Vedantu

JEE-Main-24-01-2025 (Memory Based)
[MORNING SHIFT]
Maths

Question:If the 5^{th}, 6^{th}, and 7^{th} term of the binomial expansion of $(1 + x^2)^{n+4}$ are in A.P. Then the greatest binomial coefficient in the expansion of $(1 + x^2)^{n+4}$ is
Options:
(a) 10
(b) 35
(c) 25
(d) 14
Answer: (b)

$^{N}C_4 {}^{N}C_5, {}^{N}C_6 \rightarrow AP, \quad N = n + 4$

$^{N}C_4 + {}^{N}C_6 = 2.{}^{N}C_5$

$\Rightarrow \frac{{}^{N}C_4}{{}^{N}C_5} + \frac{{}^{N}C_5}{{}^{N}C_5} = 2$

$\Rightarrow \frac{5}{N-4} + \frac{N-5}{6} = 2$

$\Rightarrow 30 + n^2 - 9N + 20 = 12N - 98$

$\Rightarrow N^2 - 21N + 98 = 0$

$\Rightarrow (N - 7)(N - 14) = 0 \Rightarrow N = 7, 14$

Greatest Binomial Coefficient $= {}^{7}C_3 = {}^{7}C_4 = \frac{7\times6\times5}{6} = 35$

or $^{14}C_7$

Question: The number of 3 digit numbers which is divisible by 2 and 3 but not divisible by 4 and 9.
Options:
(a) 150
(b) 25
(c) 125
(d) 50
Answer: (d)

Divisibe by 2 but not by 4 = 225

102, 106, 110,......998

out of this divisible by 3

102, 114, 126, ...990

$12n + 90 = n = 1, 2, ...75$

So only divisible by 3 but not by 9

n = 1, 2, 4, 5,7, 8,......i.e., 50

Question: If A is 3 × 3 matrix such that det(A) =2. Then det
$(adj(adj(adj(adjA))))$

Options:
(a) 2^{32}
(b) 2^{16}
(c) 2^{8}
(d) 2^{12}
Answer: (b)

$$|A| = 2$$

$$||adj(adj(ajd(adjA)))||$$

$$= |A|^{24} = 2^{16}$$

Question:Evaluate $\lim_{x\to 0} \cos ecx.\left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}\right)$
Options:
(a) 1
(b) 0
(c) $\frac{1}{2\sqrt{5}}$
(d) $-\frac{1}{2\sqrt{5}}$
Answer: (d)

$$\lim_{x\to 0} \frac{\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}}{\sin x}$$

$$\lim_{x\to 0} \frac{\frac{1}{2\sqrt{2\cos^2 x 3\cos x}}[(4\cos x)(-\sin x)-3\sin x]-\frac{1}{2\sqrt{\cos^2 x+\sin+4}}[(2\cos x)(\sin x)+\cos x]}{\cos x}$$

$$= 0 - \frac{1}{2\sqrt{5}} = -\frac{1}{2\sqrt{5}}$$

Question:If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c}$ is **coplanar with** $\vec{a}$ and $\vec{b}$. Also $\vec{a}.\vec{c} = 5$ and $\vec{c}$ is pependicular to $\vec{b}$. Then $|\vec{c}|$ is
Options:
(a) 18
(b) 16
(c) $\frac{\sqrt{5}}{14}$

(d) $\sqrt{\frac{11}{6}}$

Answer: (d)

$\vec{a}=(1,2,3), \vec{b}=(3,1,-1), a.c=5$

$\vec{c}=\lambda\vec{b}\times\left(\vec{a}\times\vec{b}\right)$

$=\lambda\left[b^2\vec{a}-\left(\vec{b}\cdot\vec{a}\right)\vec{b}\right]$

$=\lambda\left(11\left(\hat{i}+2\hat{j}+3\hat{k}\right)-(2)\left(3\hat{i}+\hat{j}-\hat{k}\right)\right)$

$=\lambda\left(5\hat{i}+20\hat{j}+35\hat{k}\right)$

$=5\lambda\left(\hat{i}+4\hat{j}+7\hat{k}\right)$

$\vec{a}\cdot\vec{c}=5\Rightarrow 5\lambda(1+8+21)=5$

$\Rightarrow 5\lambda=\frac{1}{6}$

$\left|\vec{c}\right|=5\lambda\sqrt{66}=\frac{\sqrt{66}}{6}=\sqrt{\frac{66}{36}}=\sqrt{\frac{11}{6}}$

