2. MATRICES

I. MCQ (2 marks each)

Q. 1. The adjoint matrix of $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ is

a) $\begin{bmatrix} 4 & 8 & 3 \\ 2 & 1 & 6 \\ 0 & 2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ c) $\begin{bmatrix} 11 & 9 & 3 \\ 1 & 2 & 8 \\ 6 & 9 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 3 \\ -2 & 3 & -3 \end{bmatrix}$

Q. 2. $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $A^{-1}$is

a)A b)-A c) adj (A) d) -adj (A)

Q. 3. The solution (x, y, z) of the equation

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$ is (x, y, z) =

a)(1, 1, 1) b) (0, -1, 2) c) (-1, 2, 2) d) (-1, 0, 2)

Q. 4. If ω is a complex cube root of unity, then the matrix $A = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{bmatrix}$ is

a) Singular matrix b) Non-symmetric matrix

c) Skew-symmetric matrix d) Non- Singular matrix

Q. 5. If A =$\begin{bmatrix} 4 & -1 \\ -1 & k \end{bmatrix}$ such that A^2 - 6A +7I = 0, then k =…

a) 1 b) 3 c) 2 d) 4

Q. 6. $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ =…..

a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q. 7. If A =$\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$, then the only correct statement about the matrix A is….

a) $A^2 = I$ b) A is a zero matrix c) A^{-1} does not exit

d) A = (-1) I, where I is a unit matrix.

Q. 8. If A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then A^{10} =…..

a) $\begin{bmatrix} \cos 10\alpha & -\sin 10\alpha \\ \sin 10\alpha & \cos 10\alpha \end{bmatrix}$ b) $\begin{bmatrix} \cos 10\alpha & \sin 10\alpha \\ -\sin 10\alpha & \cos 10\alpha \end{bmatrix}$

c) $\begin{bmatrix} \cos 10\alpha & \sin 10\alpha \\ -\sin 10\alpha & -\cos 10\alpha \end{bmatrix}$ d) $\begin{bmatrix} \cos 10\alpha & -\sin 10\alpha \\ -\sin 10\alpha & -\cos 10\alpha \end{bmatrix}$

Q. 9. The element of second row and third column in the inverse of $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is…

a) -2 b) -1 c) 1 d) 2

Q. 10. If A = $\begin{bmatrix} 4 & 5 \\ 2 & 5 \end{bmatrix}$, then $|(2A)^{-1}|$ =…..

a) $\frac{1}{30}$ b) $\frac{1}{20}$ c) $\frac{1}{60}$ d) $\frac{1}{40}$

Q. 11. If $\begin{bmatrix} x-y-z \\ -y+z \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$, then the value of x, y and z are respectively…

a) 0, -3, 3 b) 1, -2 , 3 c) 5, 2, 2 d) 11, 8, 3

Q. 12 The value of x, y, z for the following system of equations

x + y + z = 6, x - y+ 2z = 5, 2x + y -z = 1 are…

a) x = 1, y = 2, z = 3 b) x = 2, y = 1, z = 3

c) x = -1, y = 2, z = 3 d) x = y = z = 3

Q. 13. If A = $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then $|A||adjA|$=…

a) 3^3 b) 3^9 c) 3^6 a) 3^{27}

Q. 14. System of equations x + y = 2 , 2x + 2y = 3 has….

a) no solution b) only one solution

c) many finite solutions. d) infinite solutions.

Q. 15. If A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, 10B = $\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \propto \\ 1 & -2 & 3 \end{bmatrix}$ and B is the inverse of matrix A, then α = ….

a) -2 b) -1 c) 2 d) 5

II. Very Short Answers (1 mark)

Q. 1. If A = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$, then find the value of $a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$

Q. 2. For an invertible matrix A, if A· (adjA) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$,then find the value of $|A|$.

Q. 3. If the inverse of the matrix $\begin{bmatrix} \alpha & 14 & -1 \\ 2 & 3 & 1 \\ 6 & 2 & 3 \end{bmatrix}$ does not exists then find the value of α.

Q. 4. If A = $\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then find the matrix $(B^{-1}A^{-1})^{-1}$

Q. 5. A = $\begin{bmatrix} cos\theta & -sin\theta \\ -sin\theta & -cos\theta \end{bmatrix}$ then find A^{-1}.

Q. 6. If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then find the value of $|A|^{-1}$

Q. 7. If A = $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$, and AB = BA= I , then find the matrix B.

Q. 8. If A(α) = $\begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}$ then prove that $A^2(\alpha) =$ A(2α)

Q. 9. If A = $\begin{bmatrix} 1 & 2 \\ 3 & -2 \\ -1 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 3 & 2 \\ 4 & -1 & 3 \end{bmatrix}$ then find the order of AB.

