Ex 3.1 Class 11 Maths Question 1:

Find the radian measures corresponding to the following degree measures:

(i) 25°

(ii) – 47° 30′

(iii) 240°

(iv) 520° **Ans:**

(i) 25°

We know that $180° = \pi$ radian

$$\therefore 25° = \frac{\pi}{180} \times 25 \text{ radian} = \frac{5\pi}{36} \text{ radian}$$

(ii) $-47° 30'$

$$-47° 30' = -47\frac{1}{2} \text{ degree } [1° = 60']$$

$$= \frac{-95}{2} \text{ degree}$$

Since $180° = \pi$ radian

$$\frac{-95}{2} \text{ degree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{ radian} = \left(\frac{-19}{36 \times 2}\right)\pi \text{ radian} = \frac{-19}{72}\pi \text{ radian}$$

$$\therefore -47° 30' = \frac{-19}{72}\pi \text{ radian}$$

(iii) 240°

We know that $180° = \pi$ radian

$$\therefore 240° = \frac{\pi}{180} \times 240 \text{ radian} = \frac{4}{3}\pi \text{ radian}$$

(iv) 520°

We know that $180° = \pi$ radian

$$\therefore 520° = \frac{\pi}{180} \times 520 \text{ radian} = \frac{26\pi}{9} \text{ radian}$$

(iii) 240°

We know that $180° = \pi$ radian

$$\therefore 240° = \frac{\pi}{180} \times 240 \text{ radian} = \frac{4}{3}\pi \text{ radian}$$

(iv) 520°

We know that $180° = \pi$ radian

$$\therefore 520° = \frac{\pi}{180} \times 520 \text{ radian} = \frac{26\pi}{9} \text{ radian}$$

Ex 3.1 Class 11 Maths Question 2:

Find the degree measures corresponding to the following radian measures

$\left(\text{Use } \pi = \frac{22}{7}\right)$.

(i) $\frac{11}{16}$ (ii) -4 (iii) $\frac{5\pi}{3}$ (iv) $\frac{7\pi}{6}$

Ans:

(i) 1116

We know that: π radian = 180°

∴ 1116 radain = 180π×1116 × degree

= 45×11π×4 degree

= 45×11×722×4 degree

= 3158 degree

= 39 38 degree

= 39° + 3×608 minutes [1° = 60′]

= 39° + 22′ + 12 minutes = 39°22′30″ [1′ = 60°].

(ii) -4

We know that π radian $= 180°$

$$-4 \text{ radian} = \frac{180}{\pi} \times (-4) \text{ degree} = \frac{180 \times 7(-4)}{22} \text{ degree}$$

$$= \frac{-2520}{11} \text{ degree} = -229\frac{1}{11} \text{ degree}$$

$$= -229° + \frac{1 \times 60}{11} \text{ minutes} \qquad [1° = 60']$$

$$= -229° + 5' + \frac{5}{11} \text{ minutes}$$

$$= -229°5'27'' \qquad [1' = 60'']$$

(iii) $\frac{5\pi}{3}$

We know that π radian $= 180°$

$$\therefore \frac{5\pi}{3}\text{ radian} = \frac{180}{\pi} \times \frac{5\pi}{3}\ \text{degree} = 300°$$

(iv) $\frac{7\pi}{6}$

We know that π radian $= 180°$

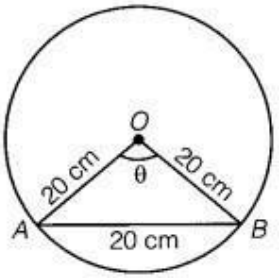
$$\therefore \frac{7\pi}{6}\text{ radian} = \frac{180}{\pi} \times \frac{7\pi}{6} = 210°$$

