मोशन है, तो भरोसा है

MOTION
18 YEARS OF LEGACY

JEE ADVANCED 2025

QUESTION PAPER WITH SOLUTIONS

MATHS [PAPER – 1]

SECTION 1 (Maximum Marks: 12)

1. Let $\mathbb{R}$ denote the set of all real numbers. Let $a_i, b_i \in \mathbb{R}$ for $i \in \{1, 2, 3\}$.

Define the functions $f : \mathbb{R} \to \mathbb{R}, g : \mathbb{R} \to \mathbb{R}$, and $h : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = a_1 + 10x + a_2x^2 + a_3x^3 + x^4$$

$$g(x) = b_1 + 3x + b_2x^2 + b_3x^3 + x^4,$$

$$h(x)=f(x+1)-g(x+2).$$

If $f(x) \neq g(x)$ for every $x \in \mathbb{R}$, then the coefficient of x^3 in $h(x)$ is

(A) 8 (B) 2 (C) – 4 (D) – 6

Sol. **C**

$$h(x)=f(x+1)-g(x+2)$$

now $h(x) = \left[a_1 + 10(x+1) + a_2(x+1)^2 + a_3(x+1)^3\right.$

$\left. +(x+1)^4\right] - \left[b_1 + 3(x+2) + b_2(x+2)^2 + b_3(x+2)^3 + (x+2)^4\right]$

coeff of x^3 in $h(x) = a_3 + 4 - b_3 + 8$

$$c = a_3 - b_3 - 4$$

Now $f(x) \neq g(x) \quad \forall x \in R$

$$a_1 - b_1 \; + (10-3)x + \; a_2 - b_2 \; x^2 + \; a_3 - b_3 \; x^3 \neq 0 \quad \forall x \in R$$

$a_1 - b_1 \; + 7x + \; a_2 - b_2 \; x^2 + \; a_3 - b_3 \; x^3 = 0$ has no solution

$$\Rightarrow a_3 - b_3 = 0$$

$$c = a_3 - b_3 - 4 = 0 - 4 = -4$$

2. Three students S_1, S_2, and S_3 are given a problem to solve. Consider the following events:

U : At least one of S_1, S_2, and S_3 can solve the problem,

V : S_1 can solve the problem, given that neither S_2 nor S_3 can solve the problem,

W : S_2 can solve the problem and S_3 cannot solve the problem,

T : S_3 can solve the problem.

For any event E, let P(E) denote the probability of E. If

$$P(U) = \frac{1}{2}, \; P(V) = \frac{1}{10} \text{ and } P(W) = \frac{1}{12}$$

Then P(T) is equal to

(A) $\frac{13}{36}$ (B) $\frac{1}{3}$ (C) $\frac{19}{60}$ (D) $\frac{1}{4}$

Sol. **C**

$$U = 1 - P \; \overline{S_1} \cap \overline{S_2} \cap \overline{S_3}$$

$$V = P \; S_1 \cap \overline{S_2} \cap \overline{S_3}$$

$$W = P \; S_2 \cap \overline{S_3}$$

$$T = P \; S_3$$

Let $P \; S_1 \; = x$

$$P \; S_2 \; = y$$

$P\ S_3 \ = z$

Now $P(U) = 1-(1-x)(1-y)(1-z) = \frac{1}{2}$

$(1\text{-}x)(1\text{-}y)(1\text{-}z) = \frac{1}{2}$ … (1)

$P(V) = x(1-y)(1-z) = \frac{1}{10}$ … (2)

$P(W) = \quad y(1-z) = \frac{1}{12}$ … (3)

By equation (1) and (3) $\frac{1-x}{x} = 5 \Rightarrow x = \frac{1}{6}$

By equation (2) and (3) $x\left(\frac{1-y}{y}\right) = \frac{6}{5} \Rightarrow y = \frac{5}{41}$

Now from (3)

$1-z = \frac{1}{12}\times\frac{41}{5}$

$z = 1-\frac{41}{60} = \frac{19}{60}$

3. Let $\mathbb{R}$ denote the set of all real numbers. Define the function $f:\mathbb{R}\to\mathbb{R}$ by

$$f(x) = \begin{cases} 2-2x^2-x^2\sin\frac{1}{x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$$

Then which one of the following statements is TRUE?

(A) The function f is NOT differentiable at x = 0

(B) There is a positive real number δ, such that f is a decreasing function on the interval (0, δ)

(C) For any positive real number δ, the function f is NOT an increasing function on the interval (–δ, 0)

(D) x = 0 is a point of local minima of f

Sol. **B**

$$f(x) = \begin{cases} 2-2x^2-x^2\sin\frac{1}{x} & x \neq 0 \\ 2 & x = 0 \end{cases}$$

(A)
$$\left.\begin{aligned} \text{RHD} &= \lim_{x\to 0^+}\frac{2-2x^2-x^2\sin\frac{1}{x}-2}{x} = 0 \\ \text{LHD} &= \lim_{x\to 0^-}\frac{2-2x^2+x^2\sin\frac{1}{x}-2}{-x} = 0 \end{aligned}\right\}\ \text{diff at } x = 0$$

