Solved Sample Paper

Time : 45 min.

CUET (Mathematics)

M.M. : 200

IMPORTANT INSTRUCTIONS:

1. The test is of 45 Minutes duration.
2. The test contains is divided into two sections.
   a. Section A contains 15 questions which will be compulsory for all candidates.
   b. Section B will have 35 questions out of which 25 questions need to be attempted.
3. Marking Scheme of the test:
   a. Correct answer or the most appropriate answer: Five marks (+5)
   b. Any incorrect option marked will be given minus one mark (-1).

Choose the correct answer :

Question ID: 481221

If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is co-factor of a_{ij}, then value of Δ is equal to

(A) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

(B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$

(D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Answer (D)

Sol. $\Delta = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Question ID: 481222

If the matrix $\begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix}$ is skew symmetric, then $6x + y$ is equal to

(A) 6

(B) 12

(C) 18

(D) 2

Answer (B)

Sol. $y = 0$ and $3x = 6$

So, $6x + y = 12$

Question ID: 481223

If $\begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 5 \\ 1 & x \end{vmatrix}$ then $|x|$ is equal to

(A) $\sqrt{\frac{5}{2}}$ (B) 4

(C) $2\sqrt{2}$ (D) 2

Question ID: 481224

Which of the following statements are true?

A. A square matrix A is said to be non-singular If $|A| = 0$

B. A square matrix A is invertible if and only if A is non-singular matrix.

C. If elements of a row are multiplied with cofactors of any other row, then their sum is zero.

D. A is square matrix of order 3 then $|\text{Adj.}(A)| = |A|^3$

Choose the correct answer from the options given below

(A) A and C only (B) B and C only

(C) C and D only (D) B and D only

Answer (B)

Sol. Statement A is incorrect as for singular matrices $|A| = 0$

Statement B is correct as A is invertible if $|A| \neq 0$

Statement C is correct

Statement D is incorrect as $|\text{Adj}(A)| = |A|^2$

Question ID: 481225

The interval in which $y = x^2 e^{2x}$ is increasing is

(A) $(-\infty, -1)$

(B) $(-1, \infty)$

(C) $(-\infty, -1) \cup (0, \infty)$

(D) $(-\infty, 0) \cup (1, \infty)$

Answer (C)

Sol. $\frac{dy}{dx} = x^2 \cdot 2e^{2x} + 2x \cdot e^{2x} = 2xe^{2x}(x+1)$

$\frac{dy}{dx} > 0$ for $x \in (-\infty, -1) \cup (0, \infty)$

Question ID: 481226

Answer (C)

Sol. $\begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 5 \\ 1 & x \end{vmatrix}$

$\Rightarrow\ 11 = 2x^2 - 5$

$\Rightarrow\ x^2 = 8$

$\Rightarrow\ |x| = 2\sqrt{2}$

If $x = t^3$, $y = t^4$ then $\frac{d^2y}{dx^2}$ at $t = 2$ is

(A) $\frac{8}{3}$

(B) $\frac{1}{9}$

(C) $\frac{2}{9}$

(D) $\frac{9}{16}$

Answer (B)

Sol. $\frac{dy}{dx} = \frac{4t^3}{3t^2} = \frac{4}{3}t$

$\frac{d^2y}{dx^2} = \frac{4}{3}\frac{dt}{dx}$

$= \frac{4}{3} \cdot \frac{1}{3t^2} = \frac{4}{9t^2}$

at $t = 2, \frac{d^2y}{dx^2} = \frac{1}{9}$

Question ID: 481227

Match List-I with List-II

List-I	List-II
A. $\int \frac{dx}{\sqrt{9x^2-16}}$	I. $\frac{1}{3}\sin^{-1}\frac{3x}{4} + C$
B. $\int \frac{dx}{\sqrt{16-9x^2}}$	II. $\log(e^x+1) + C$
C. $\int \frac{e^{2\log_e x}+1}{e^{2\log_e x}-1}dx$	III. $\frac{1}{3}\log_e\left\|3x + \sqrt{9x^2-16}\right\| + C$
D. $\int \frac{1}{1+e^{-x}}dx$	IV. $x + \log_e\left\|\frac{x-1}{x+1}\right\| + C$

Choose the correct answer from the options given below :

(A) A-I, B-III, C-II, D-IV (B) A-I, B-III, C-IV, D-II

(C) A-III, B-I, C-IV, D-II (D) A-III, B-I, C-II, D-IV

Answer (C)

Sol. (A) $I = \int \frac{dx}{\sqrt{9x^2-16}} = \frac{1}{3}\int \frac{dx}{\sqrt{x^2 - \frac{16}{9}}}$

$= \frac{1}{3}\left(\ln\left|x + \sqrt{x^2 - \frac{16}{9}}\right|\right) + C$

$= \frac{1}{3}\left(\ln\left|x + \sqrt{x^2 - \frac{16}{9}}\right| + \ln 3\right) + C$