Ex 3.1 Class 11 Maths Question 3:
A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?
Ans:
Number of revolutions made by the wheel in 1 minute = 360 ∴
Number of revolutions made by the wheel in 1 second = 3606 = 6
In one complete revolution, the wheel turns an angle of 2π radian.
Hence, in 6 complete revolutions, it will turn an angle of 6 × 2π radian, i.e., 12π radian
Thus, in one second, the wheel turns an angle of 12π radian. **Ex 3.1 Class 11 Maths Question 4:**
Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm y an arc of length 22 cm (Use π = 227). **Ans:**
We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then
θ
= lr
Therefore, for r = 100 cm, l = 22 cm, we
have

$$\theta = \frac{22}{100}\text{ radian} = \frac{180}{\pi} \times \frac{22}{100}\text{ degree} = \frac{180 \times 7 \times 22}{22 \times 100}\text{ degree}$$
$$= \frac{126}{10}\text{ degree} = 12\frac{3}{5}\text{ degree} = 12°36' \qquad [1° = 60']$$

Thus, the required angle is 12°36′.

Ex 3.1 Class 11 Maths Question 5:
In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord. **Ans:**



Given, diameter = 40 cm ∴ radius (r) = 402 = 20 cm and length of chord, AB = 20 cm Thus, ΔOAB is an equilateral triangle.
We know that, θ = Arc AB radius ⇒ Arc AB = θ × r = π3 × 20 .
= 203 π cm.

Ex 3.1 Class 11 Maths Question 6:

If in two circles, arcs of the same length s ubtend angles 60° and 75° at the centre, find the ratio of their radii.

Ans:

Let the radii of the two circles be r_1 and r_2.
Let an arc of length l subtend an angle of 60° at the centre of the circle of radius r_1, while let an arc of length l subtend an angle of 75° at the centre of t h e circle of radius r_2.
Now, 6o° = π3 radian and 75°
= 5π12 radian

a

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian t the centre, then θ = lr or l = rθ ∴ l = r1π3 and l = r25π12 ⇒ r1π3=r25π12 ⇒ r = r254 r1r2=54
Thus, the ratio of the radii is 5 : 4.

Ex 3.1 Class 11 Maths Question 7:

Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length (i) 10 cm

(ii) 15 cm
(iii) 21 cm.
Ans:

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then θ = lr.
It is given that r = 75 cm
(i) Here, l = 10 cm θ = 1075 radian = 215 radian (ii) Here, l = 15 cm θ = 1575 radian θ = 15 radian (iii) Here, l = 21 cm θ = 2175 radian = 775 radian.

NCERT Solutions for Class 11 Maths Chapter 3 Exercise 3.2

Maths Question 1:

Find the values of other five trigonometric functions if cos x = – 12 x lies in third quadrant. Ans:

$$\cos x = -\frac{1}{2}$$

$$\therefore \quad \sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \quad \sin^2 x = 1 - \cos^2 x \quad \Rightarrow \quad \sin^2 x = 1 - \left(-\frac{1}{2}\right)^2$$

$$\Rightarrow \quad \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4} \quad \Rightarrow \quad \sin x = \pm \frac{\sqrt{3}}{2}$$

Since x lies in the 3rd quadrant, the value of $\sin x$ will be negative.

$$\therefore \quad \sin x = -\frac{\sqrt{3}}{2}$$

$$\text{cosec } x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

Ex 3.2 Class 11 Maths Question 2:

Find the values of other five trigonometric functions if sin x = 35, x lies in second quadrant. Ans:

sin x = 35 cosec x = 1sinx=1(35)=53 $\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$

$\Rightarrow \cos^2 x = 1 - (35)^2$

$\Rightarrow \cos^2 x = 1 - 925$

$\Rightarrow \cos^2 x = 1625$

$\Rightarrow \cos x = \pm 45$

Since x lies in the 2nd quadrant, the value of cos x will be negative

$$\cos x = -\frac{4}{5}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$$

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

Ex 3.2 Class 11 Maths Question 3:

Find the values of other five trigonometric functions if cot x = 34, x lies in third quadrant. **Ans:**

$$\cot x = \frac{3}{4}$$

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow \quad 1 + \left(\frac{4}{3.}\right)^2 = \sec^2 x \quad \Rightarrow \quad 1 + \frac{16}{9} = \sec^2 x$$

$$\Rightarrow \quad \frac{25}{9} = \sec^2 x \quad \Rightarrow \quad \sec x = \pm \frac{5}{3}$$

Since x lies in the 3rd quadrant, the value of sec x will be negative.

$$\therefore \quad \sec x = -\frac{5}{3}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$\Rightarrow 43=\sin x-35 \Rightarrow \sin x = (43)\times(-35)=-45 \Rightarrow$ cosec x = 1sinx=−54.

