite

(c) Sodium bicarbonate, DDT

(d) DDT, Sodium chlorate

Answer: (a)

Solution: Sodium chlorate ($NaClO_3$), sodium arsinite (Na_3AsO_3) are examples of herbicides.

Question: In Haber's process, 5 g of H_2 reacts with 20 g of N_2. Find the moles of ammonia formed.

Options:

(a) 1.42

(b) 2.8

(c) 2

(d) 1

Answer: (a)

Solution:

$$N_2 + 3H_2 \rightarrow 2NH_3$$

2g 5g

$$= \left(\frac{20}{27}\right)\text{moles} \quad \left(\frac{5}{2}\right) = 2.5 \text{ moles}$$

$$= 0.714 \text{ moles}$$

N_2 is limiting reagent

1 moles N_2 forms 2 moles NH_3

0.714 mole N_2 will form 2×0.714 mole = 1.428 moles NH_3

Question: Which pair among the following is colourless?

Options:

(a) Sc^{3+}, Zn^{2+}

(b) Ti^{2-}, Cu^{2+}

(c) Fe^{3+}, Mn^{2+}

(d) Fe^{3+}, Cu^{2+}

Answer: (a)

Solution:

Sc^{3+} – [Ar]

Zn^{2+} – $3d^{10}$

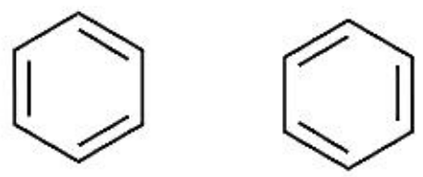
Both of them have completely filled orbitals.

Therefore, both are colourless

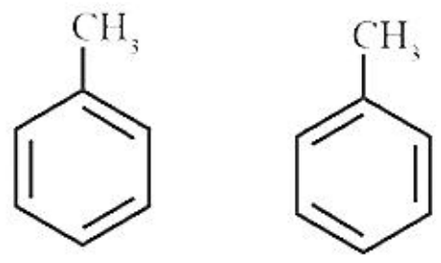
Question: Which of the following pairs will give different products on ozonolysis?

Options:

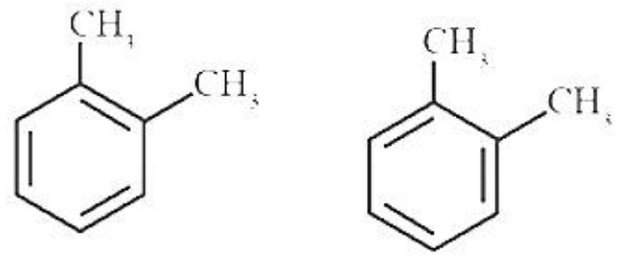
(a)



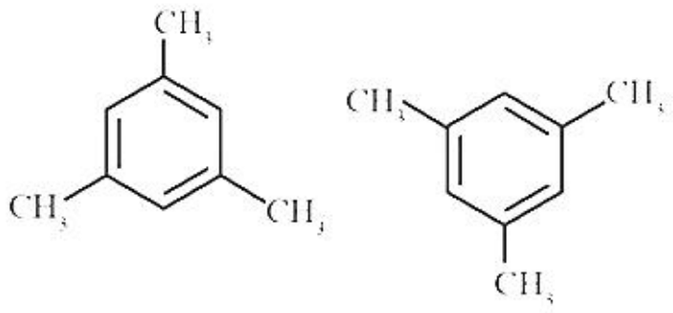
(b)



(c)