$= \frac{1}{3}\ln\left|3x + \sqrt{9x^2-16}\right| + C$

(B) $I = \int \frac{dx}{\sqrt{16-9x^2}} = \frac{1}{3}\int \frac{dx}{\sqrt{\frac{16}{9} - x^2}} = \frac{1}{3}\sin^{-1}\frac{3x}{4} + C$

(D) $I = \int \frac{e^x}{e^x+1}dx$

Put $e^x + 1 = t$, $e^x dx = dt$

$I = \int \frac{dt}{t} = \ln|t| + C$

$= \ln(e^x + 1) + C$

Question ID: 481228

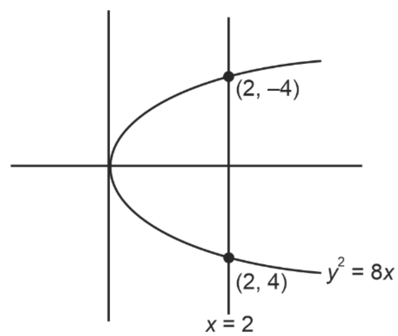
The area (in sq. units) bounded by the parabola $y^2 = 8x$ and the line $x = 2$ is

(A) $\frac{16}{3}$ (B) $\frac{32}{3}$

(C) $\frac{32\sqrt{2}}{3}$ (D) $\frac{16\sqrt{2}}{3}$

Answer (B)

Sol.



$A = 2\int_0^2 \sqrt{8x}\,dx$

$= 4\sqrt{2}\left.\frac{x^{3/2}}{3/2}\right|_0^2$

$= \frac{8\sqrt{2}}{3} \cdot 2^{3/2} = \frac{32}{3}$ sq. units

Question ID: 481229

$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{2\pi - x}}dx$ is equal to

(A) $\frac{\pi}{4}$

9

(C) $I = \int \frac{x^2+1}{x^2-1} dx = \int 1 + \frac{2}{x^2-1} dx$

$= x + \ln\left|\frac{x-1}{x+1}\right| + C$

(B) π

(C) $\frac{\pi}{2}$

(D) 1

Answer (C)

Sol. $I = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2\pi - x}} dx$ …(i)

$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{\sqrt{2\pi - x}}{\sqrt{2\pi - x}+\sqrt{x}} dx$ …(ii)

By (i) + (ii)

$2I = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1 dx$

$= \pi$

Question ID: 4812210

The solution curve $y = y(x)$ of the differential equation $ydx - xdy = 0$, passing through (2, 4), does not pass through the point

(A) (–1, –2) (B) $\left(\frac{-1}{2}, -1\right)$

(C) (–2, –4) (D) $\left(1, \frac{1}{2}\right)$

Answer (D)

Sol. $y\,dx = x\,dy$

$\Rightarrow \frac{dx}{x} = \frac{dy}{y}$

Integrating both sides

$\ln x = \ln y + \ln c$

or $x = cy$

It passes through (2, 4)

$\Rightarrow c = \frac{1}{2}$

or $y = 2x$

Question ID: 4812211

The solution curve of the differential equation $\log_e\left(\frac{dy}{dx}\right) = 3x + 4y$, passing through the origin is

(A) $4e^{3x} - 3e^{-4y} - 1 = 0$ (B) $4e^{3x} + 3e^{-4y} - 7 = 0$

(C) $3e^{3x} + 4e^{-4y} - 7 = 0$ (D) $3e^{3x} - 4e^{-4y} + 1 = 0$

Integrating both sides

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$

It passes through (0, 0)

$\frac{-1}{4} = \frac{1}{3} + C$

$\Rightarrow C = \frac{-7}{12}$

$\therefore$ Equation of curve is

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$

or $-3e^{-4y} = 4e^{3x} - 7$

Question ID: 4812212

If the probability distribution of a random variable X is

x	1	2	3	4	5
$P(X = x)$	k	$2k$	$3k$	$3k$	k

Value of $P(X > 2)$ is

(A) $\frac{5}{6}$ (B) $\frac{9}{10}$

(C) $\frac{4}{5}$ (D) $\frac{7}{10}$

Answer (D)

Sol. $k + 2k + 3k + 3k + k = 1$

$\Rightarrow k = \frac{1}{10}$

$P(x > 2) = 3k + 3k + k$

$= \frac{7}{10}$

Question ID: 4812213

The curve represented by the differential equation $\frac{dy}{dx} = \frac{2(x+1)y}{\ldots}$, $y > 0$, passes through the point

Answer (B)

Sol. $\frac{dy}{dx} = e^{3x+4y}$

$\Rightarrow \frac{dy}{e^{4y}} = e^{3x}dx$

$\overline{dx} \quad \overline{x^2 + 2x + 2}$

(0, 4). If it also passes through $P(-1, k)$ and $Q(\lambda, 10)$ then $(PQ)^2$ is equal to

(A) 148

(B) 66

(C) 68

(D) 72

Answer (C)

