

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-1}{4} \text{ and}$$

$$\frac{x+2}{7} = \frac{y-2}{8} = \frac{z+1}{2} \text{ is}$$

- (1) $\frac{88}{\sqrt{1277}}$ (2) $\frac{78}{\sqrt{1277}}$
 (3) $\frac{66}{\sqrt{1277}}$ (4) $\frac{55}{\sqrt{1277}}$

Answer (1)

Sol. $d = \frac{|(a_2 - a_1) \cdot (b_1 \times b_2)|}{|b_1 \times b_2|}$

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 7 & 8 & 2 \end{vmatrix}$$

$$= -26\hat{i} + 24\hat{j} - 5\hat{k}, \quad a_2 - a_1 = 3\hat{i} + 2\hat{k}$$

$$d = \frac{|(3\hat{i} + 2\hat{k}) \cdot (-26\hat{i} + 24\hat{j} - 5\hat{k})|}{\sqrt{26^2 + 24^2 + 5^2}}$$

$$= \frac{|-78 - 10|}{\sqrt{1277}} = \frac{88}{\sqrt{1277}}$$

2. In a bag there are 6 white and 4 black balls two balls are drawn at random, then the probability that both ball are white are

- (1) $\frac{1}{2}$ (2) $\frac{1}{3}$
 (3) $\frac{2}{3}$ (4) $\frac{1}{4}$

Answer (2)

Sol. $P(E) = \frac{{}^6C_2}{{}^{10}C_2}$
 $= \frac{15}{45} = \frac{1}{3}$

3. Let $A = \{1, 2, 3\}$ number of non-empty equivalence relations from A to A are

- (1) 4 (2) 5
 (3) 6 (4) 8

Answer (2)

Sol. The partitions for a set with 3 elements, $\{1, 2, 3\}$

- $\{\{1\}, \{2\}, \{3\}\}$ – Every element is in its own subset
- $\{\{1, 2\}, \{3\}\}$ – Two elements are together, one separate
- $\{\{1, 3\}, \{2\}\}$ – Two elements are together, one separate
- $\{\{2, 3\}, \{1\}\}$ – Two elements are together, one separate
- $\{\{1, 2, 3\}\}$ – All elements are together in one subset

∴ Therefore, total possible equivalence relation = 5

4. If $f(x) = 16(\sec^{-1} x)^2 + (\operatorname{cosec}^{-1} x)^2$. Then the maximum and minimum value of $f(x)$ is

- (1) $\frac{1001\pi^2}{33}$ and $\frac{2\pi^2}{9}$ (2) $\frac{1105\pi^2}{68}$ and $\frac{4\pi^2}{17}$
 (3) $\frac{1117\pi^2}{59}$ and $\frac{6\pi^2}{19}$ (4) $\frac{1268\pi^2}{27}$ and $\frac{3\pi^2}{16}$

Answer (2)

Sol. $f(x) = (4 \sec^{-1} x)^2 + (\operatorname{cosec}^{-1} x)^2$
 $= (4 \sec^{-1} x + \operatorname{cosec}^{-1} x)^2 - 8 \sec^{-1} x \operatorname{cosec}^{-1} x$
 $= \left(3 \sec^{-1} x + \frac{\pi}{2} \right)^2 - 8 \sec^{-1} x \left[\frac{\pi}{2} - \sec^{-1} x \right]$

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7. Let $T_r = \frac{(2r-1)(2r+1)(2r+3)(2r+5)}{64}$, then

$\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{T_r}$ is equal to

- (1) $\frac{22}{45}$ (2) $\frac{32}{35}$
 (3) $\frac{27}{45}$ (4) $\frac{32}{45}$

Answer (4)

Sol. $T_r = \frac{(2r-1)(2r+1)(2r+3)(2r+5)}{64}$

$$\Rightarrow \frac{1}{T_r} = \frac{64}{16 \left(r - \frac{1}{2}\right) \left(r + \frac{1}{2}\right) \left(r + \frac{3}{2}\right) \left(r + \frac{5}{2}\right)}$$

$$\Rightarrow \frac{1}{T_r} = \frac{\frac{4}{3} \left[\left(r + \frac{5}{2}\right) - \left(r - \frac{1}{2}\right) \right]}{\left(r - \frac{1}{2}\right) \left(r + \frac{1}{2}\right) \left(r + \frac{3}{2}\right) \left(r + \frac{5}{2}\right)}$$

$$\Rightarrow \frac{1}{T_r} = \frac{4}{3} \left[\frac{1}{\left(r - \frac{1}{2}\right) \left(r + \frac{1}{2}\right) \left(r - \frac{3}{2}\right)} - \frac{1}{\left(r + \frac{1}{2}\right) \left(r + \frac{3}{2}\right) \left(r + \frac{5}{2}\right)} \right]$$

