MHT-CET 2022 Question Paper

6^{th} August 2022 (Shift – I)

1. If matrix A = $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ is such that AX = I, where I is 2 × 2 unit matrix, then X =

   (A) $\frac{1}{5}\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$ (B) $\frac{1}{5}\begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix}$

   (C) $\frac{1}{5}\begin{bmatrix} -3 & -2 \\ -4 & -1 \end{bmatrix}$ (D) $\frac{1}{5}\begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix}$

2. $\int_{-\pi/2}^{\pi/2} f(x)dx =$

   Where f(x) = sin |x| + cos |x|, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

   (A) 0 (B) 2

   (C) 4 (D) 8

3. The principal solutions of tan 3θ = –1 are

   (A) $\left\{\frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{\pi}{16}, \frac{19\pi}{4}, \frac{23\pi}{24}\right\}$

   (B) $\left\{\frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{19\pi}{12}, \frac{23\pi}{12}\right\}$

   (C) $\left\{\frac{\pi}{4}, \frac{\pi}{12}\right\}$

   (D) $\left\{\frac{\pi}{4}, \frac{\pi}{12}, \frac{13\pi}{12}, \frac{7\pi}{4}, \frac{19\pi}{4}, \frac{23\pi}{12}\right\}$

4. For three simple statements p, q, and r, p ⟶ (q ∨ r) is logically equivalent to

   (A) (p ∨ q) ⟶ r

   (B) (p ⟶ ~q) ∧ (p ⟶ r)

   (C) (p ⟶ q) ∨ (p ⟶ r)

   (D) (p ⟶ q) ∧ (p ⟶ ~r)

5. If $\bar{a}$ and $\bar{b}$ are two vectors such that $|\bar{a}| = |\bar{b}| = \sqrt{2}$ with $\bar{a}\cdot\bar{b} = -1$, then the angle between $\bar{a}$ and $\bar{b}$ is

   (A) $\frac{2\pi}{3}$ (B) $\frac{5\pi}{6}$

   (C) $\frac{5\pi}{9}$ (D) $\frac{3\pi}{4}$

6. Argument of $\frac{1-i\sqrt{3}}{1+i\sqrt{3}}$ is

   (A) 60° (B) 210°

   (C) 120° (D) 240°

7. $\int \frac{5(x^6+1)}{x^2+1}dx =$

   (where C is a constant of integration.)

   (A) $\frac{5x^7}{7} + 5x + 5\tan^{-1}x + C$

   (B) $5\tan^{-1}x + \log(x^2+1) + C$

   (C) $5(x^7+1) + \log(x^2+1) + C$

   (D) $x^5 - \frac{5x^3}{3} + 5x + C$

8. Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is

   (A) not arithmetic mean of a and b.

   (B) the geometric mean of a and b.

   (C) the arithmetic mean of a and b.

   (D) the harmonic mean of a and b.

9. $\lim_{x\to 0}\left(\frac{1+\tan x}{1+\sin x}\right)^{\operatorname{cosec} x} =$

   (A) 0 (B) e

   (C) 1 (D) $\frac{1}{e}$

10. If $y = \sec^{-1}\left(\frac{x+x^{-1}}{x-x^{-1}}\right)$, then $\frac{dy}{dx} =$

    (A) $\frac{-2}{1+x^2}$ (B) $\frac{-1}{1+x^2}$

    (C) $\frac{2}{1-x^2}$ (D) $\frac{1}{1+x^2}$

11. If the line passing through the points (a, 1, 6) and (3, 4, b) crosses the yz – plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$, then

    (A) a = 5, b = 1 (B) a = –5, b = 1

    (C) a = –5, b = –1 (D) a = 5, b = –1

12. 20 meters of wire is available to fence of a flowerbed in the form of a circular sector. If the flowerbed is to have maximum surface area, then the radius of the circle is

    (A) 8 m (B) 5 m

    (C) 2 m (D) 4 m

13. Five letters are placed at random in five addressed envelopes. The probability that all the letters are not dispatched in the respective right envelopes is

(A) $\frac{4}{5}$ (B) $\frac{119}{120}$

(C) $\frac{1}{120}$ (D) $\frac{1}{5}$

14. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A =

(A) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

15. The general solution of the differential equation $x^2 + y^2 - 2xy\frac{dy}{dx} = 0$ is

(where C is a constant of integration.)