Sol. $\frac{dy}{dx} = \frac{2(x+1)y}{x^2+2x+2}$,

$\Rightarrow \frac{dy}{y} = \frac{2(x+1)dx}{x^2+2x+2}$

Integrating both sides

$\ln y = \ln(x^2 + 2x + 2) + \ln C$

It passes through (0, 4)

$\ln 4 = \ln (2) + \ln C$

$\Rightarrow C = 2$

Equation of curve is

$y = 2(x^2 + 2x + 2)$

$k = 2(1 - 2 + 2) = 2$

$10 = 2(\lambda^2 + 2\lambda + 2)$

$\Rightarrow \lambda^2 + 2\lambda - 3 = 0$

OR $\lambda = -3, 1$

So, $P(-1, 2)$ $Q(1, 10)$ OR $(-3, 10)$

$(PQ)^2 = 68$

Question ID: 4812214

If $\int \frac{dx}{\sqrt{x}-\sqrt{x-1}} = \lambda\left(x^{\frac{3}{2}} + (x-1)^{\frac{3}{2}}\right) + C$, then the value of λ is

(A) 2 (B) $\frac{3}{2}$

(C) 1 (D) $\frac{2}{3}$

Answer (D)

Sol. $I = \int \frac{dx}{\sqrt{x}-\sqrt{x-1}} = \int \frac{\sqrt{x}+\sqrt{x-1}}{1}dx$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}} + C$

Question ID: 4812215

The corner points of the feasible region of an LPP are (0, 0), (30, 0), (20, 30) and (0, 50). If the objective function maximize $(z) = ax + by$, where $a \neq b$ and $a, b > 0$, has alternate optimal solutions,

Sol. $z_{(0, 0)} = 0$

$z_{(30, 0)} = 30a$

$z_{(0, 50)} = 50b$

$z_{(20, 30)} = 20a + 30b$

For $a = 3b$, $20a + 30b = 30a$

Question ID: 4812251

Which of the following is not an equivalence relation on Z?

(A) $a\ \boldsymbol{R}\ b \Leftrightarrow a + b$ is an even integer

(B) $a\ \boldsymbol{R}\ b \Leftrightarrow a - b$ is an even integer

(C) $a\ \boldsymbol{R}\ b \Leftrightarrow a \leq b$

(D) $a\ \boldsymbol{R}\ b \Leftrightarrow a = b$

Answer (C)

Sol. C is not equivalence relation as it is not symmetric

$a \leq b$ then $b \geq a$

So, if $(a, b) \in R$ then $(b, a) \notin R$

Question ID: 4812252

The function $f : R - \{-1\} \rightarrow R - \{1\}$ defined by $f(x) = \frac{x}{x+1}$ is

(A) Both 1-1 and onto

(B) Only 1-1

(C) Only onto

(D) Neither 1-1 nor onto

Answer (A)

Sol. $f(x) = 1 - \frac{1}{x+1}$

$f'(x) = \frac{1}{(x+1)^2} > 0$ so $f(x)$ is one-one

$y = \frac{x}{x+1}$

then

(A) $a = 2b$ (B) $a = 3b$

(C) $2a = b$ (D) $3a = b$

Answer (B)

$\Rightarrow xy + y = x$

$\Rightarrow x(y-1) = -y$

$$\Rightarrow x = \frac{y}{1-y}$$

So, Range is $R - \{1\}$

So, $f(x)$ is onto

Question ID: 4812253

If $X+Y=\begin{bmatrix}2 & 1\\3 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}0 & 3\\7 & 9\end{bmatrix}$, then Y is equal to

(A) $\begin{bmatrix}-1 & 1\\-2 & -2\end{bmatrix}$ (B) $\begin{bmatrix}-1 & 1\\2 & 2\end{bmatrix}$

(C) $\begin{bmatrix}1 & -1\\-2 & -2\end{bmatrix}$ (D) $\begin{bmatrix}1 & -1\\2 & 2\end{bmatrix}$

Answer (C)

Sol. Adding given equations

$$2X=\begin{bmatrix}2 & 4\\10 & 14\end{bmatrix}$$

$$\Rightarrow\quad X=\begin{bmatrix}1 & 2\\5 & 7\end{bmatrix}$$

Similarly,

$$2Y=\begin{bmatrix}2 & -2\\-4 & -4\end{bmatrix}$$

$$Y=\begin{bmatrix}1 & -1\\-2 & -2\end{bmatrix}$$

Question ID: 4812254

If A and B are any two matrices such that $(A-B)^2=A^2-2AB+B^2$ then

(A) $B^2A^3+A^3B^2=0$

(B) $AB+BA=0$

(C) $B^3A^4-A^4B^3=0$

(D) $AB(B-A)=0$

Answer (C)

Sol. $A^2-AB-BA+B^2=A^2-2AB+B^2$

$\Rightarrow$ $AB=BA$

Question ID: 4812255

If A and B are two matrices such that $AB = O$ (null matrix) then

A. A may be a null matrices, but B may not be a null matrix.

B. B may be a null matrix, but A may not be a null matrix.

C. both A and B may be null matrices.

D. both A and B may be non-null matrices.

Choose the correct answer from the option given below:

(A) A, B Only

(B) A, B, C only

(C) C, D only

(D) A, B, C, D only

Answer (D)

Sol. All Statements are correct.