Question:The area of the region bounded by S(x, y) such that S= {(x, y) : $x^2+4x+2\leq y\leq|x+2|$} is (in sq. units)

Options:

(a) $\frac{24}{5}$

(b) 5

(c) $\frac{20}{3}$

(d) 7

Answer: (c)

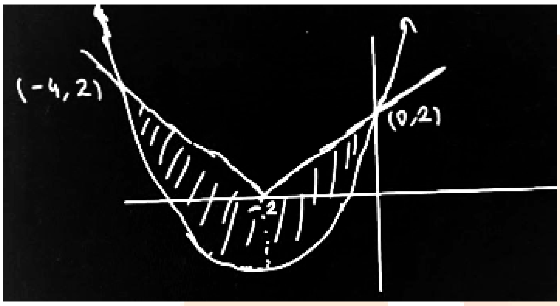
$(x+2)^2 - 2 \le y \le |x+2|$

$$A = \int_{-4}^{-2} (-x - 2 - x^2 - 4x - 2)\,dx + \int_{-2}^{0} (x + 2 - x^2 - 4x - 2)\,dx$$

$$= \int_{-4}^{-2} (-x^2 - 5x - 4)\,dx + \int_{-2}^{0} (-x^2 - 3x)\,dx$$

$$= \left(-\frac{x^3}{3} - \frac{5x^2}{2} - 4x\right)_{-4}^{-2} + \left(\frac{-x^3}{3} - \frac{3x^2}{2}\right)_{-2}^{0}$$

$$= \frac{10}{3} + \frac{10}{3} = \frac{20}{3}$$



Question: If $\dfrac{dy}{dx} + \left(\dfrac{x}{1+x^2}\right)y = \dfrac{\sqrt{x}}{\sqrt{1+x^2}}; y(0) = 0,$ **then y(1) will be**

Options:

(a) $\frac{2}{3}$

(b) $\frac{2}{\sqrt{3}}$

(c) $\frac{\sqrt{2}}{3}$

(d) $\sqrt{\frac{2}{3}}$

Answer: (c)

$\frac{dy}{dx} - \frac{x}{1+x^2}y = \frac{\sqrt{x}}{\sqrt{1+x^2}}, P = \frac{-x}{1+x^2}, Q = \sqrt{\frac{x}{1+x^2}}$

$I.F = e^{\int -\frac{x}{1+x^2}}$

Let $1 + x^2 = t, 2xdx = dt, -xdx = -\frac{dt}{2}$

So I.F $= e^{-\frac{1}{2}\int \frac{1}{2}dt} = e^{-\frac{1}{2}\log t} = \sqrt{t} = \sqrt{1+x^2}$

Now y.I.F $= \int \sqrt{\dfrac{x}{1+x^2}} \times \sqrt{1+x^2}dx$

$y.\sqrt{a+x^2} = \int \sqrt{x}dx = \dfrac{2}{3}x^{\frac{3}{2}} + c$

$y(0) = 0$ so $O = C$

$y(1) = y.\sqrt{2} = \frac{2}{3} \times 1 + 0$

$y = \frac{\sqrt{2}}{3}$

Question: **If α and β are real numbers such that $\sec^2(\tan^{-1}(\alpha)) + \mathrm{cosec}^2(\cot^{-1}(\beta)) = 36$ and $\alpha + \beta = 8$, then $(\alpha^2 + \beta)$ is $(\alpha > \beta)$**

Options:

(a) 23

(b) 28

(c) 24

(d) 27

Answer: (b)

$$\sec^2\left(\tan^{-1}\alpha\right) + \cos ec^2\left(\cot^{-1}\beta\right) = 36, \quad \alpha + \beta = 8$$

$$1 + \alpha^2 + 1 + \beta^2 = 36 \Rightarrow \alpha^2 + \beta^2 = 34$$

$$\Rightarrow \alpha^2 + (8 - \alpha)^2 = 34$$

$$\Rightarrow 2\alpha^2 - 16\alpha + 30 = 0$$

$$\alpha^2 - 8\alpha + 15 = 0 \Rightarrow \alpha = 5, \beta = 3$$

$$\alpha^2 + \beta = 28$$

Question: $f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}$. **If** $\lim_{x \to 0}\left(\frac{1}{\alpha x} + f(x)\right) = \beta$. **find $(\alpha + 2\beta)$.**

