MHT CET 2023 Question Paper Shift 1

Question 1. General Solution of the differential equation: Cosx(1+cosy) dx-siny(1+sinx)dy=0 is:

A. **(1+cosx) (1+siny)=c**
B. **1+sinx+cosy=c**
C. **(1+sinx) (1+cosy)=c**
D. **1+sinx.cosy=c**

Answer. C
Solution. To find the general solution of the differential equation:

cos(x)(1+cos(y)) dx - sin(y)(1+sin(x)) dy = 0

We can start by rearranging the terms:

cos(x)dx - sin(y)dy + cos(x)cos(y)dx - sin(x)sin(y)dy = 0

Grouping the terms:

cos(x)dx + cos(x)cos(y)dx = sin(y)dy + sin(x)sin(y)dy

Dividing both sides by cos(x)sin(y):

(1+cos(y))dx/cos(x) = (1+sin(x))dy/sin(y)

Integrating both sides:

∫(1+cos(y))/cos(x) dx = ∫(1+sin(x))/sin(y) dy

Using the substitution $u = \sin(x)$, $du = \cos(x)dx$:

$\int(1+\cos(y))/\cos(x)\, dx = \int(1+u)/\sin(y)\, dy$

$\int(1+\cos(y))/\cos(x)\, dx = -\ln|\cos(y) + 1| + \ln|\sin(y)| + C$

Simplifying using the identity $\ln(a) - \ln(b) = \ln(a/b)$:

$\ln|(\sin(y))(\cos(y) + 1)/\cos(x)| = C$

Exponentiating both sides:

$|(\sin(y))(\cos(y) + 1)/\cos(x)| = e^C$

Multiplying both sides by $\cos(x)$:

$(\sin(y))(\cos(y) + 1) = \pm c^*\cos(x)$

Where $c = e^C$. Therefore, the correct option is (C) $(1+\sin x)(1+\cos y)=c$.

Question 2. The differential equation $dy/dx=\sqrt{1-y2/y}$ determines a family of circles with

- **A. Variable radius and fixed centre at (0,1)**
- **B. Variable radius and fixed centre at (0,-1)**
- **C. Fixed radius of 1 Unit and variable centre along the X-axis**
- **D. Fixed radius of 1 Unit and variable centre along the X- axis**

Answer. D

Solution. The given differential equation is:

$dy/dx = \sqrt{(1-y^2)}/y$

We can write this equation in the form:

$dy/\sqrt{(1-y^2)} = dx/y$

Integrating both sides:

$\arcsin(y) = \ln|x| + C$

Where C is the constant of integration.

Solving for y:

$y = \sin(\ln|x| + C)$

This is the general solution of the differential equation.

We can observe that this solution represents a family of curves which are circles centered on the x-axis.

To see this, we can rewrite the solution as:

$y = \sin(\ln|x| + C) = (e^{(\ln|x|+C)} - e^{-(\ln|x|+C)})/2$

Simplifying:

$y = (x - 1/x)/2$

This is the equation of a circle centered at (0,0) with radius 1/2.

Therefore, the correct option is (D) fixed radius of 1 unit and variable center along the x-axis.

Question 3. Area of the Region bounded by the curve $y=\sqrt{49-x^2}$ and x-axis is .

A. 49 π sq. units

B. 49 π/2 sq. units
C. 49 π/4 sq. units
D. 98 π sq. units

Answer. B
Solution. The given curve is $y = \sqrt{(49 - x^2)}$.

This is the upper half of a circle with center at the origin and radius 7.

The area bounded by this curve and the x-axis is the area of the upper half of the circle.

The area of a circle with radius r is given by πr^2.

Therefore, the area of the upper half of the circle with radius 7 is:

$(1/2)\pi(7^2) = 49\pi/2$ square units.

Hence, the correct option is (B) 49π/2 sq. units.

Question 4. $\int$ **(dx/(sinx + cosx)).dx = ?**

Answer. $\log[\tan((x + \pi/4)/2)] + c$

Question 5. $\int$ **(1/7-6x-x^2).dx = ?**

Answer. 1/8 log{7+x/1-x}+c

Question 6. If the line ax+by+c=0 is a normal to the curve xy=1, then
A. a>0, b>0
B. a>0, b<0
C. a<0,b<0
D. a=0, b=0

Answer. B

Solution. The curve $xy = 1$ can be written as $y = 1/x$, which means that the derivative of y with respect to x is:

$$dy/dx = -1/x^2$$

For a normal to the curve at a given point, the slope of the tangent at that point is given by:

$$m = -1/dy/dx = x^2$$

Therefore, the equation of the tangent at the point (a, 1/a) is:

$$y - 1/a = x^2 (x - a)$$

Simplifying, we get:

$$y = a^2 x + (1 - a^3)/a$$

This is the equation of the tangent line.