Question ID: 4812256

$\int \frac{x^2+3x-1}{(x+1)^2}dx$ is equal to

(A) $\frac{1}{x+1}+\frac{1}{2}\log_e|x+1|+C$

(B) $x+\frac{3}{x+1}+\log_e|x+1|+C$

(C) $x-\frac{1}{x+1}+\log_e|x+1|+C$

(D) $x+\frac{3}{x+1}+\frac{1}{2}\log_e|x+1|+C$

Answer (B)

Sol. $\int \frac{x^2+3x-1}{}$

$A^2B = ABA$

$A^2B^2 = BAAB$

$= BABA$

$A^2B^2 = B^2A^2$

$A^3B^2 = B^2A^3$

$A^3B^3 = B^3A^3$

$A^4B^3 = A$

$I = \int \frac{}{(x+1)^2} dx$

$= \int \left(1 + \frac{x-2}{(x+1)^2}\right) dx$

$= \int \left(1 + \frac{1}{x+1} - \frac{3}{(x+1)^2}\right) dx$

$= x + \ln(x+1) + \frac{3}{(x+1)} + C$

Question ID: 4812257

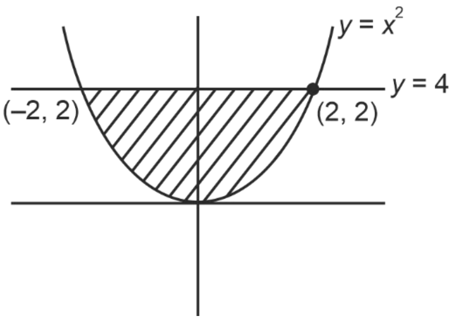
The area (in sq. units) of the region bounded by the curve $y = x^2$ and the line $y = 4$ is

(A) $\frac{16}{3}$ (B) $\frac{32}{3}$

(C) 32 (D) 24

Answer (B)

Sol.



$$A = 2\int_0^2 \left(4 - x^2\right)dx$$

$$= 2\left(4x - \frac{x^3}{3}\right)\Bigg|_0^2$$

$$= 2\left(8 - \frac{8}{3}\right) = \frac{32}{3} \text{ sq. units}$$

Question ID: 4812258

If $\int \frac{\left(x^2+1\right)\left(x^2+2\right)}{\left(x^2+3\right)\left(x^2+4\right)}dx = ax + b\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c\tan^{-1}\left(\frac{x}{2}\right) + d,$

(Where d is a constant of integration) then $\frac{a+c}{b}$ is equal to

(A) $-\sqrt{3}$ (B) $\sqrt{3}$

(C) $2\sqrt{3}$ (D) -2

Answer (A)

$\left(x^2+1\right)\left(x^2+2\right)$

$$= x + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - \frac{6}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$$

$$a = 1,\ b = \frac{2}{\sqrt{3}},\ c = -3$$

$$\frac{a+c}{b} = -\sqrt{3}$$

Question ID: 4812259

The function $f(x) = \begin{cases} \frac{\sqrt{1+px} - \sqrt{1-px}}{x}, & -1 \le x < 0, \\ \frac{2x+1}{x-2}, & 0 \le x \le 1, \end{cases}$

is continuous in the interval [–1, 1], then p is equal to

(A) 1 (B) $-\frac{1}{2}$

(C) $\frac{1}{2}$ (D) –1

Answer (B)

Sol. $\underset{x\to 0^-}{\text{lt}} f(x) = \underset{x\to 0^-}{\text{lt}} \frac{\sqrt{1+px} - \sqrt{1-px}}{x}$

$$= \underset{x\to 0^-}{\text{lt}} \frac{2px}{x.2} = p$$

$$\underset{x\to 0^+}{\text{lt}} f(x) = \underset{x\to 0^+}{\text{lt}} \frac{2x+1}{x-2} = \frac{-1}{2}$$

Question ID: 4812260

If $y = \log_e\left(\sin\left(\frac{x^2}{3} - 1\right)\right)$ then $\frac{d^2y}{dx^2}$ is equal to :

(A) $\frac{2}{3}\left(-\cot\left(\frac{x^2}{3} - 1\right) + x\,\text{cosec}^2\left(\frac{x^2}{3} - 1\right)\right)$

Sol. $I = \int \frac{dx}{\left(x^2+3\right)\left(x^2+4\right)}$

$$= \int \left(1 - \frac{2}{x^2+3}\right)\left(1 - \frac{2}{x^2+4}\right) dx$$

$$= \int \left(1 - \frac{2}{x^2+3} - \frac{2}{x^2+4} + 4\left(\frac{1}{x^2+3} - \frac{1}{x^2+4}\right)\right) dx$$

$$= \int \left(1 + \frac{2}{x^2+3} - \frac{6}{x^2+4}\right) dx$$

(B) $\frac{2}{3}\cot\left(\frac{x^2}{3}-1\right) - \frac{4x^2}{9}\operatorname{cosec}^2\left(\frac{x^2}{3}-1\right)$