(B) $f(x) = 2 - x^2\left(2+\sin\frac{1}{x}\right); x \neq 0$

$f(0) = 2$

$f(0^+) < 2,\ f(0^-) < 2$

$\Rightarrow$ f(x) has local maxima at x = 0

4. Consider the matrix

$$P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Let the transpose of a matrix X be denoted by X^T. Then the number of 3×3 invertible matrices Q with itneger entries, such that $Q^{-1} = Q^T$ and PQ = QP,

(A) 32 (B) 8 (C) 16 (D) 24

Sol. C

$$P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \text{ let } Q = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$Q^{-1} = Q^T$ and PQ = OP

$\Rightarrow Q^{-1}Q = Q^T \cdot Q$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = \begin{bmatrix} a_1 & a_1 & a_1 \\ b_1 & b_1 & a_2 \\ c_1 & c_1 & a_3 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$Q^T Q = I \Rightarrow Q$ is orthogonal

$\Rightarrow \quad \sum a_i^2 = 1 = \sum b_1^2 = \sum c_1^2$

$$\begin{bmatrix} 2a_1 & 2a_2 & 2a_3 \\ 2b_1 & 2b_2 & 2b_3 \\ 3a_1 & 3a_2 & 3c_3 \end{bmatrix} = \begin{bmatrix} 2a_1 & 2a_2 & 3a_3 \\ 2b_1 & 2b_2 & 3b_3 \\ 2a_1 & 2c_2 & 3c_3 \end{bmatrix}$$

$\sum a_i b_i = \sum b_i c_i = \sum c_i a_i = 0$

$2a_3 = 3\ a_3 \Rightarrow \dfrac{a_3 = 0}{b_3 - 0}$

$a_1^2 + a_2^2 = 1$

$a = 0$

$b_1^2 + b_2^2 = 1$

$c_2 = 0$

$c_3^2 = 1$

$$Q = \begin{bmatrix} a_1 & a_2 & 0 \\ b_1 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}$$

(I) $Q_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & \pm_1 \end{bmatrix}$ (4)

(III) $Q_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & \pm 1 \end{bmatrix}$ (4)

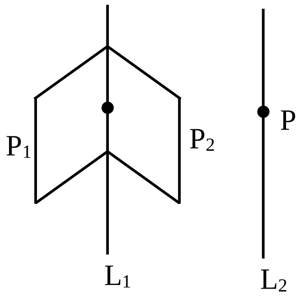
(I) $Q_2 = \begin{bmatrix} 0 & 1 & 0 \\ \pm 1 & 0 & 0 \\ 0 & 0 & \pm 1 \end{bmatrix}$ (4)

(II) $Q_2 = \begin{bmatrix} 0 & -1 & 0 \\ \pm 1 & 1 & 0 \\ 0 & 0 & \pm 1 \end{bmatrix}$ (4)

Ans 16

5. Let L_1 be the line of intersection of the planes given by the equations
$2x + 3y + z = 4$ and $x + 2y + z = 5$.
Let L_2 be the line passing through the point $P(2, -1, 3)$ and parallel to L_1. Let M denote the plane given by the equation
$2x + y - 2z = 6$.
Suppose that the line L_2 meets the plane M at the point Q. Let R be the foot of the perpendicular drawn from P to the plane M.
Then which of the following statements is (are) TRUE?
(A) The length of the line segment PQ is $9\sqrt{3}$
(B) The length of the line segment QR is 15
(C) The are of ΔPQR is $\frac{3}{2}\sqrt{234}$
(D) The acute angle between the line segments PQ and PR is $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

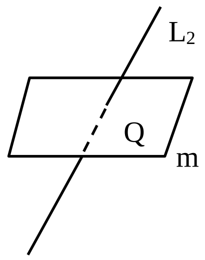
Sol. AC



$$v_{L_1} : \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{vmatrix} = \langle \hat{i} - \hat{j} + \hat{k} \rangle$$

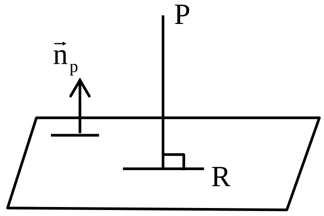
$$L_2 : \frac{x-2}{1} = \frac{y+1}{-1} = \frac{z-3}{1} = \lambda$$

m: $2x + y - 2z = 6$



Let Q on $L_2 : (\lambda + 2, -1 - \lambda, 3 + \lambda)$
lie on m
$\Rightarrow 2\lambda + 4 - 1 - \lambda - 6 - 2\lambda = 6 \Rightarrow \lambda = -9$
$Q : (-7, 8, -6)$

$$R : \frac{\alpha - 2}{2} = \frac{\beta + 1}{1} = \frac{\gamma - 3}{-2} = -\left(\frac{4 - 1 - 6 - 6}{4 + 1 + 4}\right)$$



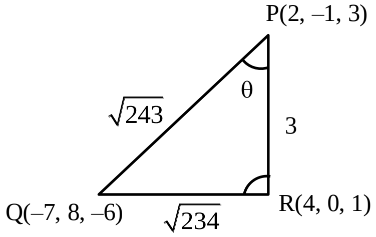
$$\frac{\alpha - 2}{2} = \frac{\beta + 1}{1} = \frac{\gamma - 3}{-2} = +1$$