Ex 3.2 Class 11 Maths Question 4:

Find the values of other five trigonometric functions if sec x = 135, x lies in fourth quadrant. **Ans:**

$$\sec x = \frac{13}{5}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \quad \sin^2 x = 1 - \cos^2 x \qquad \Rightarrow \quad \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$$

$$\Rightarrow \quad \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169} \quad \Rightarrow \quad \sin x = \pm \frac{12}{13}$$

Since x lies in the 4^{th} quadrant, the value of $\sin x$ will be negative.

$$\therefore \quad \sin x = -\frac{12}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{-12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

Ex 3.2 Class 11 Maths Question 5:

Find the values of other five trigonometric functions if tan x = 512, x lies in second quadrant. **Ans:**

tan x = – 512 cot x = 1tanx=1(−512)=−125 1 +

$\tan^2 x = \sec^2 x$

$$\Rightarrow \quad 1+\left(-\frac{5}{12}\right)^2=\sec^2 x \quad \Rightarrow \quad 1+\frac{25}{144}=\sec^2 x$$

$$\Rightarrow \quad \frac{169}{144}=\sec^2 x \quad \Rightarrow \quad \sec x=\pm\frac{13}{12}$$

Since x lies in the 2nd quadrant, the value of sec x will be negative.

$$\therefore \quad \sec x=-\frac{13}{12}$$

$$\cos x=\frac{1}{\sec x}=\frac{1}{\left(-\frac{13}{12}\right)}=-\frac{12}{13}$$

$$\tan x=\frac{\sin x}{\cos x}$$

$$\Rightarrow \quad -\frac{5}{12}=\frac{\sin x}{\left(-\frac{12}{13}\right)} \quad \Rightarrow \quad \sin x=\left(-\frac{5}{12}\right)\times\left(-\frac{12}{13}\right)=\frac{5}{13}$$

$$\text{cosec } x=\frac{1}{\sin x}=\frac{1}{\left(\frac{5}{13}\right)}=\frac{13}{5}$$

Ex 3.2 Class 11 Maths Question 6:
Find the value of the trigonometric function sin 765°. **Ans:**
It is known that the values of sin x repeat after an interval of 2π or 360°. ∴
sin 765° = sin (2 × 360° + 45°)
= sin 45° = 1

Ex 3.2 Class 11 Maths Question 7:
Find the value of the trigonometric function cosec (- 1410°) **Ans:**
It is known that the values of cosec x repeat after an interval of 2π or 360°.
∴ cosec (- 1410°) = cosec (- 1410° + 4 x 360°) = cosec (- 1410° + 1440°)
= cosec 30° = 2.

Ex 3.2 Class 11 Maths Question 8:
Find the value of the trigonometric function tan 19π3.
Ans:
It is known that the values of tan x repeat after an interval of π or 180°.
∴
tan19π3=tan613π
=
tan(6π+π3)=tanπ3
= tan 60° = √3.

Ex 3.2 Class 11 Maths Question 9:
Find the value of the trigonometric function sin (−11π3).
Ans:
It is known that the values of cot x repeat after an interval of π or 180°.

∴sin(11π3)=sin(−11π3+2×2π)
= sin(π3)=sin60∘=3√2

Ex 3.2 Class 11 Maths Question 10:
Find the value of the trigonometric function cot (−15π4).
Ans:

It is known that the values of cot x repeat after an interval of ir or 1800.