(C) $-\frac{2}{3}\cot\left(\frac{x^2}{3}-1\right) + \frac{4x^2}{9}\operatorname{cosec}^2\left(\frac{x^2}{3}-1\right)$

(D) $\frac{2}{3}\left(\cot\left(\frac{x^2}{3}-1\right) - x\operatorname{cosec}^2\left(\frac{x^2}{3}-1\right)\right)$

Answer (B)

Sol. $\frac{dy}{dx} = \frac{1}{\sin\left(\frac{x^2}{3}-1\right)}\cos\left(\frac{x^2}{3}-1\right)\frac{2x}{3}$

$\frac{dy}{dx} = \frac{2x}{3}\cot\left(\frac{x^2}{3}-1\right)$

$\frac{d^2y}{dx^2} = \left(-\text{cosec}^2\left(\frac{x^2}{3}-1\right)\cdot\left(\frac{2x}{3}\right)^2 + \cot\left(\frac{x^2}{3}-1\right)\frac{2}{3}\right)$

Question ID: 4812261

If $z = \tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$ then $6z$ is equal to ________

(A) 1/3 (B) 16

(C) 17 (D) 29

Answer (C)

Sol. $z = \tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$

$= \frac{\tan\left(\sin^{-1}\frac{3}{5}\right) + \tan\left(\cot^{-1}\frac{3}{2}\right)}{1-\tan\left(\sin^{-1}\frac{3}{5}\right)\tan\left(\cot^{-1}\frac{3}{2}\right)}$

$= \frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}$

$= \frac{17/12}{6/12} = \frac{17}{6}$

Question ID: 4812262

$\frac{d^2y}{}$

$\frac{d^2y}{dx^2} = 2x\left(-\sin\left(x^2\right)2x\right) + 2\cos\left(x^2\right)$

$= -4x^2\sin(x^2) + 2\cos(x^2)$

$a = 2, b = -4$

$a^2 + b^2 = 20$

Question ID: 4812263

If $x\sqrt{1+y} + y\sqrt{1+x} = 0, x \neq y$ then $2\frac{dy}{dx} + y$ at $x = -3$ is equal to

(A) –1 (B) 1

(C) 2 (D) –2

Answer (D)

Sol. $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 - y^2 + xy(x-y) = 0$

$\Rightarrow (x-y)(x+y+xy) = 0$

$x \neq y$ so $x + y + xy = 0$

$1 + \frac{dy}{dx} + \frac{xdy}{dx} + y = 0$

$\Rightarrow \frac{dy}{dx}(1+x) = -1-y$

$\Rightarrow \frac{dy}{dx} = -\left(\frac{1+y}{1+x}\right)$

at $x = -3$, $-3 + y - 3y = 0$

$-2y = 3$

$y = -\frac{3}{2}$

$\left(\frac{-1}{2}\right)$

If $x = \sqrt{t}$, $y = \sin t$ and $\frac{d^2y}{dx^2} = a\cos x^2 + bx^2 \sin x^2$,

then $a^2 + b^2$ is equal to

(A) 4

(B) 16

(C) 20

(D) 48

Answer (C)

Sol. $y = \sin(x^2)$

$$\frac{dy}{dx} = \cos\left(x^2\right)2x$$

$$\frac{dy}{dx} = -\frac{(\ )}{-2} = -\frac{1}{4}$$

$$\frac{2dy}{dx} + y = \frac{-1}{2} - \frac{3}{2} = -2$$

Question ID: 4812264

The radius of a sphere is increasing at the rate of 0.5 cm/minute. The rate of change of the surface area of (in cm^2/ minute) the sphere when the radius is 20 cm is

(A) 20π

(B) 40π

(C) 160π

(D) 80π

Answer (D)

Sol. $s = 4\pi r^2$

$$\frac{ds}{dt} = 8\pi r \frac{dr}{dt}$$

$\therefore \quad 8\pi \times 20 \times 5$

$= 80\pi$ cm^2/min

Question ID: 4812265

The equation of the tangent to the curve $x(\theta) = 2\sqrt{2}(\cos\theta + \theta\sin\theta)$, $y(\theta) = 2\sqrt{2}(\sin\theta - \theta\cos\theta)$, at $\theta = \frac{\pi}{4}$ is equal to

(A) $x - y = \frac{\pi}{2}$ (B) $x - y = \pi$

(C) $x - y = 2$ (D) $x - y = 4$

Answer (B)

Sol. $$\frac{dy}{dx} = \frac{2\sqrt{2}(\cos\theta + \theta\sin\theta - \cos\theta)}{2\sqrt{2}(-\sin\theta + \theta\cos\theta + \sin\theta)}$$