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{T_r} = \frac{4}{3} \left[\frac{1}{2 \cdot 2 \cdot 2} - \frac{1}{2 \cdot 2 \cdot 2} \right]$$

$$= \frac{4}{3} \left[\frac{8}{15} \right] = \frac{32}{45}$$

8. Coefficient of x^{2012} in $(1-x)^{2008} (1+x+x^2)^{2007}$

- (1) 0 (2) 1
 (3) 2 (4) 3

Answer (1)

Sol. $(1-x) [(1-x)(1+x+x^2)]^{2007}$

$$= (1-x)(1-x^3)^{2007}$$

$$= (1-x^3)^{2007} - x(1-x^3)^{2007}$$

$[(1-x^3)^{2007}$ contains 3λ types of exponents while $x(1-x^3)^{2007}$ will have $(3\lambda+1)$ type while 2012 is $(3\lambda+2)$ type] that is not possible $\Rightarrow 0$

$$\text{Coefficient of } x^{2012} \text{ in } (1-x^3)^{2007} = 0$$

$$\text{Coefficient of } x^{2011} \text{ in } (1-x^3)^{2007} = 0$$

$$\Rightarrow \text{Coefficient of } x^{2012} \text{ in } (1-x)^{2008} (1+x+x^2)^{2007} = 0$$

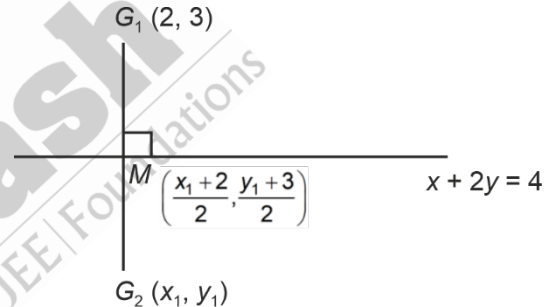
9. If the images of the points $A(1, 3)$, $B(3, 1)$ and $C(2, 4)$ in the line $x+2y=4$ are D , E and F respectively, then the centroid of the triangle DEF is

- (1) $(3, -1)$ (2) $\left(-\frac{3}{5}, -\frac{2}{5}\right)$
 (3) $\left(\frac{2}{5}, -\frac{1}{5}\right)$ (4) $\left(\frac{1}{5}, -\frac{2}{5}\right)$

Answer (3)

Sol. Centroid of the $\triangle DEF$ is the mirror image of the centroid of the $\triangle ABC$ about the line $x+2y=4$.

$G_1 =$ Centroid of $\triangle ABC \equiv (2, 3)$, $G_2 \equiv$ Centroid of $\triangle DEF$.



$$\Rightarrow \frac{y_1-3}{x_1-2} = 2, \frac{x_1+2}{2} + (y_1+3) = 4$$

$$\Rightarrow x_1 = \frac{2}{5}, y_1 = -\frac{1}{5}$$

$$\Rightarrow G_2 = \left(\frac{2}{5}, -\frac{1}{5}\right)$$

10. If $A = \{1, 2, 3, \dots, 10\}$.

$$B = \left\{ \frac{m}{n}, m, n \in A \text{ and } m < n \text{ and } \text{gcd of } (m, n) = 1 \right\}$$

Then number of elements in set B is

- (1) 30 (2) 31
 (3) 28 (4) 29

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Answer (2)

Sol. $n = 1 \quad m \in \phi \quad \dots 0$

$$n = 2 \quad m = 1 \Rightarrow \frac{m}{n} \text{ can be } \frac{1}{2} \dots 1$$

$$n = 3 \quad m = 1, 2 \Rightarrow \frac{m}{n} \text{ can be } \frac{1}{3}, \frac{2}{3} \dots 2$$

$$n = 4 \quad m = 1, 3 \Rightarrow \frac{m}{n} \text{ can be } \frac{1}{4}, \frac{3}{4} \dots 2$$

$$n = 5 \quad m = 1, 2, 3, 4 \Rightarrow \frac{m}{n} = \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5} \dots 4$$

$$n = 6 \quad m = 1, 5 \Rightarrow \frac{m}{n} = \frac{1}{6}, \frac{5}{6} \dots 2$$