Solution:

$f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}$

$6f\left(\frac{1}{x}\right) - 36f(x) = \left(\frac{35x}{3} - \frac{5}{2}\right) \times 6$

$-35f(x) = \frac{35}{3x} - \frac{5}{2} + 70x - 15$

$-35f(x) = 70x + \frac{35}{3x} - \frac{35}{2}$

$f(x) = \frac{1}{2} - 2x - \frac{1}{3x}$

$\underset{x \to 0}{\mathrm{lt}} \frac{1}{\alpha x} + \frac{1}{2} - 2x - \frac{1}{3x}$

$= \left(\frac{1}{\alpha} - \frac{1}{3}\right) + \frac{1}{2} - 2x$

$\alpha = 3$

$\beta = \frac{1}{2}$

Question: $I_{mn} = \int_0^1 x^{m-1}(1 - x)^{n-1} dx$, **then I(9, 13) is equal to**

Solution :

$$I_{m,n} = \int_0^1 x^{m-1}(1 - x)^{n-1} dx$$

$$I_{9,13} = \int_0^1 x^8(-x)^{12} dx$$

$$= x^8 \frac{(1-x)}{-13}\Big|_0^1 - \int_0^1 8x^7 \frac{(1-x)^{13}}{-13} dx$$

$$= \frac{8}{13} \int_0^1 x^7(1 - x)^{13} dx$$

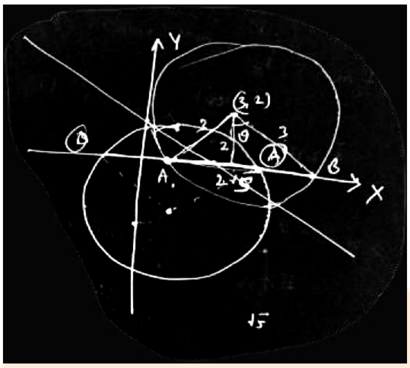
$$= \frac{8!}{13-14....20} \int_0^1 (1 - x)^{20} dx$$

$$= \frac{1}{{}^{20}C_8} \times \frac{1}{21}$$

Question:Consider the circle $x^2 + y^2 - 2x + 4y - 4 = 0$. This circle is reflected about the line $x + 2y = 2$. A chord of this reflected circle through origin and parallel to x-axis meets the circle at A and B. Find the area of region bounded by AB and circle (smaller one).

Solution :

$$(x-3)^2 + (y-2)^2 = 9$$
$$(x-3)^2 = 5$$
$$x = 3 \pm \sqrt{5}$$
$$m = \sin\theta = \frac{\sqrt{5}}{3}$$
$$A = x\theta.\,3^2 - \frac{1}{2} \times 2\sqrt{5} \times 2$$
$$= \left[9\sin^{-1}\frac{\sqrt{5}}{3} - 2\sqrt{5}\right]$$

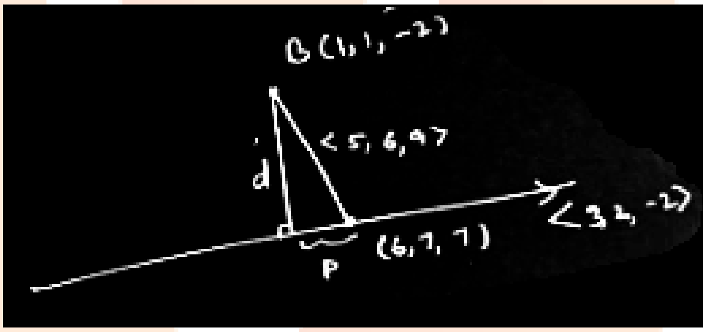


Question:A and C are two points on the line $\dfrac{x-6}{3} = \dfrac{y-7}{2} = \dfrac{z-7}{-2}$ such that AC = 6. B is (1, 1, -2). Find area of ΔABC

Solution :

$$P = \frac{15+12-18}{\sqrt{17}}$$
$$= \frac{9}{\sqrt{17}}$$
$$d = \sqrt{142 - \frac{81}{17}} = \sqrt{\frac{2333}{17}}$$
$$A = \frac{1}{2} \times 6 \times \sqrt{\frac{2333}{17}}$$

35.13



Question: Let the parabola $y = x^2 + px - 3$ cuts the coordinate axes at P, Q and R. A circle with centre (-1, -1) passes through P, Q and R, then the area of triangle PQR.