$= \tan\theta$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = 1$$

$$x(\pi/4) = 2\sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}}\right)$$

$$y(\pi/4) = 2\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{\pi}{4\sqrt{2}}\right)$$

Equation of tangent

$x - y = \pi$

Question ID: 4812266

Match List-I with List-II

	List-I		**List-II**
A.	Distance of the point (1, –1, 1) from the plane $\vec{r}\cdot(\hat{i} - \hat{j} + \hat{k}) = 2$, is	I.	$x + y + z = 2$
B.	$\vec{r}\cdot(\hat{i} + \hat{j} + \hat{k}) = 2$	II.	$-x + y + z = 5$
C.	Distance of a point (1, –1, 1) from the plane $\vec{r}\cdot(\hat{i} + \hat{j} - \hat{k}) = 5$, is	III.	$\frac{1}{3}$

Choose the correct answer from the options given below.

(A) A-IV, B-II, C-III, D-I (B) A-III, B-II, C-IV, D-I

(C) A-IV, B-I, C-III, D-II (D) A-III, B-I, C-IV, D-II

Answer (D)

Sol. (A) $d = \left|\frac{1+1+1-2}{\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$

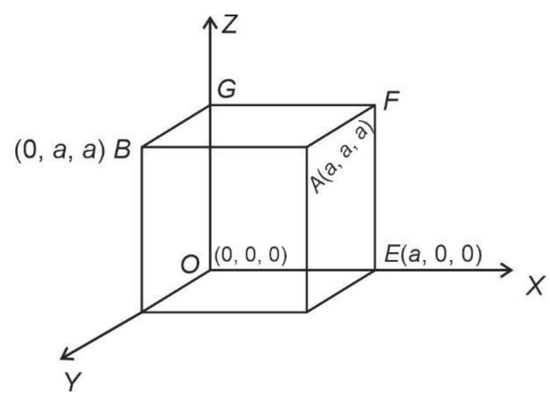
(B) $x + y + z = 2$

(C) $\left|\frac{-1-5}{\sqrt{3}}\right| = d$

$d = 2\sqrt{3}$

(D) $-x + y + z = 5$

Question ID: 4812267



Given figure is cuboid then the acute angle between the diagonal OA and BE is

(A) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(B) $\cos^{-1}\left(\frac{1}{6}\right)$

(C) $\cos^{-1}\left(\frac{2}{3}\right)$

(D) $\cos^{-1}\left(\frac{1}{3}\right)$

Answer (D)

Sol.

$\vec{OA} = a(\hat{i} + \hat{j} + \hat{k})$

$\vec{BE} = a(\hat{i} - \hat{j} - \hat{k})$

Let angle between *OA* and *BE* is θ

$$\cos\theta = \left|\frac{1-1-1}{\sqrt{3}\sqrt{3}}\right| = \left|\frac{-1}{3}\right|$$

Question ID: 4812268

A soldier fires a bullet in the air and at time '*t*' the bullet travels along the path $x = 2t$, $y = t - 2$, $z = 5 - t$. Then the path of the bullet is a

(A) Parabola

(B) Straight line with direction ratios 2, 1, –1

(C) Straight line with direction ratios 2, 1, 1

(D) Ellipse centred at (0, 2, 5)

Answer (B)

Sol. $\frac{x}{2} = \frac{y+2}{1} = \frac{5-z}{1}$

or $\frac{x}{2} = \frac{y+2}{1} = \frac{z-5}{-1}$

So straight line with direction ratios (2, 1, –1)

Question ID: 4812269

Which of the following is NOT a corner point of the feasible set $S = \{(x, y) : 3x + 4y \le 24, 8x + 6y \le 48, x \le 5, x \ge 0, y \ge 0\}$ of some LPP?

(A) $\left(\frac{24}{7}, \frac{24}{7}\right)$ (B) $\left(5, \frac{4}{3}\right)$

(C) $\left(\frac{24}{7}, \frac{33}{7}\right)$ (D) (0, 6)

Answer (C)

Sol. Clearly we can see that $\left(\frac{24}{7}, \frac{33}{7}\right)$ is not intersection point of any 2 given conditions so it cannot be a corner point.

Question ID: 4812270

If the sum of two unit vectors is a unit vector, then the magnitude of their difference is

(A) $\sqrt{2}$ **Sol.**

(C) $\sqrt{3}$

Answer (C)

$|\vec{a} + \vec{b}| = 1$

Question ID: 4812271

$\hat{i} \cdot \left(\hat{j} \times \hat{k}\right) + \hat{j} \cdot \left(\hat{k} \times \hat{i}\right) + \hat{k} \cdot \left(\hat{i} \times \hat{j}\right)$ is equal to

(A) 1 (B) 0

(C) 3 (D) –3

Answer (C)

Sol. $\hat{i}(+\hat{i}) + \hat{j}(\hat{j}) + \hat{k}(\hat{k})$

= 1 + 1 + 1

= 3

Question ID : 4812272

If $|\vec{a} - \vec{b}| = 10, |\vec{a}| = 8, |\vec{b}| = 6,$ then the angle between $\vec{a}$ and $\vec{b}$ is :

(A) 0 (B) $\cos^{-1}\left(\frac{29}{48}\right)$

(C) $\cos^{-1}\left(\frac{5}{24}\right)$ (D) $\frac{\pi}{2}$

Answer (D)

Sol.