$R : (4, 0, 1)$

(A) $PQ = \sqrt{(2+7)^2 + (8+1)^2 + (3+6)^2} = \sqrt{81 + 81 + 81} = 9\sqrt{3}$

(B) $QR = \sqrt{(-7-4)^2 + (8-0)^2 + (-6-1)^2} = \sqrt{121 + 64 + 49} = \sqrt{234}$

(C)



$|\overrightarrow{PR}| = \sqrt{4+1+4} = 3$

$|\overrightarrow{PQ}| = \sqrt{121+64+49} =$

$\Delta PQR = \frac{1}{2} \times 3 \times \sqrt{233} = +$

$\cos\theta = \frac{PR}{PQ} = \frac{3}{9\sqrt{3}} = \frac{1}{3\sqrt{3}}$

Ans AC

6. Let $\mathbb{N}$ denote the set of all natural numbers, and $\mathbb{Z}$ denote the set of all integers. Consider the functions $f : \mathbb{N} \to \mathbb{Z}$ and $g : \mathbb{Z} \to \mathbb{N}$ defined by

$$f(n) = \begin{cases} (n+1)/2 & \text{if } n \text{ is odd} \\ (4-n)/2 & \text{if } n \text{ is even} \end{cases}$$

and

$$g(n) = \begin{cases} 3+2n & \text{if } n \geq 0 \\ -2n & \text{if } n < 0 \end{cases}$$

Define $(gof)(n) = g(f(n))$ for all $n \in \mathbb{N}$, and $(fog)(n) = f(g(n))$ for all $n \in \mathbb{Z}$.

Then which of the following statements is (are) TRUE?

(A) gof is NOT one-one and gof is NOT onto

(B) fog is NOT one-one but fog is onto

(C) g is one-one and g is onto

(D) f is NOT one – one but f is onto

Sol. AD

$f(2n-1) = n \quad ; n \in N$

$f(2n) = 2 - n \quad n \in N$

$f(1) = f(2) \Rightarrow f(n)$ is not one – one also f(x) is onto

$g(n) = 2n + 3; \quad n \in w$

$g(-n) = 2n \quad n \in N$

$g(n) \neq 1 \to g(n)$ is one-one but not onto

$$gof(n) = \begin{cases} n+4; & n \to \text{odd} \\ n-4; & n \to \text{ even } -\{2,4\} \\ 5; & 2 \\ 3; & 4 \end{cases}$$

$gof(1) = gof(2)$ also $gof(n) \neq 1$

gof(n) is not one one and gof(x) is not onto

$fog(n) = n + 2; n \in Z$ one-one and onto

7. Let $\mathbb{R}$ denote the set of all real numbers. Let $z_1 = 1+2i$ and $z_2 = 3i$ be two complex numbers, where $i = \sqrt{-1}$. Let

$S = \ (x, y) \in \mathbb{R} \times \mathbb{R} : |x + iy - z_1| = 2|x + iy - z_2| \ .$

Then which of the following statements is (are) TRUE

(A) S is a circle with centre $\left(-\frac{1}{3}, \frac{10}{3}\right)$

(B) S is a circle with centre $\left(\frac{1}{3}, \frac{8}{3}\right)$

(C) S is a circle with radius $\frac{\sqrt{2}}{3}$

(D) S is a circle radius $\frac{2\sqrt{2}}{3}$

Sol. **AD**

|x+i y-1-2i|=2|x+i y-3i|

$\Rightarrow (x-1)^2 + (y-2)^2 = 4(x)^2 + 4(y-3)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 = 4x^2 + 4y^2 - 24y + 36$

$\Rightarrow 3x^2 + 3y^2 + 2x - 20y + 31 = 0$

$x^2 + y^2 + \frac{2}{3}x - \frac{20}{3}y + \frac{31}{3} = 0$

Circle with center $= \left(-\frac{1}{3}, \frac{10}{3}\right)$

Raidus = $\sqrt{\frac{1}{9} + \frac{100}{9} - \frac{31}{3}}$

$\Rightarrow \sqrt{\frac{101-93}{9}} = \frac{2\sqrt{2}}{3}$

(A), (D) is correct

8. Let the set of all relations R on the set $\{a, b, c, d, e, f\}$, such that R is reflexive and symmetric, and R contains exactly 10 elements, be denoted by S.
Then the number of elements in S is _____.

Sol. **105**

		a	b	c	d	e	f
a		✓					
b			✓				
c				✓			
d					✓		
e						✓	
f							✓

$\frac{5 \times 6}{2} = 15$ elements

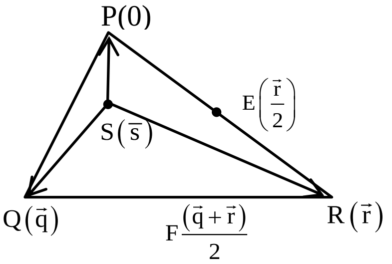
Total no of relation = ${}^{15}C_2 = \frac{15 \times 14}{2}$

= 105

9. For any two points M and N in the XY-plane, let $\overrightarrow{MN}$ denote the vector from M to N, and $\vec{0}$ denote the zero vector. Let P, Q and R be three distinct points in the XY-plane. Let S be a point inside the triangle $\triangle PQR$ such that $\overrightarrow{SP}+5\overrightarrow{SQ}+6\overrightarrow{SR}=\vec{0}$.
Let E and F be the mid-points of the sides PR and QR, respectively. Then the value of

$$\frac{\text{length of the line segment EF}}{\text{length of the line segment ES}}$$ is _____.