∴ cot(−15π4)=cot(−15π4+4π)=cotπ4 = 1.

NCERT Solutions for Class 11 Maths Chapter 3 Exercise 3.3

Ex 3.3 Class 11 Maths Question 1:

Prove that: $\sin^2 \pi 6 + \cos^2 \pi 3 - \tan^2 \pi 4 = -12$ **Ans:**

L.H.S.= $\sin^2 \pi 6 + \cos^2 \pi 3 - \tan^2 \pi 4$

= $(12)2+(12)2 - (1)^2$

= 14+14−1=−12 =

R.H.S.

Hence proved.

Ex 3.3 Class 11 Maths Question 2:

Prove that: $2 \sin^2 \pi 6 + \text{cosec}^2 7\pi 6 \cos^2 \pi 3 = 32$ **Ans:**

$$\text{L.H.S.} = 2\sin^2\frac{\pi}{6} + \text{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$$

$$= 2\left(\frac{1}{2}\right)^2 + \text{cosec}^2\left(\pi + \frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2 = 2 \times \frac{1}{4} + \left(-\text{cosec}\,\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$

$$= \frac{1}{2} + (-2)^2\left(\frac{1}{4}\right)$$

$$= \frac{1}{2} + \frac{4}{4} = \frac{1}{2} + 1 = \frac{3}{2} = \text{R.H.S.}$$

Hence proved.

Ex 3.3 Class 11 Maths Question 3: Prove that :$\cot^2 \pi 6 + \text{cosec}\, 5\pi 6 + 3 \tan^2 \pi 6 = 6$ **Ans:**

$$\text{L.H.S.} = \cot^2\frac{\pi}{6} + \text{cosec}\,\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$$

$$= (\sqrt{3})^2 + \text{cosec}\left(\pi - \frac{\pi}{6}\right) + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \text{cosec}\,\frac{\pi}{6} + 3 \times \frac{1}{3} = 3+2+1=6$$

$$= \text{R.H.S.}$$

Hence proved.

Ex 3.3 Class 11 Maths Question 4:

Prove that: $2 \sin^2 3\pi 4 + 2 \cos^2 \pi 4 + 2 \sec^2 \pi 3 = 10$ **Ans:**

L.H.S = 2sin23π4+2cos2π4+2sec2π3

= 2{sin(π−π4)}2+2(12√)2+2(2)2

= 2{sinπ4}2+2×12+8

= 2 (12√)2 + 1 + 8

= 1 + 1 + 8

= 10 = R.H.S.

Hence proved.

Ex 3.3 Class 11 Maths Question 5:

Find the value of: (i) sin 75°,

(ii) tan 15° **Ans:**

(i) sin 75° sin (45° + 30°)
= sin 45° cos 30° + cos 45° sin 30°
[∵ sin (x + y) = sin x cos y + cos x sin y]
$\frac{(12\sqrt{})(3\sqrt{2})+(12\sqrt{})(12)}{}$
=
= 3√22√+122√=3√+122√
(ii) tan 15° = tan (45° – 30°)

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} \qquad \left[\because \tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}\right]$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3+1-2\sqrt{3}}{(\sqrt{3})^2-(1)^2}$$

$$= \frac{4-2\sqrt{3}}{3-1} = 2 - \sqrt{3}$$

Ex 3.3 Class 11 Maths Question 6:
cos(π4−x)cos(π4−y)−sin(π4−x)sin(π4−y) = sin (x + y)
Ans:

$$\text{L.H.S.} = \cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)$$

Let $\frac{\pi}{4} - x = A$ and $\frac{\pi}{4} - y = B$

Then, $\text{L.H.S.} = \cos A \cos B - \sin A \sin B = \cos(A + B)$

$$= \left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$= \cos\left(\frac{\pi}{2} - x - y\right) = \cos\left[\frac{\pi}{2} - (x + y)\right]$$

$= \sin(x+y) = \text{R.H.S.}$ **Hence proved.**

Ex 3.3 Class 11 Maths Question 7:
Prove that: tan(π4+x)tan(π4−x)=(1+tanx1−tanx)2
Ans:

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B} \text{ and } \tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

$$\therefore \quad \text{L.H.S.}=\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\frac{\frac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}}{\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}}\right)=\frac{\left(\frac{1+\tan x}{1-\tan x}\right)}{\left(\frac{1-\tan x}{1+\tan x}\right)}$$

= (1+tanx1−tanx)2
= R.H.S
Hence proved.

