

INDIAN MARITIME UNIVERSITY
(A Central University, Govt. of India)

May/June 2015 End Semester Examinations

SEMESTER – I, B.TECH (MARINE ENGINEERING)

MATHEMATICS - I (T 2102 / T 1102)

Date: 09.06.2015

Time: -3 Hrs

Max. Marks: 100

Pass Marks: 50

PART – A

(3 x10 = 30 Marks)

Compulsory Questions (The Symbols have their usual meanings)

1. a) If $u = \sqrt{xy}$ find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$.
- b) If $u = \sin(ax) + \cos(ax)$ then show that $u_n = a^n [1 + (-1)^n \sin(2ax)]^{\frac{1}{2}}$.
- c) Find the maximum and minimum values of the following function $x^3 + y^3 - 12x - 3y + 15$
- d) Find the radius of curvature at any point $(a \cos t, b \sin t)$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- e) Evaluate $\int_0^{\infty} e^{-x^2} dx$
- f) Evaluate $\int_0^1 dx \int_0^x e^{\frac{y}{x}} dy$
- g) If $u = x^2 + y^2 + z^2$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then find $\text{div}(u\vec{r})$ in terms of u
- h) Find all the solutions of the following system of equations
 $x_1 + 2x_2 - x_3 = 1, 3x_1 - 2x_2 + 2x_3 = 2, 7x_1 - 2x_2 + 3x_3 = 5$
- i) Evaluate $\int_C \frac{z+1}{z^2 - 2z} dz$, where C is the circle $|z|=5$.
- j) Change the order of integration to evaluate $\int_{y=0}^1 \int_{x=y}^1 e^{x^2} dx dy$

PART – B

(5 x14 = 70 Marks)

(Answer any five of the following)

2. a) If $y = \frac{\log x}{x}$, then prove that $y_n = \frac{(-1)^n n!}{x^{n+1}} (\log x - \sum_{r=1}^n \frac{1}{r})$ (7)
- b) Find the asymptotes to the curve $x^3 - 2y^3 + 2x^2y - xy^2 + xy - y^2 + 1 = 0$ (7)

3. a) If $z = x^n f\left(\frac{y}{x}\right) + y^n \varphi\left(\frac{x}{y}\right)$ show that $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = n^2 z$ (7)

b) Find the minimum value of $u = x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = p$ (7)

4. a) Use the rule of differentiation under the sign of integration to evaluate (7)

$$\int_0^{\frac{\pi}{2}} \log(\alpha \cos^2 \theta + \beta \sin^2 \theta) d\theta$$

b) Evaluate $\iint_R \sqrt{\frac{a^2 b^2 - b^2 x^2 - a^2 y^2}{a^2 b^2 + b^2 x^2 + a^2 y^2}} dx dy$

where R is the region enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant. (7)

5. a) Find the length of an arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$

b) Find the volume and surface area of the solid generated by the revolution of the astroid

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$$
 about the x-axis.

(6+8)

6. Find the characteristic equation of the symmetric matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Express $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$ in the linear polynomial in A. (14)

7. a) Show that $\vec{v}(x,y,z) = (yz)\hat{i} + (xz)\hat{j} + (xy)\hat{k}$ is irrotational and find a scalar function $u(x,y,z)$ such that $\vec{v} = \text{grad}(u)$. (7)

b) Find the directional derivative of $f(x,y,z) = x^2yz + 4xz^2$ at the point (1,2,-1) in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$. (7)

8. a) Evaluate the following integral using residue theorem $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$

Where C is the circle $|z| = \frac{3}{2}$. (7)

b) Find the first four terms of the Taylor's series expansion of the complex variable function

$$f(z) = \frac{z+1}{(z-3)(z-4)} \quad (7)$$

9. a) Prove that $\sqrt{\pi} \Gamma(2m) = 2^{2m-1} \Gamma(m) \Gamma(m + \frac{1}{2})$ (7)

b) Test for an extremum of the functional $I[y(x)] = \int_0^1 (xy + y^2 - 2y^2 y') dx$; $y(0) = 1$, $y(1) = 2$ (7)