$$n = 7 \quad m = 1, 2, 3, 4, 5, 6 \Rightarrow \frac{m}{n} = \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7} \dots 6$$

$$n = 8 \quad m = 1, 3, 5, 7 \Rightarrow \frac{m}{n} = \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8} \dots 4$$

$$n = 9 \quad m = 1, 2, 4, 5, 7, 8 \Rightarrow \frac{m}{n} = \frac{1}{9}, \frac{2}{9}, \frac{3}{9}, \frac{4}{9}, \frac{5}{9}, \frac{7}{9}, \frac{8}{9} \dots 6$$

$$n = 10 \quad m = 1, 3, 7, 9 \Rightarrow \frac{m}{n} = \frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10} \dots 4$$

11. How many ways are there to pick 5 letters from English alphabets such that M is the middle of the letters (repetition not allowed).

(1) ${}^{26}C_5 \cdot 5!$

(2) ${}^{25}C_4 \cdot 4!$

(3) ${}^{26}C_4 \cdot 4!$

(4) ${}^{25}C_5 \cdot 5!$

Answer (2)

Sol. $\frac{A_1}{A_2} \frac{M}{\uparrow \text{fixed}} \frac{A_3}{A_4}$

$${}^{25}C_4 \times 4!$$

12. Let $|Z_i| = 1$ for $i = 1, 2, 3$ satisfying

$$|\bar{Z}_1 Z_2 + \bar{Z}_2 Z_3 + \bar{Z}_3 Z_1|^2 = a + b\sqrt{2}, \text{ where } a, b \text{ are}$$

rational numbers such that $\arg(Z_1) = \frac{\pi}{4}, \arg(Z_2) = 0$

and $\arg(Z_3) = \frac{-\pi}{4}$, then find (a, b)

(1) $(5, 2)$ (2) $(-5, -2)$

(3) $(5, -2)$ (4) $(-5, 2)$

Answer (3)

Sol. $Z_1 = |1| e^{i\frac{\pi}{4}} = \frac{1}{\sqrt{2}} + i \cdot \frac{1}{\sqrt{2}}$

$$Z_2 = |1| e^{-i0} = 1 + 0i$$

$$Z_3 = |1| e^{-i\frac{\pi}{4}} = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$$

$$\bar{Z}_1 Z_2 = \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right) (1)$$

$$\bar{Z}_2 Z_3 = 1 \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right)$$

$$\bar{Z}_3 Z_1 = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right)$$

$$\Rightarrow \bar{Z}_1 Z_2 + \bar{Z}_2 Z_3 + \bar{Z}_3 Z_1 = \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right) + \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right) + \left(\frac{1}{2} - \frac{1}{2} \right) + 2i \left(\frac{1}{2} \right)$$

$$= \sqrt{2} - \sqrt{2}i + i$$

$$\Rightarrow |\bar{Z}_1 Z_2 + \bar{Z}_2 Z_3 + \bar{Z}_3 Z_1|^2 = \left| \sqrt{2} + i(-\sqrt{2} + 1) \right|^2 = \left(\sqrt{(\sqrt{2})^2 + (1 - \sqrt{2})^2} \right)^2 = 5 - 2\sqrt{2}$$

$(a, b) = (5, -2)$

13. Let a coin is tossed thrice. Let the random variable x is tail follows head. Let the mean of x is μ and variance is σ^2 . Find $64(\mu + \sigma^2)$.

(1) 48 (2) 64

(3) 132 (4) 128

Answer (1)

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Sol.

	x_i	P_i
HHH	0	$\frac{1}{8}$
TTT	0	$\frac{1}{8}$
HHT	1	$\frac{1}{8}$
HTH	1	$\frac{1}{8}$
THH	0	$\frac{1}{8}$
TTH	0	$\frac{1}{8}$
THT	1	$\frac{1}{8}$
HTT	1	$\frac{1}{8}$

$$\mu = \sum P_i x_i = \frac{1}{2}$$

$$\sigma^2 = \sum P_i x_i^2 - \mu^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$64 \left(\frac{1}{2} + \frac{1}{4} \right) = 64 \times \frac{3}{4} = 48$$

14. Let $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x) \forall x \in (0,3)$ and $f''(x) > 0 \forall x \in (0,3)$ then $g(x)$ decreases in interval $(0, \alpha)$, then α is

