2. APPLICATIONS OF DERIVATIVE

I. MCQ (2 Marks each)

1. The slope of the tangent to the curve x = 2 $\sin^3\theta$, y = 3 $\cos^3 \theta$ at $\theta = \frac{\pi}{4}$ is

(A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

2. The slope of the normal to the curve $y = x^2 + 2e^x + 2$ at $(0,4)$ is

(A) 2 (B) -2 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

3. If the line y = 4x – 5 touches the curve $y^2 = ax^3 + b$ at the point (2, 3) then a + b is

(A) -5 (B) 2 (C) -7 (D) 9

4. If the tangent at (1, 1) on $y^2 = x(2-x)^2$ meets the curve again at P, then P is

(A) (4,4) (B)(-1, 2) (C) (3, 6) (D) $\left(\frac{9}{4}, \frac{3}{8}\right)$

5. The displacement of a particle at time *t* is given by $s = 2t^3 - 5t^2 + 4t - 3$. The time when the acceleration is 14 ft/sec^2, is

(A) 1 sec (B) 2 sec (C) 3 sec (D) 4 sec

6. Let $f(x) = x^3 - 62 + 9x + 18$, then $f(x)$ is strictly decreasing in …

(A) (-∞, 1) (B) [3,∞) (C) (-∞, 1) ∪[3,∞) (D) (1, 3)

7. A ladder 5 m in length is resting against vertical wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of 1.5 m /sec. The length of the higher point of the when foot of ladder is 4 m away from the wall decreases at the rate of ___

(A) 1 (B) 2 (C) 2.5 D) 3

8. The edge of a cube is decreasing at the rate of 0.6 cm/sec then the rate at which its volume is decreasing when the edge of the cube is 2 cm, is

(A) 1.2 cm^3 /sec (B) 3.6 cm^3 /sec (C) 4.8 cm^3 /sec (D) 7.2 cm^3 /sec

9. A particle moves along the curve $y = 4x^2 + 2$, then the point on the curve at which *y* – coordinate is changing 8 times as fast as the *x* – coordinate is

(A) (2,18) (B) $(-1,6)$ (C) (1,6) (D) (0,2)

10. The function $f(x) = x \log x$ is minimum at $x =$

(A) e (B) $\frac{1}{e}$ (C) 1 (D) $-\frac{1}{e}$

II. Very Short answer questions (1 mark each)

1. Find the slope of tangent to the curve $y = 2x^3 - x^2 + 2$ at $\left(\frac{1}{2}, 2\right)$.

2. The displacement of a particle at time t is given by $s = 2t^3 - 5t^2 + 4t - 3$. Find the velocity when $t = 2$ sec.

3. Prove that function $f(x) = x - \frac{1}{x}$, $x \in R$ and $x \neq 0$ is increasing function.

4. Show that $f(x) = x - \cos x$ is increasing for all x.

5. Show that the function $f(x) = x^3 + 10x + 7$ for $x \in R$ is strictly increasing.

III. Short answer questions (2 mark each)

1. Find the slope of normal to the curve $3x^2 - y^2 = 8$ at the point (2,2).

2. Find the slope of tangent to the curve $x = sin\theta$ and $y = cos2\theta$ at $\theta = \frac{\pi}{6}$.

3. Find the equation of normal to the curve $y = 2x^3 - x^2 + 2$ at $\left(\frac{1}{2}, 2\right)$

4. A car is moving in such a way that the distance it covers, is given by the equation $s = 4t^2 + 3t$, where s is in meters and t is in seconds. What would be the velocity and the acceleration of the car at time $t = 20$ second ?

5. A man of height 2 meters walks at a uniform speed of 6 km/hr away from a lamp post of 6 meters high. Find the rate at which the length of the shadow is increasing

6. Water is being poured at the rate of 36 m^3/sec in to a cylindrical vessel of base radius 3 meters. Find the rate at which water level is rising.