Sol. 1.2



let $p=\vec{0}$ (origin)

given

$-\vec{s}+5(\vec{q}-\vec{s})+6(\vec{r}-\vec{s})=0$

$12\vec{s}=5\vec{q}+6\vec{r}$

$\vec{s}=\frac{5}{12}\vec{a}+\frac{\vec{r}}{2}$

Asked $\frac{|\overrightarrow{EF}|}{|\overrightarrow{ES}|}=\frac{\left|\frac{\vec{q}+\vec{r}}{2}-\frac{\vec{r}}{2}\right|}{\left|\vec{s}-\frac{\vec{r}}{2}\right|}$

$=\frac{\left|\frac{\vec{q}}{2}\right|}{\left|\frac{5}{12}\vec{q}+\frac{\vec{r}}{2}-\frac{\vec{r}}{2}\right|}=\frac{\left|\frac{\vec{q}}{2}\right|}{\left|\frac{5\vec{q}}{12}\right|}\Rightarrow\frac{6}{5}=1.2$

10. Let S be the set of all seven-digit numbers that can be formed using the digits 0,1 and 2 . For example, 2210222 is in S, but 0210222 is NOT in S.
Then the number of elements x in S such that at least one of the digits 0 and 1 appears exactly twice in x, is equal to _______.

Sol. 351

Case: 1 two zeroes, no one 0 0 2 2 2 2 2

$={}^6C_2\times{}^5C_5$

Case: 2 two zeroes, one -1 0 0 1 2 2 2 2

$\Rightarrow {}^6C_2\times{}^5C_1\times{}^4C_4$

Case: 3 two zeroes, two ones

$={}^6C_2\times{}^5C_2\times{}^3C_3$

Case: 4 two one, no zero 1, 1, 2, 2, 2, 2, 2

$\frac{7!}{5!2!}=21$

Case: 5 two ones, one-zero 1, 1, 0, 2, 2, 2, 2

$\Rightarrow {}^6C_1\times{}^6C_2.$

Total = ${}^6C_2(1+5+10+6)+21=351$

11. Let α and β be the real numbers such that

$$\lim_{x\to 0}\frac{1}{x^3}\left(\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}dt+\beta x\cos x\right)=2.$$

Then the value of α + β is ______ .

Sol. **2.4**

$$\lim_{x\to 0}\frac{\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}dt+\beta x\cos x}{x^3}=2$$

$$\Rightarrow \frac{\frac{\alpha}{2}\left(1-x^2\right)^{-1}+\beta\cos x-\beta x\sin x}{3x^2}=2$$

$$\Rightarrow \lim_{x\to 0}\frac{\frac{\alpha}{2}\left(1+x^2+\cdots\right)+\beta\left(1-\frac{x^2}{2}+\cdots\right)-\beta x\left(x-\frac{x^3}{3!}\cdots\right)}{3x^2}=2$$

$\frac{\alpha}{2}+\beta=0\ \ldots(1)$ $\qquad \frac{\alpha}{2}=-\beta$

$\frac{\alpha}{2}-\frac{\beta}{2}-\beta=6\ \ldots(2)$

$\frac{\alpha}{2}-\frac{3\beta}{2}=6$

$\alpha-3\beta=12$

$-2\beta-3\beta=12$

$\beta=-\frac{12}{5}$ $\qquad \alpha+\beta=\frac{12}{5}=2.4$

$\alpha=\frac{24}{5}$

12. Let $\mathbb{R}$ denote the set of all real numbers. Let $f:\mathbb{R}\to\mathbb{R}$ be a function such that f(x) > 0 for all $x\in\mathbb{R}$, and f(x + y) = f(x)f(y) for all $x, y\in\mathbb{R}$.

Let the real numbers $a_1, a_2, \ldots, a_{50}$ be in an arithmetic progression. If $f(a_{31}) = 64f(a_{25})$, and $\sum_{i=1}^{50} f(a_i) = 3(2^{25}+1)$, then the value of $\sum_{i=6}^{30} f(a_i)$ is ______.

Sol. 96

f(x+y)=f(x)×f(y)

let $f(x)=k^x \ (k>0)$.