(A) $2(x^2 - y^2) + x = C$

(B) $x^2 + y^2 = Cy$

(C) $x^2 - y^2 = Cx$

(D) $x^2 + y^2 = Cx$

16. If the lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 sq. units, then equation of the circle is

$\left(\text{Taken } \pi = \frac{22}{7}\right)$

(A) $x^2 + y^2 - 2x - 2y - 49 = 0$

(B) $x^2 + y^2 - 2x + 2y - 49 = 0$

(C) $x^2 + y^2 - 2x - 2y - 47 = 0$

(D) $x^2 + y^2 - 2x + 2y - 47 = 0$

17. The joint equation of two lines passing through the origin and perpendicular to the lines given by $2x^2 + 5xy + 3y^2 = 0$ is

(A) $3x^2 - 5xy + 2y^2 = 0$

(B) $3x^2 - 5xy - 2y^2 = 0$

(C) $2x^2 - 5xy + 3y^2 = 0$

(D) $3x^2 + 5xy + 2y^2 = 0$

18. $\int \frac{e^x}{(2+e^x)(e^x+1)} dx =$

(where C is a constant of integration.)

(A) $\log\left(\frac{e^x+2}{e^x+1}\right) + C$

(B) $\log\left(\frac{e^x}{e^x+2}\right) + C$

(C) $\frac{e^x+1}{e^x+2} + C$

(D) $\log\left(\frac{e^x+1}{e^x+2}\right) + C$

19. The function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonically increasing in the interval

(A) $(-\infty, \infty)$ (B) $(-\infty, 1) \cup (2, \infty)$

(C) $(-\infty, 1)$ (D) $(2, \infty)$

20. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -3 \\ -1 & 2 & 3 \end{bmatrix}$, then $A_{31} + A_{32} + A_{33} =$

where A_{ij} is cofactor of a_{ij}, where $A = [a_{ij}]_{3\times3}$

(A) 0 (B) 1

(C) 10 (D) 11

21. The objective function of L.L.P. defined over the convex set attains its optimum value at

(A) none of the corner points.

(B) at least two of the corner points.

(C) all the corner points.

(D) at least one of the corner points.

22. A round table conference is to be held amongst 20 countries. If two particular delegates wish to sit together, then such arrangements can be done in ________ ways.

(A) 18! (B) $\frac{19!}{2!}$

(C) $2 \times (18)!$ (D) $19! \times 2!$

23. The general solution of differential equation $e^{\frac{1}{2}\left(\frac{dy}{dx}\right)} = 3^x$ is

(where C is a constant of integration.)

(A) $x = (\log 3)y^2 + C$

(B) $y = x^2 \log 3 + C$

(C) $y = x\log 3 + C$

(D) $y = 2x\log 3 + C$

24. If $x^y = e^{x-y}$, then $\frac{dy}{dx} =$

(A) $\frac{\log x}{(1+\log x)^2}$ (B) $\frac{\log x}{1+\log x}$

(C) $\frac{x\log x}{(1+\log x)^2}$ (D) $\frac{\log x}{x(1+\log x)^2}$

25. The vector projection of $\bar{b}$ on $\bar{a}$, where $\bar{a} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ and $\bar{b} = 7\hat{i} - 5\hat{j} - \hat{k}$ is

(A) $\frac{3(3\hat{i}+2\hat{j}+5\hat{k})}{\sqrt{38}}$ (B) $\frac{9\hat{i}+6\hat{j}+15\hat{k}}{19}$

(C) $\frac{3(3\hat{i}+2\hat{j}+5\hat{k})}{38}$ (D) $\frac{6(3\hat{i}+2\hat{j}+5\hat{k})}{\sqrt{38}}$

26. The equation of the line perpendicular to $2x - 3y + 5 = 0$ and making an intercept 3 with positive Y-axis is

(A) $3x + 2y - 6 = 0$

(B) $3x + 2y - 12 = 0$

(C) $3x + 2y - 7 = 0$

(D) $3x + 2y + 6 = 0$

27. If $\int \frac{2e^x + 3e^{-x}}{3e^x + 4e^{-x}}\,dx = Ax + B\log(3e^{2x} + 4) + C$, then values of A and B are respectively (where C is a constant of integration.)