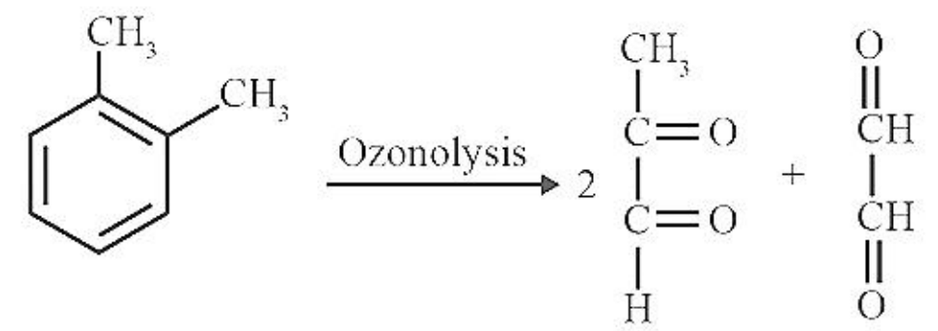


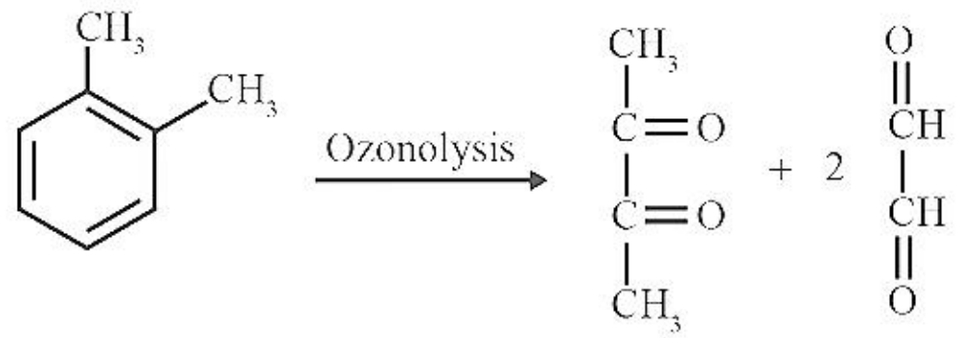
(d)



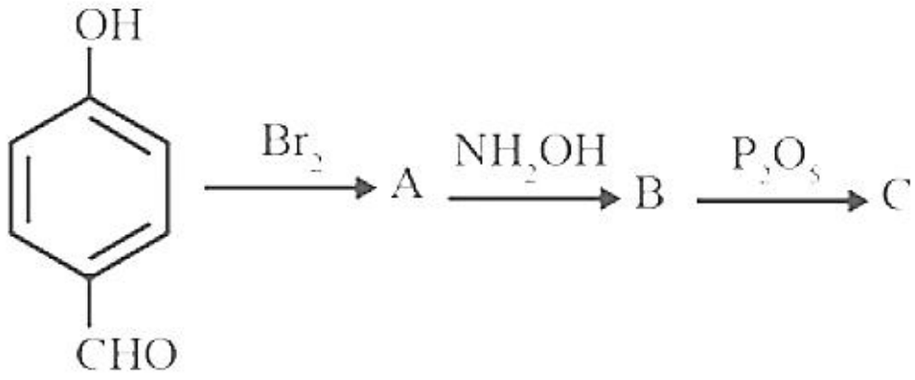
Answer: (c)

Solution:



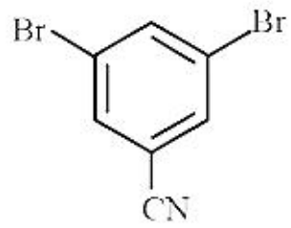


Question: Find 'C'

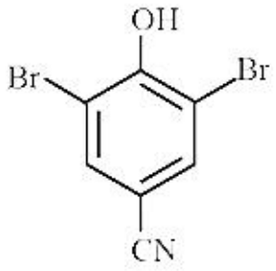


Options:

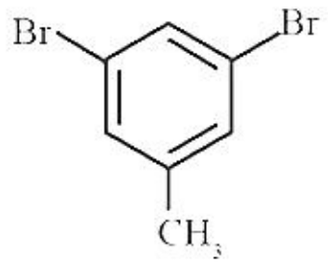
(a)



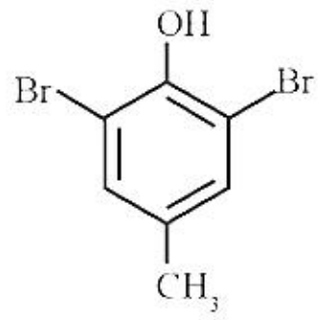
(b)



(c)