$a_1, a_2 \ldots, a_{50} \to AP$

$a_2=a_1+d$

$a_3=a_1+2d$

$a_{31}=a_1+30d$

$a_{25}=a_1+24d$

Given $f(a_{31})=64\ f(a_{25})$

$k^{a_1+30d}=64\ k^{a_1+24d}$

$k^{a_1} \cdot k^{30d} = 64\ k^{a_1}\quad k^{24}d$

$k^{6d} = 2^6 \Rightarrow\ k^d\ ^6 =\ 2\ ^6$

$\Rightarrow k^d = 2$

Given

$\sum_{i=1}^{50} f\ \ a_i\ = 3\ 2^{25} + 1$

$k^{a_1} + k^{a_2} + \cdots + k^{a_{50}} = 3\ 2^{25} + 1$

$k^{a_1} + k^{a_1+d} + k^{a_1+2d} + \cdots + k^{a_1+40d} = 3\ 2^{25} + 1$

$k^{a_1}\ \ 1 + k^d + k^{2d} + \cdots + k^{49d}\ \ = 3 \cdot\ 2^{2n} + 1$

$k^{a_1}\left(\frac{1 \cdot\ k^{50d} - 1}{k^d - 1}\right) = 3 \cdot\ 2^{25} + 1$

$k^{a_1}\left(\frac{2^{50} - 1}{2 - 1}\right) = 3 \cdot\ 2^{25} + 1$

$k^{a_1} = \frac{3}{2^{25} - 1}$

To find, $\sum_{i=6}^{30} f\ \ a_i\ = k^{a_6} + k^{a_7} + \cdots + k^{a_{30}}$

$= k^{a_1+5d} + k^{a_1+6d} + \cdots + k^{a_1+29d}$

$= k^{a_1}\ \ k^{5d} + k^{6d} + \cdots + k^{29d}$

$= k^{a_1}\ \ k^{5d}\ \ 1 + k^d + \cdots + k^{24d}$

$= \frac{3}{2^{25} - 1} \cdot\ \ 2^5\ \ 1 + 2 + 2^2 + \cdots + 2^{25}$

$= \frac{3}{2^{25} - 1} \cdot 2^5 \left(\frac{2^{25} - 1}{2 - 1}\right) = 3 \times 32$

$= 96$

13. For all $x > 0$, let $y_1(x)$, $y_2(x)$, and $y_3(x)$ be the functions satisfying

$\frac{dy_1}{dx} - (\sin x)^2 y_1 = 0, \quad y_1(1) = 5,$

$\frac{dy_2}{dx} - (\cos x)^2 y_2 = 0, y_2(1) = \frac{1}{3}$

$\frac{dy_3}{dx} - \left(\frac{2 - x^3}{x^3}\right) y_3 = 0, \quad y_3(1) = \frac{3}{5e},$

respectively. Then

$\lim_{x \to 0^+} \frac{y_1(x) y_2(x) y_3(x) + 2x}{e^{3x} \sin x}$

is equal to ______.

Sol. **2**

$$\frac{dy_1}{dx} = \sin^2 x y_1$$

$$\Rightarrow \frac{dy_1}{y_1} = \sin^2 x \cdot dx$$

Integration we get

$$\ln y_1 = \int \frac{1-\cos 2x}{2} dx$$

$$\ln y_1 = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$y_1(1)=5$

$$\therefore \quad c = \ln 5 - \frac{1}{2} + \frac{\sin 2}{4}$$

$$\therefore y_1 = e^{\frac{x}{2} - \frac{\sin 2x}{4} + \ln 5 + \frac{\sin 2}{4} - \frac{1}{2}}$$

Similarly $\frac{dy_2}{y_2} = \cos^2 x dx$

Integrating, we get

$$\ln y_2 = \frac{x}{2} + \frac{\sin 2x}{4} + c$$

$y_2(1)=\frac{1}{3}$ $\qquad c = \ln\frac{1}{3} - \frac{1}{2} - \frac{\sin 2}{4}$

$$\therefore y_2 = e^{\frac{x}{2} + \frac{\sin 2x}{4} + \ln\frac{1}{3} - \frac{1}{2} - \frac{\sin 2}{4}} = e^{\frac{x}{2} + \frac{\sin 2x}{4} - \ln 3 - \frac{1}{2} - \frac{\sin 2}{4}}$$

again $\int \frac{dy_3}{y_3} = \int \frac{2-x^3}{x^3} dx$

$$\ln y_3 = -\frac{1}{x^2} - x + c$$

$$y_3(1) = \frac{3}{5e} \quad \Rightarrow c = \ln\frac{3}{5e} + 2 = \ln 3 - \ln 5 - 1 + 2$$

$$\Rightarrow \quad \therefore y_3 = e^{\frac{-1}{x^2} - x + \ln 3 - \ln 5 + 1}$$

now, $y_1 \cdot y_2 \cdot y_3 = e^{\frac{x}{2} - \frac{\sin 2x}{4} + \ln 5 + \frac{\sin 2}{4} - \frac{1}{2} + \frac{x}{2} + \frac{\sin 2x}{4} - \ln 3 - \frac{1}{2} - \frac{\sin 2}{4} - \frac{1}{x^2} - x + \ln 3 - \ln 5 + 1}$

$= e^{-1/x^2}$

now $\lim_{x\to 0^+} \frac{y_1(x) y_2(x) y_3(x) + 2x}{e^{3x} \cdot \sin x}$

$$\Rightarrow \lim_{x\to 0^+} \frac{e^{-1/x^2} + 2x}{x}$$

let $\frac{1}{x} = t \quad \left(\text{as } x \to 0^+, t \to \infty \right)$

$$\Rightarrow \quad \lim_{t\to\infty} \left(te^{-t^2} + 2 \right)$$

$$\Rightarrow \quad \lim_{t\to\infty} \left(\frac{t}{e^{t^2}} + 2 \right)$$

= 2

14. Consider the following frequency distribution:

Value	4	5	8	9	6	12	11
Frequency	5	f_1	f_2	2	1	1	3

Suppose that the sum of the frequencies is 19 and the median of this frequency distribution is 6.
For the given frequency distribution, let α denote the mean deviation about the mean, β denote the mean deviation about the median, and σ^2 denote the the variance.
Match each entry in List-I to the correct entry in List-II and choose the correct option.