$|\vec{a} - \vec{b}| = 10$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} = 100$

$\Rightarrow \vec{a} \cdot \vec{b} = 0$

Question ID : 4812273

The linear programming problem

$\max.(z) = 5x + 7y$

Subject to

$x + y \ge 10$

$3x + 5y \le 15, x \ge 0, y \ge 0$:

(A) has unique optimal solution

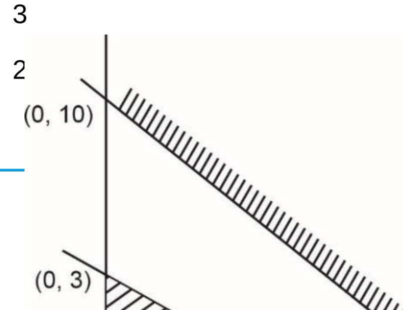
(B) is infeasible

(C) has multiple optimal solutions

(D) is feasible and unbounded

(B) 3

(D) 2



A **s** **er (B) Sol.**

n **w**

$$\left(\vec{a}+\vec{b}\right)\left(\vec{a}+\vec{b}\right)=1$$

$$\Rightarrow \quad a \cdot \vec{b} = \frac{-1}{2}$$

$$\left|\vec{a}-\vec{b}\right|^2 = |\vec{a}|^2 + \left|\vec{b}\right|^2 - 2\vec{a}\cdot\vec{b}$$

$$= 3$$

Question ID : 4812274

A bag contains 5 red and 7 black balls. Three balls are chosen at random one by one without replacement. What is the probability that the third ball is a Red ball given that first ball is red and second ball is black?

(A) $\frac{7}{33}$

(B) $\frac{14}{33}$

(C) $\frac{7}{66}$

(D) $\frac{20}{33}$

Answer (*)

Sol. After given 2 selections, 4 red and 6 black balls are present.

So required probability = $\frac{4}{10} = \frac{2}{5}$

Question ID : 4812275

If two independent events A and B are such that $P(A) = \frac{1}{2}$ and $P((A' \cap B')') = \frac{3}{5}$, then $P(B)$ is equal to

(A) $\frac{1}{10}$

(B) $\frac{1}{5}$

(C) $\frac{2}{5}$

(D) $\frac{3}{5}$

Answer (B)

Sol. $P(A \cup B) = \frac{3}{5}$

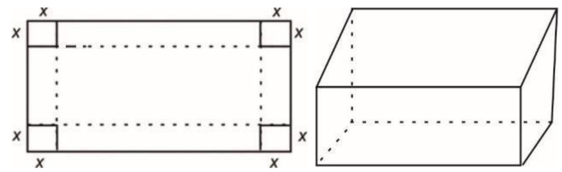
$\Rightarrow \quad P(A) + P(B) - P(A \cap B) = \frac{3}{5}$

$\Rightarrow \quad \frac{1}{2} + x - \frac{1}{2} \cdot x = \frac{3}{5}$

$\Rightarrow \quad \frac{x}{2} = \frac{1}{}$

Passage :

Case - Study :



An online retail company ships its product in cartons

Each carton is made by rectangular sheet of fiber board with dimension of 8 m by 3 m. While making the carton equal squares of side x metres are cut off from each corner of the rectangular sheet of fiber board. After that the resulting flaps are folded to form the carton (see that figure to identify the side of the carton).

Based on the above information, answer the following question.

Question ID : 4812276

The volume (v) of the carton is given by $v = f(x)$, where $f(x)$ is :

(A) $4x^3 - 24x^2 + 22x$

(B) $4x^3 + 24x^2 - 22x$

(C) $-4x^3 + 22x^2 + 24x$

(D) $4x^3 - 22x^2 + 24x$

Answer (D)

Sol. $v(x) = (8 - 2x)(3 - 2x)x$

$= x(24 + 4x^2 - 22x)$

$= 4x^3 - 22x^2 + 24x$

Question ID : 4812277

The derivative $f'(x)$ of the volume function $f(x)$ is equal to

(A) $24 + 44x - 12x^2$

(B) $24 - 44x + 12x^2$

2 10

$\Rightarrow x = \frac{1}{}$

5

(C) $-22 + 48x + 12x^2$

(D) $22 - 48x + 12x^2$

Answer (B)

Sol. $f'(x) = 12x^2 - 44x + 24$

Question ID: 4812278

The value of x (in meters) for which the volume V of carton is maximum is

(A) 3

(B) $\frac{2}{3}$

(C) 1

(D) $\frac{1}{2}$

Answer (B)