Solution :

$R(0,-3)r = \sqrt{5}$ $\quad$ $P(\alpha,0)Q(\beta,0)$

$$(x+1)^2 + (y+1)^2 = 5$$
$$(\alpha+1)^2 + 1 = 5$$
$$(\alpha+1)^2 = 4$$
$$\alpha = 1, -3$$
$$P(1,0), Q(-3,0)$$
$$\text{Area} = \frac{1}{2} \times 4 \times 3 = 6$$

Question:Find the product of all real roots of equation $(x^2 - 9x + 11)^2 - (x - 4)(x - 5) = 2$ is

Solution :

$\left(x^2-9x+11\right)^2-\left(x^2-9x+20\right)=2$
$(t+11)^2-(t+20)=2, t=x^2-9x \geq \frac{-81}{4}$
$t^2 \_ 21t+99-0 \Rightarrow t=\frac{-21\pm 3\sqrt{5}}{2}=-7.14,-13.8$
Product of roots $=99$

Question: For a distribution of 10 observations, $\sum_{i=1}^{10} x_i=55$ and $\sum_{i=1}^{10} x_i^2=328$**. If the observations 4 and 5 are replaced by 6 and 8 respectively, then the new variance is**
Options:
(a) 2.5
(b) 2.7
(c) 3.4
(d) 3.6
Answer: (b)

$$\sum x=55-4-5+6+8=60$$
$$\sum x^2=328-16-25+36+64=387$$
$$\bar{x}=6$$
$$\sigma^2=\frac{387}{10}-6^2=38.7-36=2.7$$

Question: A and B playing a game (throwing a pair of dice alternatively). A wins the game when sum = 5 and B wins the game when sum = 8. Probability of A winning given that A starts the game.
Solution :

$$P(A)=\frac{4}{36}=\frac{1}{9}, P(B)=\frac{5}{36}$$
$$P(Awins)=\frac{4}{36}+\frac{32}{36}\cdot\frac{31}{36}\cdot\frac{4}{36}+......to \propto$$
$$=\frac{\frac{4}{36}}{2-\frac{32}{36}\cdot\frac{31}{36}}=\frac{\frac{1}{9}}{1-\frac{8}{9}\cdot\frac{31}{36}}=\frac{\frac{1}{9}}{1-\frac{62}{81}}$$
$$=\frac{9}{19}$$

Question:If the images of the points A(1,3), B(3,1) and C(2,4) in the line x + 2y = 4 are D, E and F respectively, then the centroid of the triangle DEF is
Solution :

The mirror line is $x+2y-4=0$

image of A (1,3) is $\frac{x-1}{1}=\frac{y-3}{2}=-2\left(\frac{1+6-4}{5}\right)$

image of B $(3,1)$ is $\frac{x-3}{1}=\frac{y-1}{2}=-2\left(\frac{3.2.4}{5}\right)$

image of $(2,4)$ is $\frac{x-2}{1}=\frac{y-4}{2}=-2\left(\frac{2+8-4}{5}\right)$

$x=\frac{-2}{5}, y=-\frac{4}{5}$

So $D=\left(-\frac{1}{5}, \frac{3}{5}\right), E=\left(\frac{13}{5}, \frac{1}{5}\right), F=\left(\frac{-2}{5}, \frac{-4}{5}\right)$

Centroid $=\left(\frac{\frac{-1}{5}+\frac{13}{5}\frac{-2}{5}}{3}, \frac{\frac{3}{5}+\frac{1}{5}-\frac{4}{5}}{3}\right)$

$=\left(\frac{10}{15}, \frac{10}{15}\right)=\left(\frac{2}{3}, 0\right)$