Ex 3.3 Class 11 Maths Question 8:

Prove that: cos(π+x)cos(−x)sin(π−x)cos(π2+x) = cot² x

Ans:

L.H.S = cos(π+x)cos(−x)sin(π−x)cos(π2+ x)
= [−cosx][cosx](sinx)(−sinx)=−cos2x−sin2x
= cot² x

= R.H.S Hence proved.

Ex 3.3 Class 11 Maths Question 9:

$$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$$

= 1

Ans:

$$\text{L.H.S.}=\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$$

$$=\sin x\cos x[\tan x+\cot x] \qquad \left[\because \cos\left(\frac{3\pi}{2}+\theta\right)=\sin\theta,\ \cot\left(\frac{3\pi}{2}-\theta\right)=\tan\theta\right]$$

$$=\sin x\cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)=(\sin x\cos x)\left[\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right]$$

= 1 = R.H.S
Hence proved.

Ex 3.3 Class 11 Maths Question 10:

Prove that: sin (n + 1) x sin (n + 2) x + c o s (n + 1) x cos (n + 2) x = cos x **Ans:**

L.H. S. = sin (n + 1)x sin (n + 2) x + cos (n +1) x cos (n + 2) x
[By the formula, cos (A – B) = cos A cos B + sin A sin B]
= cos [(n + 2) x + (n + 1) x]
= cos (4x + 2x – 4x – x)

= cos x = R.H.S.

Hence proved.

Ex 3.3 Class 11 Maths Question 11:

Prove that: $\cos(3\pi 4+x)-\cos(3\pi 4-x)=-2-\sqrt{\sin x}$

Ans:

It is known that cos A – cos B = $-2\sin(A+B2)\circ\sin(A-B2)$

$\therefore$ L.H.S.= $=\cos(3\pi 4+x)-\cos(3\pi 4-x)$

$= -2\sin\{(3\pi 4+x)+(3\pi 4-x)2\}\circ\sin\{(3\pi 4+x)-(3\pi 4-x)2\}$

$= -2\sin(3\pi 4)\sin x$

$= -2\sin(-\pi 4)\sin x =$

$-\sqrt{2}\sin x$ = R.H.S.

Hence proved.

Ex 3.3 Class 11 Maths Question 12:

Prove that: $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Ans:

It is known that sin A + sin B = $2\sin(A-B2)\cos(A-B2)$ sin A – sin B = $2\cos(A+B2)\sin(A-B2)$ L.H.S.= $\sin^2 6x - \sin^2 4x$

$= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$

$= (2\sin 5x \cos x)(2\cos 5x \sin x)$

$= (2\sin 5x \cos 5x)(2\sin x \cos x)$

$= \sin 10x \sin 2x$ = R.H.S.

Hence proved.

Ex 3.3 Class 11 Maths Question 13:

Prove that: $\cos^2 2x \cos^2 6x = \sin 4x \sin 8x$ **Ans:**

It is known that cos A + cos B = $2\cos(A+B2)\cos(A-B2)$ cos A – cos = $2\sin(A+B2)\sin(A-B2)$

$\therefore$ L.H.S = $\cos^2 2x - \cos^2 6x$

$= (\cos 2x + \cos 6x)(\cos 2x - 6x)$

$= [2\cos(2x+6x2)\cos(2x-6x2)][-2\sin(2x+6x2)\sin(2x-6x)2]$

$\therefore$ L.H.S.= $\cos^2 2x - \cos^2 6x$

$= (\cos 2x + \cos 6x)(\cos 2x - 6x)$

$= [2\cos 4x \cos(-2x)][-2\sin 4x \sin(-2x)]$

$= [2\cos 4x \cos 2x][-2\sin 4x(-\sin 2x)]$

$= (2\sin 4x \cos 4x)(2\sin 2x \cos 2x)$

$= \sin 8x \sin 4x$ = R.H.S.