List-I	List-II
(P) $7f_1+9f_2$ is equal to	(1) 146
(Q) 19α is equal to	(2) 47
(R) 19β is equal to	(3) 48
(S) $19\sigma^2$ is equal to	(4) 145
	(5) 55

(A) (P) $\to$ (5) (Q) $\to$ (3) (R) $\to$ (2) (S) $\to$ (4)
(B) (P) $\to$ (5) (Q) $\to$ (2) (R) $\to$ (3) (S) $\to$ (1)
(C) (P) $\to$ (5) (Q) $\to$ (3) (R) $\to$ (2) (S) $\to$ (1)
(D) (P) $\to$ (3) (Q) $\to$ (2) (R) $\to$ (5) (S) $\to$ (4)

Sol. **C**

Value	4	5	6	8	9	11	12
Frequency	5	f_1	1	f_2	2	3	1
C.f	5	$(f_1 + 5)$	$(6 + f_1)$	$(6 + f_1 + f_2)$	15	18	19

$f_1+f_2=7 \qquad \sum f_i=19$

(P) $6+f_1\geq 10$

$f_1\geq 4$

$\therefore f_1=4, f_1=3$

$$\bar{x}=\frac{\sum f_i x_i}{\sum f_i}=\frac{133}{19}=7$$

$\therefore 7f_1+9f_2=28+27=55$

(Q) $d_i=|x_i-\bar{x}|$ $\qquad f_i d_i$

d_i	$f_i d_i$
3	15
2	8
1	1
1	3
2	4
4	12
5	5

Mean deviation = $=\frac{\sum f_i d_i}{\sum f_i}=\alpha$

$19\alpha=\sum f_i d_i=48$

(R) $d_i = |X_i - M|$ $\quad f_i d_i$

d_i	$f_i d_i$
2	16
1	4
0	0
2	6
3	6
5	15
6	6

Mean deviation $= \dfrac{\sum f_i d_i}{\sum f_i} = \beta$

$\therefore 19\beta = \sum f_i d_i = 47$

(S) Var(x) = $\dfrac{\sum f_i x_i^2}{\sum f_i} - (\bar{x})^2$

= 146

$\therefore$ Option (c) is correct

15. Let $\mathbb{R}$ denote the set of all real numbers. For a real number x, let [x] denote the greatest integer less than or equal to x. Let n denote a natural number.
Match each entry in List-I to the correct entry in List-II and choose the correct option.

List – I		List II	
(P)	The minimum value of n for which the function $f(x) = \left[\dfrac{10x^3 - 45x^2 + 60x + 35}{n}\right]$ is continuous on the interval [1, 2], is	(1)	8
(Q)	The minimum value of n for which $g(x) = (2n^2 - 13n - 15)(x^3 + 3x)$, $x \in \mathbb{R}$, is an increasing function on $\mathbb{R}$, is	(2)	9
(R)	The smallest natural number n which is greater than 5, such that x = 3 is a point of local minima of $h(x) = (x^2 - 9)^n (x^2 + 2x + 3)$, is	(3)	5
(S)	Number of $x_0 \in \mathbb{R}$ such that $\ell(x) = \sum_{k=0}^{4}\left(\sin\lvert x-k\rvert + \cos\left\lvert x - k + \frac{1}{2}\right\rvert\right)$, $x \in \mathbb{R}$, is NOT differentiable at x_0, is	(4)	6
		(5)	10

(A) (P) $\to$ (1) (Q) $\to$ (3) (R) $\to$ (2) (S) $\to$ (5)
(B) (P) $\to$ (2) (Q) $\to$ (1) (R) $\to$ (4) (S) $\to$ (3)
(C) (P) $\to$ (5) (Q) $\to$ (1) (R) $\to$ (4) (S) $\to$ (3)
(D) (P) $\to$ (2) (Q) $\to$ (3) (R) $\to$ (1) (S) $\to$ (5)

Sol. **B**

(P) $f(x)=\left[\frac{10x^3-45x^2+60x+35}{n}\right]$

For continuity in [1, 2]

f(1) = f(2)

$\left[\frac{60}{n}\right]=\left[\frac{55}{n}\right]$

$\therefore \quad n=9$

(Q) $g(x)=\left(2x^2-13x-15\right)\left(x^3+3x\right)$

$g'(x)=(2x-15)(x+1)\cdot\left(3x^2+3\right)>0$

$\therefore \quad n=8$

(R) $h(x)=\left(x^2-9\right)^n\cdot\left(x^2+2x+3\right)$

$h'(x)=n\left(x^2-9\right)^{n-1}\cdot(2x)\left(x^2+2x+3\right)+\left(x^2-9\right)^n(2x+2)$

$=\left(x^2-9\right)^{n-1}\left[n\cdot(2x)\left(x^2+2x+3\right)+\left(x^2-9\right)(2x+2)\right]$

x = 3 is not a factor of this expression

∴ n – 1 should be odd

So, ‘n’ should be even

smallest even number greater then 5 is 6

(S) $\ell(x)=\sum_{k=0}^{4}\left(\sin|x-k|+\cos\left|x-k+\frac{1}{2}\right|\right)$