7. Test whether the function $f(x) = x^3 + 6x^2 + 12x - 5$ is increasing or decreasing for all $x \in R$.

8. Test whether the following function $f(x) = 2 - 3x + 3x^2 - x^3$, $x \in R$ is increasing or decreasing.

9. Find the values of x for which the function $f(x) = 2x^3 - 6x^2 + 6x + 24$ is strictly increasing.

10. Find approximate value of $\sqrt[3]{27.027}$

11. Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 9$, $x \in [1, 4]$

12. Verify LMVT for the function $f(x) = \log x$, $x \in [1, e]$

IV. Short answer questions (3 mark each)

1. Find the point on the curve $y = \sqrt{x-3}$ where the tangent is perpendicular to the line 6x + 3y – 5 = 0.
2. A spherical soap bubble is expanding so that its radius is increasing at the rate of 0.02 cm/sec. At what rate is the surface area is increasing, when its radius is 5 cm?
3. The surface area of a spherical balloon is increasing at the rate of 2 cm^2/sec. At what rate the volume of the balloon is increasing when radius of the balloon is 6 cm?
4. A ladder 10 meter long is leaning against a vertical wall. If the bottom of the ladder is pulled horizontally away from the wall at the rate of 1.2 meters per seconds, find how fast the top of the ladder is sliding down the wall when the bottom is 6 meters away from the wall.
5. Find the values of x for which the function f(x) = $x^3 - 6x^2$ -36x +7 is strictly increasing.
6. Find the values of x, for which the function f (x) =$x^3 + 12x^2 + 36x + 6$ is monotonically decreasing.
7. The profit function *P(x)* of a firm, selling x items per day is given by $P(x) = (150 - x)x - 1625$. Find the number of items the firm should manufacture to get maximum profit. Find the maximum profit.
8. Divide the number 30 in to two parts such that their product is maximum.
9. A wire of length 36meters is bent in the form of a rectangle. Find its dimensions if the area of the rectangle is maximum.
10. Find approximate value of$\tan^{-1}(1.001)$
11. Find approximate value of$\cos(89^\circ 30')$ given that $1^\circ = 0.0175^c$
12. Find approximate value of$e^{1.005}$, given that $e = 2.7183$
13. Find approximate value of$log_{10}(1016)$, given that$log_{10}e = 0.4343$
14. Find approximate value of$f(x) = x^3 + 5x^2 - 7x + 10$ at $x = 1.1$

V. Long answer questions (4 mark each)

1. Find points on the curve given by $y = x^3 - 6x^2 + x + 3$ where the tangents are parallel to the line $y = x + 5$.
2. The volume of the spherical ball is increasing at the rate of 4π cc/sec. Find the rate at which the radius and the surface area are changing when the volume is 288πcc.
3. The volume of a sphere increase at the rate of 20 cm^3/sec. Find the rate of change of its surface area when its radius is 5 cm.
4. A man of height 180 cm is moving away from a lamp post at the rate of 1.2 meters per second. If the height of the lamp post is 4.5 meters, find the rate at which (i) his shadow is lengthening. (ii)the tip of the shadow is moving.
5. Find the values of x for which f(x) =$2x^3 - 15x^2 - 144x - 7$ is
   (a) Strictly increasing (b) strictly decreasing
6. Find the local maximum and local minimum value of $f(x) = x^3 - 3x^2 - 24x + 5$.
7. A wire of length 120cm is bent in the form of a rectangle. Find its dimensions if the area of the rectangle is maximum.
8. An open box is to be cut out of piece of square card board of side 18 cm by cutting of equal squares from the corners and turning up the sides. Find the maximum volume of the box.
9. A rectangular sheet of paper has it area 24 sq. Meters. The margin at the top & the bottom are 75cm each & the sides 50cm each. What are the dimensions of the paper if the area of the printed space is maximum?
10. A box with a square base is to have an open top. The surface area of the box is 192 sq. cm. What should be its dimensions in order that the volume is largest?
11. A wire of length *l* is cut into two parts. One part is bent into a circle and the other into a square. Show that the sum of the areas of the circle and the square is least, if the radius of the circle is half the side of the square.
12. Find c, if LMVT is applicable for $f(x) = (x-3)(x-6)(x-9),\ x \in [3,5]$
13. Q.2 Verify LMVT for the function $f(x) = x + \frac{1}{x}$, $x \in [1,3]$