Hence proved.

Ex 3.3 Class 11 Maths Question 14:

Prove that: $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$ **Ans:**

L.H.S.= $\sin 2x + 2\sin 4x + \sin 6x$

$= [\sin 2x + \sin 6x] + 2\sin 4x$

$= [2\sin(2x+6x2)\cos(2x-6x2)] + 2\sin 4x$

[$\because$ $\sin A + \sin B = 2\sin(A+B2)\cos(A-B2)$]

$= 2\sin 4x \cos(-2x) + 2\sin 4x$

$= 2\sin 4x \cos 2x + 2\sin 4x$

$= 2\sin 4x(\cos 2x + 1)$

$= 2\sin 4x(2\cos^2 x - 1 + 1)$

= 2 sin 4x (2 $\cos^2$ x)
= 4 $\cos^2$ x sin 4x =
R.H.S.
Hence proved.

Ex 3.3 Class 11 Maths Question 15:
cot 4x (sin 5x + sin 3x) = cot x (sin 5x – sins 3x) **Ans:**

$$\text{L.H.S.} = \cot 4x\,(\sin 5x + \sin 3x)$$
$$= \frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$
$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$
$$= \left(\frac{\cos 4x}{\sin 4x}\right)[2\sin 4x\cos x] = 2\cos 4x\cos x$$
$$\text{R.H.S.} = \cot x\,(\sin 5x - \sin 3x)$$
$$= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right.$$
$$\left[\because \sin A + \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$
$$= \frac{\cos x}{\sin x}[2\cos 4x\sin x]$$
$$= 2\cos 4x\cos x$$
$$\text{L.H.S.} = \text{R.H.S.}$$

Hence proved.

Ex 3.3 Class 11 Maths Question 16:
Prove that: cos9x−cos5xsin17x−sin3x=−sin2xcos10x **Ans:**

It is known that

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),$$
$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\therefore \quad \text{L.H.S.} = \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = \frac{-2\sin\left(\frac{9x+5x}{2}\right)\sin\frac{(9x-5x)}{2}}{2\cos\left(\frac{17x+3x}{2}\right)\sin\left(\frac{17x-3x}{2}\right)}$$
$$= \frac{-2\sin 7x\sin 2x}{2\cos 10x\sin 7x} = -\frac{\sin 2x}{\cos 10x}$$
$$= \text{R.H.S.}$$

Hence proved.

Ex 3.3 Class 11 Maths Question 17:
Prove that: sin5x+sin3xcos5x+cos3x = tan 4x **Ans:**

It is known that sin A + sin = 2 sin(A+B2)cos(A−B2) cos A + cos = 2 cos(A+B2)cos(A−B2)

∴ L.H.S = sin5x+sin3xcos5x+cos3x

= 2sin(5x+3x2)cos(5x−3x2)2cos(5x+3x2)cos(5x−3x2) = 2sin4xcosx2cos4xcosx=sin4xcos4x = tan 4x = R.H.S.

Hence proved.

Ex 3.3 Class 11 Maths Question 18:

Prove that: sinx−sinycosx+cosy=tanx−y2. **Ans:**

It is known that :

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right).$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \frac{\sin x - \sin y}{\cos x + \cos y} = \frac{2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$$

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)} = \tan\left(\frac{x-y}{2}\right) = \text{R.H.S.}$$

Hence proved.

Ex 3.3 Class 11 Maths Question 19:

Prove that: sinx+sin3xcosx+cos3x = tan 2x **Ans:**

It is known that : $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right).$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\text{L.H.S.} = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

$$= \frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)} = \frac{2\sin 2x\cos(-x)}{2\cos 2x\cos(-x)}$$

$$= \frac{\sin 2x}{\cos 2x} = \tan 2x = \text{R.H.S.}$$

Hence proved.

