MATHEMATICS

1. $\int_{-2}^{0} (x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1))\, dx =$

(A) 3 (B) 4 (C) 1 (D) 0

2. $\int_{0}^{\pi} \frac{x \tan x}{\sec x \cdot \operatorname{cosec} x}\, dx =$

(A) $\pi^2/4$ (B) $\pi/2$ (C) $\pi^2/2$ (D) $\pi/4$

3. $\int \sqrt{5 - 2x + x^2}\, dx =$

(A) $\frac{x}{2}\sqrt{5 - 2x + x^2} + 4 \log\left|(x+1) + \sqrt{x^2 - 2x + 5}\right| + C$

(B) $\frac{x-1}{2}\sqrt{5 + 2x + x^2} + 2 \log\left|(x-1) + \sqrt{5 + 2x + x^2}\right| + C$

(C) $\frac{x-1}{2}\sqrt{5 - 2x + x^2} + 2 \log\left|(x-1) + \sqrt{5 - 2x + x^2}\right| + C$

(D) $\frac{x-1}{2}\sqrt{5 - 2x + x^2} + 2 \log\left|(x+1) + \sqrt{x^2 + 2x + 5}\right| + C$

4. $\int \frac{1}{1 + 3\sin^2 x + 8\cos^2 x}\, dx =$

(A) $\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(B) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(C) $6\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$

(D) $\frac{1}{6}\tan^{-1}(2\tan x) + C$

5. If a curve passes through the point (1, 1) and at any point (x, y) on the curve, the product of the slope of its tangent and x co-ordinate of the point is equal to the y co-ordinate of the point, then the curve also passes through the point

(A) $(3, 0)$ (B) $(-1, 2)$ (C) $(\sqrt{3}, 0)$ (D) $(2, 2)$

6. The degree of the differential equation

$$1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2=\sqrt[3]{\frac{d^2y}{dx^2}+1} \text{ is}$$

(A) 3 (B) 1 (C) 2 (D) 6

7. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ then

(A) $\vec{a}$ and $\vec{b}$ are parallel.

(B) $\vec{a}$ and $\vec{b}$ are coincident.

(C) Inclined to each other at 60°.

(D) $\vec{a}$ and $\vec{b}$ are perpendicular.

8. The component of $\hat{i}$ in the direction of the vector $\hat{i}+\hat{j}+2\hat{k}$ is

(A) 6 (B) $6\sqrt{6}$ (C) $\frac{\sqrt{6}}{6}$ (D) $\sqrt{6}$

9. In the interval $(0, \pi/2)$, area lying between the curves $y = \tan x$ and $y = \cot x$ and the X-axis is

(A) 2 log 2 sq. units

(B) 4 log 2 sq. units

(C) log 2 sq. units

(D) 3 log 2 sq. units

10. The area of the region bounded by the line $y = x + 1$, and the lines $x = 3$ and $x = 5$ is

(A) $\frac{7}{2}$ sq. units

(B) $\frac{11}{2}$ sq. units

(C) 7 sq. units

(D) 10 sq. units

11. If a line makes an angle of $\frac{\pi}{3}$ with each X and Y axis then the acute angle made by Z-axis is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

12. The length of perpendicular drawn from the point (3, –1, 11) to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is

(A) $\sqrt{29}$ (B) $\sqrt{33}$ (C) $\sqrt{53}$ (D) $\sqrt{66}$

13. The equation of the plane through the points (2, 1, 0), (3, 2, – 2) and (3, 1, 7) is

(A) $2x-3y+4z-27=0$
(B) $6x-3y+2z-7=0$
(C) $7x-9y-z-5=0$
(D) $3x-2y+6z-27=0$

14. The point of intersection of the line $x+1=\frac{y+3}{3}=\frac{-z+2}{2}$ with the plane $3x+4y+5z=10$ is

(A) (2, – 6, – 4) (B) (2, 6, – 4) (C) (2, 6, 4) (D) (–2, 6, – 4)

15. If (2, 3, –1) is the foot of the perpendicular from (4, 2, 1) to a plane, then the equation of the plane is

(A) $2x+y+2z-1=0$
(B) $2x-y+2z=0$
(C) $2x+y+2z-5=0$
(D) $2x-y+2z+1=0$

16. $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=144$ and $|\vec{a}|=4$ then $|\vec{b}|$ is equal to