Q. 10. A+I = $\begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$ then find the value of (A+I)(A-I)

Q. 11. If A = $\begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$ then find A^2

Q. 12. If A = $\begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$ then find A^2

Q. 13. If A = $\begin{bmatrix} 0 & 3 & 3 \\ -3 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, find the matrix $B'(AB)$

III. Short Answers (2 marks)

Q. 1. If $f(x) = x^2 - 2x - 3$ then find $f(A)$ when A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Q. 2. If A = $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, B = $[3 \quad 1 \quad -2]$, find $B'A'$

Q. 3 If A is invertible matrix of order 3 and $|A| = 5$, then find $|adjA|$

Q. 4. If A = $\begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}$ and B = $\begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$ then find $A'B'$

Q. 5. If A = $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ then find $(A^{-1}B^{-1})$

Q. 6. If A = $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ then find the matrix X such that $A^{-1}X = B$

Q. 7. Find the matrix X such that AX = I where A = $\begin{bmatrix} 6 & 17 \\ 1 & 3 \end{bmatrix}$

Q. 8. Find A^{-1} using adjoint method, where A = $\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$

Q. 9. Find A^{-1} using column transformations :

i) A = $\begin{bmatrix} 5 & 3 \\ 3 & -2 \end{bmatrix}$ ii) A = $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Q. 10. Find the adjoint of matrix A = $\begin{bmatrix} 6 & 5 \\ 3 & 4 \end{bmatrix}$

Q. 11. Transform $\begin{bmatrix}1 & 2 & 4\\3 & -1 & 5\\2 & 4 & 6\end{bmatrix}$ into an upper triangular matrix by using suitable row transformations.

IV. Short answers (3 Marks)

Q. 1. If A = $\begin{bmatrix}0 & 4 & 3\\1 & -3 & -3\\-1 & 4 & 4\end{bmatrix}$, then find A^2 and hence find A^{-1}

Q. 2 If A = $\begin{bmatrix}0 & 1\\2 & 3\\1 & -1\end{bmatrix}$ and B = $\begin{bmatrix}1 & 2 & 1\\2 & 1 & 0\end{bmatrix}$, then find $(AB)^{-1}$

Q. 3. If A = $\begin{bmatrix}-4 & -3 & -3\\1 & 0 & 1\\4 & 4 & 3\end{bmatrix}$, find adj(A)

Q. 4. Solve the following by inversion method $2x + y = 5$, $3x + 5y = -3$

Q. 5. If A=$\begin{bmatrix}1 & 2 & -1\\3 & -2 & 5\end{bmatrix}$, apply $R_1 \leftrightarrow R_2$ and then $C_1 \rightarrow C_1 + 2C_3$ on A.

Q. 6. Three chairs and two tables costs ₹ 1850. Five chairs and three tables costs ₹2850. Find the cost of four chairs and one table by using matrices.

Q. 7. If A = $\begin{bmatrix}4 & 5\\2 & 1\end{bmatrix}$, show that $A^{-1} = \frac{1}{6}(A - 5I)$

Q. 8. Find the adjoint of matrix A = $\begin{bmatrix}2 & 0 & -1\\3 & 1 & 2\\-1 & 1 & 2\end{bmatrix}$

Q. 9. Find the matrix X such that $\begin{bmatrix}1 & 2 & 3\\2 & 3 & 2\\1 & 2 & 2\end{bmatrix} X = \begin{bmatrix}2 & 2 & -5\\-2 & -1 & 4\\1 & 0 & -1\end{bmatrix}$,

Q. 10. Find the inverse of A = $\begin{bmatrix}\sec\theta & \tan\theta & 0\\\tan\theta & \sec\theta & 0\\0 & 0 & 1\end{bmatrix}$

Q. 11. Transform $\begin{bmatrix}1 & 2 & 4\\3 & -1 & 5\\2 & 4 & 6\end{bmatrix}$ into an upper triangular matrix by using suitable row transformations.

Q. 12. If A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$, then find the matrix X such that XA = B

V. Long answers (4 Marks)

Q. 1. Find the inverse of A = $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$ by using elementary row transformations.

Q. 2. If A = $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$, find A^{-1} by the adjoint method.

Q. 3. Solve the following equations by using inversion method.

x + y + z = -1 , x - y + z = 2 and x + y - z = 3

Q. 4. If three numbers are added, their sum is 2. If 2 times the second number is subtracted from the sum of first and third numbers, we get 8. If three times the first number is added to the sum of second and third numbers, we get 4. Find the numbers using matrices.

Q. 5. Find the inverse of A= $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ by elementary column transformations.

Q. 6. If A = $\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is a non-singular matrix, then find A^{-1} by using elementary row transformations. Hence, write the inverse of $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

Q. 7. Find the inverse of A = $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ by using elementary row transformations.

Q. 8. If A = $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$, and C = $\begin{bmatrix} 24 & 7 \\ 31 & 9 \end{bmatrix}$, then find the matrix X such that AXB = C.

Q. 9. If A = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$, verify that A(adj A) = (adj A)A.

Q. 10. If A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$, find AB and $(AB)^{-1}$.

Q. 11. Solve the following system of equations by using inversion method.

$x + y = 1$, $y + z = \frac{5}{3}$ and $z + x = \frac{4}{3}$

Q12. Express the following equations in the matrix form and solve them by the method of reduction.

$x + y + z = 6$, $3x - y + 3z = 6$, $5x + 5y - 4z = 3$.

Q. 13. The cost of 4 dozen pencils, 3 dozen pens and 2 dozen erasers is ₹ 60. The cost of 2 dozen pencils, 4 dozen pens and 6 dozen erasers is ₹ 90. Whereas the cost of 6 dozen pencils, 2 dozen pens and 3 dozen erasers is ₹ 70. Find the cost of each item per dozen by using matrices.

Q14. The total cost of 3 televisions and 2 VCR's is Rs.35,000. The shopkeeper wants profit of Rs.1000 per television and Rs.500 per VCR. He sells 2 televisions and 1 VCR and gets the total revenue as Rs.21,500. Find the cost of a television and a VCR by using reduction method.