$\ell(x)=\sin|x|+\sin|x-1|+\sin|x-2|+\sin|x-3|$

$+\sin|x-4|+\cos\left|x+\frac{1}{2}\right|+\cos\left|x-\frac{1}{2}\right|+\cos\left|x-\frac{3}{2}\right|$

$+\cos\left|x-\frac{5}{2}\right|+\cos\left|x-\frac{7}{2}\right|$

Since cos|x| is differentiable every where only non-differentiate point comes from sin|x| where x=0

∴ x = 0, 1, 2, 3, 4 are point of none differentiable

16. Let $\vec{w}=\hat{i}+\hat{j}-2\hat{k}$, and $\vec{u}$ and $\vec{v}$ be two vectors, such that $\vec{u}\times\vec{v}=\vec{w}$ and $\vec{v}\times\vec{w}=\vec{u}$. Let α,β,γ, and t be real numbers such that $\vec{u}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k},\ -t\alpha+\beta+\gamma=0,\ \alpha-t\beta+\gamma=0,$ and $\alpha+\beta-t\gamma=0$.
Match each entry in List-I to the correct entry in List-II and choose the correct option.

List – I		List II	
(P)	$\lvert\vec{v}\rvert^2$ is equal to	(1)	0
(Q)	If $\alpha=\sqrt{3}$, the γ^2 is equal to	(2)	1
(R)	If $\alpha=\sqrt{3}$, then $(\beta+\gamma)^2$ is equal to	(3)	2
(S)	If $\alpha=\sqrt{2}$, then $t+3$ is equal to	(4)	3
		(5)	5

(A) $(P) \to (2)$ $(Q) \to (1)$ $(R) \to (4)$ $(S) \to (5)$
(B) $(P) \to (2)$ $(Q) \to (4)$ $(R) \to (3)$ $(S) \to (5)$
(C) $(P) \to (2)$ $(Q) \to (1)$ $(R) \to (4)$ $(S) \to (3)$
(D) $(P) \to (5)$ $(Q) \to (4)$ $(R) \to (1)$ $(S) \to (3)$

Sol. C

$\vec{u} \times \vec{v} = \vec{w}$ … (1)

$\vec{v} \times \vec{w} = \vec{u}$ … (2)

$(\vec{v} \times \vec{w}) \times \vec{v} = \vec{u} \times \vec{v}$

$v^2\vec{w} \cdot -(\vec{w} \cdot \vec{v})\vec{v} = \vec{w}$

$v^2\vec{w} \cdot \vec{v} - (\vec{w} \cdot \vec{v})v^2 = \vec{w} \cdot \vec{v} \Rightarrow \vec{w} \cdot \vec{v} = 0$

Now $v^2\vec{w} = \vec{w} \Rightarrow |v^2| = 1$

and from (1) $\vec{u} \times \vec{v} = \vec{w}$

$(\vec{u} \times \vec{v}) \times \vec{w} = \vec{w} \times \vec{w}$

$(\vec{u} \cdot \vec{w})\vec{v} - (\vec{v} \cdot \vec{w})\vec{u} = 0$

$(\vec{u} \cdot \vec{w})\vec{v} = 0 \Rightarrow \vec{u} \cdot \vec{w} = 0$

Hence $\alpha + \beta - 2\gamma = 0$ … (1)

$-t\alpha + \beta + \gamma$ … (2) $\alpha(2 - t) = 0$

$\alpha - t\beta + \gamma = 0$ … (3)

$\alpha + \beta - t\gamma = 0$ … (4)

(1) – (4)