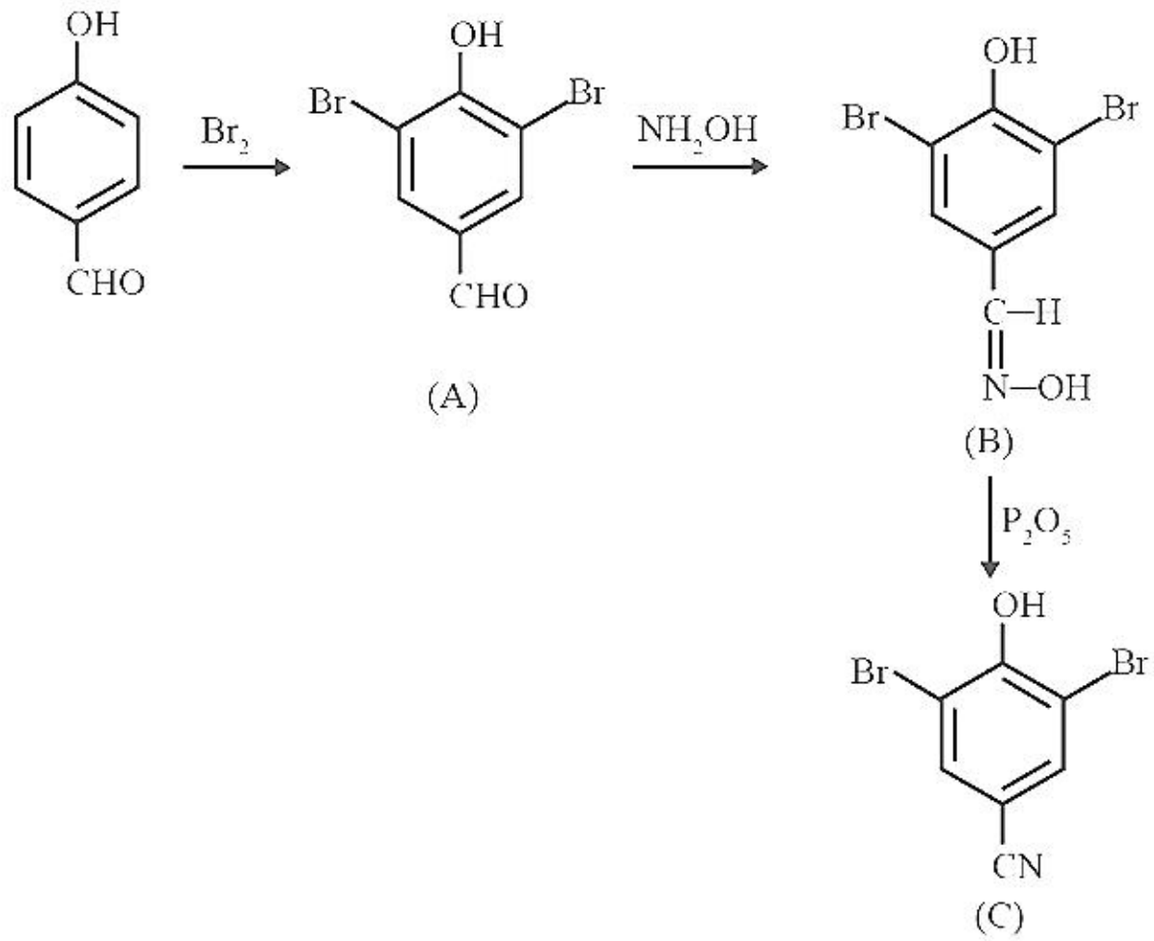


(d)

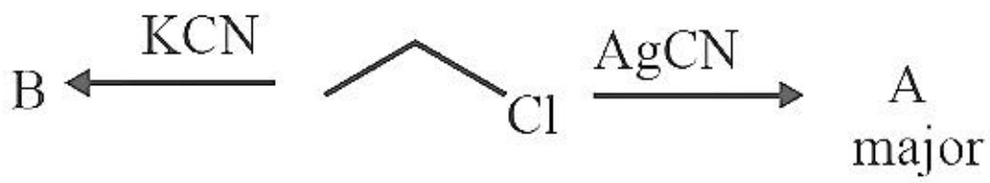


Answer: (b)

Solution:

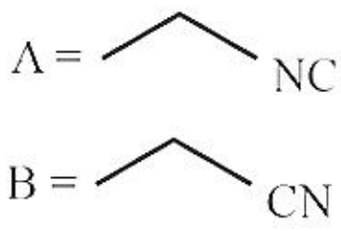


Question: Find A and B respectively?