(A) $\frac{3}{4}, \frac{1}{24}$ (B) $\frac{4}{3}, -24$
(C) $\frac{1}{4}, \frac{1}{24}$ (D) $\frac{3}{4}, \frac{-1}{24}$

28. If the slope of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is two times the other, then

(A) $8h^2 = 9ab$ (B) $8h = 9ab$
(C) $8h^2 = 9ab^2$ (D) $8h = 9ab^2$

29. Two numbers are selected at random from the first six positive integers. If X denotes the larger of two numbers, then Var (X) =

(A) $\frac{14}{3}$ (B) $\frac{14}{9}$
(C) $\frac{1}{3}$ (D) $\frac{70}{3}$

30. The ratio in which the plane $\bar{r}\cdot(\hat{i} - 2\hat{j} + 3\hat{k}) = 17$ divides the line joining the points $-2\hat{i} + 4\hat{j} + 7\hat{k}$ and $3\hat{i} - 5\hat{j} + 8\hat{k}$ is

(A) 5 : 3 (B) 4 : 5
(C) 3 : 10 (D) 10 : 3

31. If surrounding air is kept at 20 °C and body cools from 80 °C to 70 °C in 5 minutes, then the temperature of the body after 15 minutes will be

(A) 54.7 °C (B) 51.7 °C
(C) 52.7 °C (D) 50.7 °C

32. A random variable X has the following probability distribution

X	0	1	2	3	4	5	6
P (X)	k	3k	5k	7k	9k	11k	13k

then P (X ≥ 2) =

(A) $\frac{1}{49}$ (B) $\frac{45}{49}$
(C) $\frac{40}{49}$ (D) $\frac{15}{49}$

33. Give that $f(x) = \frac{1 - \cos 4x}{x^2}$ if $x < 0$

$= a$ if $x = 0$

$= \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}$ if $x > 0$,

is continuous at $x = 0$, then a =

(A) 16 (B) 2
(C) 4 (D) 8

34. The area of the region bounded by the y-axis, $y = \cos x$, $y = \sin x$, when $0 \le x \le \frac{\pi}{4}$, is

(A) $\sqrt{2}$ sq. units
(B) $2(\sqrt{2} - 1)$ sq. units
(C) $(\sqrt{2} - 1)$ sq. units
(D) $(\sqrt{2} + 1)$ sq. units

35. Given three vectors $\bar{a}$, $\bar{b}$, $\bar{c}$, two of which are collinear. If $\bar{a} + \bar{b}$ is collinear with $\bar{c}$ and $\bar{b} + \bar{c}$ is collinear with $\bar{a}$ and $|\bar{a}| = |\bar{b}| = |\bar{c}| = \sqrt{2}$, then $\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a} =$

(A) –3 (B) 5
(C) 3 (D) –1

36. In a triangle ABC, with usual notations $\angle A = 60°$, then $\left(1 + \frac{a}{c} + \frac{b}{c}\right)\left(1 + \frac{c}{b} - \frac{a}{b}\right) =$

(A) 3 (B) $\frac{1}{2}$
(C) $\frac{3}{2}$ (D) 1

37. If $y = 4x - 5$ is tangent to the curve $y^2 = px^3 + q$ at (2, 3), then

(A) p = –2, q = 7 (B) p = 2, q = –7
(C) p = 2, q = 7 (D) p = –2, q = –7

38. Which of the following statement pattern is a contradiction?

(A) $S_4 \equiv (\sim p \wedge q) \vee (\sim q)$
(B) $S_2 \equiv (p \rightarrow q) \vee (p \wedge \sim q)$
(C) $S_1 \equiv (\sim p \vee \sim q) \vee (p \vee \sim q)$
(D) $S_3 \equiv (\sim p \wedge q) \wedge (\sim q)$

39. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$, then $\tan 2\alpha =$

(A) $\frac{20}{7}$ (B) $\frac{56}{33}$
(C) $\frac{19}{12}$ (D) $\frac{25}{16}$

40. If the position vectors of the points A and B are $3\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} - 2\hat{j} - 4\hat{k}$ respectively, then the equation of the plane through B and perpendicular to AB is

(A) $2x + 3y + 6z + 28 = 0$
(B) $2x + 3y + 6z - 11 = 0$
(C) $2x - 3y - 6z - 32 = 0$
(D) $2x + 3y + 6z + 9 = 0$

41. The particular solution of the differential equation $\frac{dy}{dx} - e^x = ye^x$, when $x = 0$ and $y = 1$ is
(A) $\log\left(\frac{y+1}{2}\right) = e^x - 1$
(B) $\log(y - 1) = e^x - 1$
(C) $\log 2(y - 1) = e^x - 1$
(D) $\log\left(\frac{y+1}{2}\right) = \frac{e^x}{2} - \frac{1}{2}$

42. If the standard deviation of first n natural numbers is 2, then the value of n is
(A) 6 (B) 7
(C) 5 (D) 4

43. If $\bar{a}, \bar{b}, \bar{c}$ are position vectors of points A, B, C respectively, with $2\bar{a} + 3\bar{b} - 5\bar{c} = \bar{0}$, then the ratio in which point C divides segment AB is
(A) 3:2 externally (B) 2:3 externally
(C) 3:2 internally (D) 2:3 internally

44. The second derivative of a $\sin^3 t$ w.r.t. a $\cos^3 t$ at $t = \pi/4$ is
(A) $\frac{-4\sqrt{2}}{3a}$ (B) $\frac{4\sqrt{2}}{3a}$
(C) $\frac{4\sqrt{3}}{3a}$ (D) $\frac{1}{12a}$

45. $\int_2^3 \frac{\log x}{x} dx =$
(A) $\frac{1}{2}\log 6 \log 3$ (B) $\log 6 \log \frac{3}{2}$
(C) $\frac{1}{2}\log 6 \log \frac{3}{2}$ (D) $2\log 6 \log \frac{3}{2}$

46. With reference to the principal values, if $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then $x^{100} + y^{100} + z^{100} =$
(A) 1 (B) 2 (C) 3 (D) 6

47. For the differential equation $\left[1-\left(\frac{dy}{dx}\right)^2\right]^{\frac{5}{2}} = 8\frac{d^2y}{dx^2}$ has the order and degree _______ respectively.
(A) 2 and 6 (B) 2 and 3
(C) 2 and 2 (D) 2 and 1

48. The angle between two lines $\frac{x+1}{2} = \frac{y+3}{2} = \frac{z-4}{-1}$ and $\frac{x-4}{1} = \frac{y+4}{2} = \frac{z+1}{2}$ is
(A) $\cos^{-1}\left(\frac{4}{9}\right)$ (B) $\cos^{-1}\left(\frac{1}{9}\right)$
(C) $\cos^{-1}\left(\frac{2}{9}\right)$ (D) $\cos^{-1}\left(\frac{5}{9}\right)$

49. If $f(x) = \frac{a^x - a^{-x}}{a^x + a^{-x}}$, where a, x satisfy the necessary conditions, then $f^{-1}(x) =$
(A) $\frac{1}{2}\log_a\left(\frac{x}{1-x}\right)$ (B) $\frac{1}{2}\log_a\left(\frac{1+x}{x}\right)$
(C) $\frac{1}{2}\log_a\left(\frac{1+x}{1-x}\right)$ (D) $\frac{1}{2}\log_a\left(\frac{2+x}{2-x}\right)$

50. For a Binomial distribution, n = 6, if $9P(X = 4) = P(X = 2)$, then q =
(A) $\frac{2}{5}$ (B) $\frac{3}{4}$
(C) $\frac{1}{4}$ (D) $\frac{1}{2}$