Ex 3.3 Class 11 Maths Question 20:

Prove that: sinx−sin3xsin2x−cos2x = 2 sin x **Ans:**

It is known that

sin A – sin B = 2 cos(A+B2)sin(A−B2) cos² A
– sin² A = cos 2A

∴ L.H.S. = =sinx−sin3xsin2x−cos2x
= 2cos(x+3x2)sin(x−3x2)−cos2x
= 2cos2xsin(−x)−cos2x
= – 2 × (- sin x) = 2 sin x

= R.H.S

Hence proved.

Ex 3.3 Class 11 Maths Question 21:
Prove that: cos4x+cos3x+cos2xsin4x+sin3x+sin2x = cot 3x **Ans:**

$$\text{L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

$$\left[\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right).\right.$$

$$\left.\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{2\cos 3x\cos x + \cos 3x}{2\sin 3x\cos x + \sin 3x}$$

$$= \frac{\cos 3x\,(2\cos x + 1)}{\sin 3x\,(2\cos x + 1)} = \cot 3x = \text{R.H.S.}$$ **Hence proved.**

Ex 3.3 Class 11 Maths Question 22:
Prove that : cot x cot 2x – cot 2x cot 3x – cot 3x cot x = 1 **Ans:**
L.H.S.= cot x cot 2x – cot 2x cot 3x – cot 3x cot x
= cot x cot 2x – cot 3x (cot 2x + cot x)
= cot x cot 2x – cot (2x + x) (cot 2x + cot x)
= cot x cot 2x – [cot2xcotx−1cotx+cot2x] (cot 2x + cot x)
[∵ cot (A + B) = cotAcotB−1cotA+cotB] =
cot x cot 2x – (cot 2x cot x – 1) = 1 = R.H.S.
Hence proved.

Ex 3.3 Class 11 Maths Question 23:
Prove that: tan 4x = 4tanx(1−tan2x)1−6tan2x+tan4x.
Ans:
It is known that: tan 2A
= 2tanA1−tan2A

$$\text{L.H.S.} = \tan 4x = \tan 2(2x) = \frac{2\tan 2x}{1-\tan^2(2x)}$$

$$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2} = \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\frac{4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[\frac{(1-\tan^2 x)^2-4\tan^2 x}{(1-\tan^2 x)^2}\right]} = \frac{4\tan x(1-\tan^2 x)}{(1-\tan^2 x)^2-4\tan^2 x}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x} = \text{R.H.S.}$$ **Hence proved.**

Ex 3.3 Class 11 Maths Question 24:

Prove that: $\cos 4x = 18 \sin^2 x \cos^2 x$ **Ans:**

L.H.S. = $\cos 4x = \cos 2(2x)$

$= 1 - 2\sin^2 2x$ [$\because \cos 2A = 1 - 2\sin^2 A$]

$= 1 - 2(2 \sin x \cos x)^2$ [$\because \sin 2A = 2 \sin A \cos A$] =

$1 - 8 \sin^2 x \cos^2 x$

= R.H.S.

Hence proved.

Ex 3.3 Class 11 Maths Question 25:

Prove that: $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$ **Ans:**

We know that: $\cos 3x = 4\cos^3 x - 3\cos x$ On

replacing x by 2x, we get $\cos 3(2x) = 4\cos^3$

$(2x) - 3\cos 2x \Rightarrow \cos 6x = 4(2\cos^2 x - 1)^3 -$

$3(2\cos^2 x - 1)$

[$\because \cos 2x = 2\cos^2 x - 1$]

$= 4[8\cos^6 x - 12\cos^4 x + 6\cos^2 x - 1] - 6\cos^2 x + 3$

[$\because (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$]

$= 32\cos^6 x - 48\cos^4 x + 24\cos^2 x - 4 - 6\cos^2 x + 3 \Rightarrow$

$\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Hence proved.