Sol. $f'(x) = 12x^2 - 44x + 24$

$= 4(3x^2 - 11x + 6)$

$= 4(3x^2 - 9x - 2x + 6)$

$= 4(3x(x - 3) - 2(x - 3))$

$= 4(3x - 2)(x - 3)$

$f'(x) = 0$ at $x = \frac{2}{3}, 3$

So, $f(x)$ attains maxima at $x = \frac{2}{3}$

Question ID: 4812279

The length (in meters) of carton box having maximum volume is

(A) 6

(B) $\frac{5}{3}$

(C) $\frac{20}{3}$

(D) $\frac{22}{3}$

Answer (C)

Sol. Length = $8 - 2x$

Question ID: 4812280

The maximum volume (in m^3) of carton box is

(A) $\frac{200}{9}$

(B) 6

(C) $\frac{154}{27}$

(D) $\frac{200}{27}$

Answer (D)

Sol. $V(x) = 4x^3 - 22x^2 + 24x$

at $x = \frac{2}{3}$

$$V = 4\cdot\frac{8}{27} - 22\cdot\frac{4}{9} + 16$$

$$= \frac{32}{27} - \frac{88}{9} + 16$$

$$= 16 - \frac{232}{27}$$

$$= \frac{200}{27}$$

Passage:

By using equations of the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z - 3 = 0$, answer the following questions.

Question ID: 4812281

The acute angle between the line and the plane is

(A) $\cos^{-1}\left(\frac{8}{21}\right)$

(B) $\sin^{-1}\left(\frac{8}{21}\right)$

(C) $\sin^{-1}\left(\frac{6}{7}\right)$

(D) $\cos^{-1}\left(\frac{6}{7}\right)$

Answer (B)

$$= 8 - 2 \cdot \frac{2}{3}$$

$$= \frac{20}{3}$$

Sol. $\sin\theta = \frac{10 \cdot 2 + 2 \cdot 3 - 11 \cdot 6}{15 \cdot 7}$

$$= \frac{-40}{105}$$

Question ID: 4812282

Co-ordinates of point of intersection of line and plane

(A) $\left(\frac{-33}{10}, \frac{69}{20}, \frac{39}{10}\right)$

(B) $\left(\frac{-33}{10}, \frac{-69}{20}, \frac{-39}{10}\right)$

(C) $\left(0, \frac{3}{2}, 6\right)$

(D) $\left(-2, -\frac{3}{2}, 0\right)$

Answer (B)

Sol. Point on line ≡ $(2\lambda - 1, 3\lambda, 6\lambda + 3)$

So, $10(2\lambda - 1) + 2(3\lambda) - 11(6\lambda + 3) - 3 = 0$

$\Rightarrow 20\lambda - 10 + 6\lambda - 66\lambda - 33 - 3 = 0$

$\Rightarrow -40\lambda - 46 = 0$

$\Rightarrow \lambda = \frac{-23}{20}$

Point $\equiv \left(\frac{-46}{20} - 1, \frac{-69}{20}, \frac{-138}{20} + 3\right)$

$\equiv \left(\frac{-33}{10}, \frac{-69}{20}, \frac{-39}{10}\right)$

Question ID: 4812283

Distance of plane $10x + 2y - 11z - 3 = 0$, from the origin is:

(A) 1

(B) $\frac{1}{10}$

(C) $\frac{1}{15}$

1

Question ID: 4812284

The foot of perpendicular from the origin to the given plane is

(A) $\left(\frac{2}{75}, \frac{2}{15}, -\frac{11}{75}\right)$

(B) $\left(\frac{2}{15}, \frac{2}{75}, -\frac{11}{75}\right)$

(C) $\left(\frac{2}{5}, \frac{2}{25}, \frac{11}{25}\right)$

(D) $\left(\frac{2}{5}, \frac{2}{25}, -\frac{11}{25}\right)$

Answer (B)

Sol. Let foot of perpendicular be (α, β, γ)

$$\frac{\alpha - 0}{10} = \frac{\beta - 0}{2} = \frac{\gamma - 0}{-11} = \frac{-(-3)}{225}$$

$$(\alpha, \beta, \gamma) = \left(\frac{30}{225}, \frac{6}{225}, \frac{-33}{225}\right)$$

$$= \left(\frac{2}{15}, \frac{2}{75}, \frac{-11}{75}\right)$$

Question ID: 4812285

The plane $10x + 2y - 11z - 3 = 0$ intersect the Z-axis at the point

(A) $\left(0, 0, \frac{3}{10}\right)$

(B) $\left(0, 0, \frac{3}{2}\right)$

(C) $\left(0, 0, \frac{3}{11}\right)$

(D) $\left(0, 0, -\frac{3}{11}\right)$

(D) $-\frac{}{5}$

Answer (D)

Sol. $d=\left|\frac{-3}{\sqrt{10^2+2^2+11^2}}\right|=\frac{1}{5}$

$\left(\quad \right)$

Answer (D)

Sol. At *Z*-axis, *x* and *y* coordinates are zero

So, $z=-\frac{3}{11}$