For this line to be a normal to the curve $xy = 1$, it must be perpendicular to the curve at the point (a, 1/a).

The slope of the curve at this point is:

$$dy/dx = -1/x^2 = -a^2$$

Therefore, the slope of the line perpendicular to the curve is:

$$m = 1/a^2$$

This means that the product of the slopes of the tangent and the normal at the point (a, 1/a) is:

$$m * (-a^2) = -1$$

Solving for a, we get:

$a = \pm 1$

Substituting $a = \pm 1$ in the equation of the tangent line, we get:

$y = \pm x + 1$

These are the equations of the two lines that are normal to the curve at the points (1, 1) and (-1, -1).

The normal at (1, 1) has a positive slope, and the normal at (-1, -1) has a negative slope.

Therefore, the correct option is (B) $a > 0$, $b < 0$.

Question 7. The Points (1,3), (5,1) are Opposite vertices of a diagonal of a rectangle. If the order two vertices lie on the line $y=2x+c$, then one of the vertex on the other diagonal is?

- **A. (1,-2)**
- **B. (0,-4)**
- **C. (2,0)**
- **D. (3,2)**

Answer. C

Question 8. The number of solutions of $\tan x+\sec x=2\cos x$, n$(0,2\pi)$ are?

- **A. 6**
- **B. 4**
- **C. 3**

D. 2

Answer. D

Solution. We can start by simplifying the given equation using the identities:

$\tan(x) = \sin(x)/\cos(x)$

$\sec(x) = 1/\cos(x)$

Substituting these expressions, we get:

$\sin(x)/\cos(x) + 1/\cos(x) = 2\cos(x)$

Multiplying through by $\cos(x)$, we get:

$\sin(x) + 1 = 2\cos^2(x)$

Using the identity $\sin^2(x) + \cos^2(x) = 1$, we can write $\cos^2(x) = 1 - \sin^2(x)$. Substituting this, we get:

$\sin(x) + 1 = 2(1 - \sin^2(x))$

Simplifying, we get:

$2\sin^2(x) + \sin(x) - 1 = 0$

Using the quadratic formula, we get:

$\sin(x) = (-1 \pm \sqrt{9})/4$

$\sin(x) = -1$ or $\sin(x) = 1/2$

For $\sin(x) = -1$, $x = 3\pi/2$, which is not in the range $(0,2\pi)$.

For $\sin(x) = 1/2$, we have $x = \pi/6$ and $x = 5\pi/6$.

Therefore, the given equation has 2 solutions in the range (0,2π), which is option D.

Question 9. Find k if ∫0 1/2 [x2 dx / (1 - x2) 3/2] = k/6.

Question 10. Find the coordinates of the point where the line through A (9, 4 , 1) and B (5, 1, 6) crosses X axis ?

Question 11. What is the number of solutions of tanx + secx = 2 cosx if x belongs to (0, 2π)?

Question 12.Three vectors a, b and c are given. Find the equation of a vector that lies in the plane of vector a and vector b and whose projection on vector c is 1/√3.

Question 13. Find the general solution of the differential equation: cosx (1 + cosy) dx - siny (1 + sinx) dy = 0

Question 14. Considering only the principal value of an inverse function, the set: A= { x ≥ 0, tan-1x + tan-16x = π/4}, then A is...

- **A. an empty set**
- **B. a singleton se**
- **C. consists of two elements**
- **D. contains more than two elements**

Question 15. If ∫0 π/2 log(cosx) dx = π/2 (log(1/2)), then find ∫0 π/2 log(secx) dx.

Question 16. If ax + by + c = 0 is normal to xy = 1, then determine if a and b are less than, greater than, or equal to zero.

Question 17. If a matrix A = [(1 m 2) (1 2 2) (1 3 3)] is adjoint of matrix B and | B | = 5, then find the value of m.

Question 18. $f(x) = 2x - 3$, $g(x) = x3 + 5$, then find $[fog]-1\ (-9) = ?$

Question 19. Out of five siblings, what is the probability that the eldest and youngest children have the same gender?

Question 20. Find bond order; N_2^{+}, N_2^{-} , N_2^{+2}, CO.
Arrange the given molecules in increasing order of their acidic strength.

Question 21. Which of the following is a biodegradable polymer?

Question 22. Find the density of a given molecule (solid state).

Question 23. Which of the following is the correct representation of Haber process?

Question 24. Identify the one differentiating characteristics between Homoleptic complex and Heteroleptic complex.

Question 25. Identify the product question Sandmeyer/Gattermann/BalzSchiemann reactions (any one).