(A) 3 (B) 8 (C) 4 (D) 12

17. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ and

$$(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})=\lambda(\vec{b}\times\vec{c})$$

then the value of λ is equal to

(A) 3 (B) 4 (C) 6 (D) 2

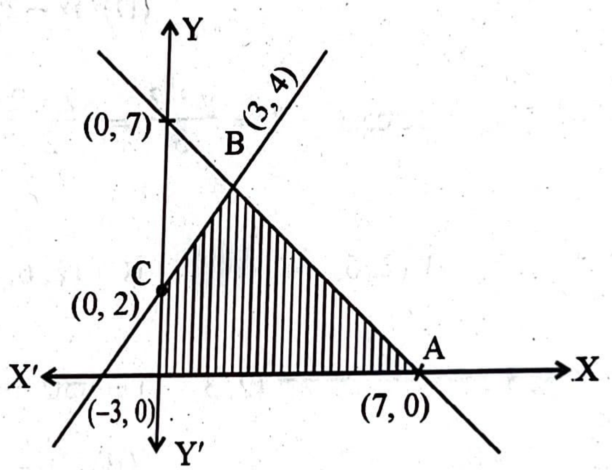
18. A bag contains 2n + 1 coins. It is known that n of these coins have head on both sides whereas the other n + 1 coins are fair. One coin is selected at random and tossed. If the probability that toss results in heads is $\frac{31}{42}$, then the value of n is

(A) 6 (B) 8 (C) 10 (D) 5

19. Let A = {x, y, z, u} and B = {a, b}. A function f : A → B is selected randomly. The probability that the function is an onto function is

(A) $\frac{1}{8}$ (B) $\frac{5}{8}$ (C) $\frac{1}{35}$ (D) $\frac{7}{8}$

20. The shaded region in the figure given is the solution of which of the inequations ?



(A) $x + y \geq 7,\ 2x - 3y + 6 \leq 0,\ x \geq 0,\ y \geq 0$

(B) $x + y \geq 7,\ 2x - 3y + 6 \geq 0,\ x \geq 0,\ y \geq 0$

(C) $x + y \leq 7,\ 2x - 3y + 6 \leq 0,\ x \geq 0,\ y \geq 0$

(D) $x + y \leq 7,\ 2x - 3y + 6 \geq 0,\ x \geq 0,\ y \geq 0$

21. If A and B are events such that $P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{2}$ and $P(B/A) = \frac{2}{3}$ then P(B) is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{6}$

22. The value of $e^{\log_{10}\tan 1° + \log_{10}\tan 2° + \log_{10}\tan 3° + \ldots + \log_{10}\tan 89°}$ is

(A) 3
(B) $\frac{1}{e}$
(C) 1
(D) 0

23. The value of $\begin{vmatrix} \sin^2 14° & \sin^2 66° & \tan 135° \\ \sin^2 66° & \tan 135° & \sin^2 14° \\ \tan 135° & \sin^2 14° & \sin^2 66° \end{vmatrix}$

is

(A) 0
(B) 1
(C) 2
(D) –1

24. The modulus of the complex number $\frac{(1+i)^2(1+3i)}{(2-6i)(2-2i)}$ is

(A) $\frac{2}{\sqrt{2}}$
(B) $\frac{1}{\sqrt{2}}$
(C) $\frac{\sqrt{2}}{4}$
(D) $\frac{4}{\sqrt{2}}$

25. Given that a, b and x are real numbers and $a < b$, $x < 0$ then

(A) $\frac{a}{x} \geq \frac{b}{x}$
(B) $\frac{a}{x} < \frac{b}{x}$
(C) $\frac{a}{x} \leq \frac{b}{x}$
(D) $\frac{a}{x} > \frac{b}{x}$

26. Ten chairs are numbered as 1 to 10. Three women and two men wish to occupy one chair each. First the women choose the chairs marked 1 to 6, then the men choose the chairs from the remaining. The number of possible ways is

(A) ${}^6P_3 \times {}^4P_2$
(B) ${}^6C_3 \times {}^4P_2$
(C) ${}^6P_3 \times {}^4C_2$
(D) ${}^6C_3 \times {}^4C_2$

27. Which of the following is an empty set ?

(A) $\{x : x^2 + 1 = 0, x \in R\}$
(B) $\{x : x^2 - 9 = 0, x \in R\}$
(C) $\{x : x^2 = x + 2, x \in R\}$
(D) $\{x : x^2 - 1 = 0, x \in R\}$

28. If $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$ then a and b are respectively

(A) 2, –3
(B) 0, 2
(C) 2, 3
(D) –3, –1

29. If $p\left(\frac{1}{q}+\frac{1}{r}\right)$, $q\left(\frac{1}{r}+\frac{1}{p}\right)$, $r\left(\frac{1}{p}+\frac{1}{q}\right)$ are in A.P., then p, q, r

(A) are in G.P.