$r(t - 2) = 0 \Rightarrow r = 0$ or $t = 2$

$\Downarrow$

$\alpha + \beta = 0$

Hence (P) $|\vec{v}|^2 = 1 \to P \to (2)$

(Q) $\alpha = \sqrt{3}$

$\Rightarrow \gamma^2 = 0 \to Q \to (1)$

(R) $\alpha = \sqrt{3}$

$\Rightarrow (\beta + \gamma)^2 = (\beta)^2 = (-\alpha)^2 = 3 \to R \to (4)$

(S) $\alpha = \sqrt{2} \Rightarrow t = -1 \Rightarrow t + 3 = 2 \to s \to (3)$

$\Rightarrow r = 0$

$\beta = -\sqrt{2}$

मोशन है तो भरोसा है
Physics
Chemistry
Math
Name | Kota Exp. | Rank Under 100 | Selection in IIT | Selection in NIT (Approx)
Chandra S. Sharma (CSS Sir) 20 yrs. AIR-2,4,5 (65+) 11000+ 25000+
Ankush Jain (AKJ Sir) 15 yrs. AIR-4,5 (5) 8000+ 15000+
Amit Kumar Singh (AKS Sir) 16 yrs. (10) 2500+ 12000+
Amit Kumar Rathore (AR Sir) 16 yrs. AIR-1 (5) 2500+ 12000+
Sharad Kothari (SK Sir) 18 yrs. AIR-1 (12) 10000+ 18000+
Sanjay Sodhani (Sodhani Sir) 16 yrs. AIR-1 (16) 10000+ 17000+
Shubham Soni (SBS Sir) 12 yrs. NA 200+ 1000+
Mr.Nitin Viijay (NV SIR)
Now Bigger & Stronger
We Welcome Top Educators of Kota
ACADEMIC PILLAR OF JEE MOTION KOTA
PHYSICS
Nitin Viijay (NV Sir) Founder & CEO Exp. : 22
Avinash Kishore (AVN Sir) Sr. Faculty Exp. : 11
Akshay Jain (AJ Sir) Sr. Faculty Exp. : 9
Rohit Malav (RM Sir) Sr. Faculty Exp. : 13
Sanjay Gandhi (SG Sir) Sr. Faculty Exp. : 17
Rahul Kumar (RK Sir) Sr. Faculty Exp. : 9
Devashish Potar (Dev Sir) Faculty Exp. : 9
Vikram Singh Meena (VKM Sir) Faculty Exp. : 7
Deepak Jain (D. Jain Sir) Faculty Exp. : 9
MATH
Nikhil Srivastava (NS sir) Deputy Director Exp. : 22
Amarnath Anand (ANNA Sir) Sr. Faculty Exp. : 21
Atish Agarwal (AA sir) Sr. Faculty (VP-Outside Centres) Exp. : 22
Jayant Chittora (JC sir) Sr. Faculty Exp. : 21
Arjun Gupta (Arjun sir) Sr. Faculty Exp. : 18
Vipin Sharma (VS sir) Sr. Faculty Exp. : 13
Deepak Singh (DPS Sir) Sr. Faculty Exp. : 19
Krishan Kumar Soan (KKS Sir) Sr. Faculty Exp. : 19
Indrajeet Chourasiya (IC Sir) Sr. Faculty Exp. : 16
Ashutosh Vishvkarma (Ashutosh Sir) Faculty Exp. : 6
Anirudha Yadav (ANY Sir) Faculty Exp. : 5
PHYSICAL CHEMISTRY
Ram Ratan Dwivedi (RRD sir) Sr. Faculty & Joint Director Exp. : 24
Akhilesh Kanther (AKK sir) Sr. Faculty Exp. : 22
Jitendra Hirwani (JH Sir) Sr. Faculty Exp. : 20
Deepak Sharma (DS Sir) Sr. Faculty Exp. : 13
Shravan Choudhary (SC Sir) Sr. Faculty Exp. : 9
Shaleen Singhwal (Shaleen Sir) Faculty Exp. : 7
Mukesh Suman (MSS Sir) Faculty Exp. : 14
INORGANIC CHEMISTRY
Vishal Joshi (VJ sir) Sr. Faculty Exp. : 23
Dr. Deepak Joshi (DJ Sir) Sr. Faculty Exp. : 30
Devki Nandan (DN sir) Sr. Faculty Exp. : 22
Ritesh Kumar (RTK Sir) Sr. Faculty Exp. : 14
Amit Singh Sisodiya (AMS Sir) Faculty Exp. : 12
ORGANIC CHEMISTRY
Surendra K. Mishra (SKM sir) Sr. Faculty Exp. : 24
Anurag Garg (AG sir) Sr. Faculty Exp. : 22
Sachin Srivastava (SCS Sir) Sr. Faculty Exp. : 15
Ankur Chourasia (AC Sir) Faculty Exp. : 12
Stronger Team, Better Learning!
The Best Teachers, Now Under One Roof!

मोशन है, तो भरोसा है

MOTION
18 YEARS OF LEGACY

ADMISSION OPEN

DROPPER BATCH

(12th-13th Moving Students)

Batch Starts From: 21st May & 02nd June 2025

Get scholarships on the basis of JEE Advanced 2025 Marks

JEE Adv. 2025 Marks	Program Fee	Final Fee
Above 125	~~1,65,000~~	FREE
124 - 105	~~1,65,000~~	25,000
104 - 95	~~1,65,000~~	45,000
94 - 85	~~1,65,000~~	65,000
Below 85	~~1,65,000~~	85,000

VSTAR Batch (100% Selection Batch)

JEE Adv. 2025 Marks	PROGRAM FEE	FINAL FEE
Above 125	~~1,65,000/-~~	FREE
124 - 105	~~1,65,000/-~~	25,000

Exam	Eligibility
In Main	< 20,000
JEE Advanced	IIT Selected any category / Rank
CBSE Board	95% +Written Test
Other Board	District / State Board Merit List + Written Test

Scan the **QR CODE** to get scholarships on the basis of your past achievements

Want to take a Drop? Watch this video

OUR KOTA CAMPUSES

CORPORATE OFFICE

394 Rajeev Gandhi Nagar, Kota

DAKSH CAMPUS

B-84, Road No.- 5 RIICO, IPIA, Kota

DRONA CAMPUS

E-5-II Road No.1 IPIA, Kota

DIGITAL CAMPUS

50, Rajeev Gandhi Nagar, Kota

DRONA -2 CAMPUS

E4 (II),IPIA, Kota

EXPERIENCE CENTER

Kunhadi, Kota

KOTA RESIDENTIAL CAMPUSES

DHRUV CAMPUS

BOYS & GIRLS

Kuber Industrial Area, Ranpur, Kota

DRONA RESIDENCY

BOYS ONLY

E-4 I B - IPIA, Kota

DAKSH RESIDENCY

BOYS ONLY

C-159, IPIA Road No.5, Kota

DRONA RESIDENCY

GIRLS ONLY

E-5 I, IPIA, Kota

DRONA RESIDENCY

BOYS ONLY

E-5-II, Road Number 1, IPIA, Kota