

MHT CET 2023 Question Paper with Answers and Solution May 11 Shift 2 (Memory-based)

Question 1. The value of $\int (1 - \cos x) \operatorname{cosec}^2 x dx$ is?

Answer. $\cos(x) + \sin(x) + \cot(\cos(x)) + C$

Solution. To evaluate the integral $\int (1 - \cos x) \operatorname{cosec}^2(x) dx$, we can simplify the integrand using trigonometric identities.

Recall that $\operatorname{cosec}^2(x)$ is equal to $1 + \cot^2(x)$, where $\cot(x)$ is the cotangent of x .

$$\int (1 - \cos x) \operatorname{cosec}^2(x) dx = \int (1 - \cos x) (1 + \cot^2(x)) dx$$

Expanding the expression:

$$= \int (1 - \cos x + \cot^2(x) - \cos x * \cot^2(x)) dx$$

Now, let's evaluate each term separately:

$$\int (1 - \cos x) dx = x - \sin(x) + C_1$$

$\int \cot^2(x) dx$ can be integrated by using the formula $\int \cot^2(x) dx = -x - \cot(x) + C_2$

$\int \cos x * \cot^2(x) dx$ can be integrated by substitution. Let's denote $\cos(x)$ as u :

$$u = \cos(x) \quad du = -\sin(x) dx$$

Replacing dx and $\cos(x)$ with du and u , respectively, we have:

$$\int u * \cot^2(x) (-du/\sin(x)) = -\int u \cot^2(x) du = -\int \cot^2(x) du$$

Using the formula mentioned earlier, we know that $\int \cot^2(x) dx = -x - \cot(x) + C_2$. Hence, the integral of $-\int \cot^2(x) du$ will be $-(-u - \cot(u) + C_2) = u + \cot(u) - C_2$.

Putting it all together, the integral becomes:

$$x - \sin(x) + C_1 + (-x - \cot(x) + C_2) + (\cos(x) + \cot(\cos(x))) -$$

C_2) Simplifying:

$$= x - x + \cos(x) + \sin(x) + \cot(\cos(x)) + C_1 - C_2$$

The final result is $\cos(x) + \sin(x) + \cot(\cos(x)) + C$, where $C = C_1 - C_2$ is the constant of integration.

Question 2. The function of $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonically increasing in the interval

- A. $(-\infty, 1)$
- B. $(-\infty, 1) \cup (2, \infty)$
- C. $(-\infty, -\infty)$
- D. $(2, \infty)$

Answer. B

Solution. To determine whether the function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonically increasing in an interval, we need to analyze the first derivative of the function, which is given by:

$$f'(x) = 6x^2 - 18x + 12$$

To find the critical points of the function (where the derivative is equal to zero), we need to solve the equation $f'(x) = 0$:

$$6x^2 - 18x + 12 = 0$$

Dividing both sides by 6, we get:

$$x^2 - 3x + 2 = 0$$

Factoring the left-hand side, we get:

$$(x - 1)(x - 2) = 0$$

So the critical points are $x = 1$ and $x = 2$.

Now we need to analyze the sign of the derivative in the different

intervals: Interval $(-\infty, 1)$:

For $x < 1$, we can choose $x = 0$ as a test point. Plugging this into the derivative, we get:

$$f'(0) = 6(0)^2 - 18(0) + 12 = 12$$

Since $f'(0) > 0$, the derivative is positive in the interval $(-\infty, 1)$. This means that the function is monotonically increasing in this interval.

Interval $(1, 2)$:

For $1 < x < 2$, we can choose $x = 1.5$ as a test point. Plugging this into the derivative, we get:

$$f'(1.5) = 6(1.5)^2 - 18(1.5) + 12 = -3$$

Since $f'(1.5) < 0$, the derivative is negative in the interval $(1, 2)$. This means that the function is not monotonically increasing in this interval.

Interval $(2, \infty)$:

For $x > 2$, we can choose $x = 3$ as a test point. Plugging this into the derivative, we get:

$$f'(3) = 6(3)^2 - 18(3) + 12 = 30$$

Since $f'(3) > 0$, the derivative is positive in the interval $(2, \infty)$. This means that the function is monotonically increasing in this interval.

Therefore, the function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonically increasing in the interval $(-\infty, 1) \cup (2, \infty)$, which corresponds to option B.

Question 3. The equation of the tangent to the curve $y = \sqrt{9-2x^2}$, at the point where the ordinates and abscissa are equal, is?

Answer. $y > 0$

Question 4. For all real x , the minimum value of function $f(x)=1-x+x^2/1-x+x$

A. 1/3

B. 0

C. 3

D. 1

Answer.1/3

Question 5. The minimum value of function $(1 - x + x^2) / (1 + x + x^2)$

- A. $\frac{1}{3}$**
- B. 0**
- C. 3**
- D. 1**

Answer. A

Question 6. $\int \sin(\log x) dx$

- A. $(x/2)[\sin(\log x) - \cos(\log x)]$**
- B. $\cos(\log x) - x$**
- C. $\int (x-1)e^x / (x+1)^3$**
- D. $-\cos \log x$**

Answer. A

Question 7. If the line $(x - 1)/2 = (y+1)/3 = (z-2) /4 = \diamond\diamond$ meets the plane, $x+2y+3z = 15$ at a point P, then the distance of P from the origin is?

- A. $7/2$**
- B. $9/2$**
- C. $\sqrt{5}/2$**
- D. $2\sqrt{5}$**

Answer. B

Question 8. If $\cos x + \cos y - \cos(x+y) = 3/2$ then,

A. $x+y = 0$

B. $x = 2y$

C. $x = y$

D. $2x = y$

Answer.

C

Question 9. If the vertices of a triangle are $(-2,3)$, $(6,-1)$ and $(4,3)$, then the co-ordinates of the circumcentre of the triangle are? A. $(1,1)$

B. $(-1,-1)$

C. $(-1,1)$

D. $(1,-1)$

Answer. D

Question 10. If $\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{(1-p)} + \cos^{-1}\sqrt{(1-a)} = 3\pi/4$, then q is? A. $1/2$

B. $1/\sqrt{2}$

C. 1

D. 1/3

Answer. A

Question 11. In $\triangle ABC$ $b=\sqrt{3}$, $c=1$ angle $A = 30$, then largest angle?

Answer. 120

Question 12. If the area of the parallelogram with a and b as two adjacent sides 16 sq. units, then the area of the Parallelogram having $3a+2b$ and $a+3b$ as two adjacent sides in sq.units is

A. 96

B. 112

C. 144

D. 128

Answer. B

Question 13. $dy/dx + y/x = \sin x$

Answer. $xy + \cos y - \sin x = c$

Question 14. $x=5/1-21$, value of x^3+x^2-x-22

Answer. $x^2-2x+1 = -4$

Question 15. Equation of tangent to the curve $y = \sqrt{9-2x^2}$ where $x=y$.