(B) are in A.P.

(C) are not in G.P.

(D) are not in A.P.

30. A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its y-intercept is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) $\frac{1}{3}$

31. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is

(A) $\frac{x^2}{4} - \frac{y^2}{9} = 1$ (B) $2x^2 - 3y^2 = 7$ (C) $y^2 - x^2 = 32$ (D) $x^2 - y^2 = 32$

32. If $\lim_{x \to 0} \frac{\sin(2+x) - \sin(2-x)}{x} = A \cos B$, then the values of A and B respectively are

(A) 1, 2 (B) 2, 1 (C) 1, 1 (D) 2, 2

33. If n is even and the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^n$ is $924\, x^6$, then n is equal to

(A) 14 (B) 12 (C) 8 (D) 10

34. n^{th} term of the series

$1 + \frac{3}{7} + \frac{5}{7^2} + \frac{1}{7^2} + \ldots$ is

(A) $\frac{2n+1}{7^n}$ (B) $\frac{2n-1}{7^n}$ (C) $\frac{2n+1}{7^{n-1}}$ (D) $\frac{2n-1}{7^{n-1}}$

35. $f : R \to R$ and $g : [0, \infty) \to R$ are defined by $f(x) = x^2$ and $g(x) = \sqrt{x}$. Which one of the following is not true ?

(A) $(fog)(-4) = 4$ (B) $(fog)(2) = 2$ (C) $(gof)(-2) = 2$ (D) $(gof)(4) = 4$

36. Let $f : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g : R \to R$ by $g(x) = \frac{x}{x^2+1}$ then gof is

(A) $\frac{3x^2-5}{9x^4-6x^2+26}$ (B) $\frac{3x^2}{x^4+2x^2-4}$ (C) $\frac{3x^2}{9x^4+30x^2-2}$ (D) $\frac{3x^2-5}{9x^4-30x^2+26}$

37. Let the relation R be defined in N by aRb if $3a + 2b = 27$ then R is

(A) $\left\{\left(0, \frac{27}{2}\right), (1, 12), (3, 9), (5, 6), (7, 3)\right\}$

(B) $\{(1, 12), (3, 9), (5, 6), (7, 3), (9, 0)\}$

(C) $\{(2, 1), (9, 3), (6, 5), (3, 7)\}$

(D) $\{(1, 12), (3, 9), (5, 6), (7, 3)\}$

38. Let $f(x) = \sin 2x + \cos 2x$ and $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain

(A) $x \in \left[\frac{-\pi}{8}, \frac{\pi}{8}\right]$ (B) $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (C) $x \in \left[0, \frac{\pi}{4}\right]$ (D) $x \in \left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

39. The contrapositive of the statement

"If two lines do not intersect in the same plane then they are parallel." is

(A) If two lines are parallel then they intersect in the same plane.

(B) If two lines are not parallel then they do not intersect in the same plane.

(C) If two lines are parallel then they do not intersect in the same plane.

(D) If two lines are not parallel then they intersect in the same plane.

40. The mean of 100 observations is 50 and their standard deviation is 5. Then the sum of squares of all observations is

(A) 252500 (B) 250000 (C) 255000 (D) 50000

Space For Rough Work

41. If $x\begin{bmatrix} 3 \\ 2 \end{bmatrix} + y\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \end{bmatrix}$ then the value of x and y are

(A) $x = 4, y = -3$ (B) $x = -4, y = -3$

(C) $x = -4, y = 3$ (D) $x = 4, y = 3$

42. If A and B are two matrices such that AB = B and BA = A then $A^2 + B^2 =$

(A) 2 AB (B) AB (C) 2 BA (D) A + B

43. If $A = \begin{bmatrix} 2-k & 2 \\ 1 & 3-k \end{bmatrix}$ is singular matrix, then the value of $5k - k^2$ is equal to