- (1) $\frac{7}{4}$ (2) $\frac{2}{3}$
 (3) $\frac{9}{4}$ (4) $\frac{7}{3}$

Answer (3)

Sol. $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x)$

$$g'(x) = 3 \cdot \frac{1}{3} f'\left(\frac{x}{3}\right) - f'(3-x)$$

$$= f'\left(\frac{x}{3}\right) - f'(3-x)$$

$$g''(x) = \frac{f''(x)}{3} + f''(3-x)$$

$$\Rightarrow g'(x) > 0$$

$$f'\left(\frac{x}{3}\right) - f'(3-x) > 0$$

$$f'(x) > 0 \Rightarrow f'(x) \text{ is increasing}$$

15. Let $\vec{b} = \lambda \hat{i} + 4\hat{k}$, $\lambda > 0$ and the projection vector of \vec{b} on $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$ is \vec{c} . If $|\vec{a} + \vec{c}| = 7$, then the area of the parallelogram formed by vector \vec{b} and \vec{c} is (in square units)

- (1) 8
 (2) 16
 (3) 32
 (4) 64

Answer (3)

Sol. $\vec{c} = (\vec{b} \cdot \hat{a}) \hat{a} = \frac{2\lambda - 4}{6} \vec{a}$

$$\therefore |\vec{a} + \vec{c}| = 7 \Rightarrow \left| \vec{a} \left(1 + \frac{2\lambda - 4}{9} \right) \right| = 7$$

$$\left| \frac{5 + 2\lambda}{9} \right| \times 3 = 7 \Rightarrow |5 + 2\lambda| = 21$$

$$\therefore \lambda > 0 \Rightarrow \lambda = 8$$

$$\Rightarrow \vec{c} = \frac{4}{3} \vec{a} \text{ and } \vec{b} = 4(2\hat{i} - \hat{k})$$

$$\Rightarrow \vec{b} \times \vec{c} = \frac{16}{3} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 2 & 2 & -1 \end{vmatrix} = \frac{16}{3} (-2\hat{i} + 4\hat{j} + 4\hat{k})$$

$$\Rightarrow |\vec{b} \times \vec{c}| = \frac{32}{3} |- \hat{i} + 2\hat{j} + 2\hat{k}| = 32$$

\Rightarrow Area of parallelogram formed by \vec{b} and \vec{c}

$$\Rightarrow |\vec{b} \times \vec{c}| = 32$$

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. If A be a 3×3 square matrix such that $\det(A) = -2$. If $\det(3 \operatorname{adj}(-6 \operatorname{adj}(3A))) = 2^n \cdot 3^m$, where $m \geq n$, then $4m + 2n$ is equal to

Answer (104)

Sol. Concept: $A. \operatorname{adj}(A) = |A|I$, $\det(\lambda A) = \lambda^n \det(A)$

$$\Rightarrow \det(A) = |A|^{n-1}, \text{ where } n \text{ is order}$$

$$\Rightarrow \det(3 \operatorname{adj}(-6 \operatorname{adj}(3A)))$$

$$= 3^3 \cdot \det(\operatorname{adj}(-6 \operatorname{adj}(3A)))$$

$$= 3^3 \cdot (-6 \operatorname{adj}(3A))^2$$

$$= 3^3 \cdot (-6)^6 |3A|^4$$

$$= 3^9 \cdot 2^6 \cdot 3^{12} \cdot (-2)^4$$

$$= 3^{21} \cdot 2^{10}$$

$$\therefore n = 10, m = 21$$

$$\therefore 4m + 2n = 104$$

22. If $a_1, a_2, a_3, \dots, a_n$ are in geometric progression such that $a_1 a_5 = 28$, $a_2 + a_4 = 29$, then the value of a_6 is

(1) 635

(2) 784

(3) 872

(4) 898

Answer (2)

Sol. $a_1 a_5 = 28 \Rightarrow a^2 r^4 = 28$

$$a_2 + a_4 = 29 \Rightarrow ar + ar^3 = 29$$

$$ar, ar^3 \text{ are roots of } k^2 - 29k + 28 = 0$$

$$\Rightarrow k = 1, k = 28$$

$$\Rightarrow ar = 1, ar^3 = 28$$

$$\Rightarrow r^2 = 28, a^2 = \frac{1}{28}$$

$$a_6 = ar^5 \Rightarrow a_6^2 = a^2 r^{10} = \frac{1}{28} \times (28)^5 = (28)^4$$

$$\Rightarrow a_6 = (28)^2 = 784$$

23.

24.

25.



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