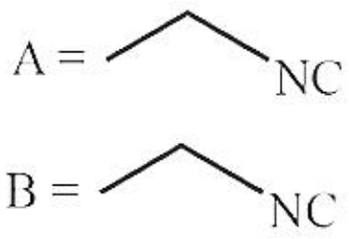


Options:

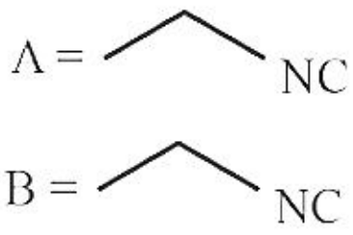
(a)



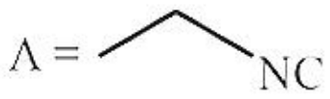
(b)



(c)



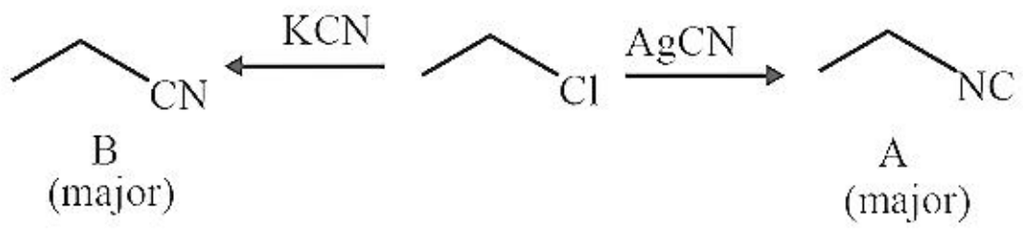
(d)

A = 

B = — C ≡ N

Answer: (a)

Solution:



Question: Which of the following is a hypnotic drug?

Options:

(a) Seldane

(b) Terpineol

(c) Amytal

(d) Histamine

Answer: (c)

Solution: Derivatives of barbituric acid viz, veronal, amytal, nembutal, luminal and seconal constitute an important class of tranquilizers. These are hypnotic.

Question: K_{sp} of PbS is given as 9×10^{-30} at a given temperature. Its solubility is $x \times 10^{-15}$. Find the value of x

Answer: 3.00

Solution: $PbS \rightleftharpoons \underset{s}{Pb^{2+}} + \underset{s}{S^{2-}}$

$K_{sp} = S^2$

$9 \times 10^{-30} = S^2$

$S = \sqrt{9 \times 10^{-30}} = 3 \times 10^{-15}$

Question: Ionic radius for A^+ and B^- are 281 pm and 180 pm respectively forming a ccp structure. If B^- forms a ccp lattice and A^- fills the octahedral voids, then what is the value of edge length in pm?

Answer: 778.00

Solution:

$$r^{+} + r^{-} = \frac{a}{2}$$

$$281 + 180 = \frac{a}{2}$$

$a = 778$ pm

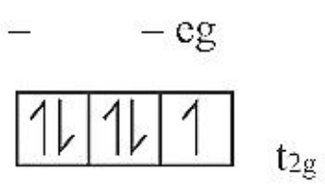
Question: Consider a complex $[Fe(OH)_6]^{3-}$ which act as an inner orbital complex. If the CFSE value after ignoring pairing energy is represented as $-x\ \Delta_o$, then x is:

(Δ_o is splitting energy in octahedral complex)

Answer: 2.00

Solution: Charge on Fe in $[Fe(OH)_6]^{3-}$ is +3

$Fe^{+3} - 3d^5$



$CFSE = (-0.4 \times 5)\Delta_o = -2\Delta_o$

Question: The magnitude of change in oxidation state of manganese in $KMnO_4$ in faintly alkaline or neutral medium is:

Answer: 3.00

Solution:

$$\overset{(+7)}{K\,Mn\,O_4} \xrightarrow[\text{Medium}]{\text{Neutral}} \overset{(+4)}{Mn\,O_2}$$

Change in oxidation state of Mn = 7 – 4 = 3

JEE-Main-29-07-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: $\int_0^{\frac{\pi}{2}} \frac{dx}{3+2\sin x+\cos x}$ is equal to:

Options:

(a) $\tan^{-1}(2)$

(b) $\tan^{-1}(2)-\frac{\pi}{4}$

(c) $\frac{1}{2}\tan^{-1}(2)-\frac{\pi}{8}$

(d) $\frac{\pi}{3}-\tan^{-1}(2)$

Answer: (b)

Solution:

$$\int_0^{\frac{\pi}{2}} \frac{dx}{3+2\sin x+\cos x}=\int_0^{\frac{\pi}{2}} \frac{\left(1+\tan^2\frac{x}{2}\right)dx}{3+3\tan^2\frac{x}{2}+4\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}$$

Let $\tan\frac{x}{2}=t \Rightarrow \sec^2\frac{x}{2}dx=2dt$

$$=\int_0^{\frac{\pi}{2}} \frac{2dt}{2t^2+4t+4}=\int_0^{\frac{\pi}{2}} \frac{dt}{(t+1)^2+1}$$

$$\Rightarrow\left[\tan^{-1}(t+1)\right]_0^1=\tan^{-1}(2)-\tan^{-1}(1)$$

$$=\tan^{-1}(2)-\frac{\pi}{4}$$

Question: Let $z=2+3i$, then value of $(z)^5+(\overline{z})^5$ is:

Options:

(a) 246

(b) 244

(c) 248

(d) 234

Answer: (b)

Solution:

$$(z)^5+(\overline{z})^5=(2+3i)^5+(2-3i)^5$$

$$= 2\left[{}^5C_0 \cdot 2^5 + {}^5C_2 \cdot 2^3 (3i)^2 + {}^5C_4 \cdot 2^1 \cdot (3i)^4 \right]$$

$$= 2[32 - 720 + 810]$$

$$= 244$$

Question: Let $\vec{a} = 3\hat{i} + \hat{j}, \vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{a} \times (\vec{b} \times \vec{c})$, $\vec{b}$ is non parallel to $\vec{c}$, then value of λ is:

Options:

(a) 5

(b) –5

(c) 1

(d) –1

Answer: (b)

Solution:

Given, $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

Also,

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \lambda \vec{c}$$

$$\Rightarrow (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \vec{b} + \lambda \vec{c}$$

$$\therefore \lambda = -(\vec{a} \cdot \vec{b}) = -(2+3) = -5$$

Question: If $\lim\limits_{x \to 0} \dfrac{\alpha e^x + \beta e^{-x} + \gamma \sin x}{x \sin^2 x} = \dfrac{2}{3}$, then which of the following option is incorrect?

Options:

(a) $\alpha^2 + \beta^2 + \gamma^2 = 1$

(b) $\alpha\beta + \beta\gamma + \gamma\alpha + 1 = 0$

(c) $\alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2 + 3 = 0$

(d) $\alpha^2 - \beta^2 + \gamma^2 + 4 = 0$

Answer: (b)

Solution:

$$\lim_{x \to 0} \frac{\alpha e^x + \beta e^{-x} + \gamma \sin x}{x \sin^2 x} = \frac{2}{3}$$

For indeterminacy, $\alpha + \beta = 0 \qquad ....(i)$

$$\Rightarrow \lim_{x \to 0} \frac{\alpha e^x + \beta e^{-x} + \gamma \sin x}{x^3} \times \frac{x^2}{\sin^2 x} = \frac{2}{3}$$

Apply L-Hospital rule,

$$\Rightarrow \lim_{x \to 0} \frac{\alpha e^x - \beta e^{-x} + \gamma \cos x}{3x^2} = \frac{2}{3}$$

$\because \alpha - \beta + \gamma = 0 \qquad ...(ii)$

$$\Rightarrow \lim_{x\to 0}\frac{\alpha e^{x}+\beta e^{-x}-\gamma \sin x}{6x}=\frac{2}{3}$$

$$\Rightarrow \lim_{x\to 0}\frac{\alpha e^{x}-\beta e^{-x}-\gamma \cos x}{6}=\frac{2}{3}$$

$$\Rightarrow \alpha-\beta-\gamma=4 \quad \ldots.(\text{iii})$$

$$\Rightarrow \beta=-1,\alpha=1,\gamma=-2$$

Question: If $A=\{1,2,...,60\}$ and B is relation on A defined as $B=\{(x,y):y=pq$ where p and q are primes $\geq 3\}$ then number of elements in B is:

Options:

(a) 720

(b) 660

(c) 540

(d) 600

Answer: (b)

Solution:

Given $y=pq$ $\{p,q$ are prime numbers $\geq 3\}$

$\therefore y$ can be generated from

$3\times 3,3\times 5,3\times 7,3\times 11,3\times 13,3\times 17,3\times 19,5\times 5,5\times 7,5\times 11,7\times 7$

$\Rightarrow$ Total 11 possibilties

x can be $\{1,2,....,60\}$

Number of relations $=60\times 11=660$

Question: If $f(x)=3^{(x^2-2)^3}+4$ and

$P: f(x)$ attains maximum value at $x=0.$

$Q: f(x)$ have point of inflection at $x=\sqrt{2}.$

$R: f(x)$ is increasing for $x>\sqrt{2},$ then which of the following statement are correct?

Options:

(a) P and R

(b) Q and R

(c) P and Q

(d) P, Q and R all

Answer: (b)

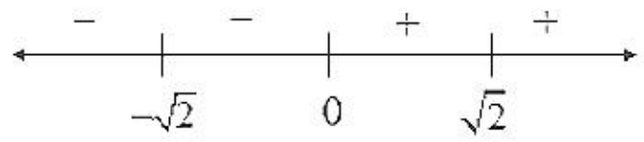
Solution:

Given $f(x)=3^{(x^2-2)^3}+4$

$\therefore f'(x)=3^{(x^2-2)^3}\cdot \ln 3\cdot 3(x^2-2)^2\,2x$

Now, $f'(x)=0,$ we get $x=0,\ x=\pm\sqrt{2}$

$\therefore$ $f(x)$ will have inflation point at $x=\sqrt{2}$



So, $f(x)$ is increasing for $x>\sqrt{2}$

And will make minimum at $x=0$

Question: Let $f(x)=|(x-1)|\cos|x-2|\sin|x-1|+|x-3||x^2-5x+4|$. The number of points where the function is not differentiable is:

Options:

(a) 3

(b) 4

(c) 5

(d) 6

Answer: (a)

Solution:

$f(x)=|(x-1)|\cos|x-2|\sin|x-1|+|x-3||x^2-5x+4|$

$f(x)=|(x-1)|\cos|x-2|\sin|x-1|+|x-3||(x-1)(x-4)|$

$f(x)=|(x-1)|\sin|x-1|\cdot\cos|x-2|+|x-3||(x-1)||(x-4)|$

We know, $|(x-a)|g(|x-a|)$ is differentiable when $x-a=0$

$\therefore$ $f(x)$ is non-differentiable at $x=1,3,4$

Question: Let A and B are two 3×3 non-zero real matrices and $AB=0$, then which of the following options is correct?

Options:

(a) $AX=B$ has unique solution

(b) $AX=B$ has infinite solutions

(c) B is invertible

(d) $(adj(A))B$ is invertible

Answer: (b)

Solution:

$\because$ $AB=0\Rightarrow|A|=0=|B|$

So, B is not invertible as $|B|=0$

$(adj(A))B$ is not invertible as $|adj(A)B|=|adj(A)||B|=0$

$AX=B$ has either no solution nor infinitely many solutions.

Question: If $|x-1| \le y \le \sqrt{5-x^2}$, then the area of region bounded by the curves is:

Options:

(a) $\frac{5\pi}{4}-\frac{1}{2}$

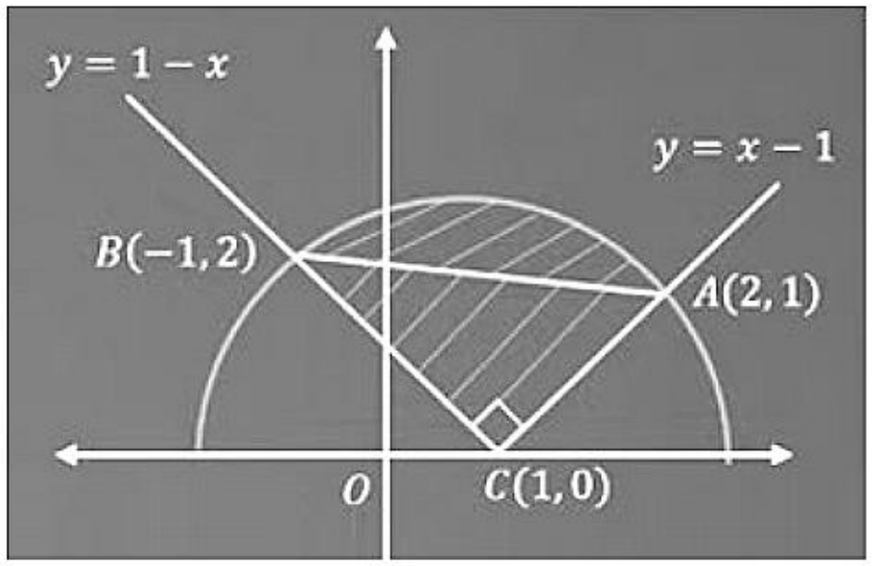
(b) $\frac{5\pi}{4}-\frac{3}{2}$

(c) $\frac{3\pi}{4}-\frac{1}{2}$

(d) $\cos^{-1}\frac{1}{3}-\frac{1}{2}$

Answer: (a)

Solution:



Clearly chord AB subtends a right angle at centre.

Required area = area of $\Delta ABC +$ area of segment of circle on chord AB

$= AC \cdot BC +$ [area of quarter circle – area of ΔAOB]

$= \frac{1}{2}\sqrt{2}\cdot 2\sqrt{2}+\left(\frac{5\pi}{4}-\frac{1}{2}\sqrt{5}\cdot\sqrt{5}\right)$

$= \frac{5\pi}{4}-\frac{1}{2}$

Question: A matrix of 3×3 order, should be filled either by 0 or 1 and sum of all elements should be prime number. Then the number of such matrix is equal to _____.

Answer: 282.00

Solution:

Let $A=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Now, sum of all elements to be prime numbers.

So, for 2, number of ways $= {}^{9}C_{2} = 36$

For 3, number of ways $= {}^{9}C_{3} = 84$

For 5, number of ways $= {}^9C_5 = 126$

For 7, number of ways $= {}^9C_7 = 36$

That's all prime number we can get

So total number of such matrix = 282.

Question: Let $a_1, a_2, a_3, ..., a_n$ are in A.P. and $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$, then $4a_2$ is equal to ____.

Answer: 16.00

Solution:

Given, $\sum_{r=1}^{\infty} \frac{a^r}{2^r} = 4$

$$\Rightarrow 4 = \frac{a_1}{2} + \frac{a^2}{2^2} + \frac{a^3}{2^3} + ...$$

$$\frac{4}{2} = \frac{a_1}{2^2} + \frac{a_2}{2^2} + ...$$

$$2 = \frac{a_1}{2} + \left(\frac{d}{2^2} + \frac{d}{2^3} + ... \right)$$

$$2 = \frac{a_1}{2} + \frac{\frac{d}{4}}{1 - \frac{1}{2}}$$

$$a_1 + d = 4$$

$$\Rightarrow 4a_2 = 4(a_1 + d) = 4 \times 4 = 16$$

Question: If $\frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + ... + \frac{1}{100 \cdot 101 \cdot 102} = \frac{k}{101}$, then $34k$ is equal to ____.

Answer: 286.00

Solution:

$$\frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + ... + \frac{1}{100 \cdot 101 \cdot 102} = \frac{k}{101}$$

$$\Rightarrow \frac{1}{2}\left(\frac{4-2}{2 \cdot 3 \cdot 4} + \frac{5-3}{3 \cdot 4 \cdot 5} + ... + \frac{102-100}{100 \cdot 101 \cdot 102} \right) = \frac{k}{101}$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{2 \cdot 3} - \frac{1}{3 \cdot 4} + \frac{1}{3 \cdot 4} - \frac{1}{4 \cdot 5} + ... + \frac{1}{100 \cdot 101} - \frac{1}{101 \cdot 102} \right) = \frac{k}{101}$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{2 \cdot 3} - \frac{1}{101 \cdot 102} \right) = \frac{k}{101}$$

$$\Rightarrow k = \frac{1}{2}\left(\frac{101}{2 \cdot 3} - \frac{1}{102} \right) = \frac{1}{2}\left(\frac{10296}{2 \cdot 3 \cdot 102} \right) = \frac{858}{102}$$

$$\therefore 34k = 286$$