(A) -6 (B) -4 (C) 6 (D) 4

44. The area of a triangle with vertices $(-3, 0)$, $(3, 0)$ and $(0, k)$ is 9 sq. units, the value of k is

(A) -9 (B) 6 (C) 3 (D) 9

45. If $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a & b & c \end{vmatrix}$ then

(A) $\Delta_1 = 3\Delta$ (B) $\Delta_1 \neq \Delta$ (C) $\Delta_1 = -\Delta$ (D) $\Delta_1 = \Delta$

46. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ where a, $x \in (0, 1)$ then the value of x is

(A) $\frac{a}{2}$ (B) $\frac{2a}{1+a^2}$ (C) $\frac{2a}{1-a^2}$ (D) 0

47. The value of

$$\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right] \text{ where } x \in \left(0, \frac{\pi}{4}\right)$$

is

(A) $\frac{x}{2} - \pi$ (B) $\pi - \frac{x}{3}$ (C) $\pi - \frac{x}{2}$ (D) $\frac{x}{2}$

48. The function $f(x) = \cot x$ is discontinuous on every point of the set

(A) $\{x = 2n\pi; n \in Z\}$

(B) $\left\{x = (2n+1)\frac{\pi}{2}; n \in Z\right\}$

(C) $\left\{x = \frac{n\pi}{2}; n \in Z\right\}$

(D) $\{x = n\pi; n \in Z\}$

49. If the function is $f(x) = \frac{1}{x+2}$, then the point of discontinuity of the composite function $y = f(f(x))$ is

(A) $\frac{5}{2}$

(B) $\frac{2}{5}$

(C) $\frac{1}{2}$

(D) $\frac{-5}{2}$

50. If $y = a \sin x + b \cos x$, then $y^2 + \left(\frac{dy}{dx}\right)^2$ is a

(A) function of y

(B) function of x and y

(C) constant

(D) function of x

51. If $f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \ldots\ldots + x^n$ then $f''(1) =$

(A) $n(n-1)2^{n-2}$

(B) $n(n-1)\,2^n$

(C) 2^{n-1}

(D) $(n-1)2^{n-1}$

52. If $A = \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}$ and $AB = I$ then $B =$

(A) $\cos^2 \alpha/2 \cdot A$

(B) $\cos^2 \alpha/2 \cdot I$

(C) $\sin^2 \alpha/2 \cdot A$

(D) $\cos^2 \alpha/2 \cdot A^T$

53. If $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ then $\frac{du}{dv}$ is

(A) 2

(B) $\frac{1-x^2}{1+x^2}$

(C) 1

(D) $\frac{1}{2}$

54. The distance 's' in meters travelled by a particle in 't' seconds is given by $s = \frac{2t^3}{3} - 18t + \frac{5}{3}$.

The acceleration when the particle comes to rest is

(A) 10 m^2/sec. (B) 12 m^2/sec. (C) 18 m^2/sec. (D) 3 m^2/sec.

55. A particle moves along the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$. When the rate of change of abscissa is 4 times that of its ordinate, then the quadrant in which the particle lies is

(A) II or IV (B) III or IV (C) II or III (D) I or III

56. An enemy fighter jet is flying along the curve given by $y = x^2 + 2$. A soldier is placed at (3, 2) wants to shoot down the jet when it is nearest to him. Then the nearest distance is

(A) $\sqrt{6}$ units (B) 2 units (C) $\sqrt{5}$ units (D) $\sqrt{3}$ units

57. $\int_2^8 \frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}} + 5^{\sqrt{10-x}}} dx =$

(A) 6 (B) 4 (C) 3 (D) 5

58. $\int \sqrt{\operatorname{cosec} x - \sin x}\, dx =$

(A) $\frac{\sqrt{\sin x}}{2} + C$ (B) $2\sqrt{\sin x} + C$ (C) $\frac{2}{\sqrt{\sin x}} + C$ (D) $\sqrt{\sin x} + C$

59. If f(x) and g(x) are two functions with $g(x) = x - \frac{1}{x}$ and $fog(x) = x^3 - \frac{1}{x^3}$ then $f'(x) =$

(A) $3x^2 + \frac{3}{x^4}$ (B) $x^2 - \frac{1}{x^2}$ (C) $1 - \frac{1}{x^2}$ (D) $3x^2 + 3$

60. A circular plate of radius 5 cm is heated. Due to expansion, its radius increases at the rate of 0.05 cm/sec. The rate at which its area is increasing when the radius is 5.2 cm is

(A) 27.4 π cm^2/sec (B) 5.05 π cm^2/sec

(C) 0.52 π cm^2/sec (D) 5.2 π cm^2/sec